“Precision Radiative Corrections for Next Generation Experiments”

JLab: May 16-19, 2016.

MODELS FOR FRAGMENTATION

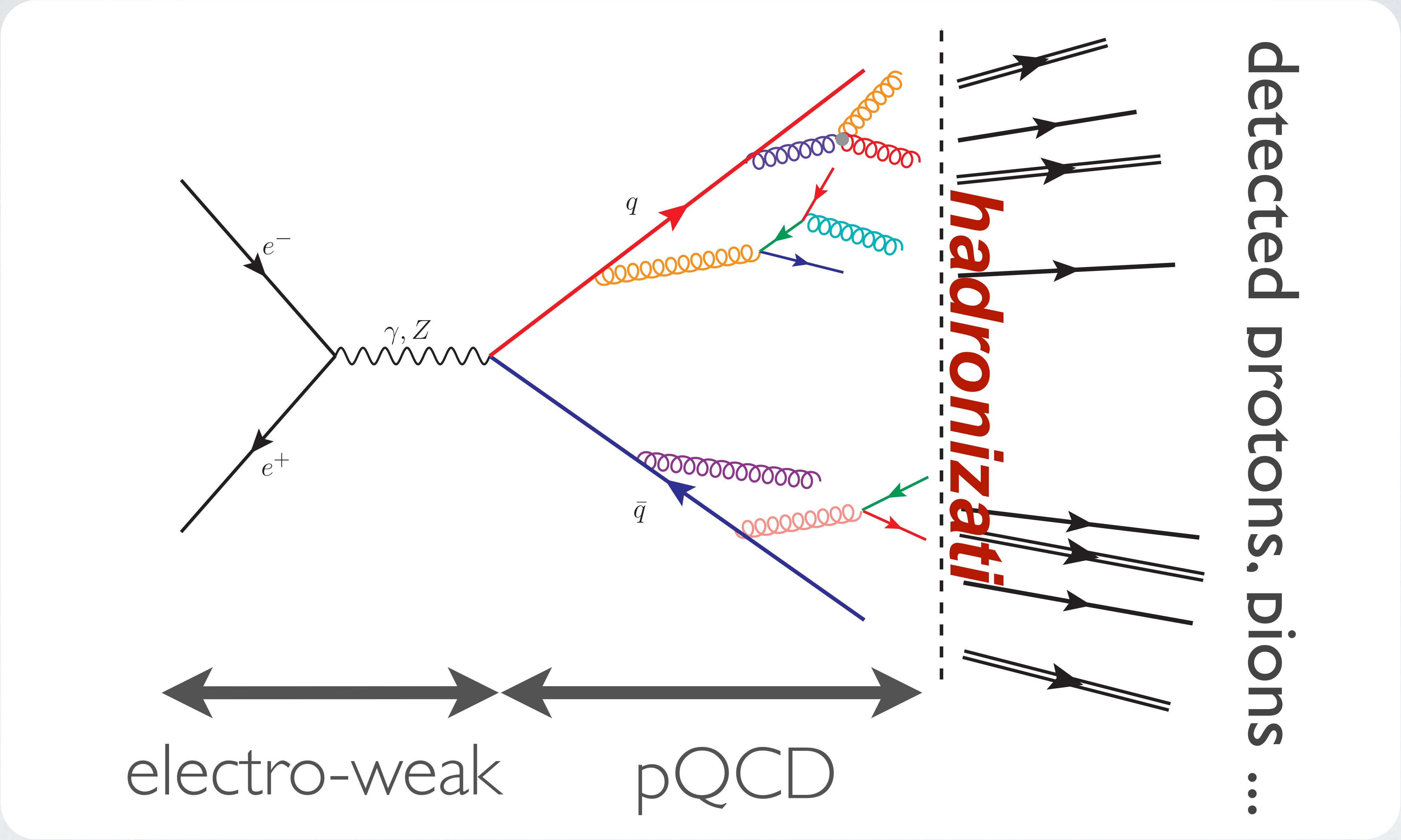
Hrayr Matevosyan

Outlook

- *Introduction and Motivation.*
- *Overview of models for fragmentation functions.*
- *The recent progress on modelling <u>polarised</u> quark hadronisation.*
- *Conclusions.*

HADRONIZATION: $e^-e^+ \to hX$

- **Factorization:** *pQCD "hard" partonic scattering separated from "soft", universal fragmentation functions* at renormalization scale.



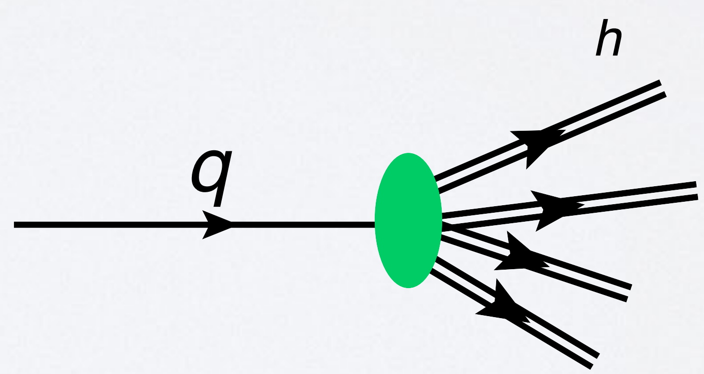
FRAGMENTATION FUNCTIONS

- The cross-sections of DIS processes can be factorized into ***"hard scattering"*** parts calculable in pQCD and ***"soft"***, non-perturbative universal functions encoding ***parton distribution in hadrons (PDFs)*** and parton hadronization: ***Fragmentation Functions (FF)***.

$$\boxed{\frac{1}{\sigma}\frac{d}{dz}\sigma(e^-e^+ \to hX) = \sum_i \mathcal{C}_i(z,Q^2) \otimes D_i^h(z,Q^2)}$$

- Unpolarized FF is the number density of hadron h with LC momentum fraction z, produced by quark q:

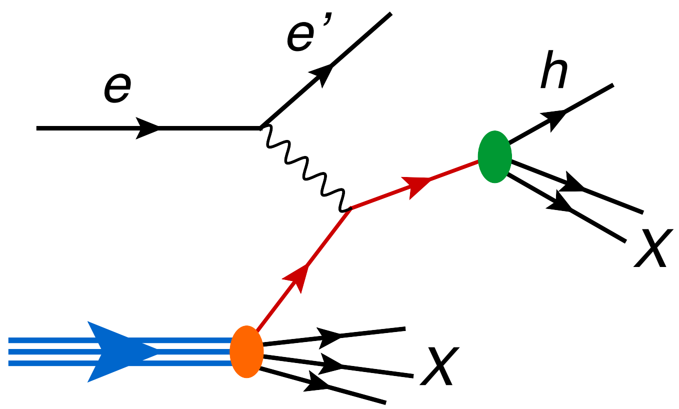
$$\boxed{D_q^h(z,Q^2)}$$



- z is the light-cone mom. fraction of the parton carried by the hadron

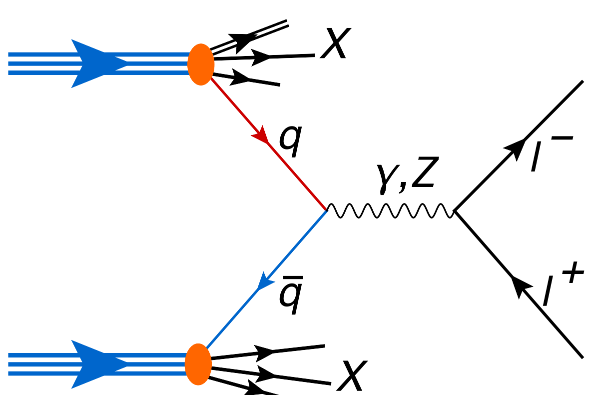
$$z = \frac{p^-}{k^-} \approx z_h = \frac{2E_h}{Q} \qquad a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$$

FACTORIZATION AND *UNIVERSALITY*



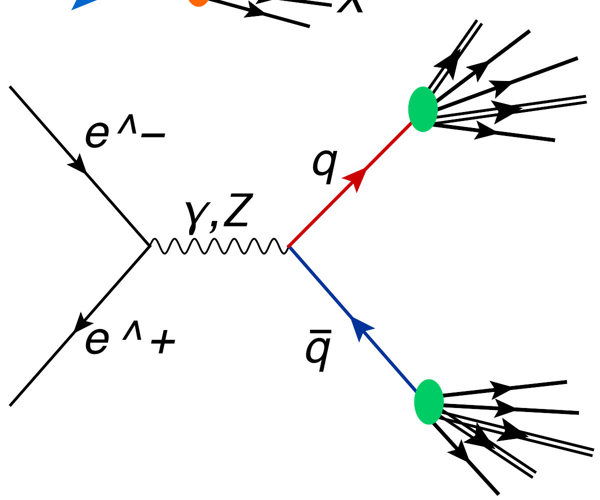
- SEMI INCLUSIVE DIS (SIDIS)

$$\boxed{\sigma^{eP\to ehX} = \sum_q f_q^P \otimes \sigma^{eq\to eq} \otimes D_q^h}$$



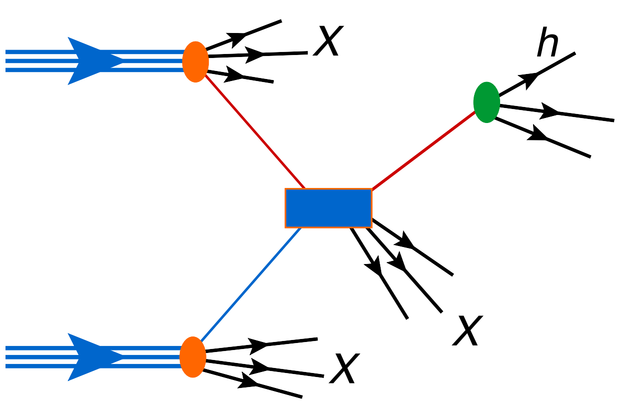
- DRELL-YAN (DY)

$$\boxed{\sigma^{PP\to l^+l^-X} = \sum_{q,q'} f_q^P \otimes f_{\bar{q}}^P \otimes \sigma^{q\bar{q}\to l^+l^-}}$$



- e^+e^-

$$\boxed{\sigma^{e^+e^-\to hX} = \sum_q \sigma^{e^+e^-\to q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)}$$



- Hadron Production

$$\boxed{\sigma^{PP\to hX} = \sum_{q,q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq'\to qq'} \otimes D_q^h}$$

How to obtain (TMD) FFs?

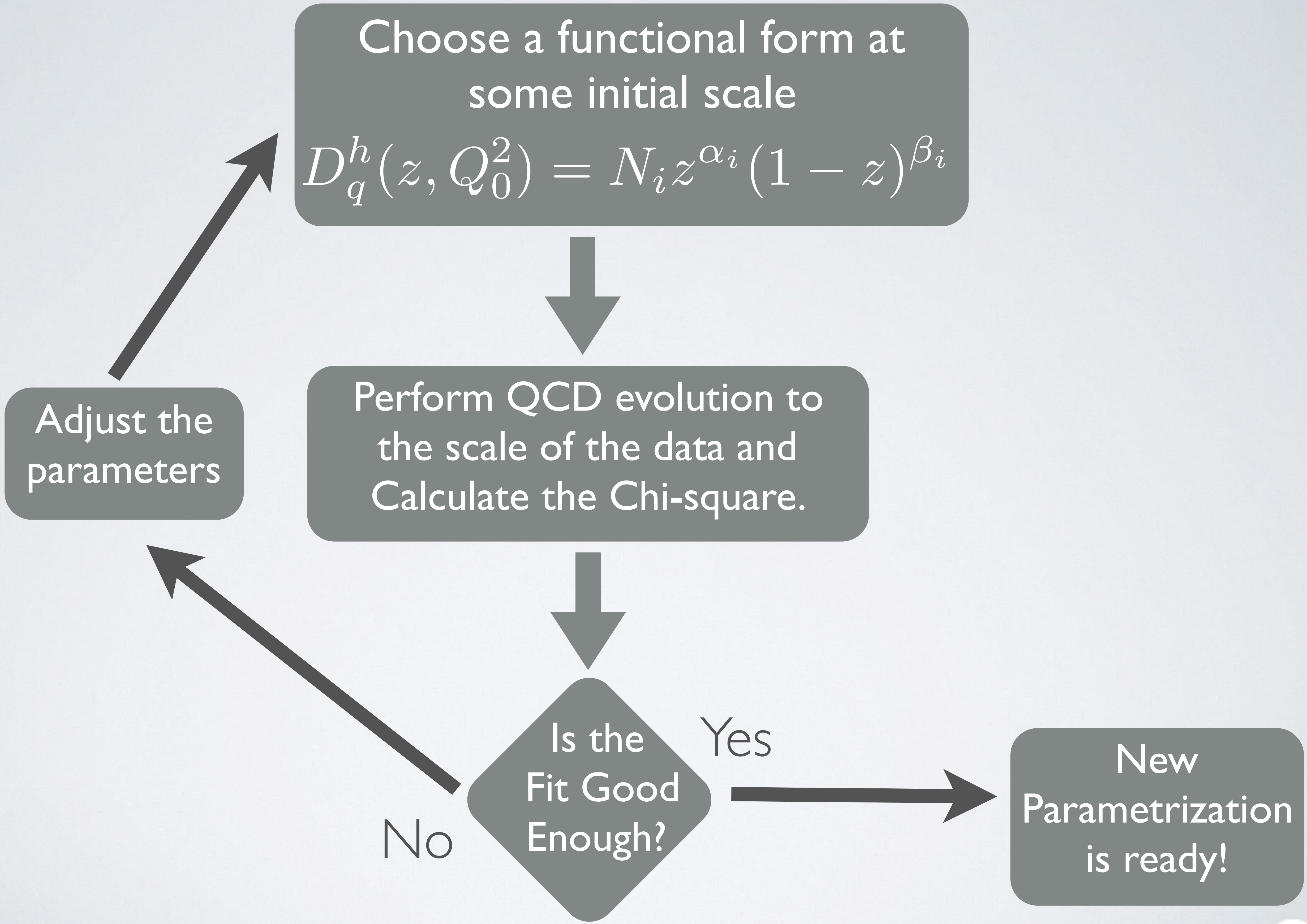
✦ Phenomenological Extractions from Experiment.

- **Use phenomenological parametrizations of *PDFs/FFs* to fit the *SIDIS/e+e-* x-sections for producing *h*.**
- ***Limited physical insight into the hadronization process.***
- ***Still have to model the contributions of non-DIS processes, etc.***

✦ Models

- **Non-quantifiable model approximations.**
- **Only applicable in certain scenarios (**when Jupiter aligns with Mars**).**
- ***Often provide only partial information: only leading hadron/ favoured FFs, etc.***

EMPIRICAL PARAMETRIZATIONS OF DATA



EMPIRICAL PARAMETRIZATIONS OF DATA

Choose a functional form at some initial scale

$$D_q^h(z, Q_0^2) = N_i z^{\alpha_i}(1-z)^{\beta_i}$$

Perform QCD evolution to the scale of the data and Calculate the Chi-square.

Is the Fit Good Enough?

No

Adjust the parameters

Yes

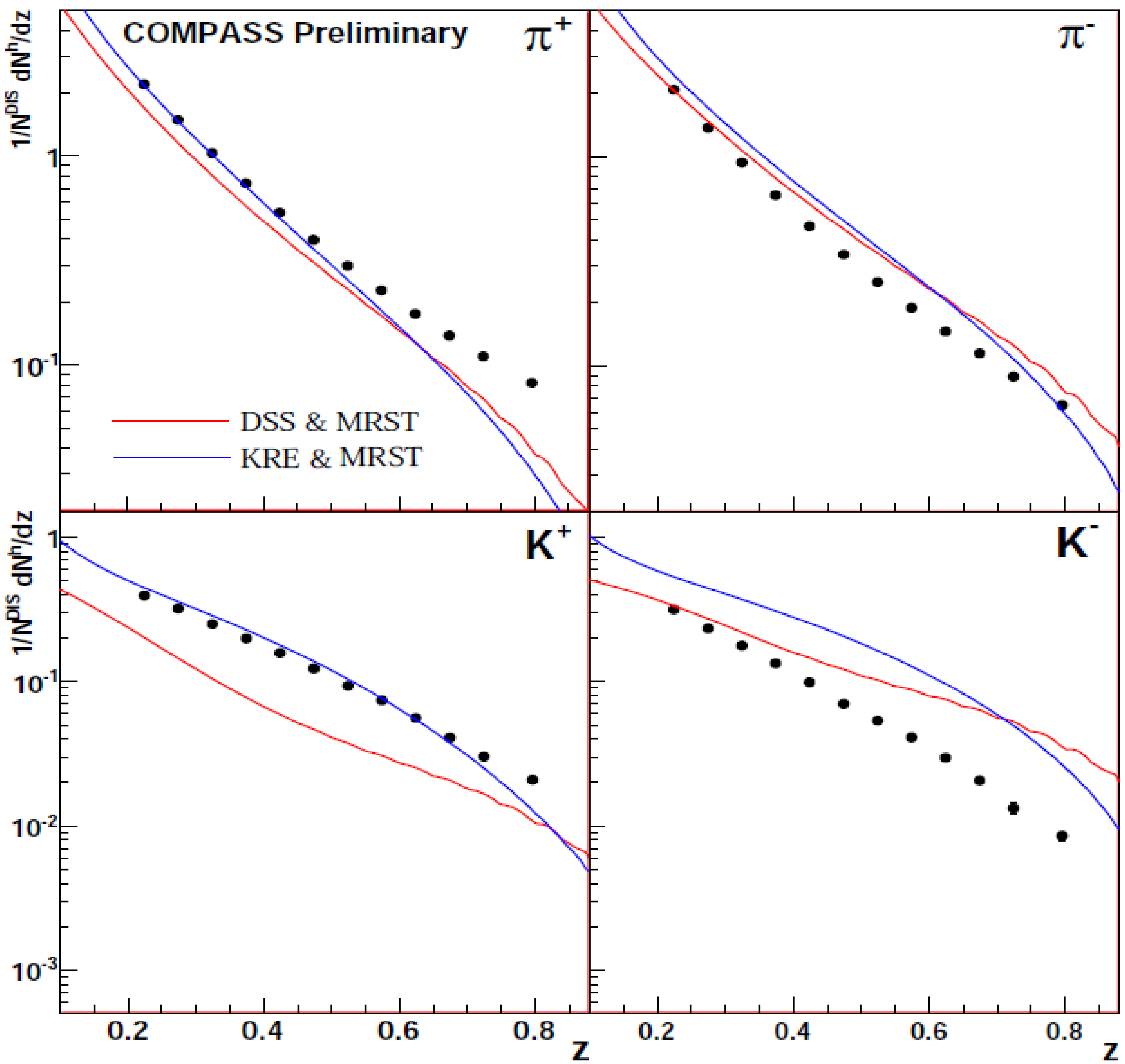
New Parametrization is ready!

Unfavored FFs NOT well known!

Hadron Multiplicities

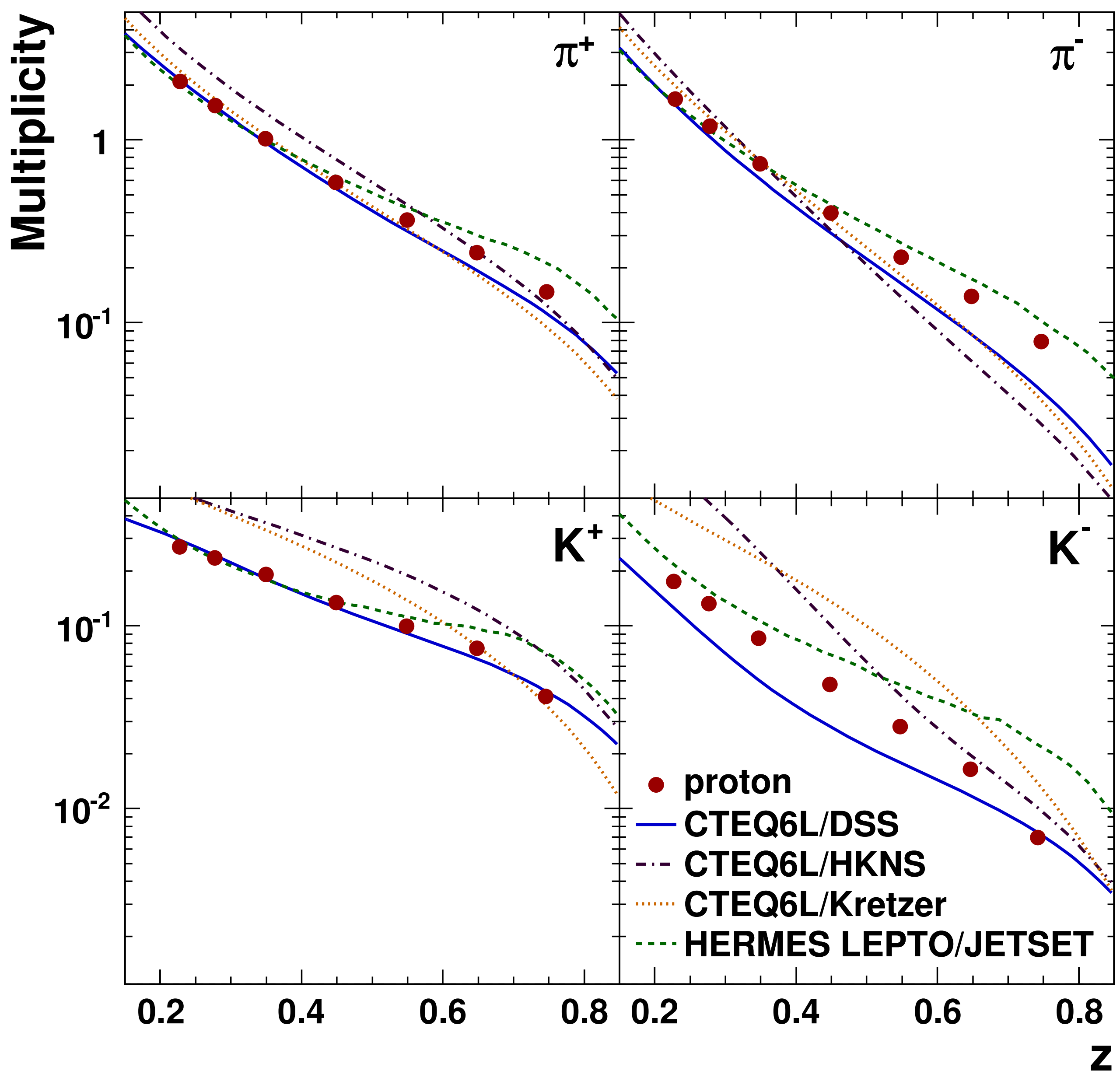
- Preliminary from COMPASS

Talk by C.Franco at CIPANP 2012.



- Also results from HERMES

Phys. Rev. D 87, 074029 (2013)

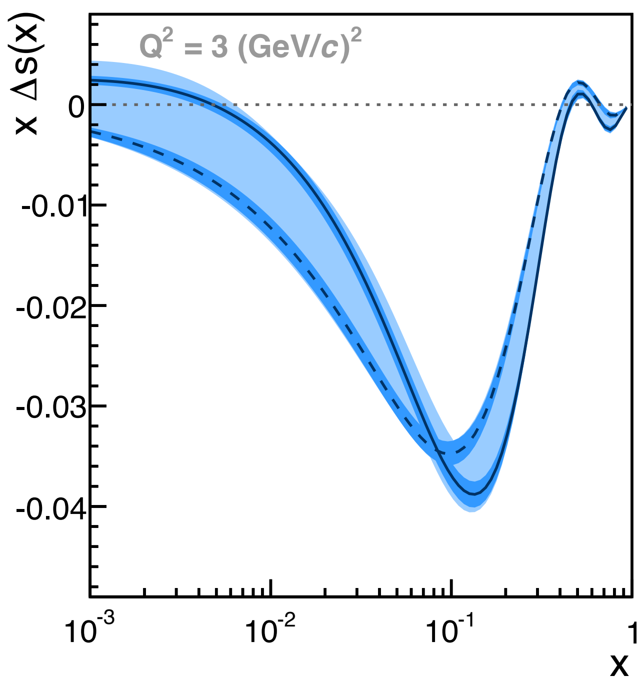


Impact of FF uncertainties on extracted PDFs

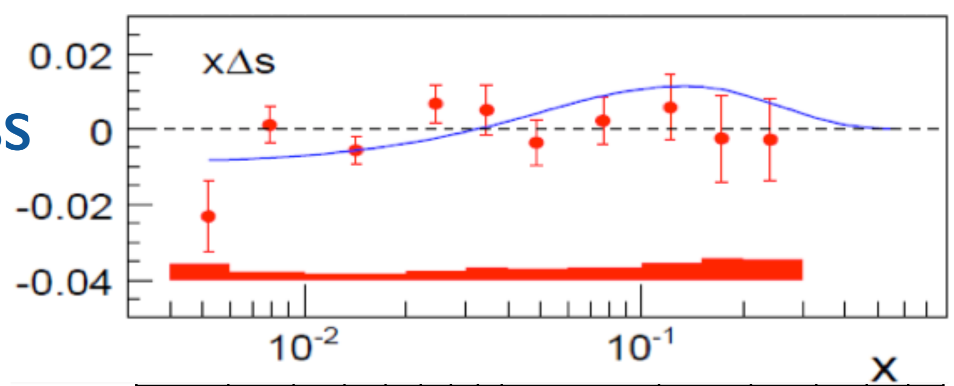
‣ Δs puzzle: DIS vs SIDIS.

Platchkov: Talk in Chile, 2016.

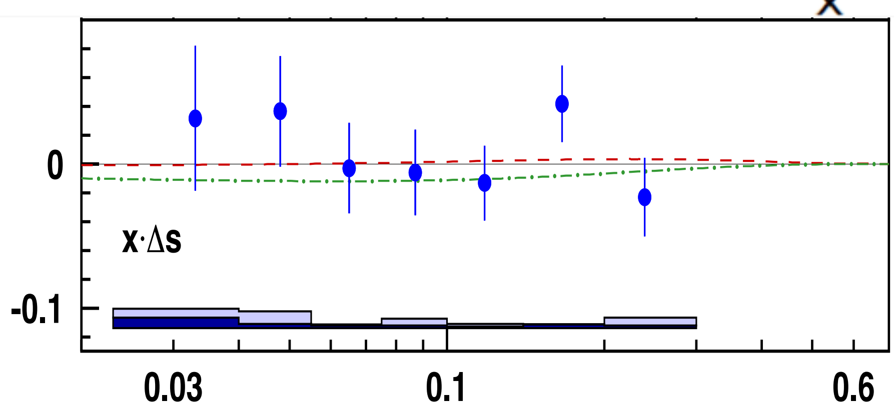
DIS COMPASS



SIDIS COMPASS



SIDIS HERMES

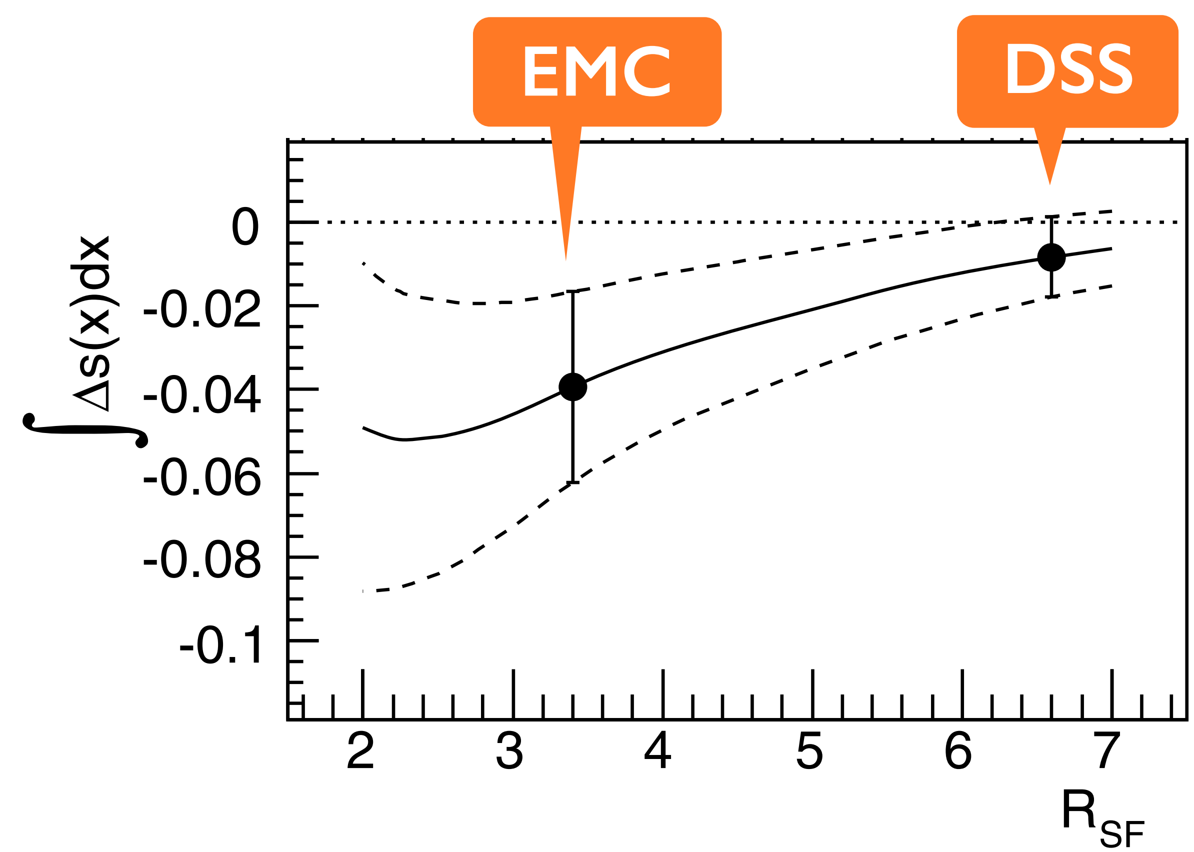


‣ Impact on extracted Δs

COMPASS: PLB 693 (2010) 227–235.

$$A_1^h(x,z) = \frac{\sum_q e_q^2 (\Delta q(x) D_q^h(z) + \Delta\bar{q}(x) D_{\bar{q}}^h(z))}{\sum_q e_q^2 (q(x) D_q^h(z) + \bar{q}(x) D_{\bar{q}}^h(z))}.$$

$$R_{UF} = \frac{\int D_d^{K^+}(z)\,dz}{\int D_u^{K^+}(z)\,dz}, \qquad R_{SF} = \frac{\int D_{\bar{s}}^{K^+}(z)\,dz}{\int D_u^{K^+}(z)\,dz}.$$



TMD Fragmentation Functions

✦ TMD Polarized Fragmentation Functions at LO.

- *Only **two** for unpolarised final state hadrons.*

PDFs

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\, h_{1T}^\perp$

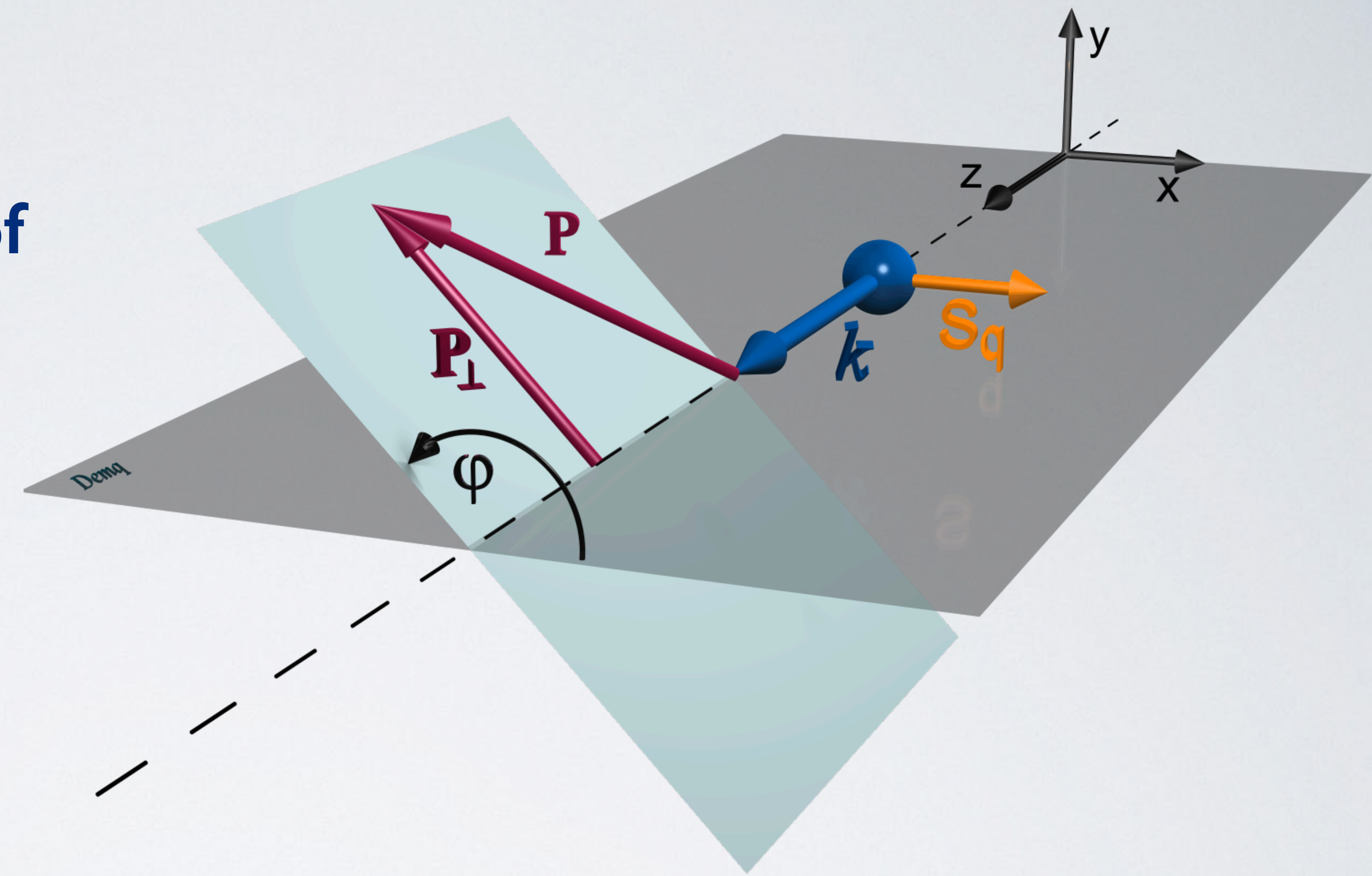
Fragment. Functions

h/q	U	L	T
U	D_1		$H_1^\perp$

COLLINS FRAGMENTATION FUNCTION

- **Collins Effect:**

  Azimuthal Modulation of Transversely Polarized Quark' Fragmentation Function.



Unpolarized

$$D_{h/q^{\uparrow}}(z, P_{\perp}^2, \varphi) = D_1^{h/q}(z, P_{\perp}^2) - H_1^{\perp h/q}(z, P_{\perp}^2)\frac{P_{\perp} S_q}{z m_h}\sin(\varphi)$$

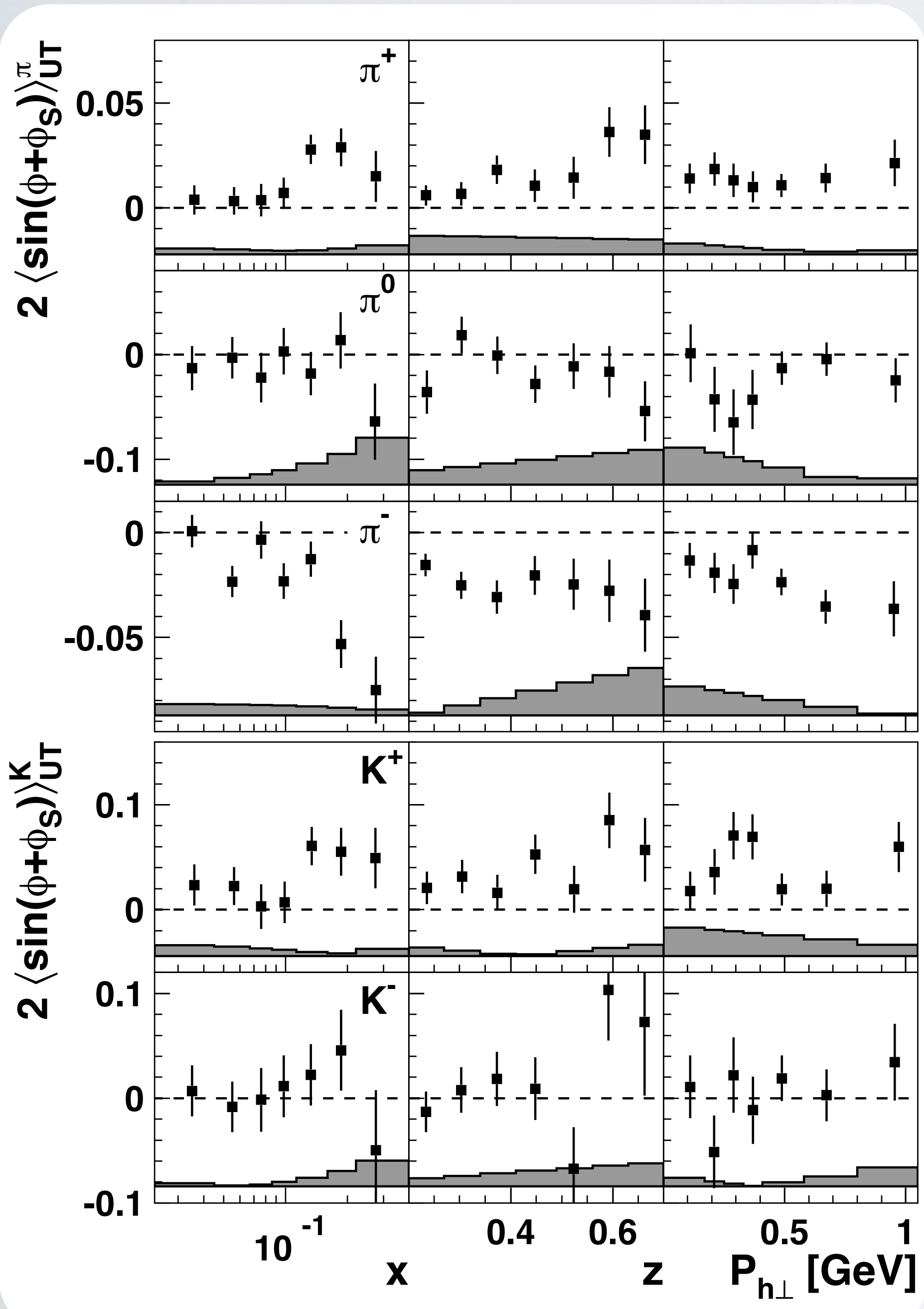
Collins

- **Chiral-ODD**: Needs to be coupled with another chiral-odd quantity to be observed.

EMPIRICAL EXTRACTIONS OF TRANSVERSITY

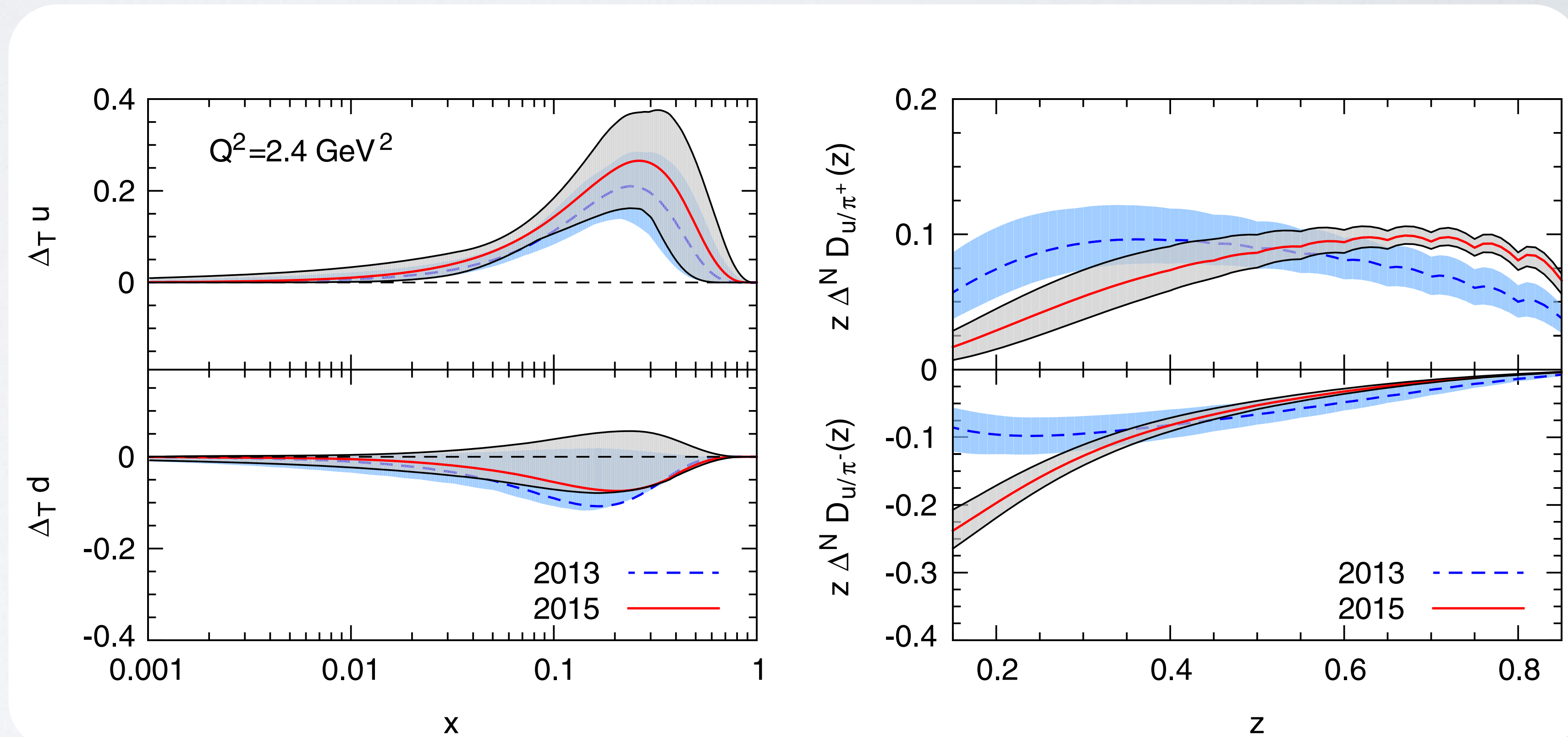
- ***SIDIS at HERMES***
  PLB693 (2010) 11-16.

$$\langle \sin(\phi + \phi_S) \rangle^h_{UT} \sim \frac{\mathcal{C}[h_1^q \; H_{1q}^{\perp h/q}]}{\mathcal{C}[f_1^q \; D_1^{h/q}]}$$



- Opposite sign for the charged pions.
- Large positive signal for K^+.
- Consistent with 0 for π^0 and K^-.

❖ ***Fits to HERMES, COMPASS and BELLE/BaBar:*** PRD 92, 114023 (2015).

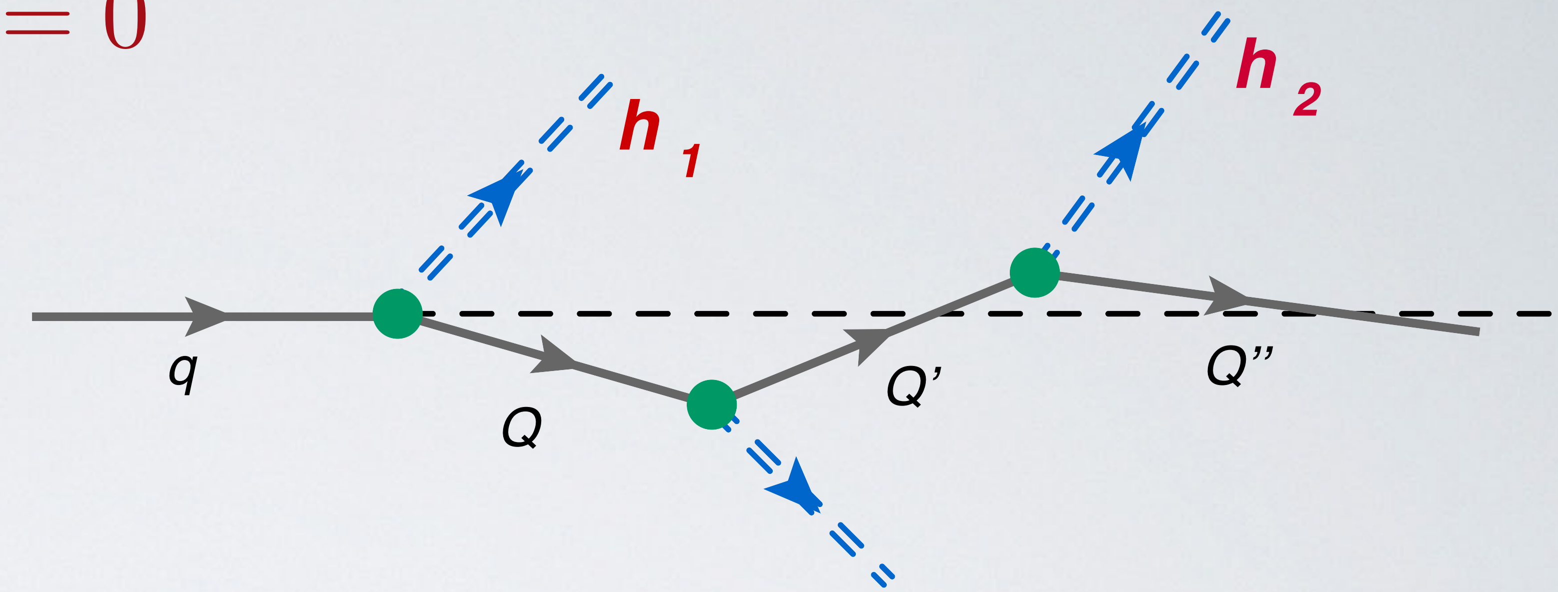


- **Still Large Uncertainties!**
- **Simplistic Approximations !**

TWO-HADRON FRAGMENTATION

✦ **Transformation to frame $\mathbf{k}_T = 0$**

$$k = (k^-, k^+, \mathbf{0})$$
$$\mathbf{k}_T = -\mathbf{P}_T / z_h$$



$$\mathbf{P}_T = \mathbf{P}_{h_1}^{\perp} + \mathbf{P}_{h_2}^{\perp}$$
$$\mathbf{R} = (\mathbf{P}_{h_1}^{\perp} - \mathbf{P}_{h_2}^{\perp})/2$$

✦ Integrate over one or other momentum:

$$D_{q^\uparrow}^{h_1 h_2}(\varphi_R) = D_{1,q}^{h_1 h_2} + \sin(\varphi_R - \varphi_S)\mathcal{F}[H_1^{\sphericalangle}, H_1^{\perp}]$$
$$D_{q^\uparrow}^{h_1 h_2}(\varphi_T) = D_{1,q}^{h_1 h_2} + \sin(\varphi_T - \varphi_S)\mathcal{F}'[H_1^{\sphericalangle}, H_1^{\perp}]$$

✦ The IFF surviving after $\mathbf{k}_T$ integration is redefined as

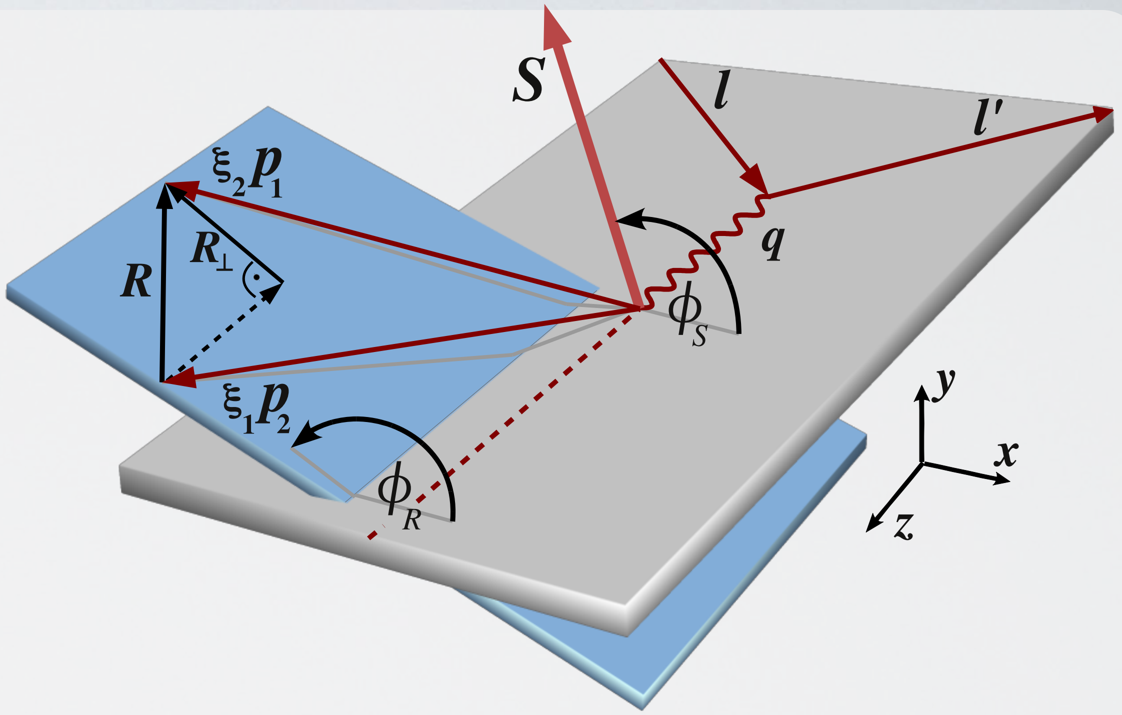
A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

$$H_1^{\sphericalangle}(z_h, \xi, M_h^2) \equiv \int d^2\mathbf{k}_T \left[H_1^{\sphericalangle' e}(z_h, \xi, M_h^2, k_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) + \frac{k_T^2}{2M_h^2} H_1^{\perp o}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \right]$$

ACCESS TO TRANSVERSITY PDF From DiFF in *SIDIS*

M. Radici, et al: PRD 65, 074031 (2002).

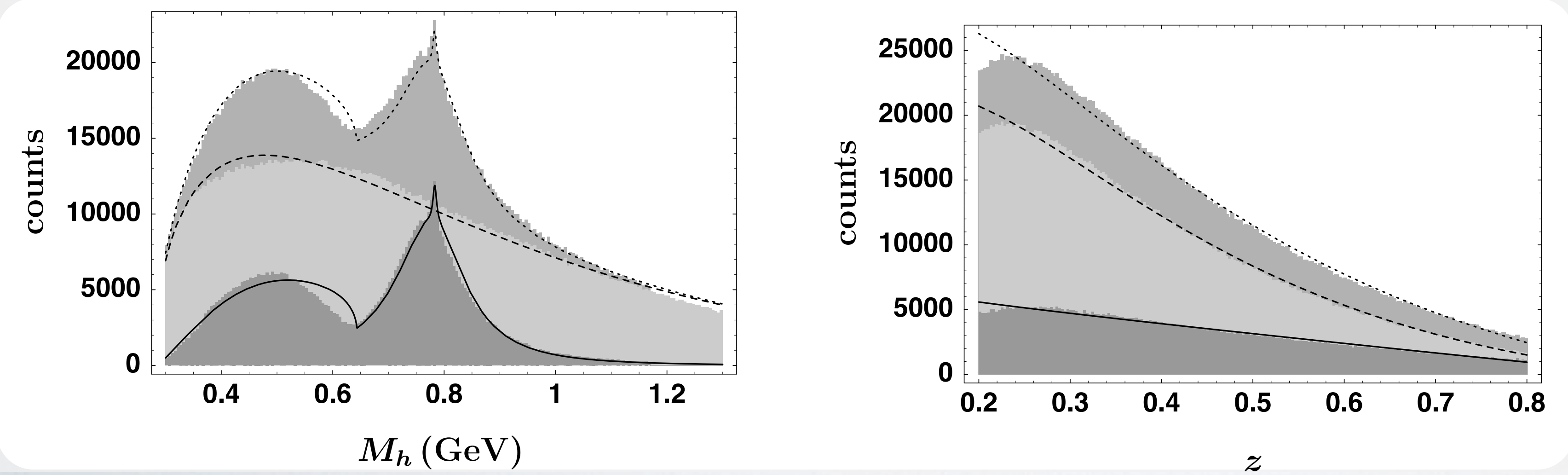
- In two hadron production from polarized target the cross section factorizes ***collinearly* - no TMD!**
- Allows clean access to ***transversity***.
- ***Unpolarized*** and ***Interference*** Dihadron FFs are needed!



$$\frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2 \; h_1^q(x)/x \;\; H_1^{\sphericalangle q}(z, M_h^2)}{\sum_q e_q^2 \; f_1^q(x)/x \;\; D_1^q(z, M_h^2)}$$

- **Empirical Model for D_1^q has been *fitted to PYTHIA simulations*.**

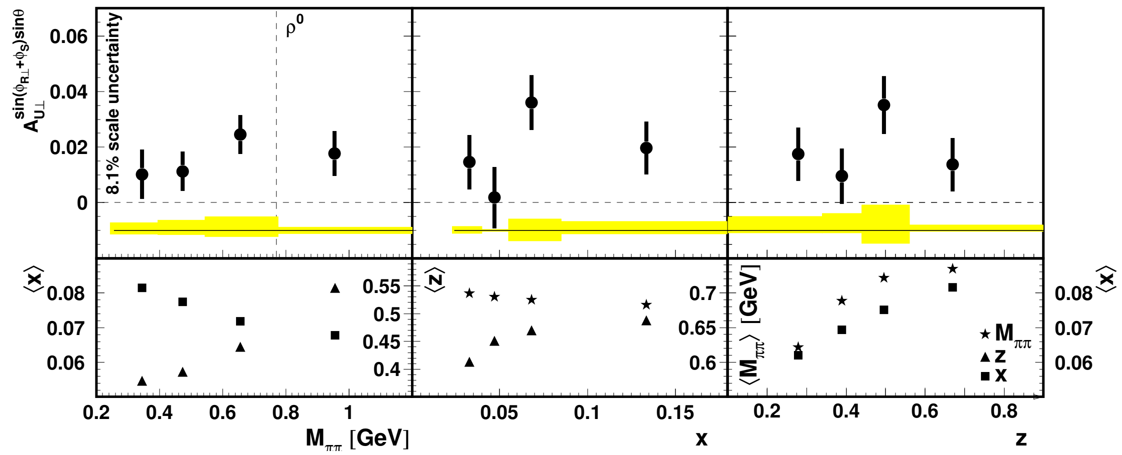
A. Bacchetta and M. Radici, PRD 74, 114007 (2006).



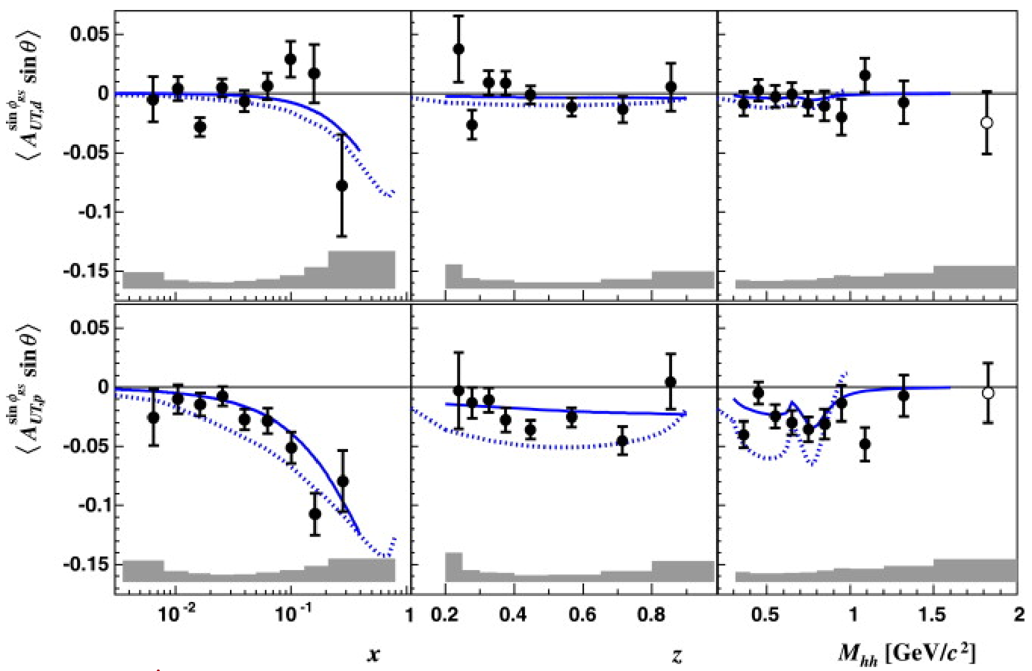
Experiments:
BELLE,
HERMES,
COMPASS.

Empirical Extractions of Transversity from 2h Data

- *SIDIS at HERMES*

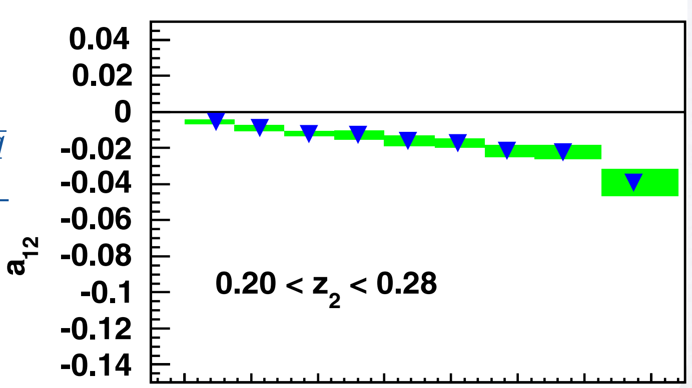


- *SIDIS at COMPASS*

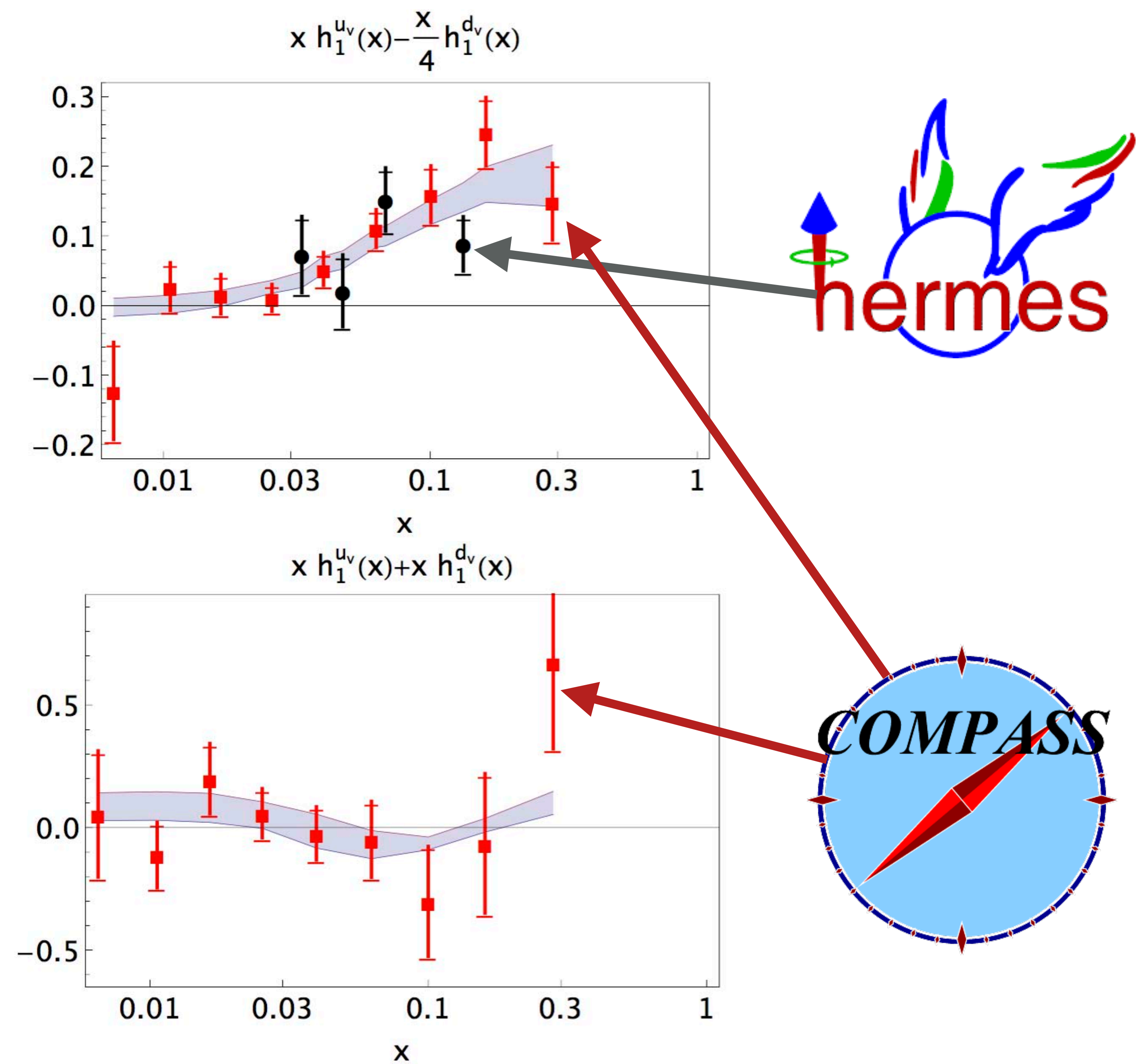


- e^+e^- at BELLE for *two* 2h

$$a_{12} \sim \frac{\sum_q H^{\sphericalangle q}\ H^{\sphericalangle \bar{q}}}{\sum_q D_1^q\ D_1^{\bar{q}}}$$



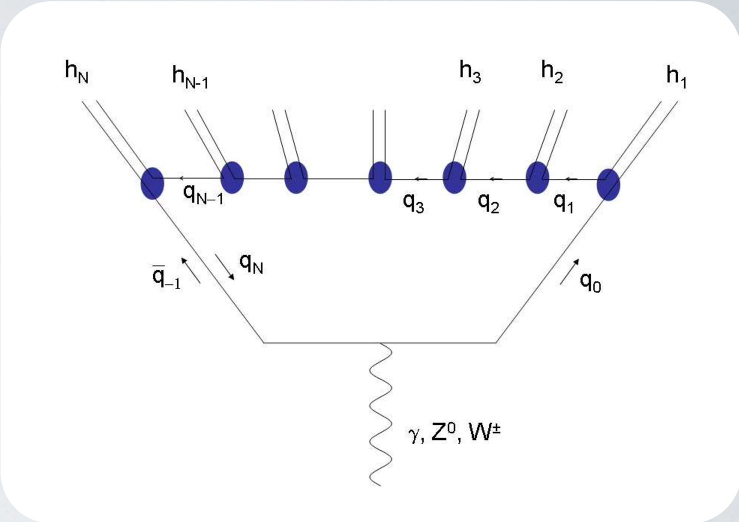
- *Fits to HERMES, COMPASS* ***using*** *BELLE DiFF:* Radici et al: JHEP 1505 (2015) 123.



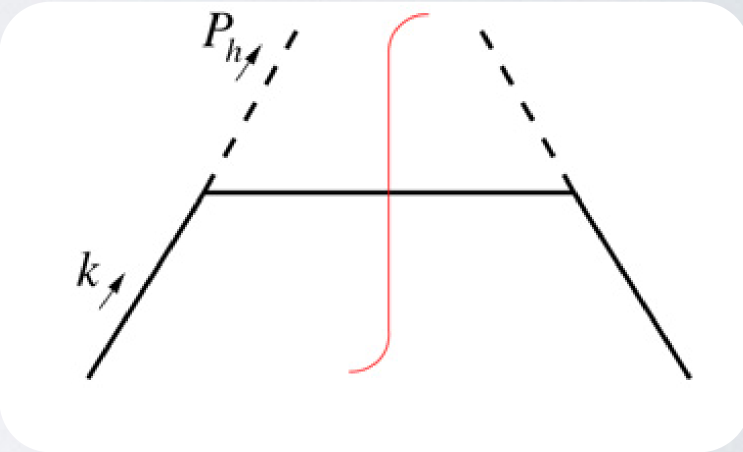
- **Empirical parametrizations of IFF.**
- **Rely on unpolarised DiFF Model!**

(SOME of the) MODELS FOR FRAGMENTATION

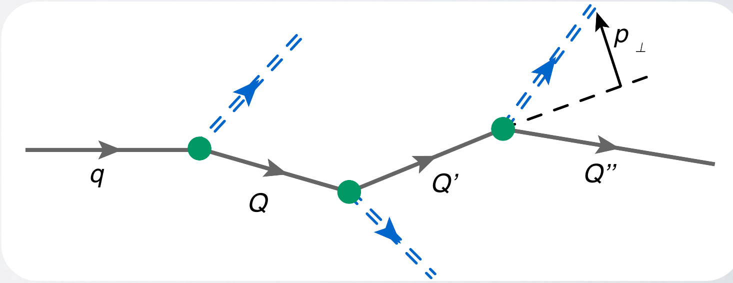
- *Complete Hadronization:*
  - **Lund String, Cluster Hadronization.**
  - *Very Successful:* *PYTHIA, HERWIG, SHERPA, …*
  - Highly Tunable - Limited Predictive Power.
  - **No Spin Effects** - Formalism X. Artru for Lund model!



- *Spectator Models*
  - Quark model calculations with empirical form factors.
  - No unfavored fragmentations.
  - Need to tune parameters for small z dependence.



- *NJL-jet Model*
  - *Multi-hadron* emission framework.
  - Effective quark model input.
  - *Monte-Carlo framework:* flexibility in including the transverse momentum, spin effects, two-hadron correlations, etc.



The Original
Lund String
(courtesy of D. Sivers)

Overwhelmed
Physicist

THE LUND STRING MODEL

SLIDE STOLEN FROM P. SKANDS

Andersson - Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol. 7 (1997) 1-471

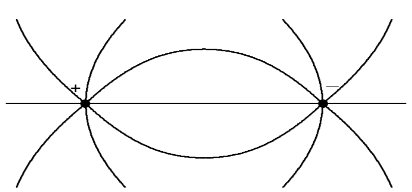
The Ultimate Limit: Wavelengths > 10^{-15} m

Quark-Antiquark Potential

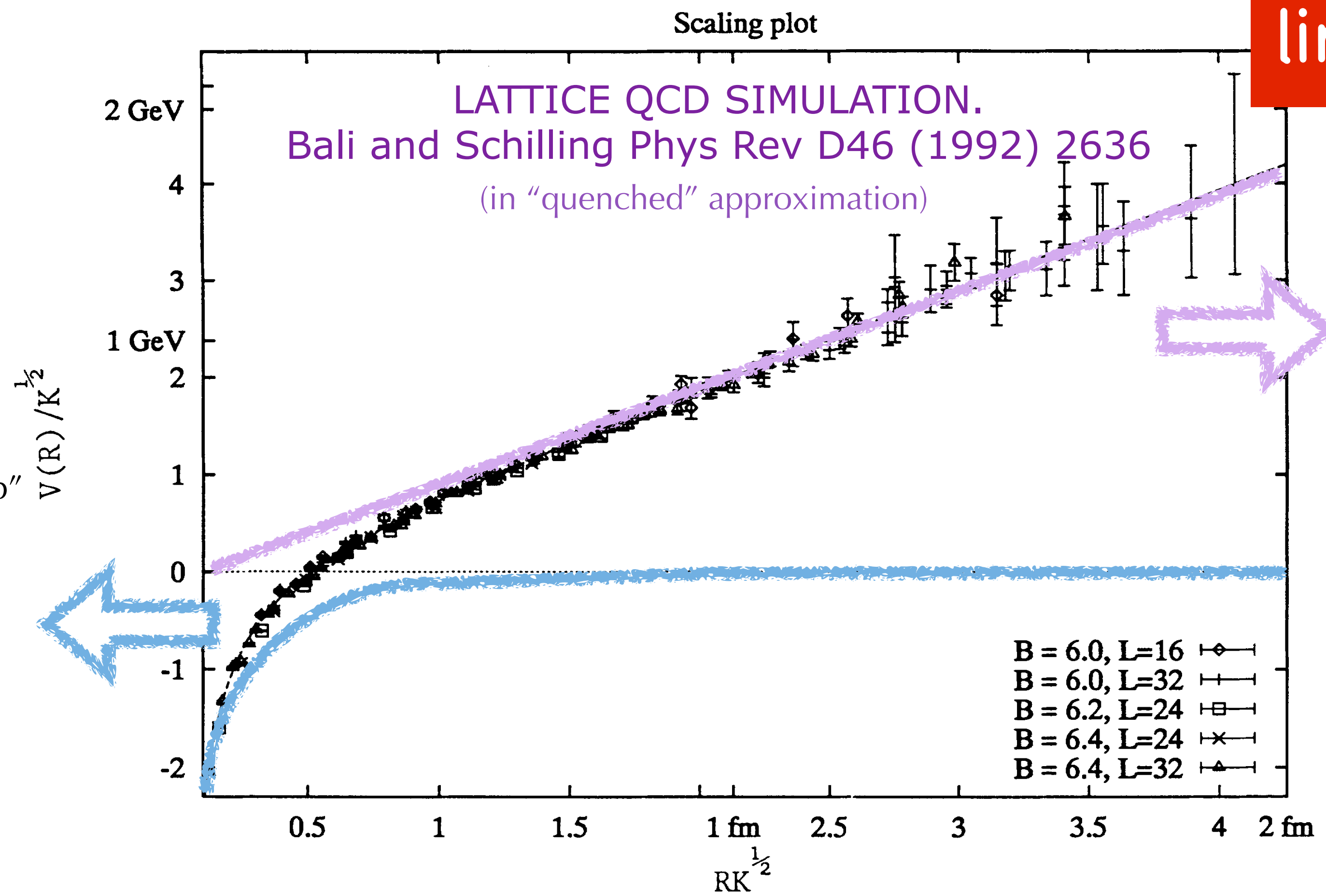
As function of separation distance

What physical system has a linear potential?

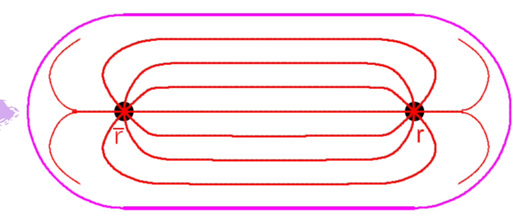
Short Distances ~ "Coulomb"



"Free" Partons



Long Distances ~ Linear Potential



"Confined" Partons (a.k.a. Hadrons)

$$F(r) \approx \text{const} = \kappa \approx 1\ \text{GeV/fm} \quad \Longleftrightarrow \quad V(r) \approx \kappa r$$

~ Force required to lift a 16-ton truck

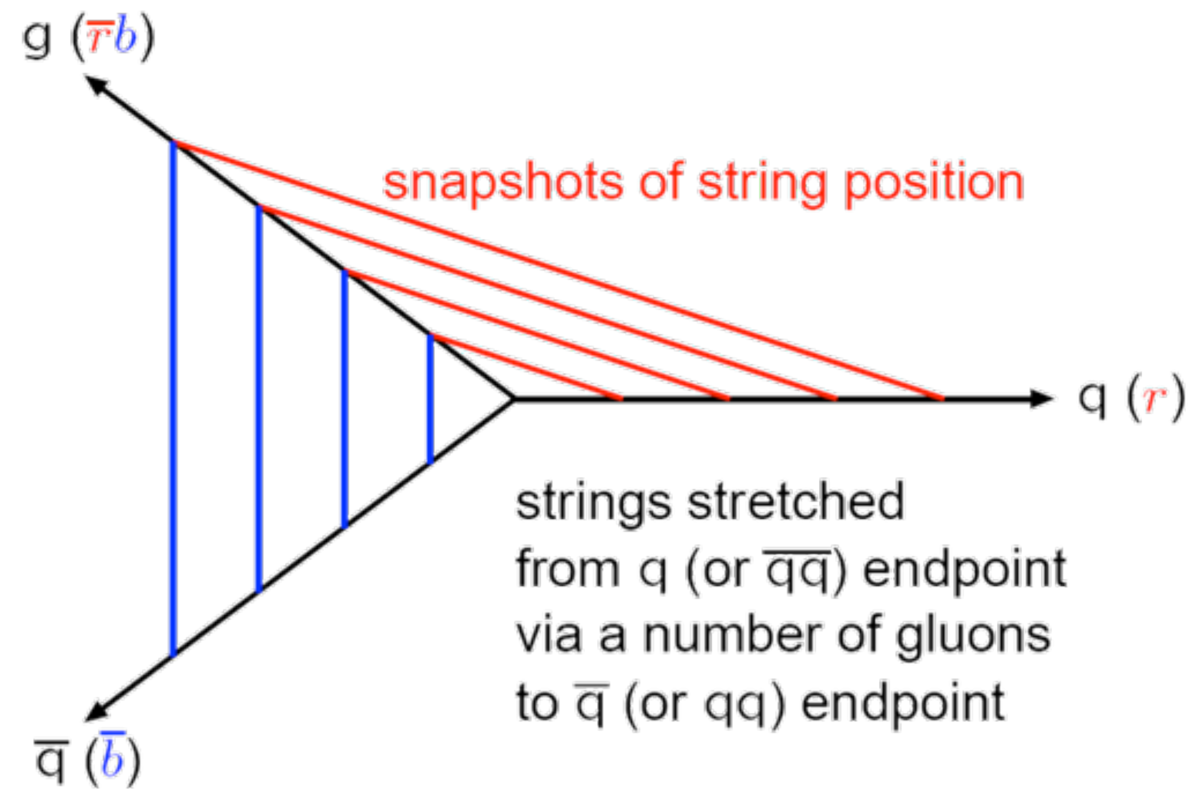
SLIDE STOLEN FROM P. SKANDS

The (Lund) String Model

See also Yuri's 2nd lecture

Map:

- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)
- Physics then in terms of string worldsheet evolving in spacetime
- Probability of string break (by quantum tunneling) constant per unit area → **AREA LAW**



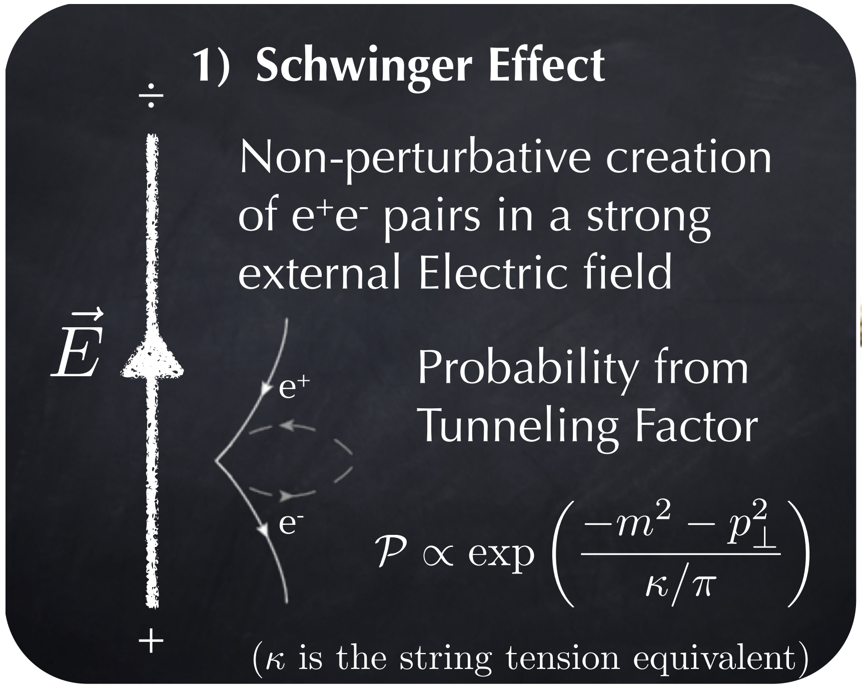
Gluon = kink on string, carrying energy and momentum

→ **STRING EFFECT**

Physics now in terms of strings, with kinks, evolving in spacetime
Very simple space-time picture, few parameters at this point

LUND SYMMETRIC FF

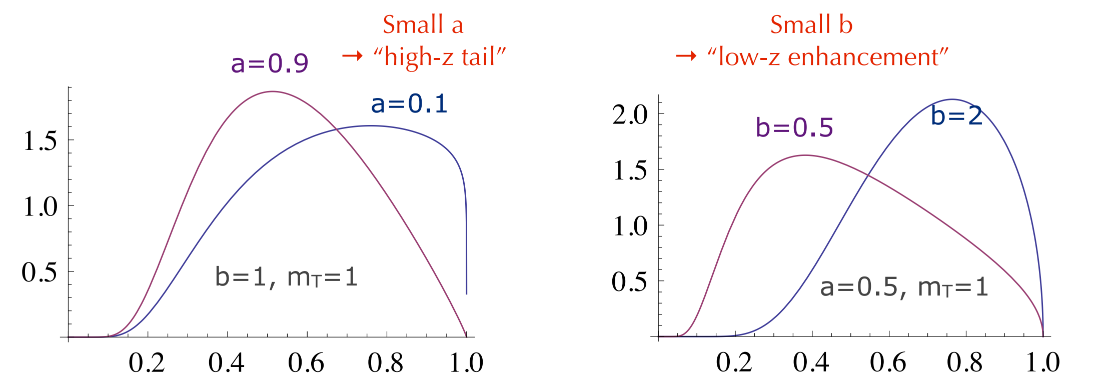
- String breaks: quark-antiquark pair creation via tunnelling in strong "chromoelectric" field.
  - ✦ Does NOT depend on the type of produced hadron!



- Causality: independent breaking of the string:

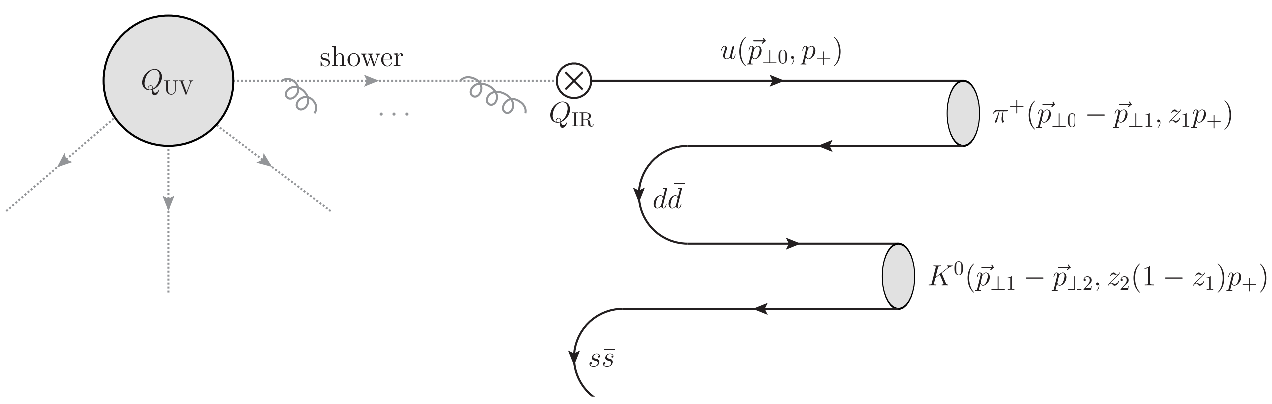
❖ Constrained form of FF

$$f(z) \propto \frac{1}{z}(1-z)^a \exp\left(-\frac{b\,(m_h^2 + p_{\perp h}^2)}{z}\right)$$



Note: In principle, a can be flavour-dependent. In practice, we only distinguish between baryons and mesons

❖ May produce h in *any* order.



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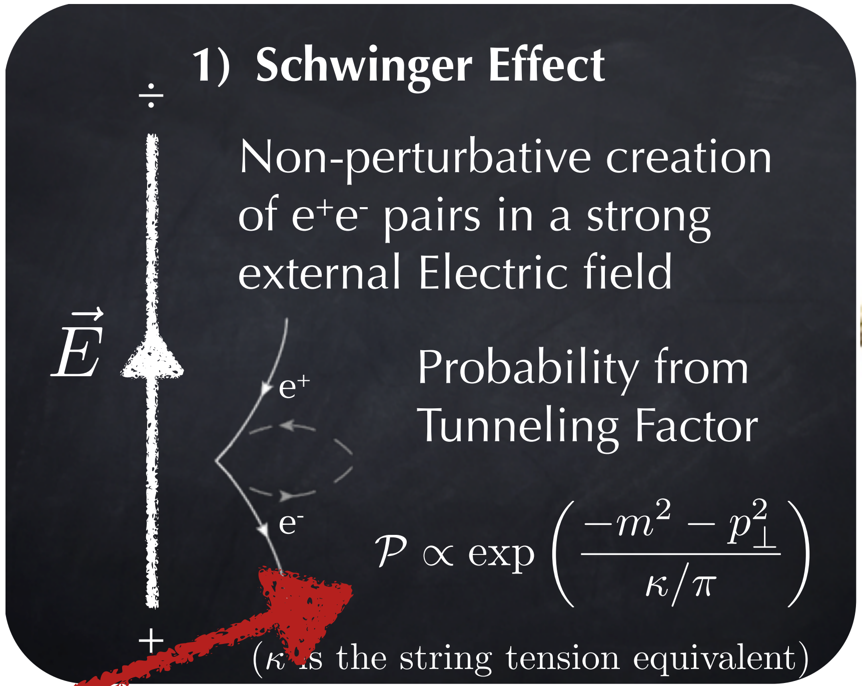

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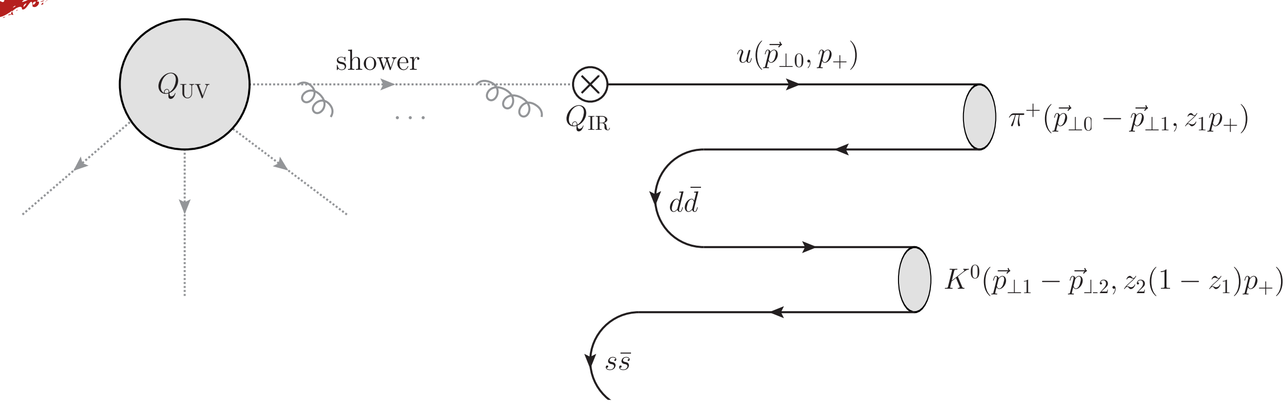
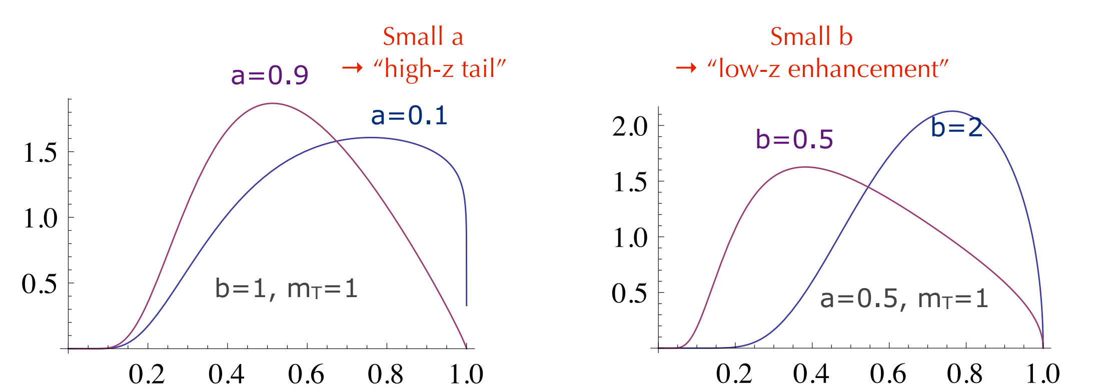




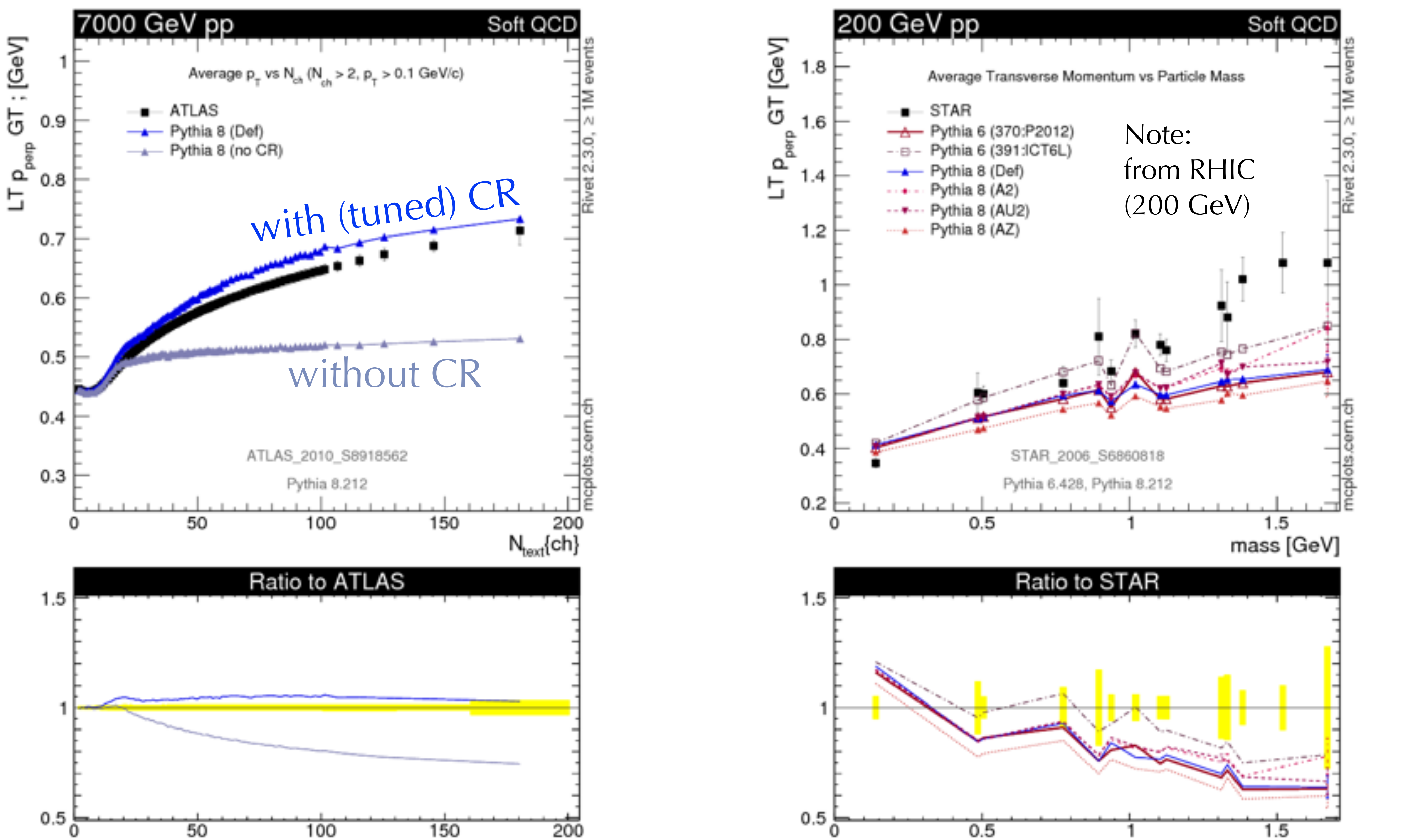
Note: In principle, *a* can be flavour-dependent. In practice, we only distinguish between baryons and mesons

The hadron z *depends* on combined TM of antiquark and a quark from previous string break!

What do we see?

<pT> vs Number of Particles

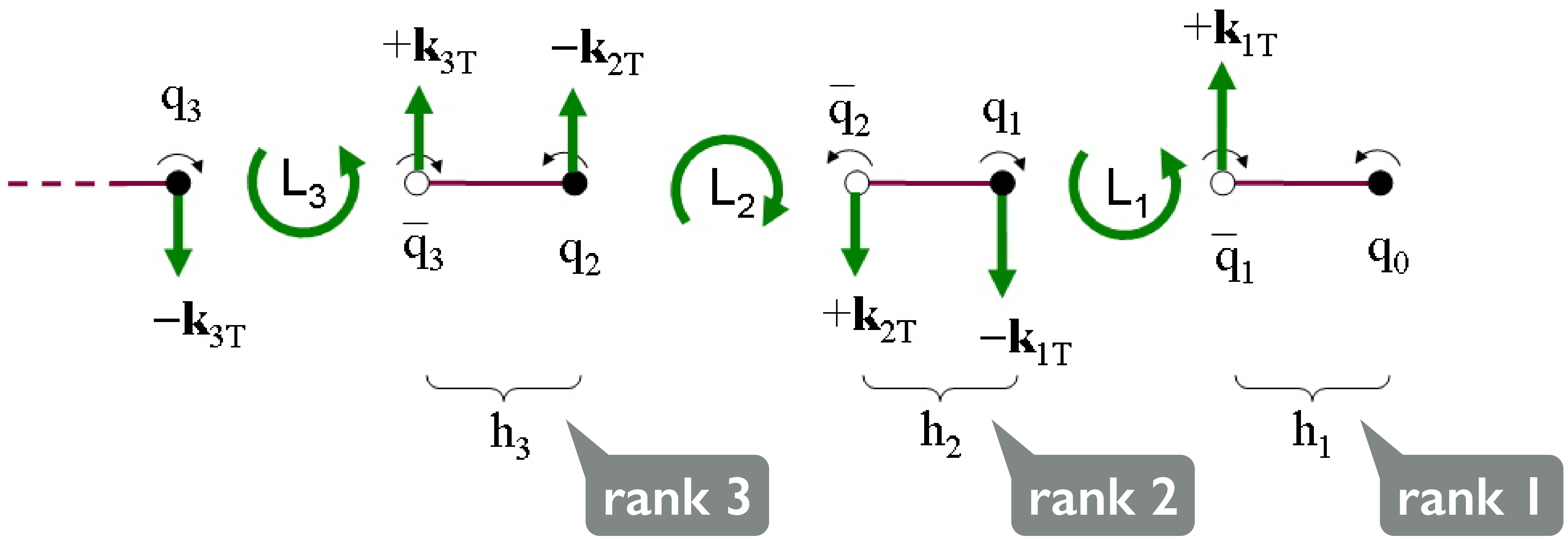
<pT> vs Particle Mass



Average pT increases with particle multiplicity and (faster than predicted) with particle mass

Artru Model

- ✦ $q\bar{q}$ **created in** $^3P_0$ **state.**
- ✦ **Local compensation of TM.**



- ✦ No quantitative results for Collins FFs: implies opposite signs for favoured and unfavored. (Omitting complications from favoured production at rank 2, etc .)
- ✦ *Simple* and intuitive *quantum-mechanical* picture.

SPECTATOR MODELS

SPECTATOR MODELS

E.G. - Bacchetta et al, PLB 659:234, 2008

✦ *Use Field-theoretical definition of FFs from a Correlator.*

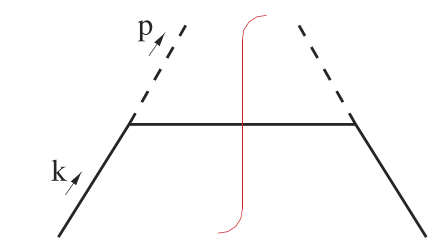
$$\boxed{\Delta(z,k_T) = \frac{1}{2z}\int dk^+\,\Delta(k,P_h) = \frac{1}{2z}\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3} e^{ik\cdot\xi} \langle 0|\mathcal{U}^{n_+}_{(+\infty,\xi)}\psi(\xi)|h,X\rangle\langle h,X|\bar{\psi}(0)\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\big|_{\xi^-=0}}$$

$$D_1(z, z^2\vec{k}_T^2) = \mathrm{Tr}[\Delta(z,\vec{k}_T)\gamma^-]. \qquad \frac{\epsilon_T^{ij}k_{Tj}}{M_h} H_1^\perp(z,k_T^2) = \frac{1}{2}\mathrm{Tr}[\Delta(z,k_T)i\sigma^{i-}\gamma_5]$$

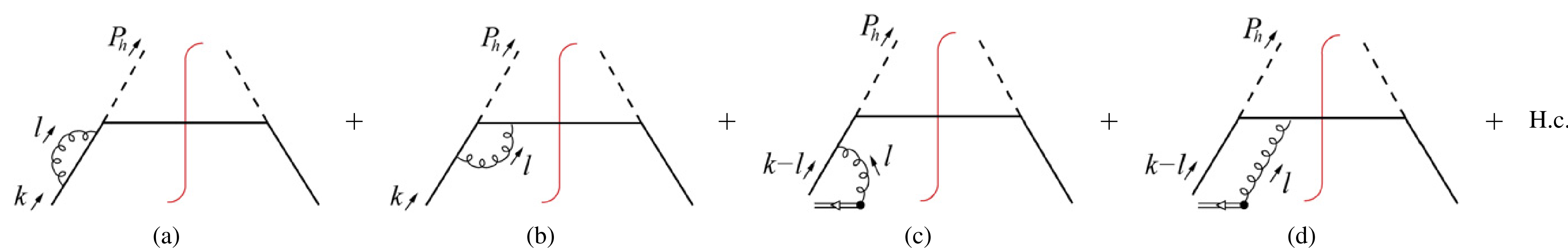
✦ *Approximate the remnant X as a "spectator" (quark).*

✦ *Calculate the FFs at leading-order in favourite quark model.*

$D_1(z, p_\perp^2)$



$H_1^\perp(z, p_\perp^2)$

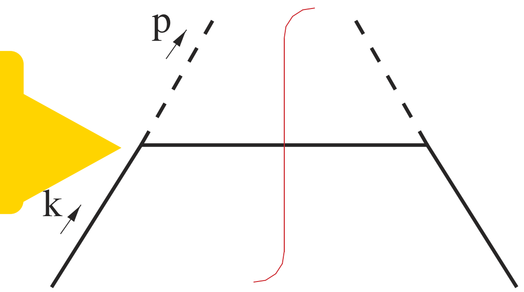


SPECTATOR MODELS

E.G. - Bacchetta et al, PLB 659:234, 2008

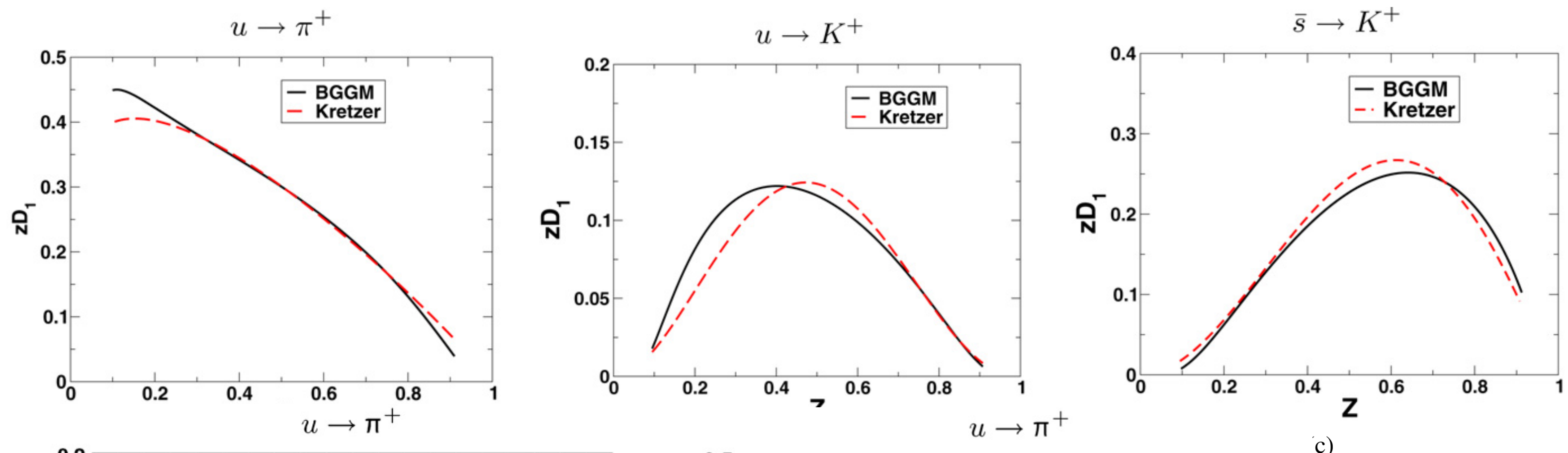
- ***Empirical meson-quark coupling:***

$$g_{q\pi} \mapsto \frac{g_{q\pi}}{z} e^{-\frac{k^2}{\Lambda^2}}$$

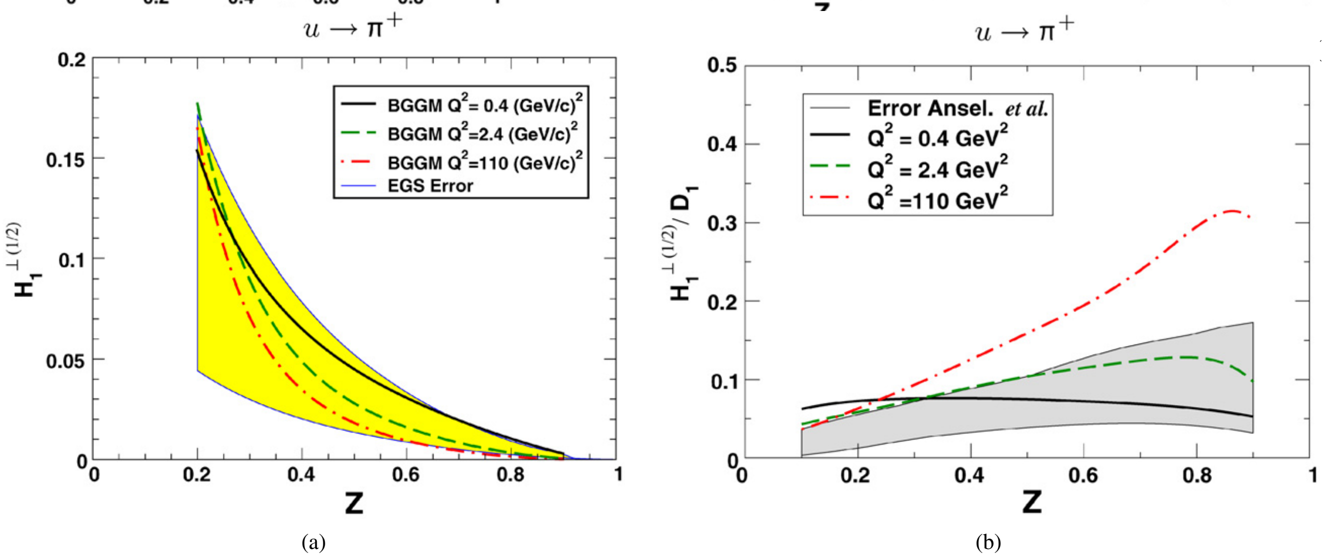


fitted to unpolarised FFs:

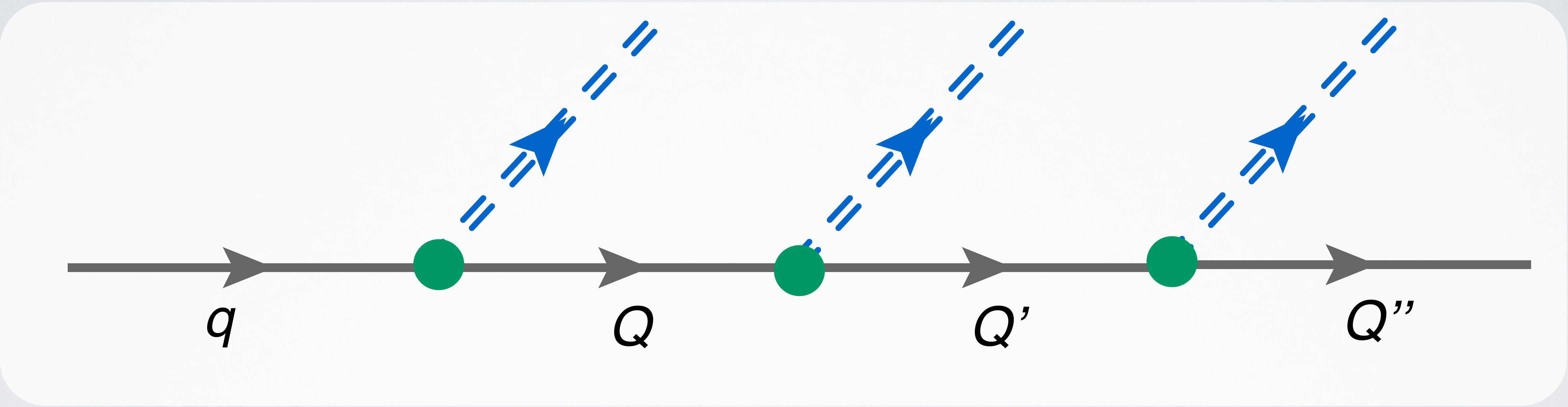
$$\Lambda^2 = \lambda^2 z^\alpha (1-z)^\beta$$



- ***Calculated Collins FF.***



- ***Can calculate TM-dependence of FF, Dihadron FFs, etc.***
- ***Missing multi-hadron emission effect:***
  - *No direct access to unfavored FFs.*
  - *Description of small-z region.*

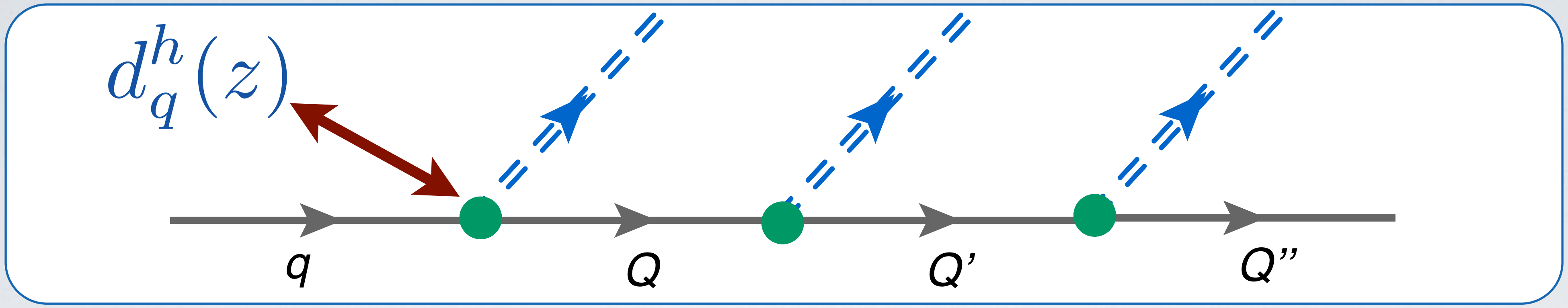


THE NJL-jet MODEL

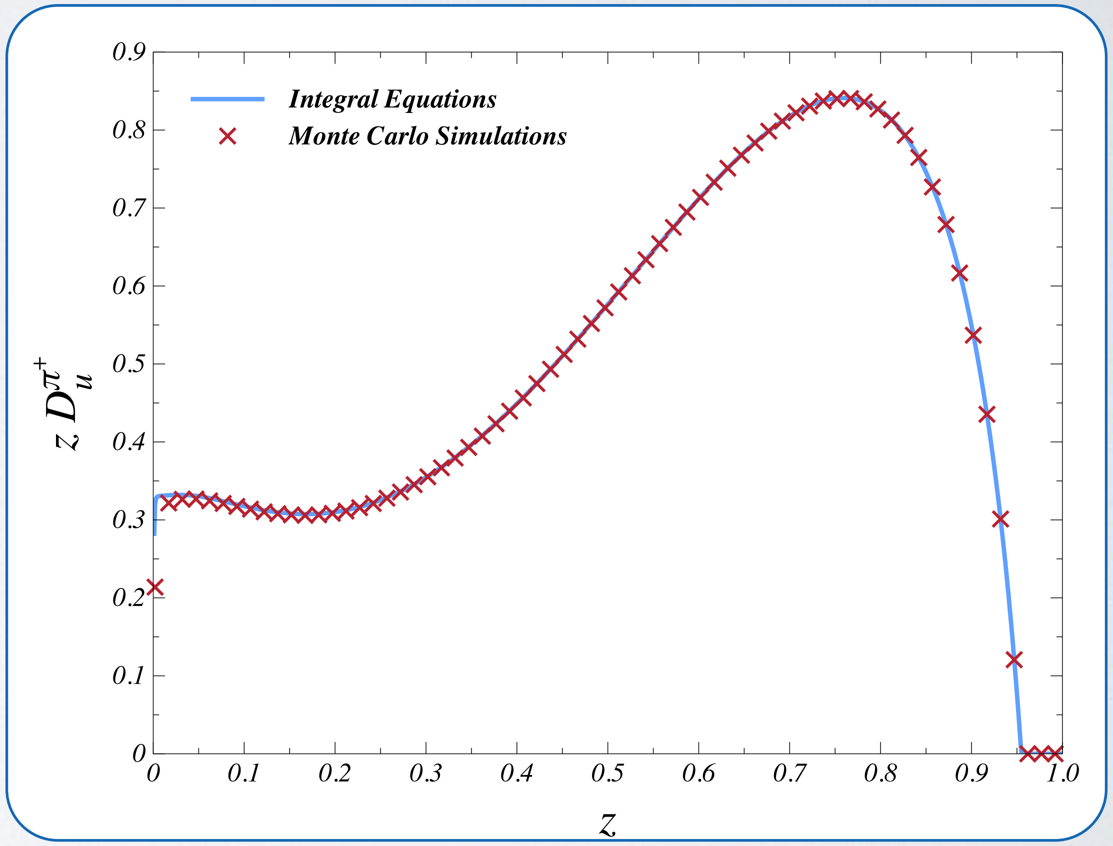
COLLINEAR FRAGMENTATIONS FROM MC

H.M., Thomas, Bentz, PRD. 83:07400; PRD.83:114010, 2011.

- Input: One hadron emission probability



- Sample the emitted hadron type and z according to input splitting.
- ***CONSERVE***: Momentum and Quark Flavor in each step.
- Repeat for decay chains with the same initial quark.



$$D_q^h(z)\Delta z = \langle N_q^h(z, z+\Delta z)\rangle \equiv \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z)}{N_{Sims}}$$

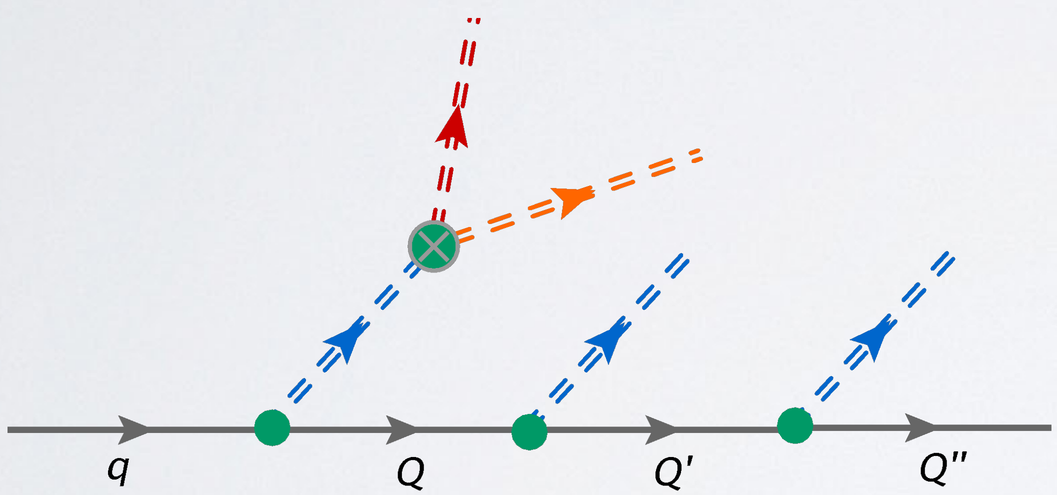
MORE CHANNELS

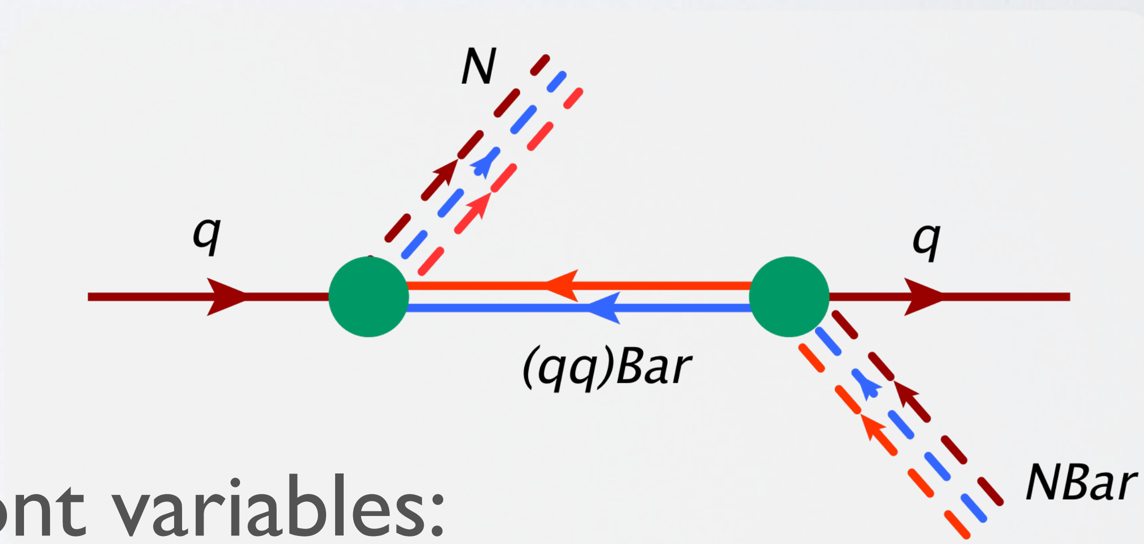
H.M., Thomas, Bentz, PRD. 83:074003, 2011

- Calculate quark splittings to vector mesons, Nucleon Anti-Nucleon: $d_q^h(z)$

$$h = \rho^0, \rho^\pm, K^{*0}, \overline{K}^{*0}, K^{*\pm}, \phi, N, \bar{N}$$

- Add the decay of the resonances:



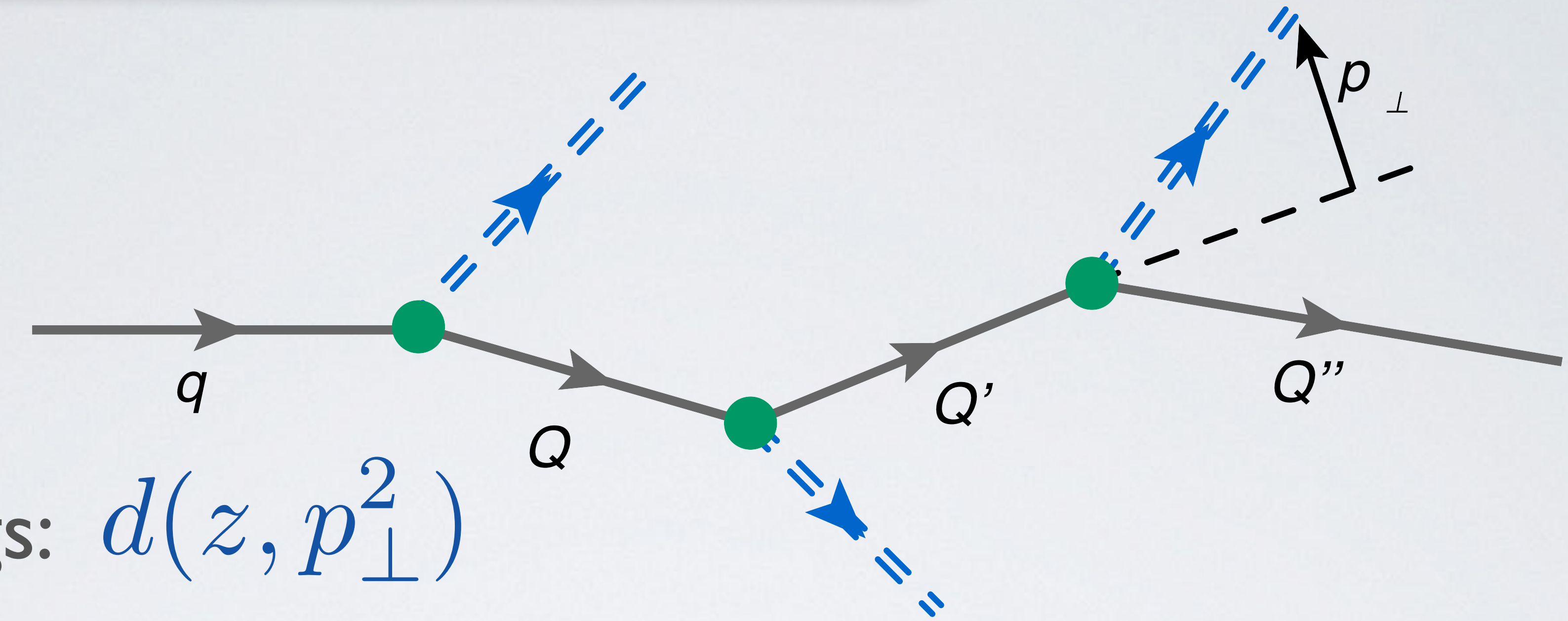


- Decay cross-section in light-front variables:

$$dP^{h\to h_1,h_2}(z_1) = \begin{cases} \frac{C_h^{h_1 h_2}}{8\pi}\, dz_1 & \text{if } z_1 z_2\, m_h^2 - z_2 m_{h1}^2 - z_1 m_{h2}^2 \geq 0;\ z_1+z_2=1, \\ 0 & \text{otherwise.} \end{cases}$$

INCLUDING THE TRANSVERSE MOMENTUM

H.M.,Bentz, Cloet, Thomas, PRD.85:014021, 2012

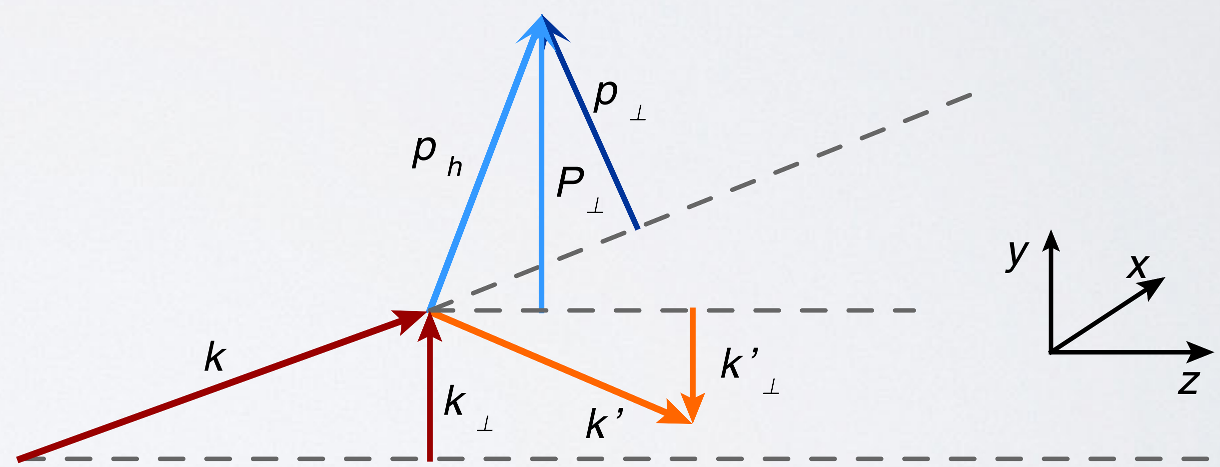


- TMD splittings: $d(z, p_\perp^2)$

- Conserve transverse momenta at each link.

$$\boxed{\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp}$$

$$\boxed{\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp}$$



- Calculate the Number Density

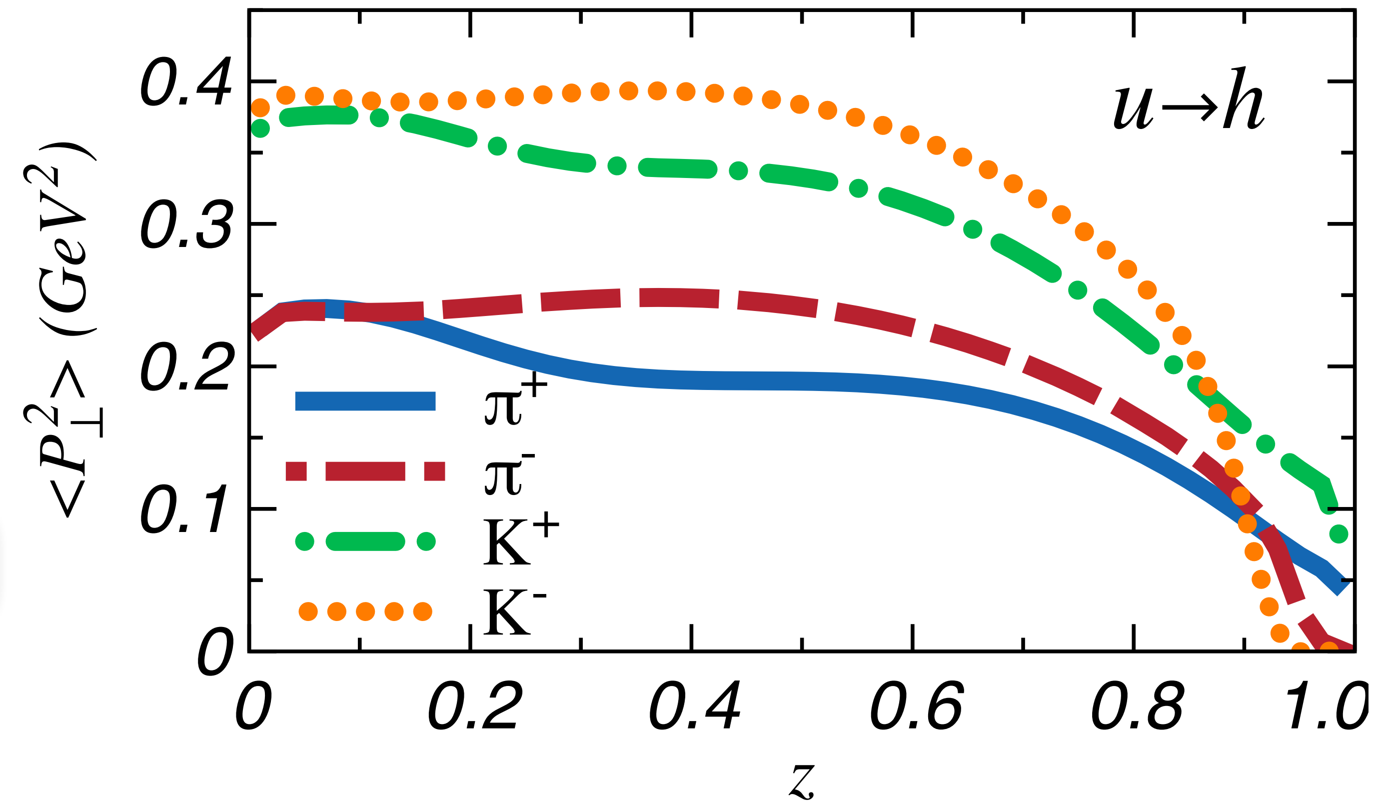
$$D_q^h(z, P_\perp^2)\Delta z\, \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2)}{N_{Sims}}.$$

AVERAGE Transverse Momenta vs z

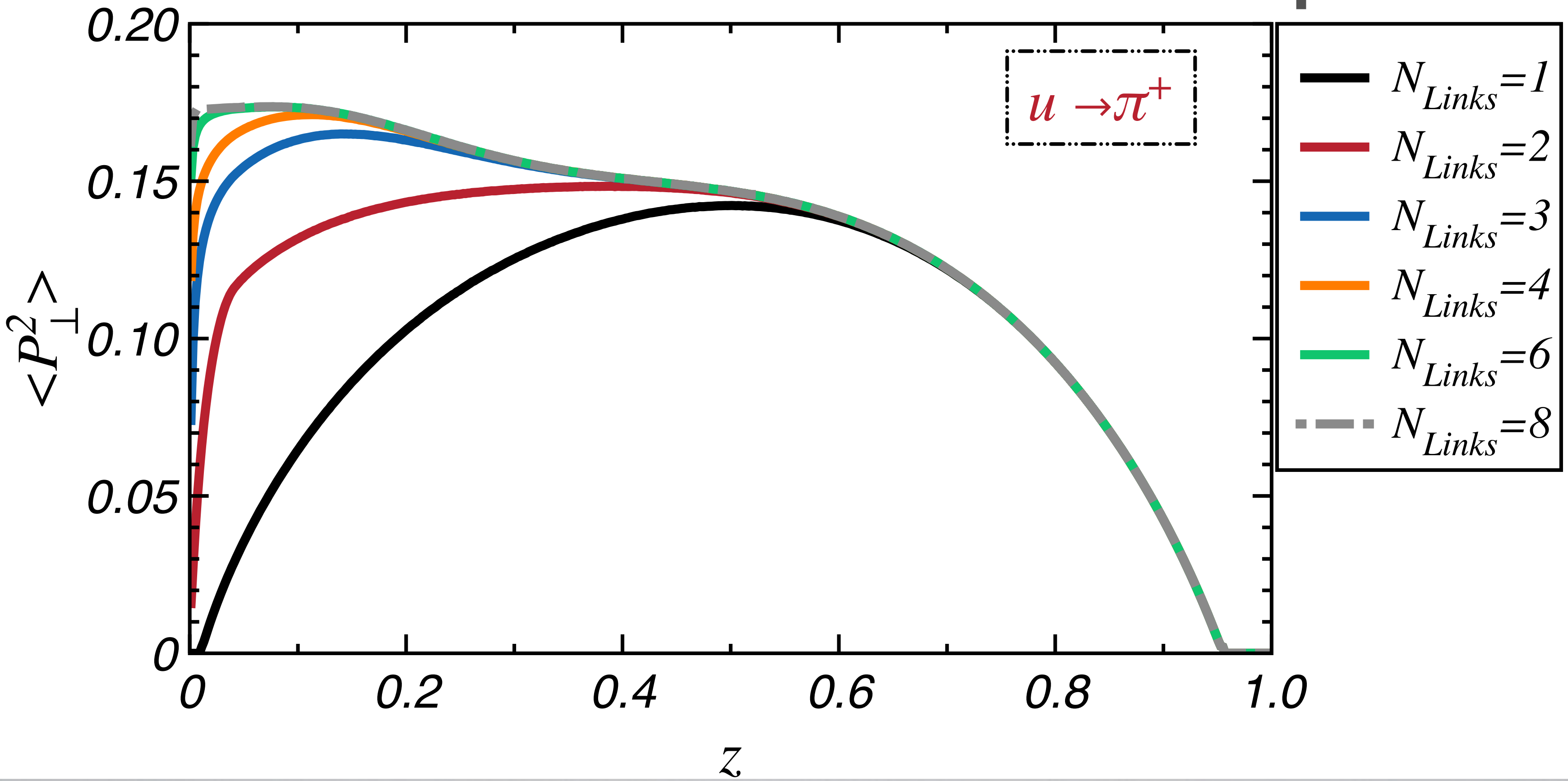
FRAGMENTATION

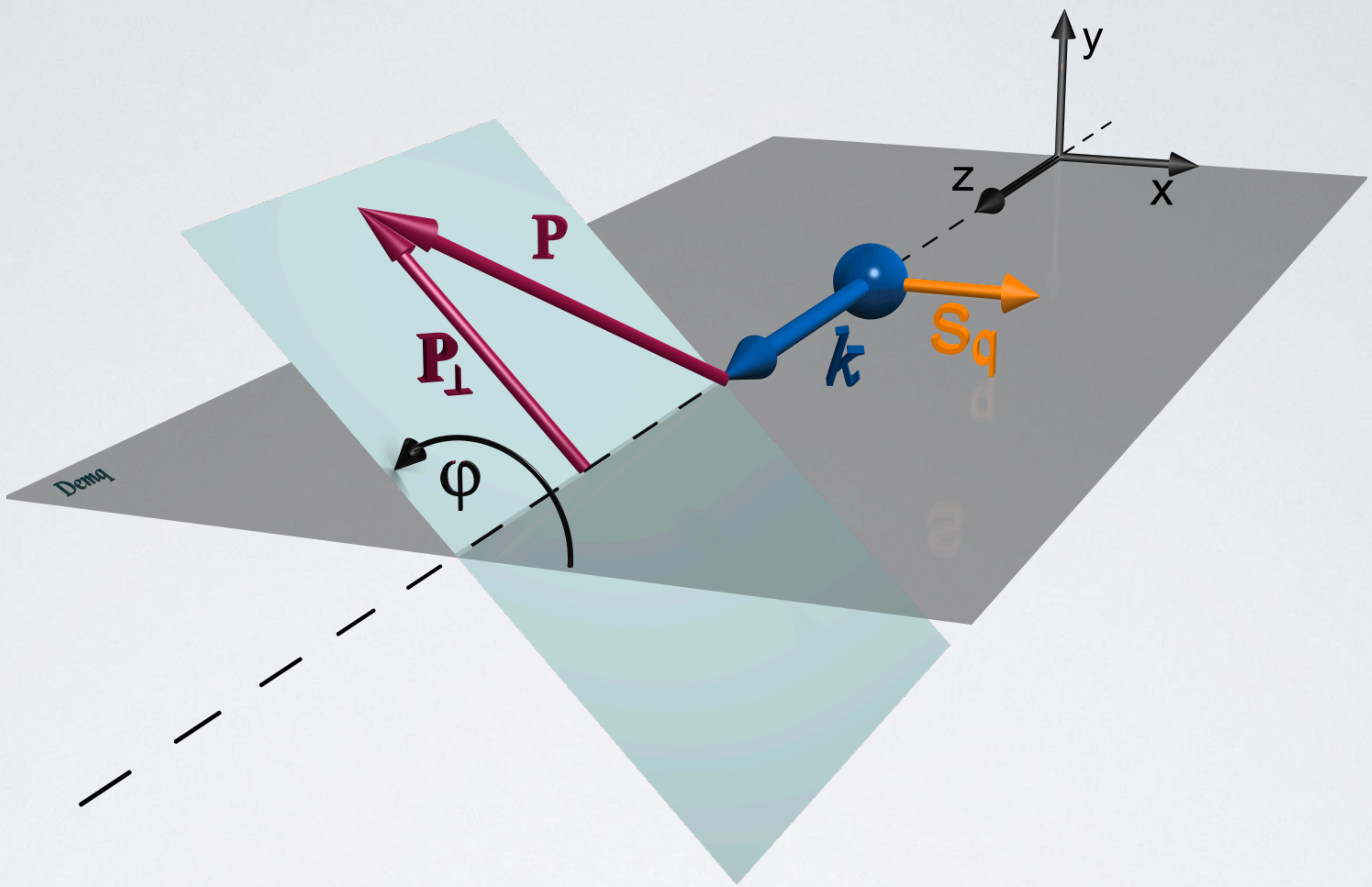
$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_f$$

✦Indications from HERMES data: A. Signori, et al: JHEP 1311, 194 (2013)



✓Multiple hadron emissions: ***broaden*** the TM dependence at ***low z***!



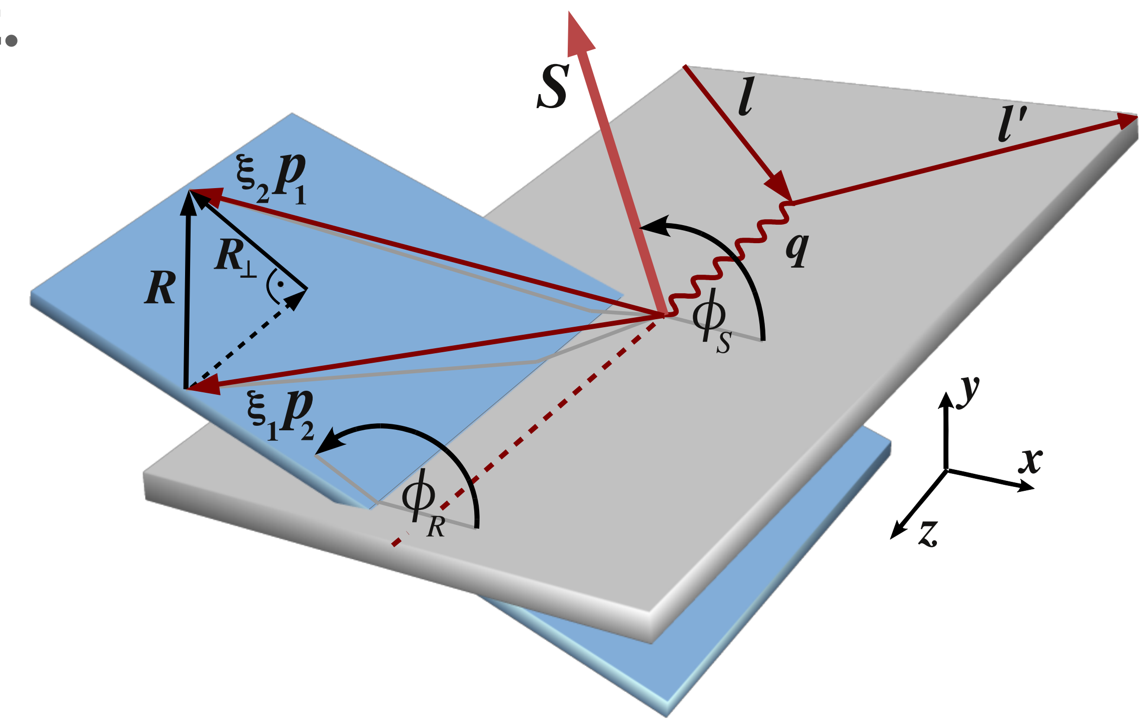


TRANSVERSELY POLARIZED QUARK FRAGMENTATION:
COLLINS EFFECT AND TWO-HADRON CORRELATIONS

RECENT COMPASS RESULTS

COMPASS, PLB736, 124-131 (2014).

✦ SIDIS with transversely polarized target.



✦ Collins single spin asymmetry:

$$A_{Coll} = \frac{\sum_q e_q^2\, h_1^q \otimes H_1^{\perp h/q}}{\sum_q e_q^2\, f_1^q \otimes D_1^{h/q}}$$

✦ Two hadron single spin asymmetry:

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}_1 - \boldsymbol{p}_2|}{2M_{h^+h^-}} \frac{\sum_q e_q^2 \cdot h_1^q(x) \cdot H_{1,q}^{\sphericalangle}(z,\, M_{h^+h^-}^2,\, \cos\theta)}{\sum_q e_q^2 \cdot f_1^q(x) \cdot D_{1,q}(z,\, M_{h^+h^-}^2,\, \cos\theta)}$$

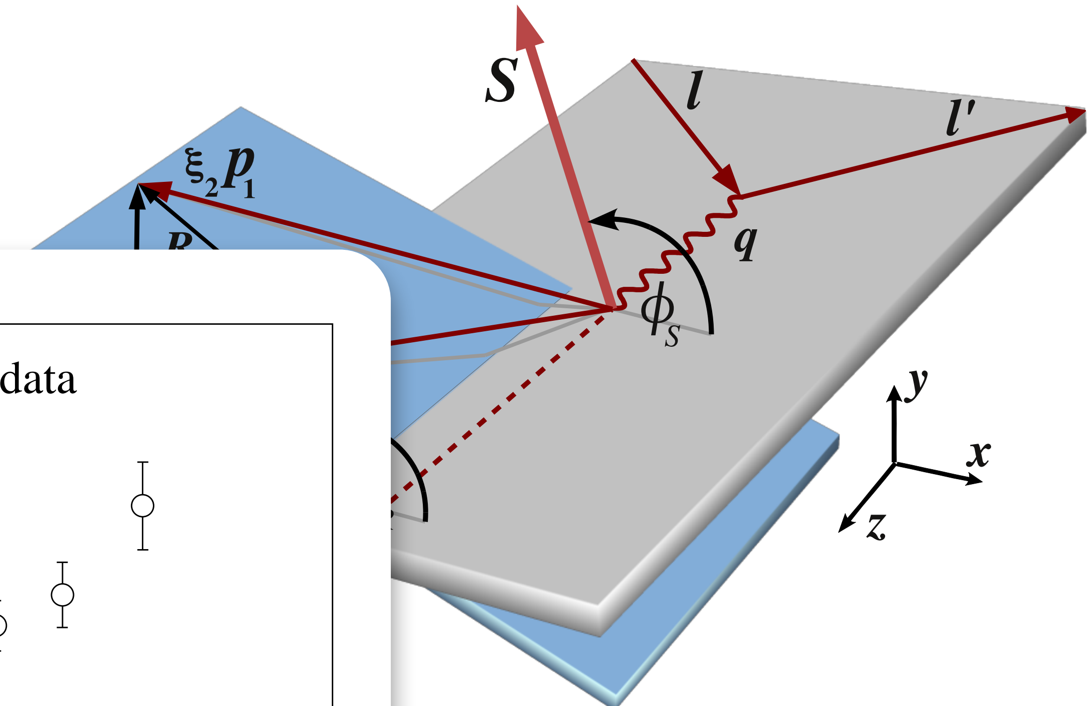
✦ Note the choice of the vector

$$\boldsymbol{R}_{Artru} = \frac{z_2 \boldsymbol{P}_1 - z_1 \boldsymbol{P}_2}{z_1 + z_2}$$

RECENT COMPASS RESULTS

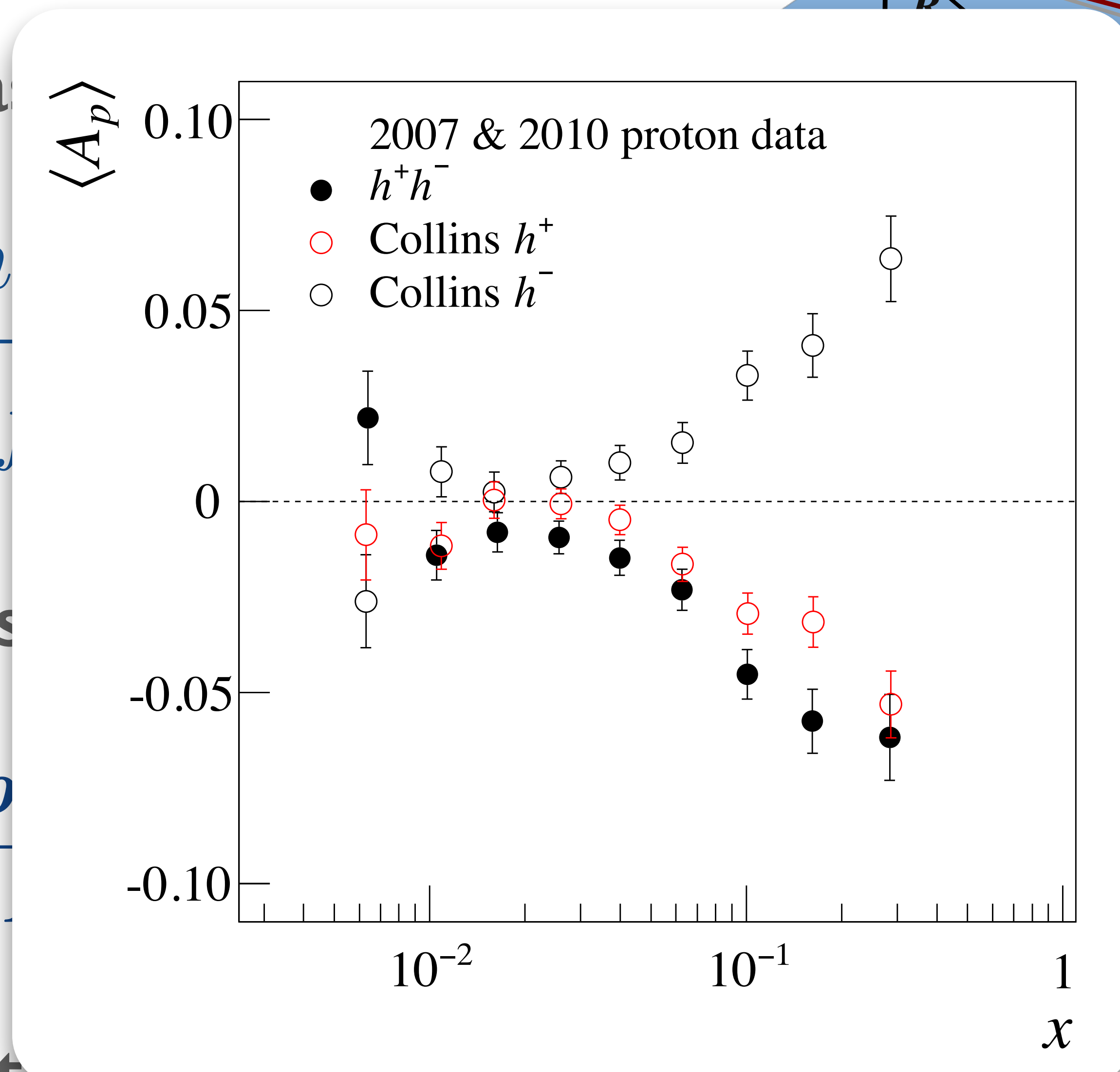
COMPASS, PLB736, 124-131 (2014).

✦SIDIS with transversely polarized target.



✦Collins single spin a

$$A_{Coll} = \frac{\sum_q e_q^2 \, h}{\sum_q e_q^2 \;\; j}$$



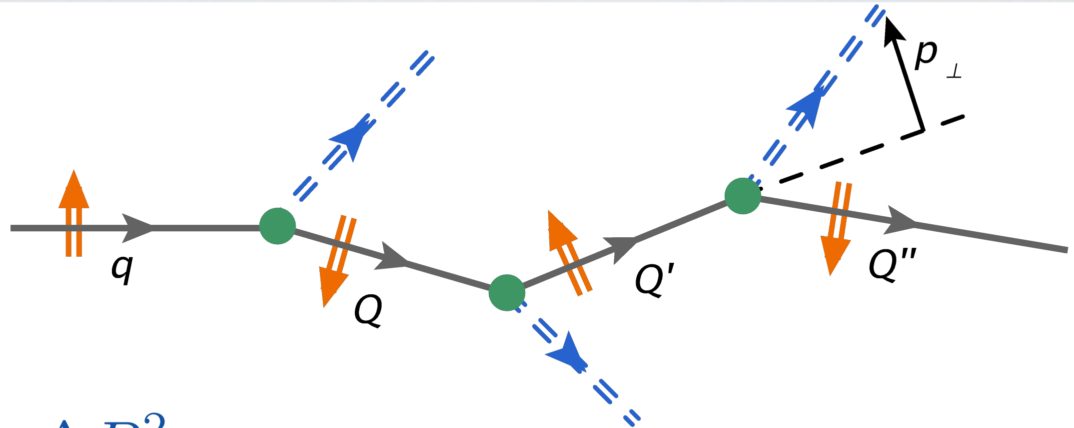
✦Two hadron single s

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}}{2} \quad \frac{M^2_{h^+h^-},\ \cos\theta)}{M^2_{h^+h^-},\ \cos\theta)}$$

✦Note the choice of the vector

$$\boldsymbol{R}_{Artru} = \frac{z_2 \boldsymbol{P}_1 - z_1 \boldsymbol{P}_2}{z_1 + z_2}$$

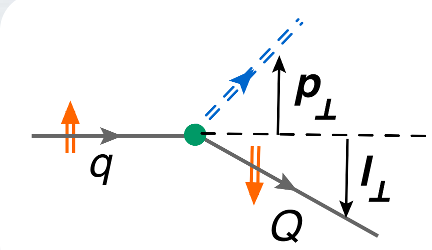
COLLINS EFFECT - NJL-jet MKII



$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi)\, \Delta z\, \frac{\Delta P_\perp^2}{2}\, \Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z+\Delta z; P_\perp^2, P_\perp^2 + \Delta P^2; \varphi, \varphi+\Delta\varphi) \right\rangle$$

MKII Model Assumptions: **H.M., Kotzinian, Thomas, PLB731 208-216 (2014).**

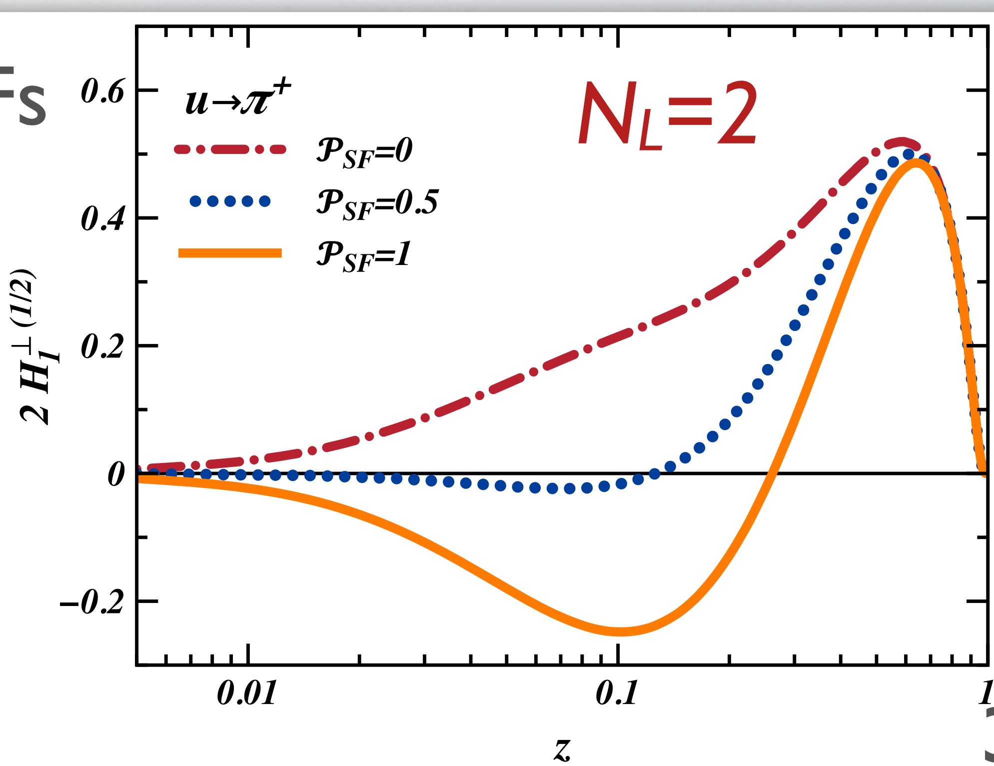
1. Allow for Collins Effect only in a SINGLE emission vertex (N_L^{-1} scaling of the resulting Collins function).
2. Use constant values for spin flip probability: $\mathcal{P}_{SF}$.



✦ Use fit form to extract unpol. and Collins FFs from $D_{h/q^\uparrow}$.

$$\boxed{F(c_0, c_1) = c_0 - c_1 \sin(\varphi)}$$

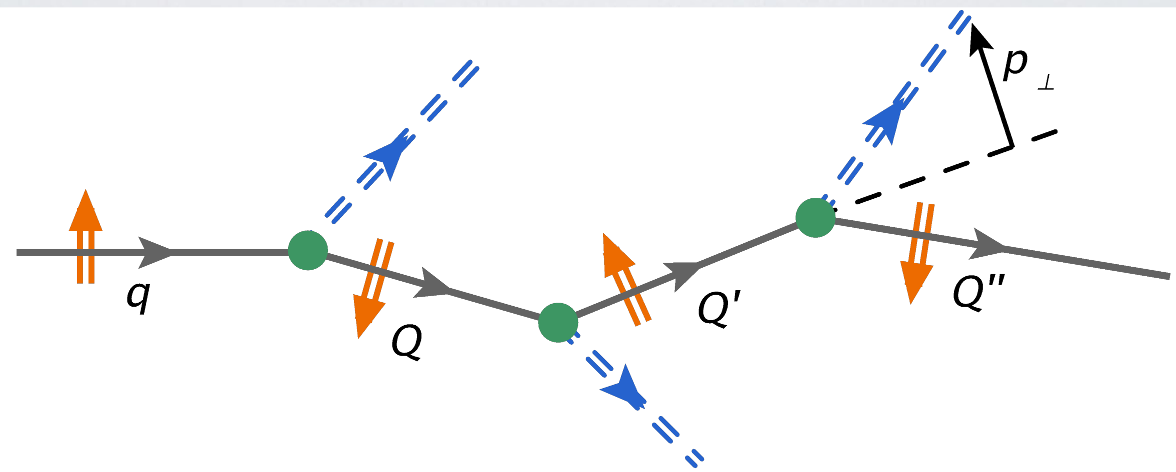
✦ The results for N_L=2, $\mathcal{P}_{SF} = 1$.



POLARIZED QUARK DIFF IN QUARK-JET.

H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

- Use the NJL-jet Model including Collins effect (MKII) to study DiFFs.

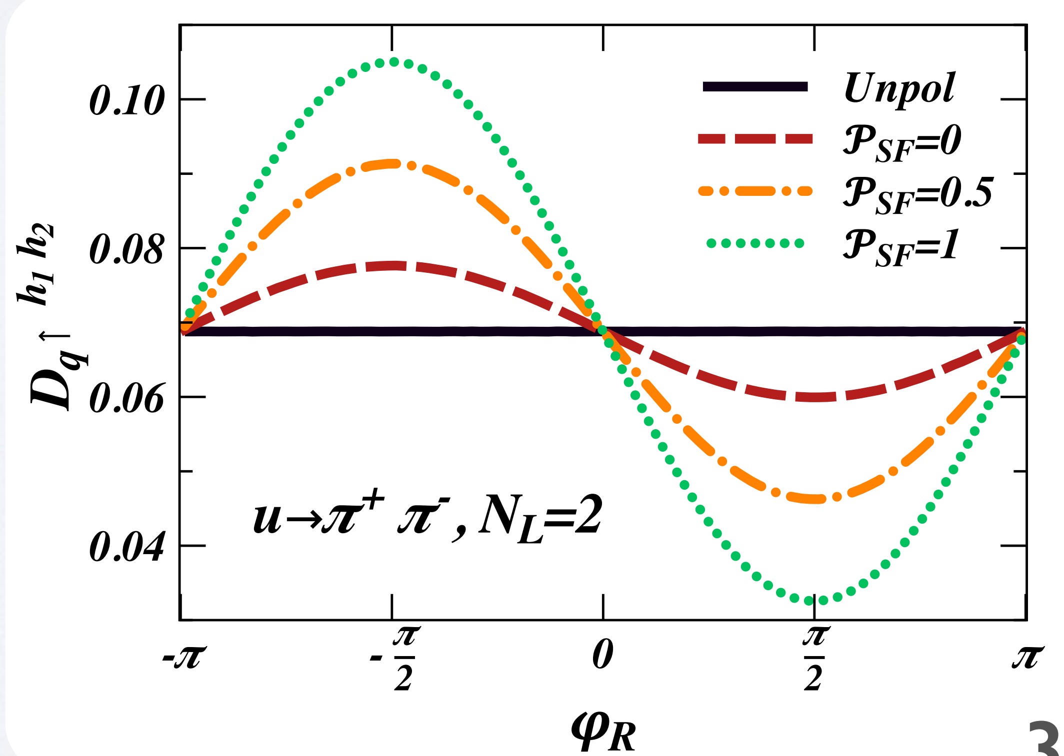


$$D_{q^\uparrow}^{h_1 h_2}(z, M_h^2, \varphi_R)\ \Delta z\ \Delta M_h^2\ \Delta\varphi_R = \left\langle N_{q^\uparrow}^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2+\Delta M_h^2; \varphi_R, \varphi_R+\Delta\varphi_R)\right\rangle .$$

- *Choose a constant Spin flip probability:* $\mathcal{P}_{SF}$

- *Simple model to start with:*
  Only pions and extreme ansatz for the Collins term in elementary function.

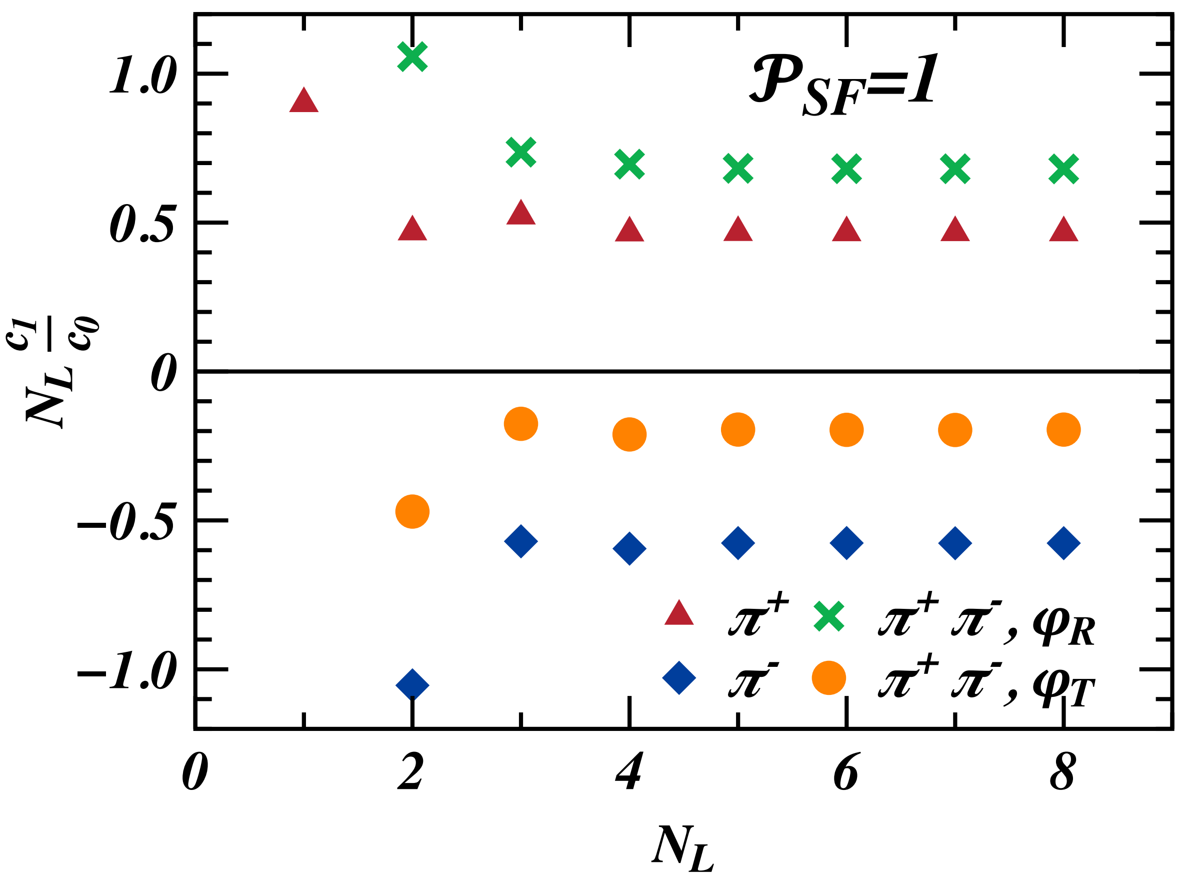
$$\boxed{d_{h/q^\uparrow}(z, \mathbf{p}_\perp) = d_1^{h/q}(z, p_\perp^2)(1 - 0.9\sin\varphi)}$$

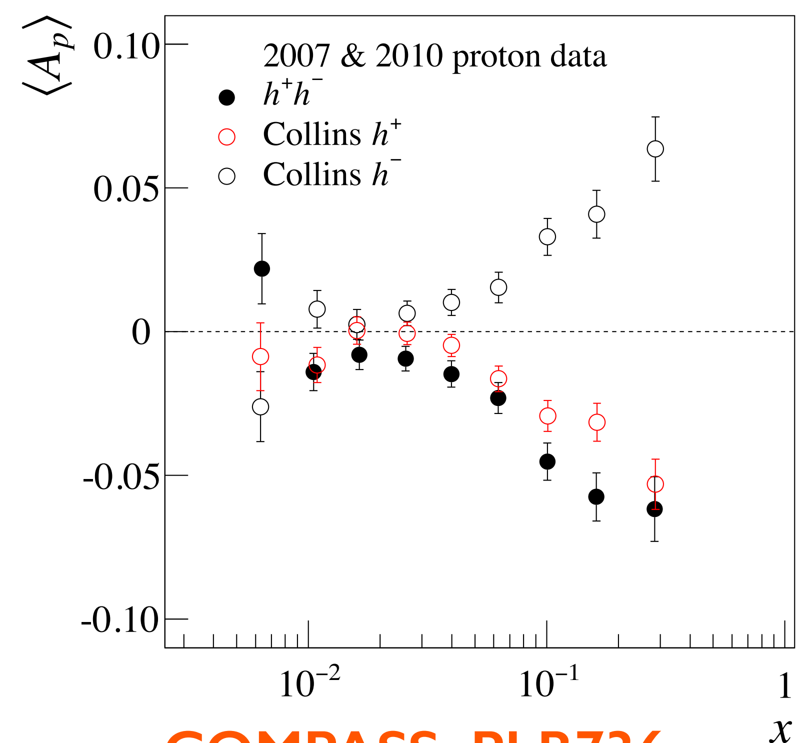


INTEGRATED ANALYZING POWERS

H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

$$z_{1,2} > 0.2, z > 0.2$$

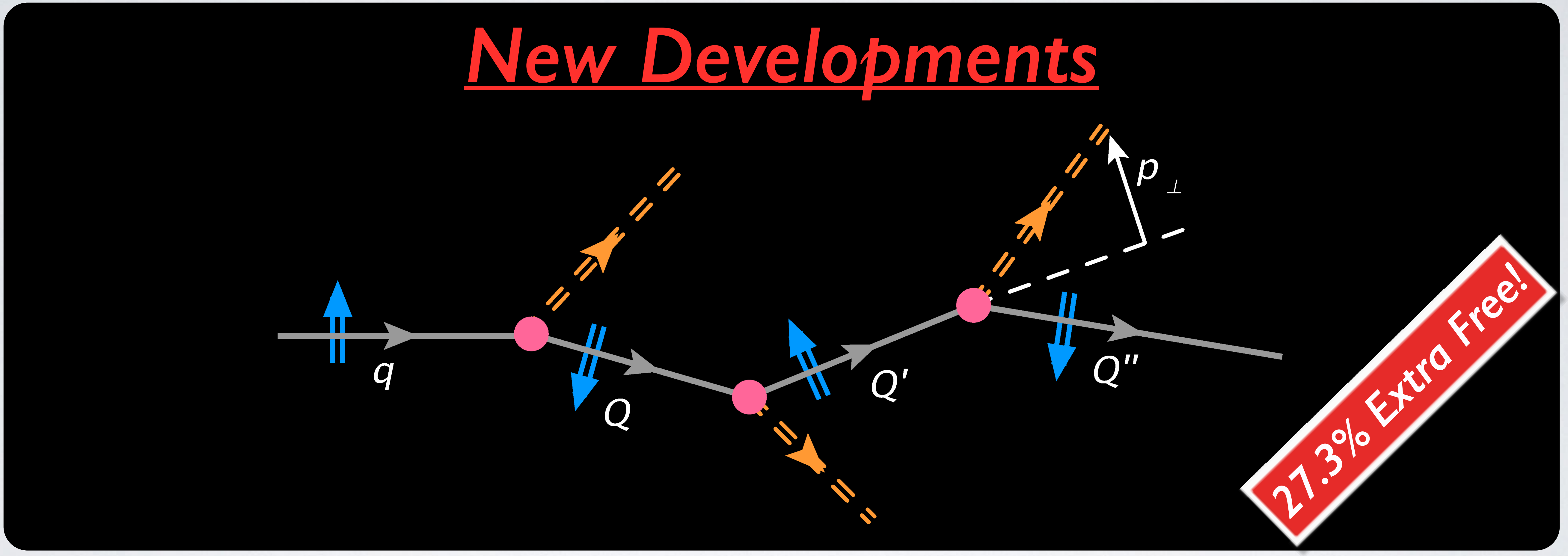




COMPASS: PLB736, 124-131 (2014).

✓ ***NJL-jet** model results are **consistent** with **COMPASS** measurements on interplay between one- and two- hadron SSAs.*

NJL-jet MKIII



Modelling Hadronization with Spin: The Objectives.

NJL-jet Complete Hadronization Model with Spin. Input from QCD-inspired effective Quark Model.

1) **Predictions for full set of polarised FFs.**
   - *Quantitative* extract. of fav. and unfav. polarised TMD FF.
   - Include resonance productions and decays.
   - Should explain possible connections between single and dihadron FFs!
   - *The correspondence to FFs in limited z region ($z > z_0$).*

2) **Interpretation in Full Event Generators:**
   - Number density interpretation.
   - Iterative picture: spin transfer! Should be adaptable to the MC framework.
   - *Should not break any of the unpolarised observables! (PYTHIA fits to existing data, etc.)*

TMD FFs for Spin 1/2 Particles

✦ TMD Polarized Fragmentation Functions at LO.

- *Only two for unpolarised final state hadrons.*

PDFs

N/q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	$g_{1T}^{\perp}$	$h_1 \, h_{1T}^{\perp}$

Fragment. Functions

h/q	U	L	T
U	D_1		$H_1^{\perp}$

TMD FFs for Spin 1/2 Particles

✦ TMD Polarized Fragmentation Functions at LO.

- *Only **two** for unpolarised final state hadrons.*

PDFs

N/q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	$g_{1T}^{\perp}$	$h_1\, h_{1T}^{\perp}$

Fragment. Functions

h/q	U	L	T
U	D_1		$H_1^{\perp}$
L		G_{1L}	$H_{1L}^{\perp}$
T	$D_{1T}^{\perp}$	G_{1T}	$H_{1T} H_{1T}^{\perp}$

- *8 for spin 1/2 final state (including quark). Similar to TMD PDFs.*

Field-Theoretical Definitions

- *We can use the same "spectator" type calculations as for pion.*

$$\Delta^{[\Gamma]}(z,\vec{p}_T) \equiv \frac{1}{4}\int \frac{dp^+}{(2\pi)^4} Tr[\Delta\Gamma]|_{p^-=zk^-}$$

$$= \frac{1}{4z}\sum_X \int \frac{d\xi^+ d^2\vec{\xi}_T}{2(2\pi)^3} e^{i(p^-\xi^+/z-\vec{\xi}_T\cdot\vec{p}_T)} \langle 0|\psi(\xi^+,0,\vec{\xi}_T)|p,S_h,X\rangle\langle p,S_h,X|\bar{\psi}(0)\Gamma|0\rangle$$

- *The definitions of FFs from the quark correlator*

$$\Delta^{[\gamma^+]} = D(z,p_\perp^2) - \frac{1}{M}\epsilon^{ij} k_{Ti} S_{Tj} D_T^\perp(z,p_\perp^2)$$

$$\Delta^{[\gamma^+\gamma_5]} = S_L G_L(z,p_\perp^2) + \frac{\boldsymbol{k}_T\cdot S_T}{M} G_T(z,p_\perp^2)$$

$$\Delta^{[i\sigma^{i+}\gamma_5]} = S_T^i H_T(z,p_\perp^2) + \frac{S_L}{M} k_T^i H_L^\perp(z,p_\perp^2) + \frac{k_T^i(\boldsymbol{k}_T\cdot S_T)}{M^2} H_T^\perp(z,p_\perp^2) - \frac{\epsilon^{ij}k_{Tj}}{M} H^\perp(z,p_\perp^2)$$

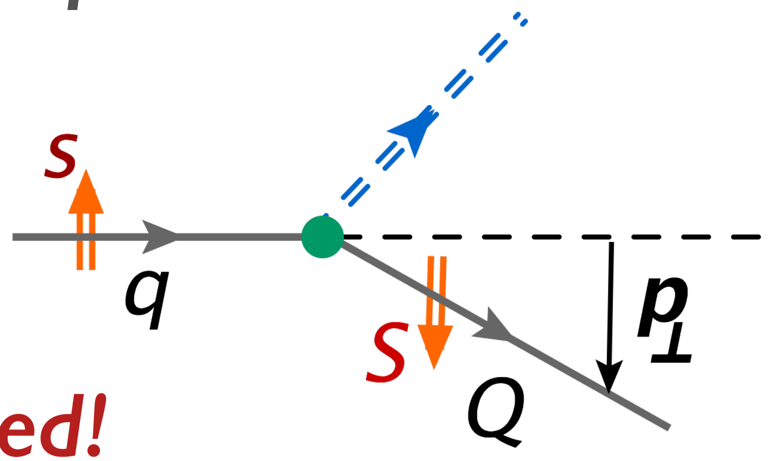
Spin Transfer in quark-jet Framework.

✦NJL-jet MKIII:

Bentz et al, arXiv:1603.08333 (2016).

- *The probability for the process* $q \to Q$, *initial spin* $\mathbf{s}$ *to* $\mathbf{S}$

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) = \alpha_s + \beta_\mathbf{s} \cdot \mathbf{S}$$

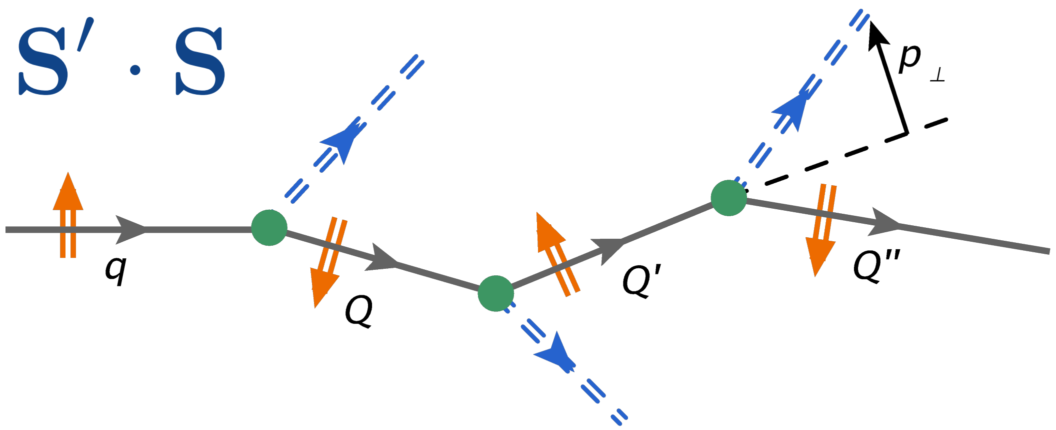


- *Intermediate quarks in quark-jet are unobserved!*

Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii: **QUANTUM ELECTRODYNAMICS (1982).**

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) \sim \mathrm{Tr}[\rho^{\mathbf{S}'} \rho^{\mathbf{S}}] \sim 1 + \mathbf{S}' \cdot \mathbf{S}$$

$$\mathbf{S}' = \frac{\beta_\mathbf{s}}{\alpha_\mathbf{s}}$$



- *Remnant quark's* $\mathbf{S}'$ *uniquely determined by* $z, \mathbf{p}_\perp$ *and* $\mathbf{s}$ *!*

- *Process probability is the same as transition to unpolarized state.*

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{0}) = \alpha_s$$

Field-Theoretical Definitions

- *The probability for the process* $q \to Q$, *initial spin* $\mathbf{s}$ *to* $\mathbf{S}$

$$
\begin{aligned}
F(z,\boldsymbol{p}_\perp;\boldsymbol{S},\boldsymbol{s}) = D(z,\boldsymbol{p}_\perp^2) &+ \frac{1}{z\,\mathcal{M}}(\boldsymbol{p}_\perp \times \boldsymbol{S}_T)\cdot\hat{\boldsymbol{z}} D_T^\perp(z,\boldsymbol{p}_\perp^2) \\
&+ (s_L S_L)\; G_L(z,\boldsymbol{p}_\perp^2) - \frac{1}{z\mathcal{M}} s_L(\boldsymbol{p}_\perp\cdot\boldsymbol{S}_T) G_T(z,\boldsymbol{p}_\perp^2) \\
&+ (\boldsymbol{s}_T\cdot\boldsymbol{S}_T)\; H_T(z,\boldsymbol{p}_\perp^2) - \frac{1}{z\mathcal{M}} S_L(\boldsymbol{p}_\perp\cdot\boldsymbol{s}_T) H_L^\perp(z,\boldsymbol{p}_\perp^2) \\
&+ \frac{1}{z^2\mathcal{M}^2}(\boldsymbol{p}_\perp\cdot\boldsymbol{s}_T)(\boldsymbol{p}_\perp\cdot\boldsymbol{S}_T)\; H_T^\perp(z,\boldsymbol{p}_\perp^2) + \frac{1}{z\mathcal{M}}(\boldsymbol{p}_\perp\times\boldsymbol{s}_T)\cdot\hat{\boldsymbol{z}}\; H^\perp(z,\boldsymbol{p}_\perp^2).
\end{aligned}
$$

- ***Rewriting in terms of*** $\boxed{F^{q\to Q}(z,\mathbf{p}_\perp;\mathbf{s},\mathbf{S}) = \alpha_s + \beta_{\mathbf{s}}\cdot\mathbf{S}}$

$$
\alpha_q \equiv D(z,\boldsymbol{p}_\perp^2) + (\boldsymbol{p}_\perp\times\boldsymbol{s}_T)\cdot\hat{\boldsymbol{z}}\frac{1}{z\mathcal{M}}\; H^\perp(z,\boldsymbol{p}_\perp^2)
$$

$$
\beta_{q\|} \equiv s_L\; G_L(z,\boldsymbol{p}_\perp^2) - (\boldsymbol{p}_\perp\cdot\boldsymbol{s}_T)\frac{1}{z\mathcal{M}} H_L^\perp(z,\boldsymbol{p}_\perp^2)
$$

$$
\begin{aligned}
\beta_{q\perp} \equiv \boldsymbol{p}'_\perp \frac{1}{z\,\mathcal{M}} D_T^\perp(z,\boldsymbol{p}_\perp^2) &- \boldsymbol{p}_\perp\frac{1}{z\mathcal{M}} s_L G_T(z,\boldsymbol{p}_\perp^2) \\
&+ \boldsymbol{s}_T\; H_T(z,\boldsymbol{p}_\perp^2) + \boldsymbol{p}_\perp(\boldsymbol{p}_\perp\cdot\boldsymbol{s}_T)\frac{1}{z^2\mathcal{M}^2}\; H_T^\perp(z,\boldsymbol{p}_\perp^2)
\end{aligned}
$$

Example: Pion production.

Fragment. Functions

$$F^{q\to Q}(z,\boldsymbol{p}_\perp;\boldsymbol{s},\boldsymbol{S})$$

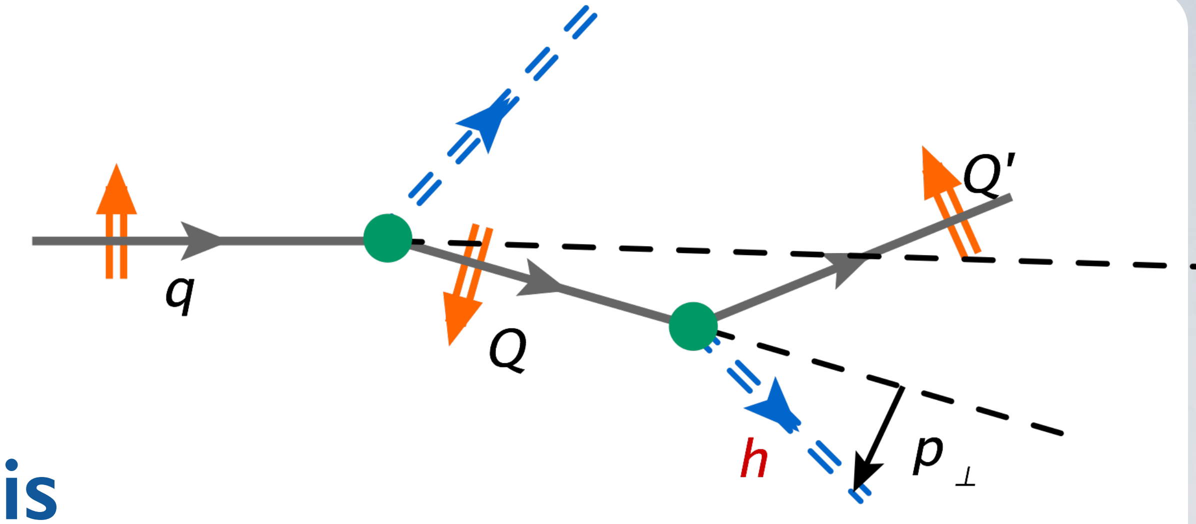
Q/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

$$F^{q\to\pi}(z,\boldsymbol{p}_\perp;\boldsymbol{s})$$

π/q	U	L	T
U	D_1		$H_1^\perp$

Example: Pion prod. up to Rank 2

✦ **Only consider pion produced in the first two emission steps!**



✦**Then the polarised number density is**

1st rank | 2nd rank

$$F^{(2)q\to\pi} = \boxed{f^{q\to\pi}} + \boxed{f^{q\to Q} \otimes f^{Q\to\pi}}$$

✦ **"Elementary" number densities:** *only favoured types are non-zero.*

$$\boxed{f^{q\to\pi} = d^{q\to\pi} - \frac{p_\perp}{zM_h} s_T h_1^{\perp q\to\pi}} \qquad \boxed{f^{u\to\pi^-} = 0}$$

✦*It is shown <u>analytically</u> that only Collins modulations appear!*

$$\boxed{F^{(2)q\to\pi}(z, p_\perp^2, \varphi_C) = F_0^{(2)}(z, p_\perp^2) - \sin(\varphi_C) F_1^{(2)}(z, p_\perp^2)}$$

Example: Pion prod. up to Rank 2

✦*It is shown **analytically** that only Collins modulations appear!*

$$F^{(2)q\to\pi}(z,p_\perp^2,\varphi_C) = F_0^{(2)}(z,p_\perp^2) - \sin(\varphi_C)F_1^{(2)}(z,p_\perp^2)$$

✦*Up to **unspecified** coefficients, using.*

Unpolarised term:

From TM-induced Spin of intermediate quark

$$F_0^{(2)q\to\pi} = d^{q\to\pi} + (d^{q\to Q}\otimes d^{Q\to\pi} + d_T^{\perp q\to Q}\otimes h^{\perp Q\to\pi})$$

Collins term:

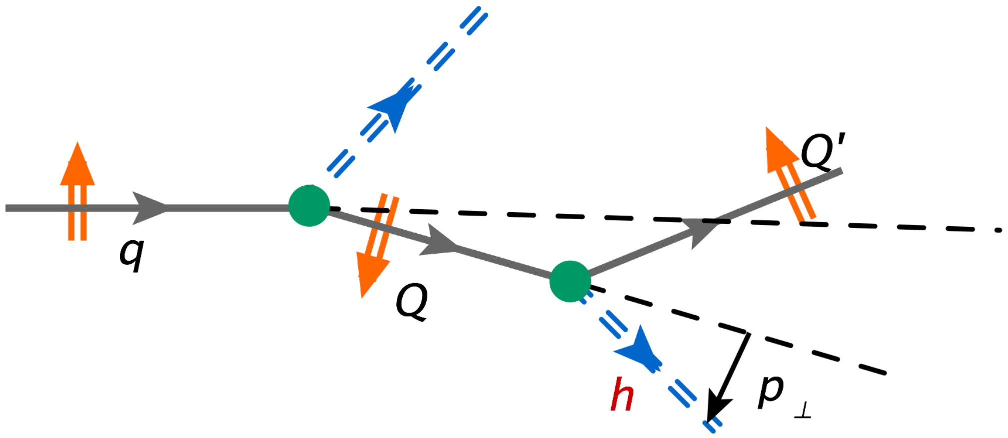
"Recoil" TM contribution

$$F_1^{(2)q\to\pi} \sim h^{\perp q\to\pi} + [h^{\perp q\to Q}\otimes d^{Q\to\pi} + (h_T^{q\to Q} + h_T^{\perp q\to Q})\otimes h^{\perp Q\to\pi}]$$

Transferred Spin of intermediate quark

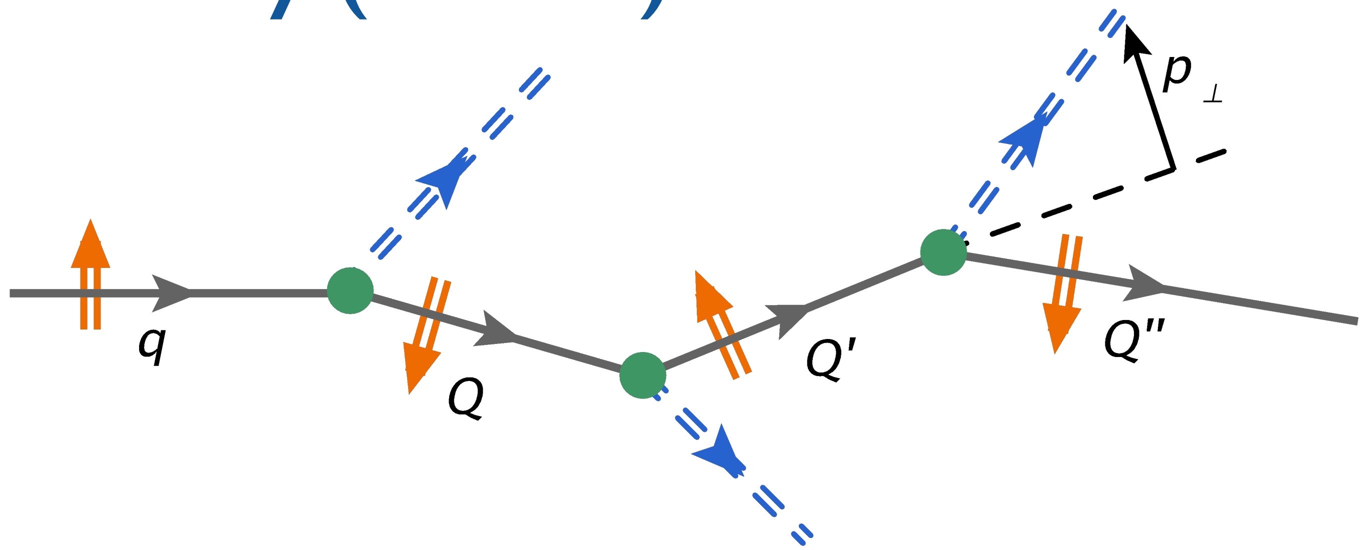
✦ *Reminder*

Q/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T}H_{1T}^\perp$



Full Hadronization

- ✦ We can consider many (infinite) number of emissions.



- ✦ Then the polarised number density is

$$\boxed{F^{q\to h} = f^{q\to h} + f^{q\to Q} \otimes F^{Q\to h}}$$

- ✦ *Again, only Collins modulations appear for unpolarised h!*

$$F^{q\to h}(z, p_\perp^2, \varphi_C) = F_0(z, p_\perp^2) - \sin(\varphi_C) F_1(z, p_\perp^2)$$

- ✦ Also applicable for *polarised* hadron production.
- ✦ We can also employ *MC approach*.
- ✦ We only need the "elementary" splittings.

$$\boxed{f^{q\to h}} \qquad \boxed{f^{q\to Q}}$$

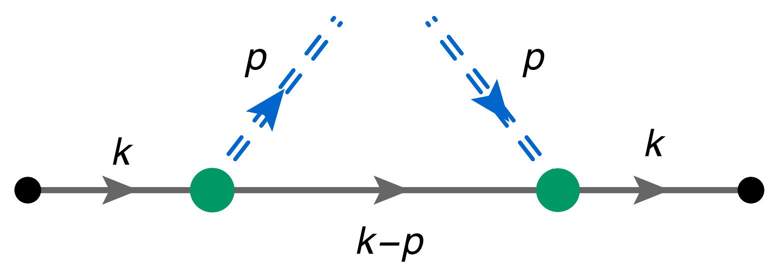
Model Calculations of $q \to Q$ Splittings

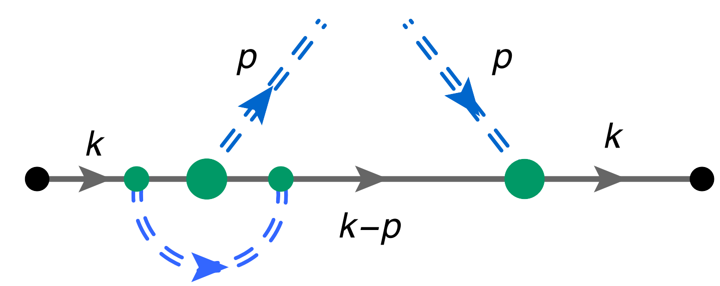
E.G. - Meissner et al, PLB 690, 296 (2010).

✦We can use the same “spectator” type calculations as for pion.

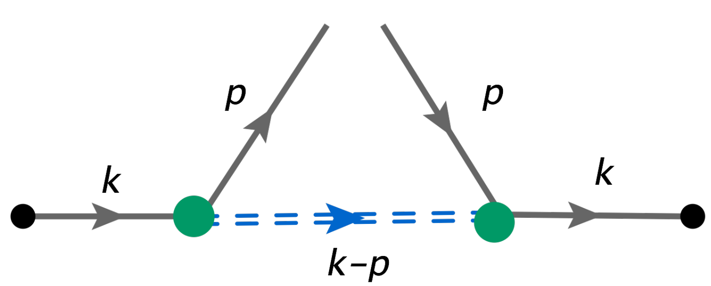
T-even ***T-odd***

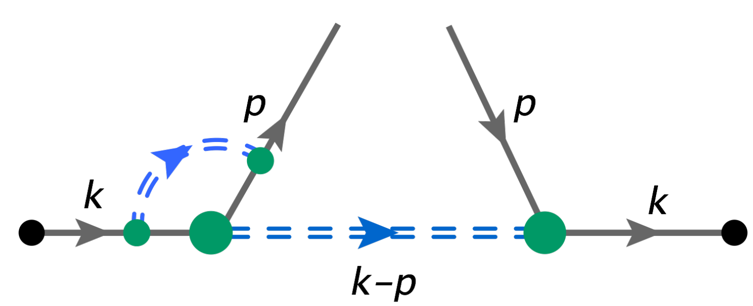
$$q \to h$$





$$q \to Q$$





Positivity and Polarisation of Quark

Bacchetta et al, PRL 85, 712 (2000) .

✦ The probability density is Positive Definite: constraints on FFs.

✦ Leading-order T-Even functions FULLY Saturate these bounds!

✦ For non-vanishing $H^{\perp}$ and $D_T^{\perp}$, need to calculate T-Even FFs at next order!

✦ Average value of remnant quark's spin.

$$\langle \boldsymbol{S}_T \rangle_Q = \boldsymbol{s}_T \frac{\int dz \left[h_T^{(q\to Q)}(z) + \frac{1}{2z^2 M_Q^2} \; h_T^{\perp[1](q\to Q)}(z) \right]}{\int dz \; d^{(q\to Q)}(z)}$$

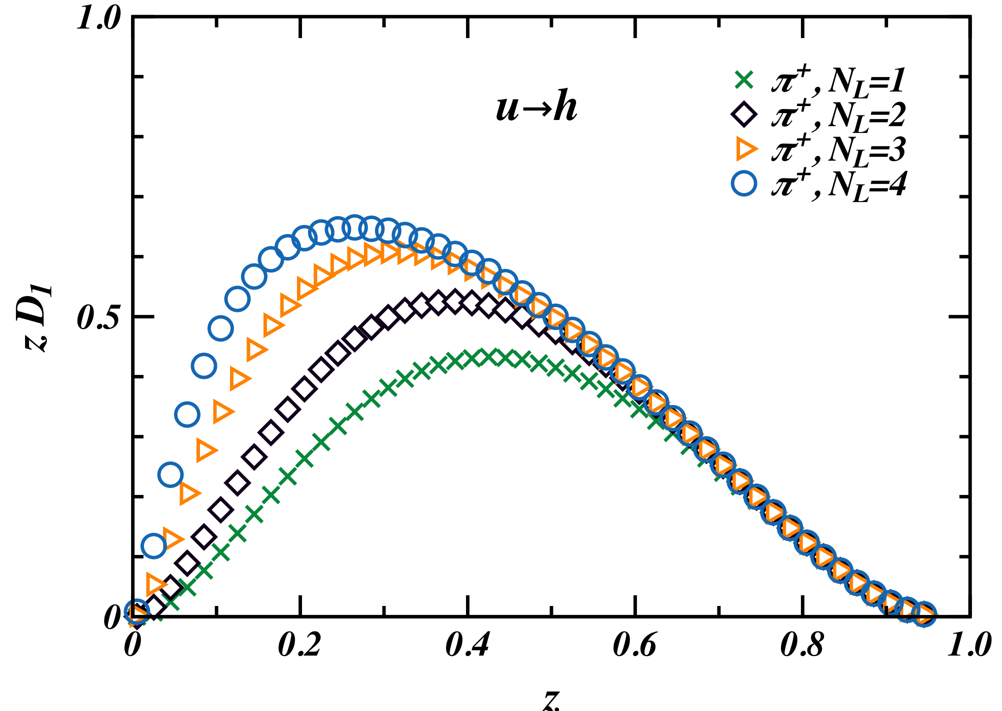
✦ In spectator model, at leading order: $h_T(z) = -d(z)$

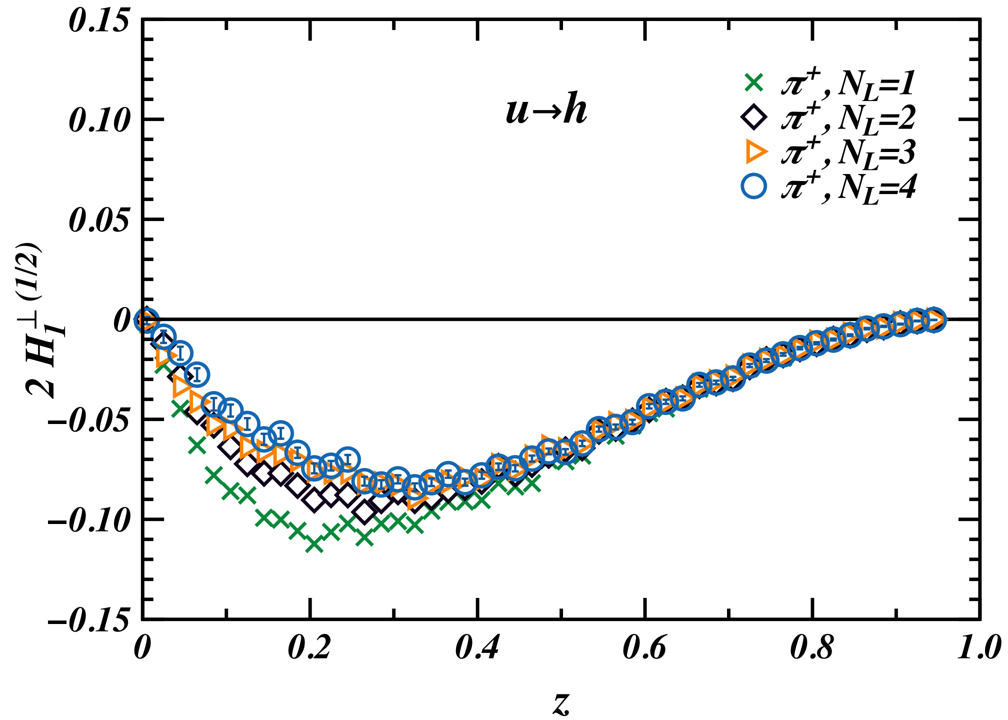
✦ Non-zero $h_T^{\perp}$ means $\langle \boldsymbol{S}_T \rangle_Q \neq -\boldsymbol{s}_T$ (full flip of the spin)!

MC Simulation in Toy Model

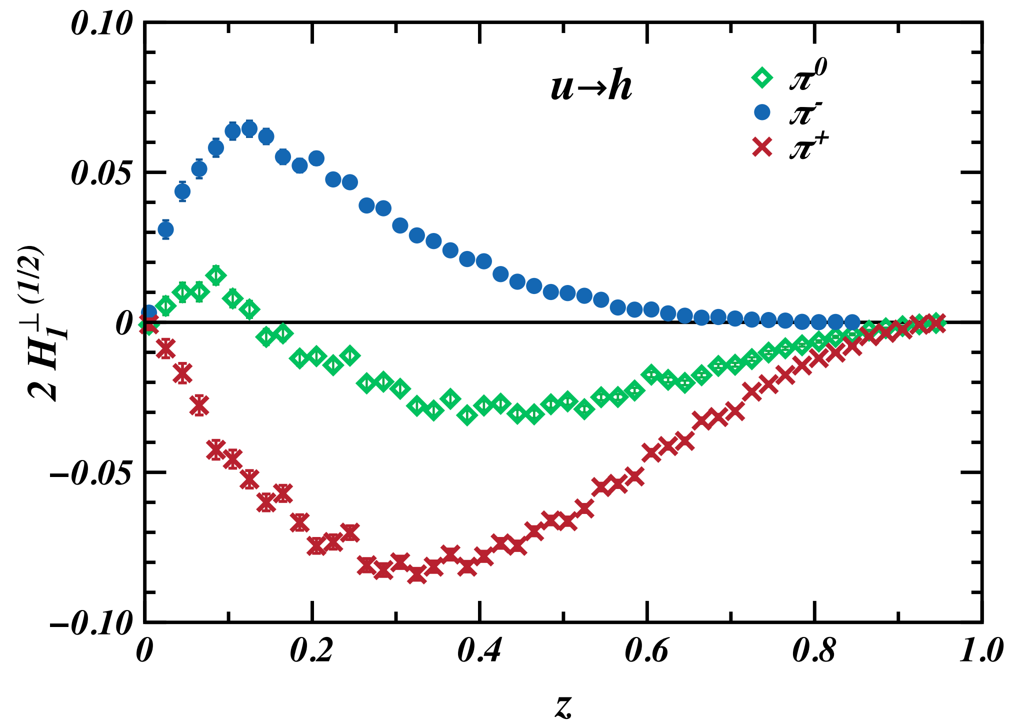
✦ **Allow** $h^{\perp} = -d_T^{\perp} = 0.1\ d_{Tree}\,, \quad d = 1.2\ d_{Tree}$

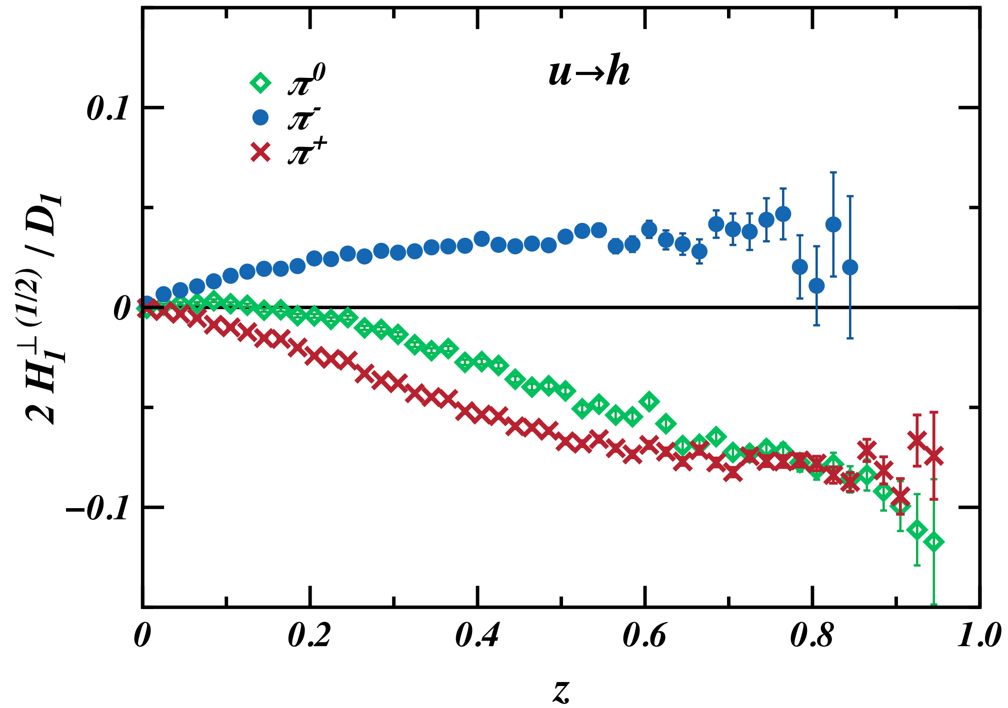
✦ **FFs vs N_L**





✦ *Results for $N_{L=4}$.*



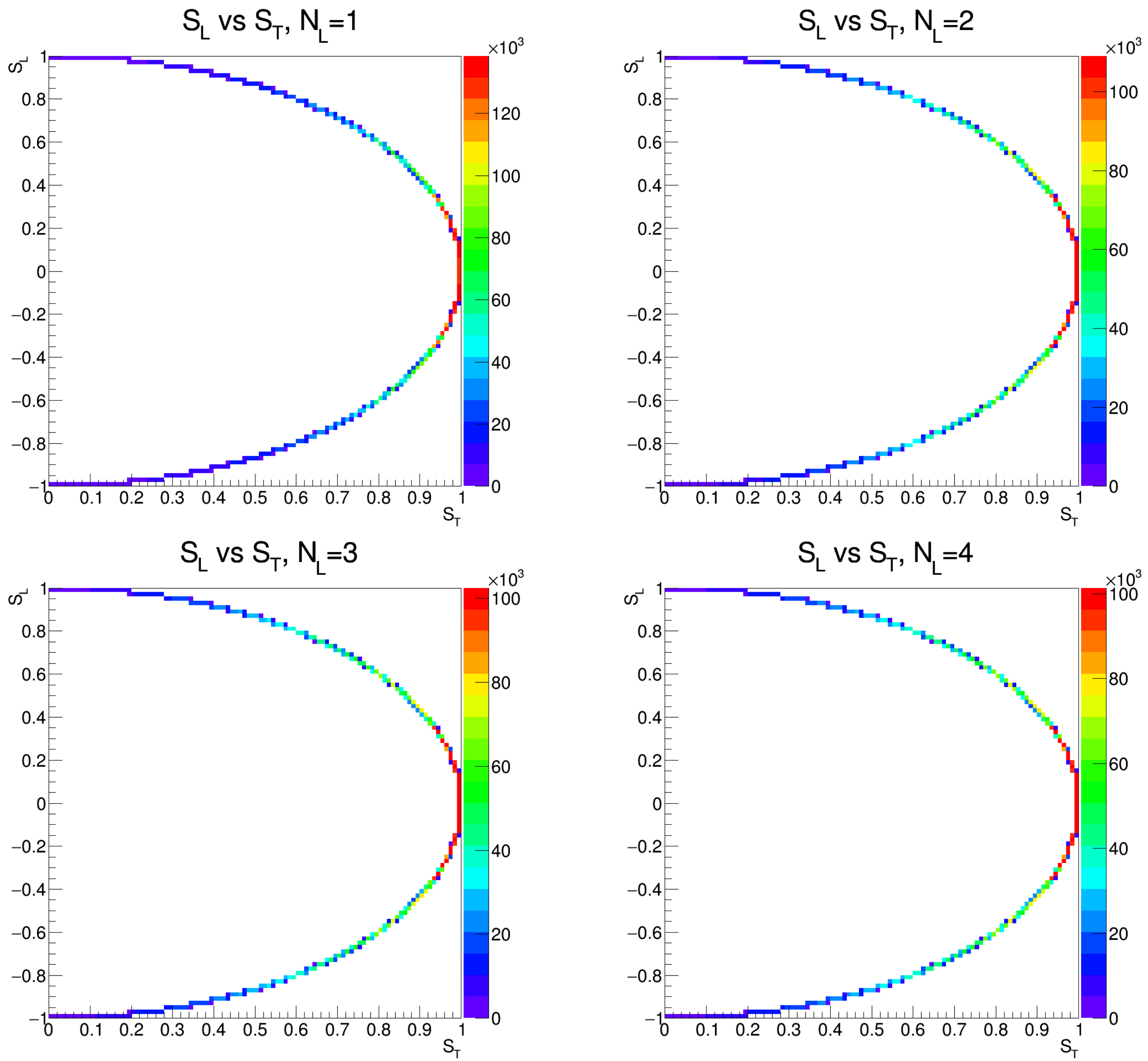


MC Simulation - Polarisation Evolution

- ✦ **Number of Event in S_L vs S_T, after N_L emissions: NO T-ODD.**
  - **Start with S_T=1, S_L=0.**

N_L =1 ***N_L =2***

N_L =3 ***N_L =4***



MC Simulation - Polarisation Evolution

✦ **Number of Event in S_L vs S_T, after N_L emissions: WITH T-ODD.**

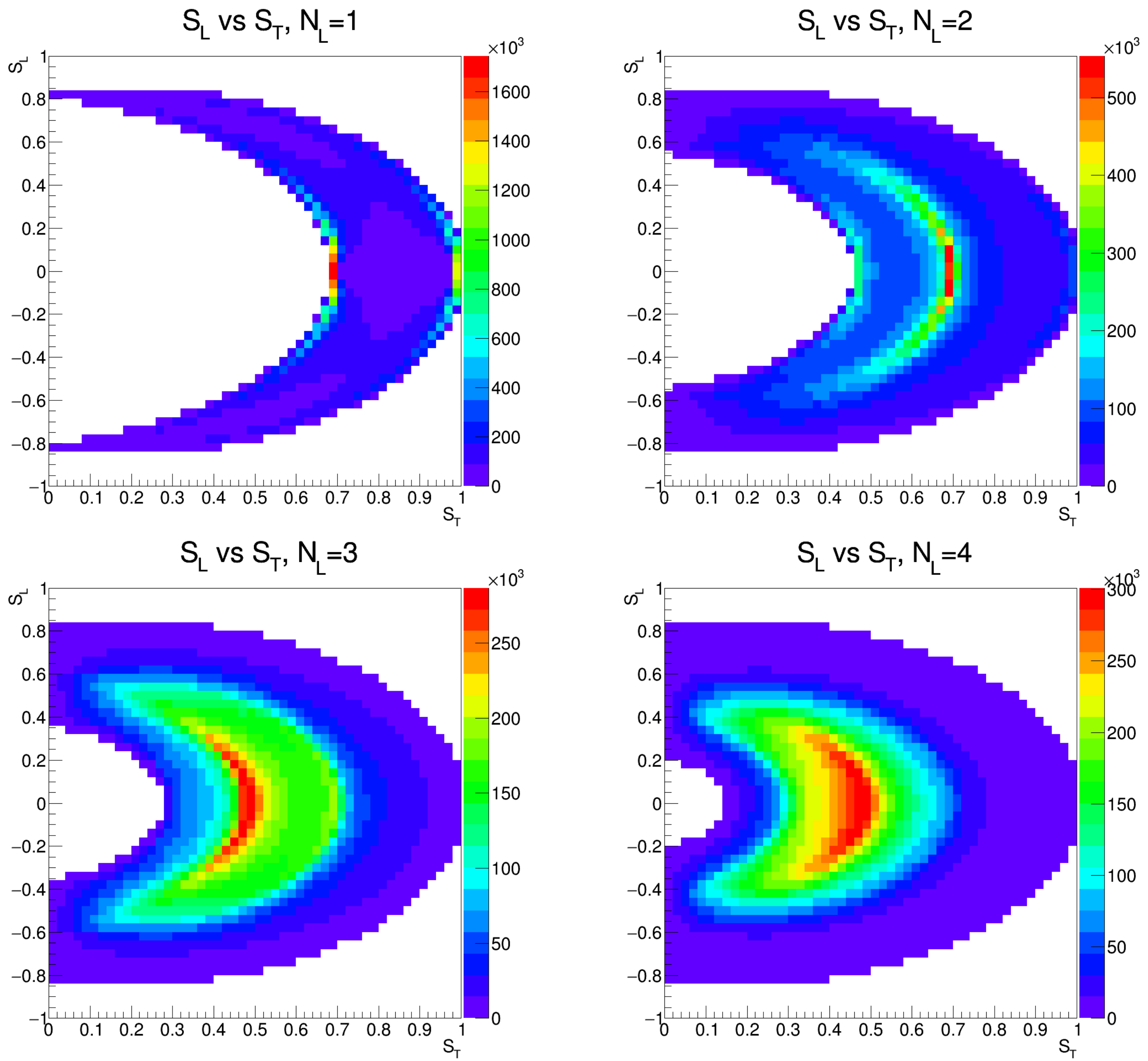
- **Start with S_T=1, S_L=0.**

N_L =1

N_L =2

N_L =3

N_L =4



Conclusions

- (Polarised) TMD FFs provide a wealth of information about the *spin-spin* and *spin-momentum* correlations in hadronisation.

- They are *essential* in describing DIS *structure functions* with hadronic final states.

- Modelling (Polarised) Quark Hadronization is needed for both calculations of polarised FFs and phenomenological studies of various correlations (Collins and IFF, etc).

- Incorporating polarised parton hadronisation into *MC generators* is needed for *supporting future experiments* in mapping out the 3D structure of nucleon (*JLab12, BELLE II, EIC*).

- The *NJL-jet* model provides a robust and extendable framework for *microscopic* description of various fragmentation phenomena using MC simulations: *TMD, Collins, DiHadron*.

- The extension of the underlying *quark-jet* mechanism to include polarisation can be readily incorporated in other MC frameworks.