Coulomb Corrections in Deep Inelastic Scattering from Nuclei

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*Precision Radiative Corrections for Next Generation Experiments*

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1. Coulomb corrections in electron nucleus scattering
2. Application to inelastic/DIS processes
3. Examples
   → EMC effect measurements
   → Re-analysis of SLAC E140
4. JLab Experiment 12-14-002: Nuclear Dependence of $R$
Heavy Nuclei and Coulomb Distortion

Electrons scattering from nuclei can be accelerated/decelerated in the Coulomb field of the nucleus. This effect is in general NOT included in most radiative corrections procedures. Important to remove/correct for apparent changes in the cross section due to Coulomb effects.

In a very simple picture – Coulomb field induces a change in kinematics in the reaction:

\[ E_e \rightarrow E_e + V_0 \]
\[ E_e' \rightarrow E_e' - V_0 \]

\[ V_0 = 3\alpha (Z-1)/2R \]

Electrostatic potential energy at center of nucleus
Coulomb Corrections in QE Processes

Importance of Coulomb Corrections in quasi-elastic processes well known

Gueye et al., PRC60, 044308 (1999)

Distorted Wave Born Approximation calculations are possible – but difficult to apply to experimental cross sections

→ Instead use Effective Momentum Approximation (EMA) tuned to agree with DWBA calculations

EMA:

\[ E_e \rightarrow E_e + V_0 \]
\[ E_e' \rightarrow E_e' - V_0 \]

with “focusing factor” \( F^2 = (1-V_0/E) \)

\[ V_0 \rightarrow (4/5)V_0, \quad V_0 = 3\alpha(Z-1)/2R \]

\( V_0 = 10 \text{ MeV for Cu, } 20 \text{ MeV for Au} \)

Coulomb Corrections in Inelastic Scattering

  - Perturbative expansion in powers of strength of Coulomb field
  - Effect of order $\mathcal{O}(Z^2 (Q^2)^2 (E_e + E'_e) / E_e E'_e < r >)$
  - “For any reasonable kinematics, this is completely negligible”

  - Estimates non-zero effect using Eikonal approximation
  - Applies estimates to vector meson production, not DIS

  - Coulomb Corrections for neutrino reactions
  - DWBA calculation that results in modifications to structure functions
    “at most 5%” effects for energies $> 1$ GeV
  - Final state particle only
Application: EMC Effect

JLab E03-103 measured $\sigma_A/\sigma_D$ for light and heavy nuclei
→ Study modification of quark distributions in nuclei → **EMC effect**

$\sigma_A/\sigma_D$ for Gold
A=197 Z=79

SLAC E-139
$E_e \sim 8\text{-}25$ GeV
$E_{e'} \sim 4\text{-}8$ GeV

JLab E03-103
$E_e \sim 6$ GeV
$E_{e'} \sim 1\text{-}2$ GeV

No Coulomb Corrections applied
Application: EMC Effect

Coulomb corrections significantly larger for JLab data → 5-10%, SLAC → 1-2%

\[ \frac{\sigma_A}{\sigma_D} \] for Gold

A=197 Z=79

SLAC E-139

\[ E_e \sim 8-25 \text{ GeV} \]

\[ E_{e'} \sim 4-8 \text{ GeV} \]

JLab E03-103

\[ E_e \sim 6 \text{ GeV} \]

\[ E_{e'} \sim 1-2 \text{ GeV} \]

with Coulomb Corrections (both data sets)
DIS/Inelastic cross section:

\[
\frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2(E')^2}{Q^4 \nu} \left[ F_2(\nu, Q^2) \cos^2 \theta - \frac{2}{M\nu} F_1(\nu, Q^2) \sin^2 \theta \right]
\]

\[
F_2(x) = \sum_i e_i^2 xq_i(x)
\]

Quark distribution functions

\[
\frac{d\sigma}{d\Omega dE'} = \Gamma \left[ \sigma_T(\nu, Q^2) + \varepsilon \sigma_L(\nu, Q^2) \right]
\]

\[F_1 \propto \sigma_T \quad F_2 \text{ linear combination of } \sigma_T \text{ and } \sigma_L\]

Measurements of EMC effect often assume \( \sigma_A/\sigma_D = F_2^A/F_2^D \)

\[\Rightarrow \text{this is true if } R=\sigma_L/\sigma_T \text{ is the same for A and D}\]

SLAC E140 set out to measure \( R=\sigma_L/\sigma_T \) in deuterium and the nuclear dependence of \( R \), i.e., measure \( R_A - R_D \)
$R_A - R_D$: E140 Re-analysis

E140 measured $\epsilon$ dependence of cross section ratios $\sigma_A/\sigma_D$ for $x=0.2, 0.35, 0.5$  
$Q^2 = 1.0, 1.5, 2.5, 5.0$ GeV$^2$  
Iron and Gold targets

$R_A - R_D$ consistent with zero within errors

No Coulomb corrections were applied

Large $\epsilon$ data: $E_e \sim 6-15$ GeV $E_{e'} \sim 3.6-8$ GeV  
Low $\epsilon$ data: $E_e \sim 3.7-10$ GeV $E_{e'} \sim 1-2.6$ GeV

[E140 Phys. Rev. D 49 5641 (1993)]
Re-analyzed E140 data using Effective Momentum Approximation for published “Born”-level cross sections.

→ Total consistency requires application to radiative corrections model as well.

Including Coulomb Corrections yields result $1.5 \sigma$ from zero when averaged over $x$.
Interesting result from E140 re-analysis motivated more detailed study
→ $x=0.5$, $Q^2=5$ GeV$^2$

→ Include E139 Fe data
→ Include JLab data
   Cu, $Q^2=4-4.4$ GeV$^2$

Normalization uncertainties between experiments treated as extra point-to-point errors

No Coulomb Corrections → combined analysis still yields
$R_A-R_D \sim 0$

$R_A-R_D = -0.035 +/- 0.042$

No Coulomb Corrections
Interesting result from E140 re-analysis motivated more detailed study
→ $x=0.5$, $Q^2=5$ GeV$^2$

→ Include E139 Fe data
→ Include JLab data
  → Cu, $Q^2=4-4.4$ GeV$^2$

Normalization uncertainties between experiments treated as extra point-to-point (between data sets) errors

Application of Coulomb Corrections → $R_A-R_D$ 2 $\sigma$ from zero
2007 Nuclear target ratios
→ 300 LT separations for $R_A - R_D$ for $Q^2 > 1.5$ GeV$^2$

Additional beam energies from 2005 not shown

→ Precision extraction of separated structure functions on D, Al, C, Fe/Cu
→ Search for nuclear effects in $F_L$, $R$
→ Neutron and p-n moment extractions (compare to lattice calculations)
→ Allow study of quark-hadron duality for neutron, nuclei separated structure functions

$F_2$, $F_L$, $R$ on Deuterium and heavier targets
Prefatory text: Preliminary results from E02-109/E04-001/E06-009 also suggest $R_A - R_D < 0$ at large $x$.

→ Resonance region

→ Similar results for heavier targets

\[ \Delta R \text{ in Resonance Region} \]

\[ \xi_{TM} = \frac{Q^2}{M \nu [1 + \sqrt{1 + Q^2 / \nu^2}]} \]

A. Bodek, PoS DIS2015 (2015) 026
**$R_A - R_D$ at Large $x$**

- Evidence is suggestive that $R_A - R_D < 0$ at large $x$
  - Effect is not large – depends on precision of the experimental data
  - Coulomb corrections are crucial to observation/existence of this effect $\rightarrow$ CC has significant dependence on electron energy, varies between $\epsilon$ settings
- Implications of $R_A - R_D < 0$
  - $F_1, F_2$ not modified in the same way in nuclei
  - What does this mean for our understanding of the EMC effect?
- Additional data (dedicated measurement) in DIS region required
Precision Measurements and Studies of a Possible Nuclear Dependence of $R = \sigma_L / \sigma_T$
(S. Malace, M.E. Christy, D. Gaskell, C. Keppel, P. Solvignon)

Measurements of nuclear dependence of structure functions, $R_A - R_D$ via direct L-T separations

Detailed measurements of $x$ and $Q^2$ dependence for Copper target
→ A dependence at select kinematics using C and Au
Experiment will study $R_A - R_D$ in both the EMC effect and anti-shadowing regions.

Overlap previous L-T separated data but will extend to both smaller and larger $x$.
Coulomb corrections play important role in extraction of $R_A - R_D$

The procedure we use currently is *ad-hoc*

→ Application of modified EMA not rigorously justified for inelastic scattering

For new, precision data from Hall C, we would like to be on firmer theoretical footing

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**Coulomb Corrections and $R_A - R_D$**

\[ R_A - R_D = -0.035 \pm 0.042 \]

\[ \epsilon' = \frac{\epsilon}{1 + \epsilon R_D} \]

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**No Coulomb Corrections**

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**With Coulomb Corrections**

\[ R_A - R_D = -0.084 \pm 0.04 \]

\[ \epsilon' = \frac{\epsilon}{1 + \epsilon R_D} \]
Summary

• Coulomb acceleration from “extra” protons in nucleus often neglected in analysis of DIS/Inelastic electron scattering data

• Formalism for applying Coulomb corrections in QE processes under control (EMA vs. DWBA comparison)
  – No simple prescription rigorously demonstrated for inelastic scattering

• Re-analysis of SLAC E140, combined analysis with E139+JLab data suggests non-zero $R_A-R_D$
  – JLab/Hall C data from 2005/2007 will address this with superior systematic and statistical errors
  – Non-zero $R_A-R_D$ has important ramifications for understanding the origins of the EMC effect

• JLab Experiment 12-14-002 will provide new, precise data to measure $R_A-R_D$ at large $x$
  – Correct/tested Coulomb Corrections procedure needed to properly interpret this data
Extra
Heavy Targets - Coulomb Corrections

• Initial (scattered) electrons are accelerated (decelerated) in Coulomb field of nucleus with $Z$ protons
  – Not accounted for in typical radiative corrections
  – Usually, not a large effect at high energy machines – **not true at JLab (6 GeV!)**

• E03-103 uses modified Effective Momentum Approximation (**EMA**), *Aste and Trautmann, Eur, Phys. J. A26, 167-178(2005)*
  – $E \rightarrow E+\Delta, E' \rightarrow E'+\Delta$
  – $\Delta = -\frac{3}{4} V_0$, $V_0 = 3\alpha(Z-1)/(2r_c)$

![Graph showing the comparison of $\sigma_{\text{Born}}/\sigma_{\text{CC}}$ for Au at 50 and 40 degrees](image)
New measurement of the EMC effect in light nuclei in Hall C

- Both A and density dependent fits fail
- **Be structure suggests “local density” picture**
  - Cluster structure dominated by $2\alpha + n$
  - Ave. density low, but all protons in $\alpha$-like clusters

Linear fit between $x=0.3-0.7$

[J. Seely et al, 103:202301 (2009)]