

# Charge asymmetry of elastic $e^\pm p$ -scattering due to radiation

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## Radiative Corrections to the exclusive process

$$A_1(p_1) + A_2(p_2) \rightarrow \sum_{i=1}^n B_i(p'_i)$$

Contribution of additional virtual particles calculated exactly or in ultrarelativistic approximation

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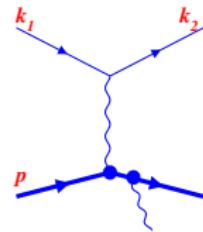
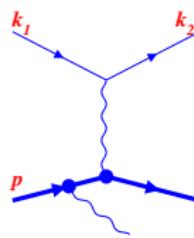
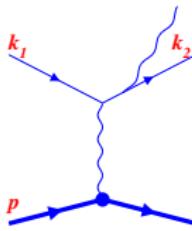
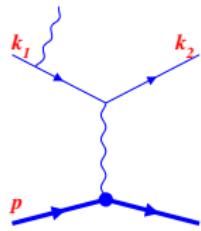
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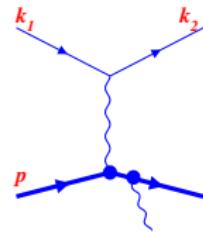
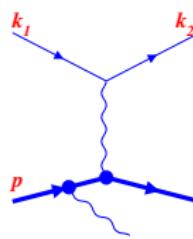
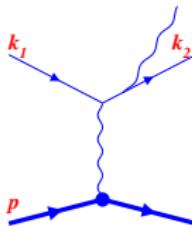
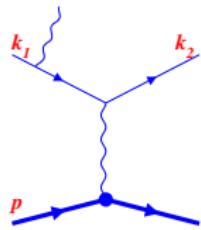
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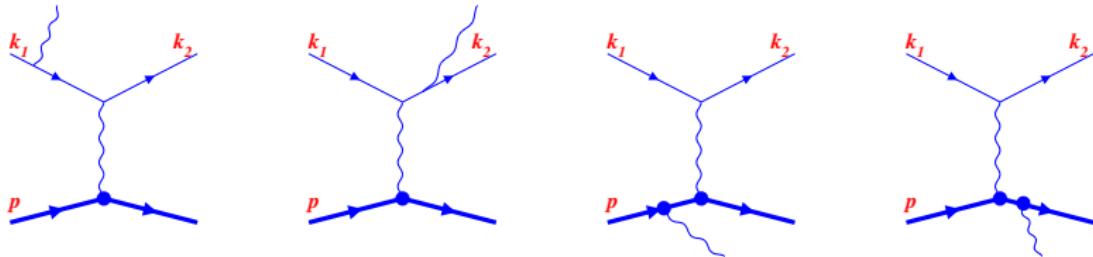
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$i e \gamma_\mu$  – electron-photon vertex

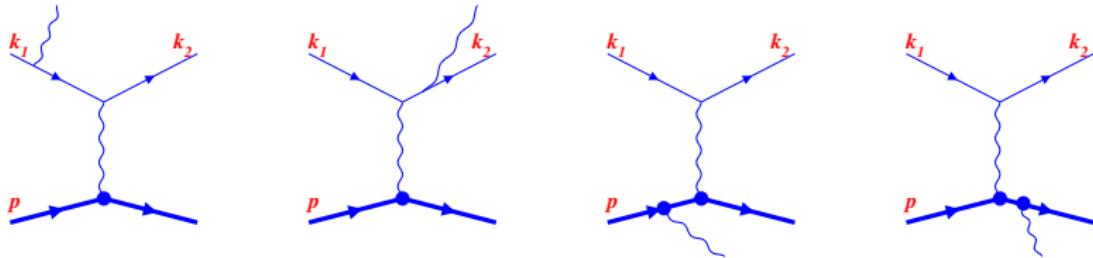


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$$q = p' - p, \quad p^2 = p'^2 = M, \quad \tau = Q^2/4M^2, \quad Q^2 = -q^2$$

$$F_d(q^2) = \frac{G_E(q^2) + \tau G_M(q^2)}{1 + \tau}, \quad F_p(q^2) = \frac{G_M(q^2) - G_E(q^2)}{1 + \tau},$$



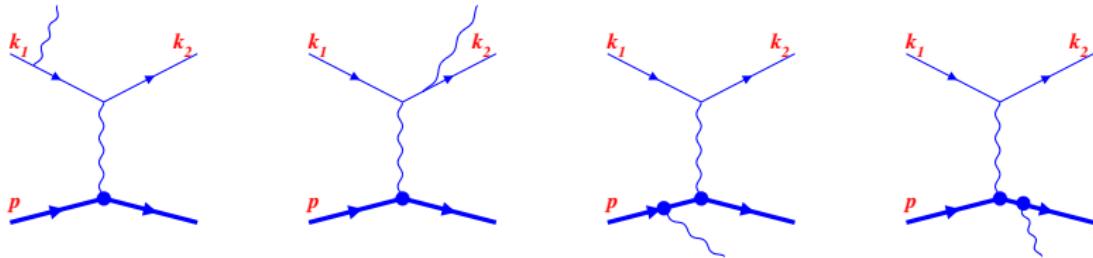
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$$J^{\mu\alpha} \sim \underbrace{\left[ \frac{k_1^\alpha}{kk_1} - \frac{k_2^\alpha}{kk_2} \right]}_{\text{Soft part}} \gamma^\mu - \frac{\gamma^\mu \hat{k} \gamma^\alpha}{2kk_1} - \frac{\gamma^\alpha \hat{k} \gamma^\mu}{2kk_2} \quad \text{– BH current}$$



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$$d\sigma_i = \frac{1}{2S} \left( \mathcal{M}_{BH}\mathcal{M}_h^\dagger + \mathcal{M}_h\mathcal{M}_{BH}^\dagger \right) d\Gamma$$

$$d\Gamma = \frac{dQ^2 dv dt d\phi_k}{2^8 \pi^4 S \sqrt{Q^2(Q^2 + 4M^2)}}$$

$$S = 2k_1 k, Q^2 = -q^2 = -(k_1 - k_2)^2, t = -(q - k)^2 = -(p - p')^2$$

$$v = (p + q)^2 - M^2 \text{ -- inelasticity}$$

$\phi_k$  angle between  $(\vec{q}, \vec{k})$  and  $(\vec{k}_1, \vec{k}_2)$  for  $p = (M, 0, 0, 0)$

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Infrared free part

$$d\sigma_i^F = d\sigma_i - d\sigma_i^{IR} = \int_0^{v_{cut}} dv \sum_{i,j,k=d,p} \theta_{ijk} F_i(Q^2) F_j(t) F_k(0)$$

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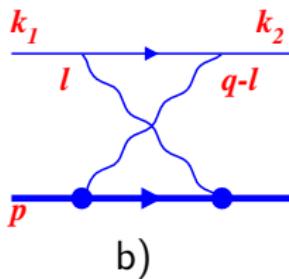
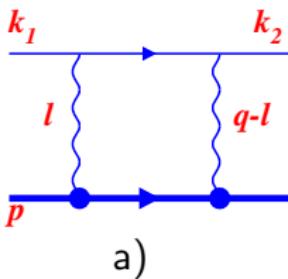
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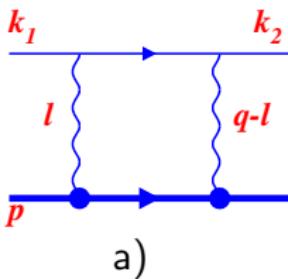
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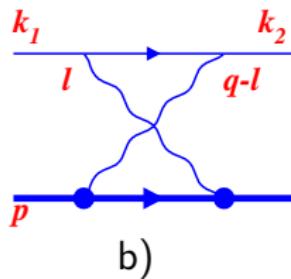
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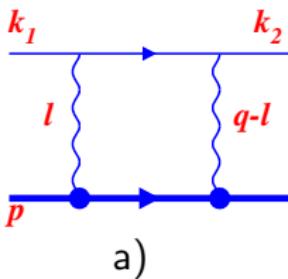
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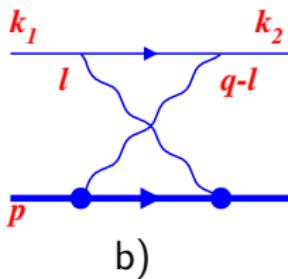
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Take into account only infrared divergence part i.e.  $l \rightarrow 0, q$



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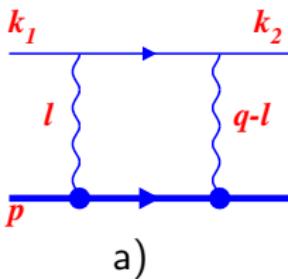


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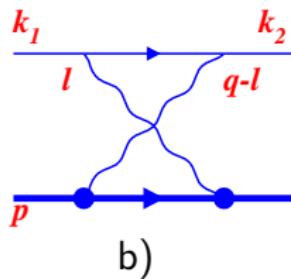
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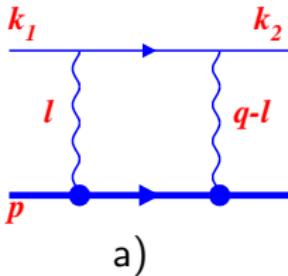
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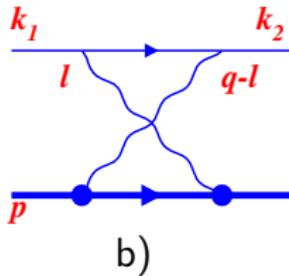
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The sum  $\frac{d\sigma_i^{IR}}{dQ^2} + \frac{d\sigma_{2\gamma}^{IR}}{dQ^2}$  are infrared free



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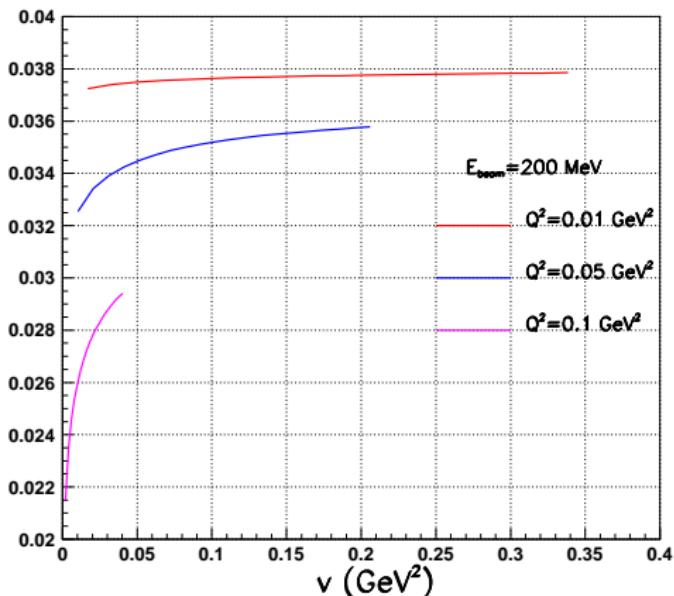
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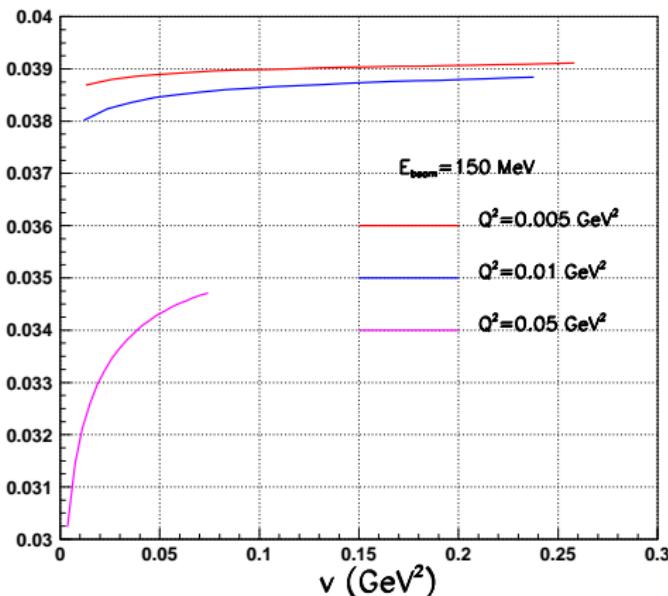
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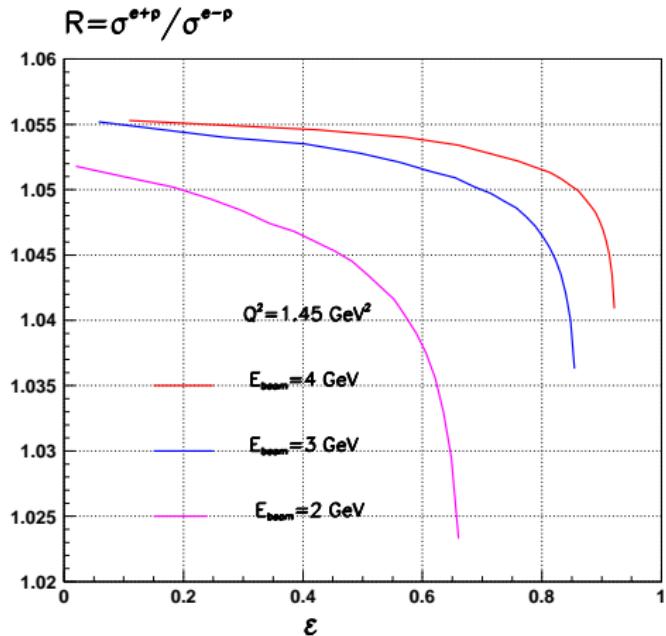
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$$A = (\sigma_{e^+p} - \sigma_{e^-p}) / 2\sigma^B$$



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$$\varepsilon^{-1} = 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2}$$

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