Precision Radiative Corrections for Next Generation Experiments May 16-19, 2016

Thomas Jefferson National Accelerator Facility Newport News, VA

Exploring GPDs: energy-momentum tensor & applications*

Peter Schweitzer (University of Connecticut)

Outline

- **Introduction**
  hard-exclusive reactions: GPDs → ?
  crucial quantity: energy-momentum tensor

- **Energy-momentum tensor**
  Ji sum rule & mechanical properties
  last unknown global nucleon property(!)

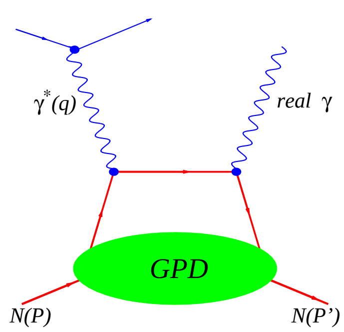
- **Applications**
  motivation & vision: mechanical stability
  practical use: from hard-exclusive reactions at JLab ...
  ... to charmonium pentaquark spectroscopy at LHCb

- **Outlook**
  cool and promising future!

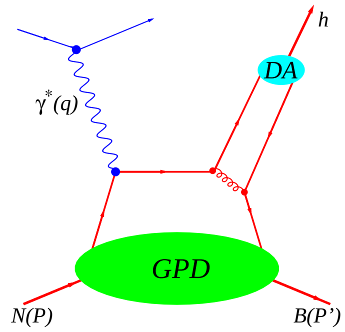
* in collaboration with Irina Perevalova, Maxim Polyakov
based on work supported by JLab, DOE, NSF

Introduction

- hard-exclusive reactions
  factorization, access to **GPDs**
  Ji; Radyushkin; Collins, Frankfurt, Strikman



DVCS

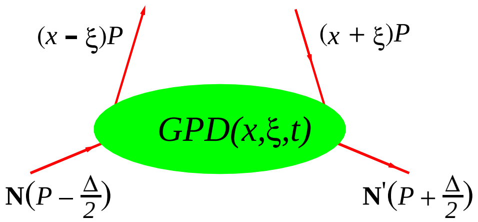


hard meson production

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle N'(p')|\overline{\psi}_q(-\tfrac{\lambda n}{2})\, n_\mu \gamma^\mu \,[-\tfrac{\lambda n}{2}, \tfrac{\lambda n}{2}]\psi_q(\tfrac{\lambda n}{2})\,|N(p)\rangle$$

$$= \overline{u}(p')\left[n_\mu \gamma^\mu\, H^q(x,\xi,t) + \overline{u}(p')\,\frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M_N}\, E^q(x,\xi,t)\right] u(p)$$

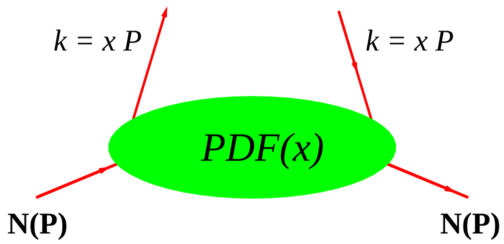
- meaning: "microsurgery"

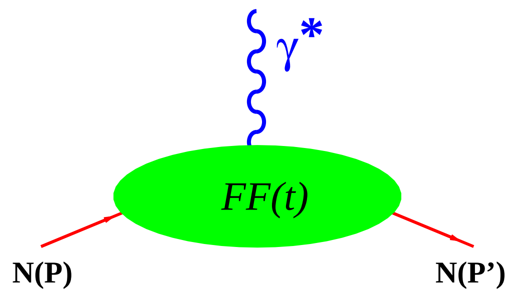


definitions for completeness:
$\xi = (n\cdot\Delta)/(n\cdot P)$, $t = \Delta^2$
$P = \frac{1}{2}(p'+p)$, $\Delta = p' - p$
$n^2 = 0$, $n\cdot P = 2$, $k = xP$
renormalization scale μ
analog gluon GPDs

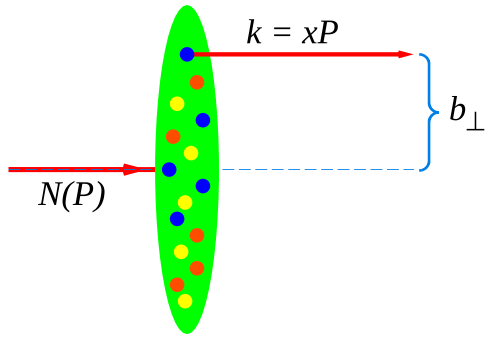
- what do we learn?

We learn a lot, because GPDs . . .



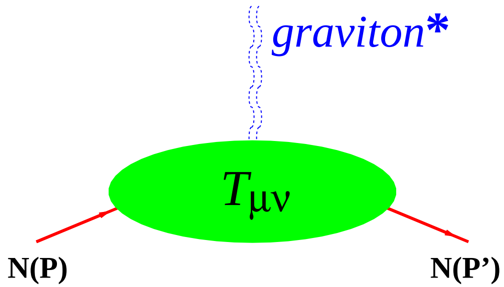


- generalize PDFs &
  em or axial form factors
  $\int \mathrm{d}x\ H^q(x,\xi,t) = F_1^q(t)$
  $\lim\limits_{\Delta\to 0} H^q(x,\xi,t) = f_1^q(x)$



- explore impact parameter space
  & tomography (M. Burkardt, . . .)

$$H^q(x,b_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\left[\lim_{\xi\to 0} H^q(x,\xi,t)\right] e^{i\boldsymbol{\Delta}_T\boldsymbol{b}_T}$$



- allow to access (polynomiality)
  gravitational form factors
  $\int \mathrm{d}x\ x\,H^q(x,\xi,t) = A^q(t) + \frac{4}{5}\,\xi^2 d_1^q(t)$
  $\int \mathrm{d}x\ x\,E^q(x,\xi,t) = B^q(t) - \frac{4}{5}\,\xi^2 d_1^q(t)$

- and gravity couples to
  energy momentum tensor
  probably most fundamental quantity

Energy-momentum tensor (EMT)

- instead of arguing how important EMT is, question:
  are you aware of introductory QFT text books
  which *do not* discuss EMT in first chapters?*

- if a theory can be solved:
  construct $T_{\mu\nu}$ and generators of Poincaré group
  learn what is **mass**, **spin**, **D-term (?)** of the particles
  (in introductory QFT text books: free fields, so it can be done & is instructive)

- even if a theory cannot be solved, studies of EMT insightfull
  3 examples (in QCD)
  (in chronological order)

---

* interestingly, advanced QFT books discuss EMT in later chapters: *trace anomaly*
$\hat{T}^{\mu}_{\mu} \equiv \frac{\beta}{2g} F^{\mu\nu}F_{\mu\nu} + (1+\gamma_m)\sum_q m_q \bar{\psi}_q \psi_q$ Adler, Collins, Duncan, PRD15 (1977) 1712;
Nielsen, NPB 120, 212 (1977); Collins, Duncan, Joglekar, PRD 16, 438 (1977)

- **mass**: celebrated Higgs mechanism explains $m_u \approx 2\,\text{MeV}$, $m_d \approx 5\,\text{MeV}$

  $\hookrightarrow$ $\mathbf{2\,MeV + 2\,MeV + 5\,MeV \overset{!?}{=} 940\,MeV}$ 2 orders of magnitude mismatch! Cool!!

  trace anomaly $\to$ glue! Can we do better? Ji, PRL 74, 1071 & PRD 52, 271 (1995)

  nucleon mass decomposition: how do E_{kin}, E_{pot} of quarks & gluons contribute?

- **spin** structure: **Ji sum rule** X.-D. Ji, Phys. Rev. Lett. 78 (1997) 610

  prominent motivator since its appearance!

$$\int \mathrm{d}x\, x \left(H^q(x,\xi,t) + E^q(x,\xi,t) \right) = A^q(t) + B^q(t) \xrightarrow{t \to 0} 2J^q(0)$$

- **D-term**, last unknown global property M. V. Polyakov, C. Weiss, PRD60 (1999) 114017

  stress tensor & **mechanical properties** M. V. Polyakov, PLB555 (2003) 57

Energy momentum tensor $T^{\mu\nu}$

- definition
$$\langle P'|\hat{T}^{q,g}_{\mu\nu}|P\rangle = \bar{u}(p')\Big[\; A^{q,g}(t)\,\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B^{q,g}(t)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M_N} + d_1^{q,g}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t)g_{\mu\nu}\Big]u(p)$$

- $\hat{T}^q_{\mu\nu}$ and $\hat{T}^g_{\mu\nu}$ separately gauge-invariant (but not conserved)
- total EMT $\hat{T}_{\mu\nu} = \hat{T}^q_{\mu\nu} + \hat{T}^g_{\mu\nu}$ is conserved $\partial_\mu \hat{T}^{\mu\nu} = 0$
- constraints: **mass** $\Leftrightarrow$ $A^q(0) + A^g(0) = 1$ (100 % of nucleon momentum carried by quarks + gluons)

  spin $\Leftrightarrow$ $J^q(0) + J^g(0) = \frac{1}{2}$ (100 % of nucleon spin carried by quarks + gluons)
- property: **D-term** $\Leftrightarrow$ $d_1^q(0) + d_1^g(0) \equiv d_1 \rightarrow$ also conserved Noether charge!

  but unconstrained! *Unknown!*

$\hookrightarrow$ *last unknown global property(!?) of the nucleon*

- How do we learn about nucleon?

$|N\rangle$ = **strong** interaction particle. Use other forces to probe it!

em:	$\partial_\mu J^\mu_{\mathbf{em}} = 0$	$\langle N'\|J^\mu_{\mathbf{em}}\|N\rangle$	$\longrightarrow$	$Q,\ \mu,\ \ldots$
weak:	PCAC	$\langle N'\|J^\mu_{\mathbf{weak}}\|N\rangle$	$\longrightarrow$	$g_A,\ g_p,\ \ldots$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\mathbf{grav}}\|N\rangle$	$\longrightarrow$	$M,\ J,\ d_1,\ \ldots$

1st global properties:

$$
\begin{aligned}
Q_{\text{prot}} &= 1.602176487(40) \times 10^{-19}\text{C} \\
\mu_{\text{prot}} &= 2.792847356(23)\mu_N \\
g_A &= 1.2694(28) \\
g_p &= 8.06(0.55) \\
M &= 938.272013(23)\,\text{MeV} \\
J &= \tfrac{1}{2} \\
\boldsymbol{d_1} &= \textbf{???}
\end{aligned}
$$

$\hookrightarrow$ d_1 = "last" global unknown

2nd partonic structure:

...	...	...	...
...	...	...	
...	...		
...			

- **interpretation** (M.V.Polyakov, PLB 555 (2003) 57)

Breit frame with $\Delta^\mu = (0, \vec{\Delta})$: static EMT $\boldsymbol{T_{\mu\nu}(\vec{r}, \vec{s})} = \int \frac{\mathrm{d}^3\vec{\Delta}}{2E(2\pi)^3}\, e^{i\vec{\Delta}\,\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle$

interpretation okay for large-N_c: $M_N \sim N_c$, $t \sim N_c^0$ $\Rightarrow$ recoil corrections $t/M_N^2 \sim 1/N_c^2$ (formulae correct $\forall N_c$)

$$\int \mathrm{d}^3r\, T_{00}(\vec{r}) = M_N \qquad \text{known}$$

$$\int \mathrm{d}^3r\, \varepsilon^{ijk}\, s_i\, r_j\, T_{0k}(\vec{r}) = \frac{1}{2} \qquad \text{known}$$

$$-\frac{M_N}{2}\int \mathrm{d}^3r\, \left(r^i r^j - \frac{r^2}{3}\,\delta^{ij}\right) T_{ij}(\vec{r}) \equiv \boldsymbol{d_1} \qquad \textbf{new!}$$

with: $$\boldsymbol{T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij}} \qquad \textbf{stress tensor}$$

$\boldsymbol{s(r)}$ related to distribution of *shear forces*
$\boldsymbol{p(r)}$ distribution of *pressure* inside hadron

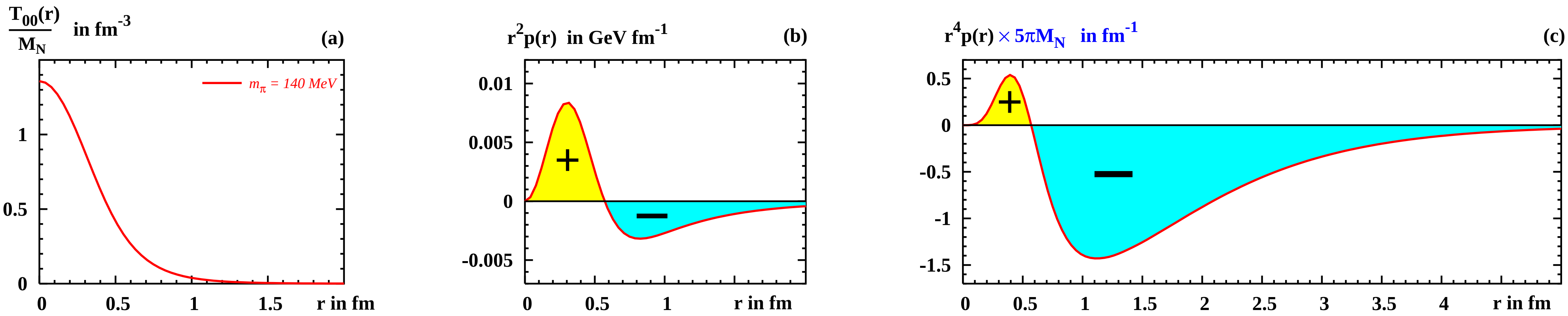
$\Big\} \longrightarrow$ **"mechanical properties"**

- **relation to stability**: EMT conservation $\Leftrightarrow$ $\partial^\mu \hat{T}_{\mu\nu} = 0$ $\Leftrightarrow$ $\nabla^i T_{ij}(\vec{r}) = 0$

$\hookrightarrow$ necessary condition for stability $\int\limits_0^\infty \mathrm{d}r\, \boldsymbol{r^2\, p(r)} = 0$ (von Laue, 1911)

$$d_1 = -\frac{4\pi}{3} M_N \int\limits_0^\infty \mathrm{d}r\; r^4 s(r) = 5\pi M_N \int\limits_0^\infty \mathrm{d}r\; \boldsymbol{r^4\, p(r)}$$

$\hookrightarrow$ d_1 shows how internal forces balance



energy density | pressure: $r^2 p(r)$ | $r^4 p(r)$ $\rightsquigarrow$ sign of d_1 negative

insights:

$T_{00}(0) = 1.70\,\mathrm{GeV/fm}^3 \approx 3\times 10^{15}\,\rho(\mathrm{H_2O}) \approx 13\times$ (nuclear density)

$p(0) = 0.23\,\mathrm{GeV/fm}^3 \approx 4\times 10^{34}\,\mathrm{N/m}^2 \gtrsim 100\times$(pressure in center of neutron star)

in chiral quark soliton model (Goeke et al, PRD75 (2007) 094021)

. . . and in ***QCD?***

wouldn't it be fascinating to know??

- **investigating forces**

  prominent property of proton:
  life time $\tau_{\text{prot}} > 2.1 \times 10^{29}$ years!

  question: how do strong forces balance to produce stability?

- answer in **model:** strong cancellation of repulsive forces due to quark core, and attractive forces from pion cloud

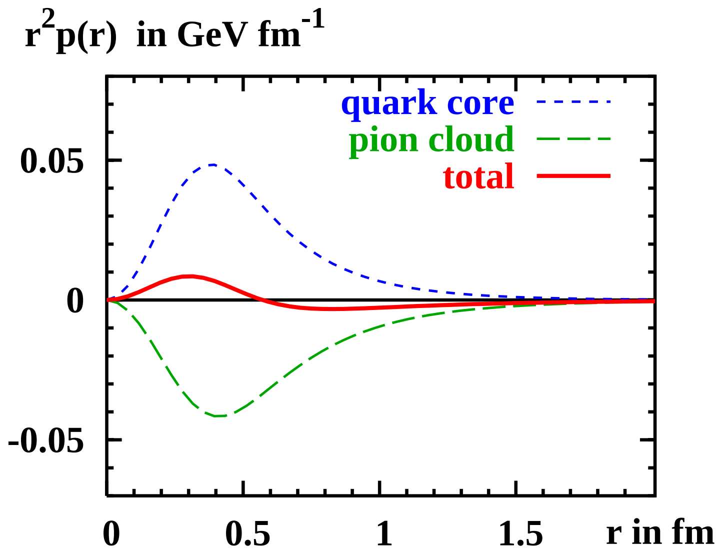
- answer in **QCD:** we do not know

  nice pictures(!), attractive insights(!)
  underexplored propaganda(?)

  (be aware: same picture for neutron, $\tau_{\text{neut}} = 14\,\text{min}\,40\,\text{sec} \gg 10^{-23}\,\text{sec}$)

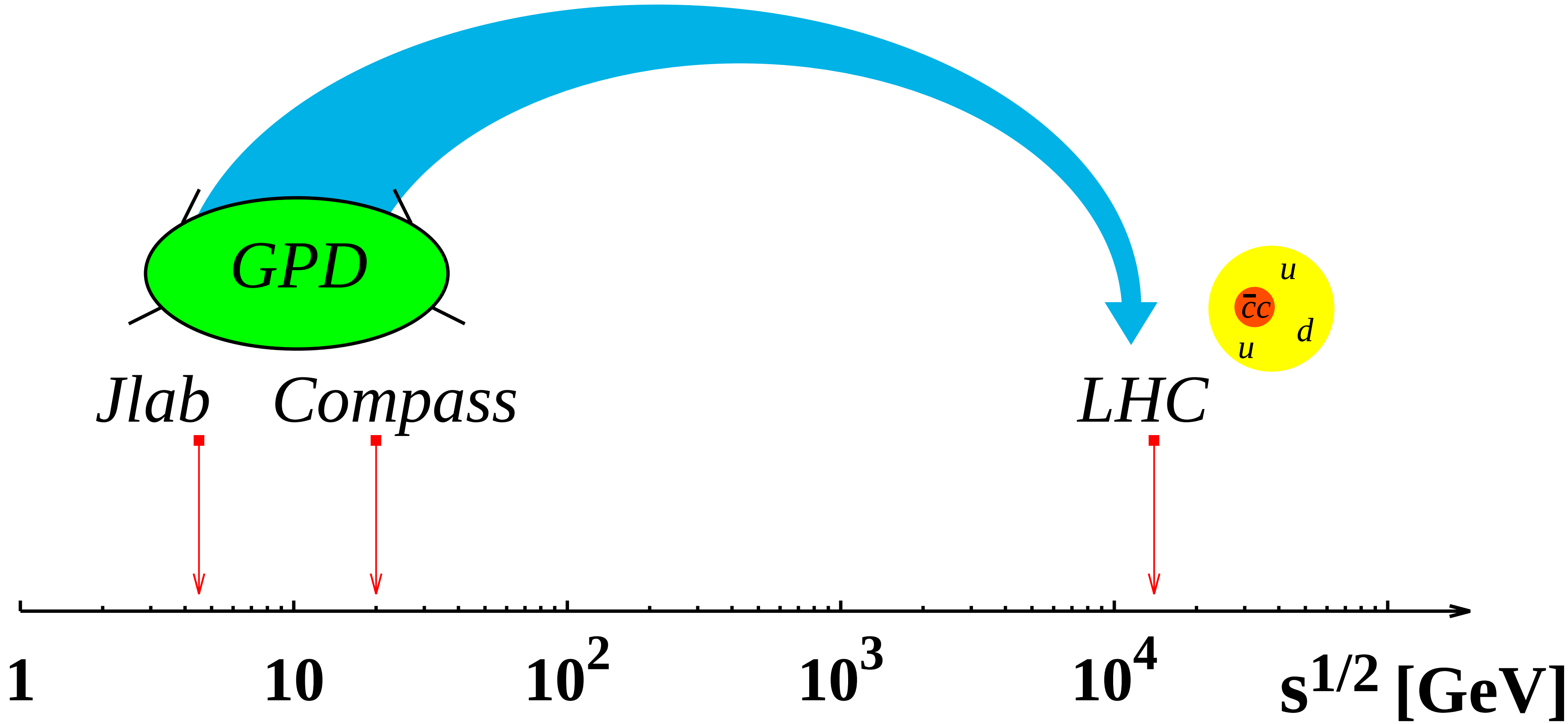
- as mental support for GPD program: okay

  ... but is there any practical use of that?
  answer before: *not really ...*
  answer today: *Yes!*



in chiral quark soliton model
chiral symmtry breaking ✓
realization of QCD in large-N_c ✓
built on instanton vacuum calculus ✓
not bad, but after all a model ...
Goeke et al, PRD75 (2007)

- amazing application



from hard-exclusive reactions at JLab, COMPASS ...

... to spectroscopy of $\bar{c}c$-pentaquarks at LHCb

not usual hadrons, not just any exotic hadron

only $\bar{c}c$-baryon bound states $\rightarrow$ rich enough!

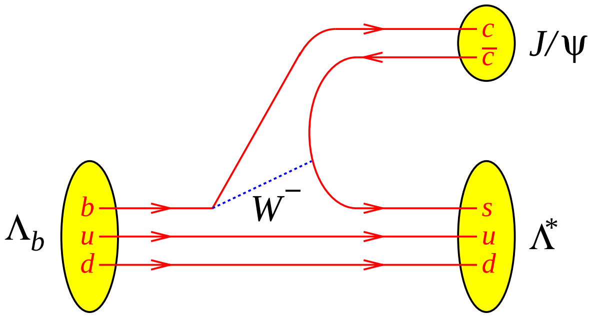
- **discovery of charmonium pentaquarks in Λ_b^0 decays at LHCb**

Aaij *et al.* PRL 115, 072001 (2015)

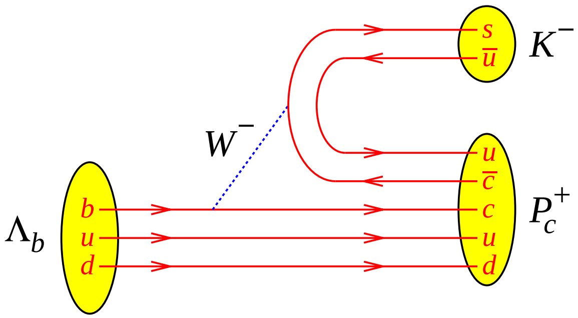
Λ_b^0	$m = 5.6$ GeV,	$\tau = 1.5$ ps
J/Ψ	$m = 3.1$ GeV,	$\Gamma = 93$ keV, $\Gamma_{\mu^+\mu^-} = 6\,\%$
Λ^*	$m = 1.4$ GeV or more,	$\Lambda^* \to K^- p$ in 10^{-23}s

$\Lambda_b^0 \longrightarrow J/\Psi\, p\, K^-$ seen

$\longrightarrow J/\Psi\, \Lambda^*$



$\longrightarrow J/\Psi\, P_c^+$



state	m [MeV]	Γ [MeV]	Γ_{rel}	mode	J^P
$P_c^+(4380)$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$(4.1 \pm 0.5 \pm 1.1)\,\%$	$J/\psi\, p$	$\frac{3}{2}^{\mp}$ or $\frac{5}{2}^{+}$
$P_c^+(4450)$	$4450 \pm 2 \pm 3$	$39 \pm 5 \pm 19$	$(8.4 \pm 0.7 \pm 4.2)\,\%$	$J/\psi\, p$	$\frac{5}{2}^{\pm}$ or $\frac{3}{2}^{-}$

• how can GPDs help? M. I. Eides, V. Y. Petrov and M. V. Polyakov, PRD93, 054039 (2016)

$R_{J/\psi} \ll R_N$ $\Rightarrow$ non-relativistic multipole expansion Gottfried, PRL 40 (1978) 598

baryon-quarkonium interaction dominated by 2 virtual chromoelectric dipole gluons

$V_{\text{eff}} = -\frac{1}{2}\,\alpha\,\vec{\boldsymbol{E}}^2$ Voloshin, Sov. J. Nucl. Phys. **36**, 143 (1982) [Yad. Fiz. **36**, 247 (1982)]

$$\alpha(1S) \approx \quad 0.2\,\text{GeV}^{-3}\ \text{(pert)},$$
$$\alpha(2S) \approx \quad 12\,\text{GeV}^{-3}\ \text{(pert)},$$
$$\alpha(2S\to 1S) \approx \begin{cases} -0.6\,\text{GeV}^{-3}\ \text{(pert)}, \\ \pm 2\,\text{GeV}^{-3}\ \text{(pheno)}, \end{cases}$$

α = chromoelectric polarizability
heavy quark mass limit & large-N_c limit
$\leadsto$ "perturbative result" Peskin, NPB 156 (1979) 365

value for $2S \to 1S$ transition from
phenomenological analysis of $\psi' \to J/\psi\,\pi\,\pi$ data
Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455

$$\text{(chromoelectric field)}^2 = \vec{\boldsymbol{E}}^2 = \tfrac{1}{2}(\vec{\boldsymbol{E}}^2 + \vec{\boldsymbol{H}}^2) + \tfrac{1}{2}(\vec{\boldsymbol{E}}^2 - \vec{\boldsymbol{H}}^2) = g^2\left(\frac{8\pi^2}{bg^2}\,T^{\mu}{}_{\mu} + T^G_{00}\right)$$

$b = \frac{11}{3}\,N_c - \frac{2}{3}\,N_F$ leading coeff. of β-function

g = strong coupling at low (nucleon) scale $\lesssim 1\,\text{GeV}$

g_s = strong coupling at scale of heavy quark (e.g. $m_c \sim 1.5\,\text{GeV}$, $g_s \neq g$)

$T^G_{00} = \xi T_{00}$ with ξ = fractional contributions of gluon to nucleon mass

$T^{\mu}{}_{\mu} = T^{00} - T^{ii}$
with stress tensor $T^{ij} = \left(\frac{r^i}{r}\frac{r^j}{r} - \frac{1}{3}\,\delta^{ij}\right)s(r) + \delta^{ij}\,p(r)$ M. V. Polyakov, PLB **555**, 57 (2003)

$$\boldsymbol{V_{\text{eff}} = -\tfrac{1}{2}\,\alpha\,\frac{8\pi^2}{b}\,\frac{g^2}{g_s^2}\Big[\nu\,T_{00}(r) + 3p(r)\Big]\,,\quad \nu = 1 + \xi_s\,\frac{b\,g_s^2}{8\pi^2}}$$

universal potential
$\nu \approx 1.5$ Eides et al, op. cit.
Novikov & Shifman, Z.Phys.C8, 43 (1981);
X. D. Ji, Phys. Rev. Lett. **74**, 1071 (1995)

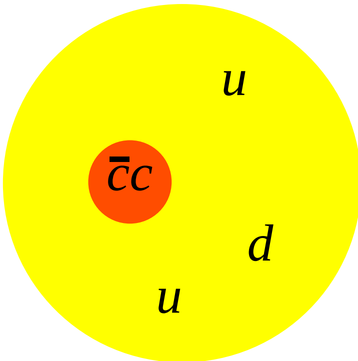
- **in future GPDs can help:** GPDs $\Rightarrow$ EMT form factors $\Rightarrow$ EMT densities
  $\Rightarrow$ universal potential V_{eff} for quarkonium-baryon interaction!

- **currently:** use e.g. chiral quark soliton model (done in Eides et al, op. cit.)

- **compute quarkonium-nucleon bound state**

  solve $\left(-\dfrac{\vec{\nabla}^2}{2\mu}+V_{\text{eff}}(r)\right)\psi = E_{\text{bind}}\,\psi$

  (here $\mu =$ reduced quarkonium-baryon mass)



- **results:**

  nucleon and J/ψ form no bound state

  nucleon and $\psi(2S)$ form 2 bound states with nearly degenerate masses around 4450 MeV
  in $L=0$ channel, $J^P=\frac{1}{2}^+$ and $\frac{3}{2}^+$ if $\alpha(2S)\approx 17\ \text{GeV}^{-3}$ (consistent with guideline from pert. calc.)

- **decay**

  cannot decay directly to $\psi(2S)$ and nucleon, as $M_{\psi(2S)} + M_N > 4450\,\text{MeV}$

  instead transition $(2S) \to (1S)$ governed by the same $\boldsymbol{V_{\text{eff}}}$
  but with small $\alpha(2S \to 1S)$ transition polarizability
  $\Rightarrow$ it "takes time"

  after transition "completed,"
  prompt decay to $J/\psi +$ nucleon (as observed)

  estimated width is tens of MeV $\to$ compatible with data!

- based on χQSM (Eides et al)
  model-dependent? Need to confirm in different models!
  (Perevalova, Polyakov, PS, in progress)

- Skyrme model (Cebulla et al 2007)
  incorporates chiral symmetry & also soliton
  however, different model, different way to realize stability (Skyrme term)
  ideal to provide an independent cross-check

- result:
  same conclusions!
  Very robust predictions for mass and decay width of $P_c^+(4450)$
  important confirmation!

- what about $P_c^+(4380)$?
  much broader
  more possibilities
  under investigation

Summary & Outlook

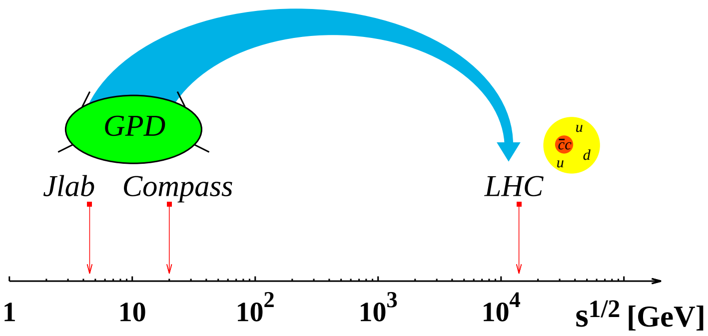
- GPDs important objects, but what do we learn?

- crucial quantity: form factors of energy momentum tensor
  mass decomposition, spin decomposition, and D-term!

- what's D-term? Last unknown global property, related to forces
  attractive and physically appealling $\rightarrow$ "motivation"

- recent development: knowledge of internal forces and energy density
  $\rightarrow$ quarkonium-baryon interaction V_{eff}

- naturally explains properties of $P_c^+(4450)$ observed at LHCb
  rich potential, ongoing work



Thank you!