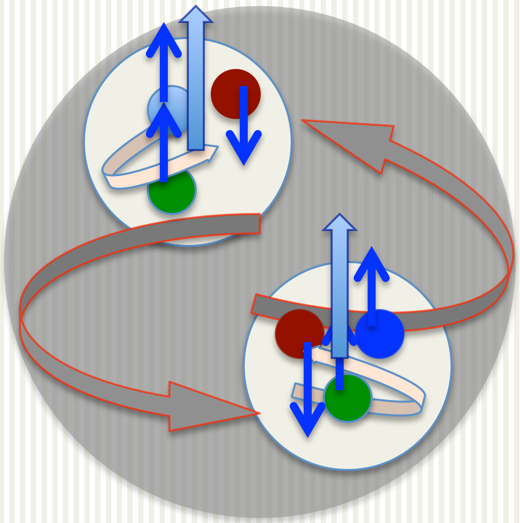
# New Framework for Extracting GPDs from Experiment



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Radiative Corrections Meeting
Jefferson Lab, May 17th, 2016

# Outline

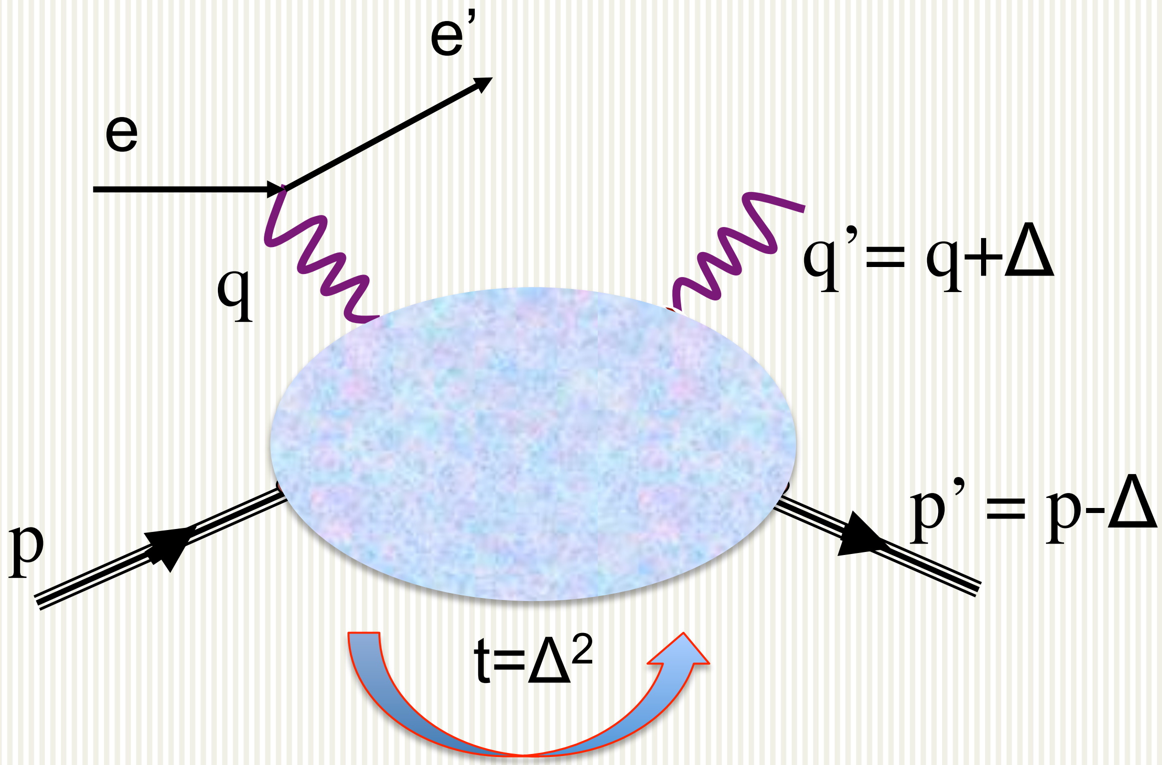
- ✓ Motivation:
  - ✓ Accessing Angular Momentum and Orbital Angular Momentum in the Proton (Aurore Courtoy, Michael Engelhardt, Abha Rajan)
  - ✓ 3D Structure of the Proton, Wigner Distributions and Color Entanglement
- ✓ Helicity Amplitude Formalism:
  - ✓ New Analysis (Gary Goldstein, Osvaldo Gonzalez, Abha Rajan)

✓ DVCS, DVMP, TCS and related processes as probes of Orbital Angular Momentum (OAM) in the proton
X. Ji, PRL78 (1997), Feshbach Prize (2016)

- It is the total angular momentum that appears as a conserved quantity through the QCD Energy Momentum Tensor
- ➔Ji Sum Rule: $J_q+J_g=1$

- The total angular momentum is measurable in DVCS; the observables are GPDs

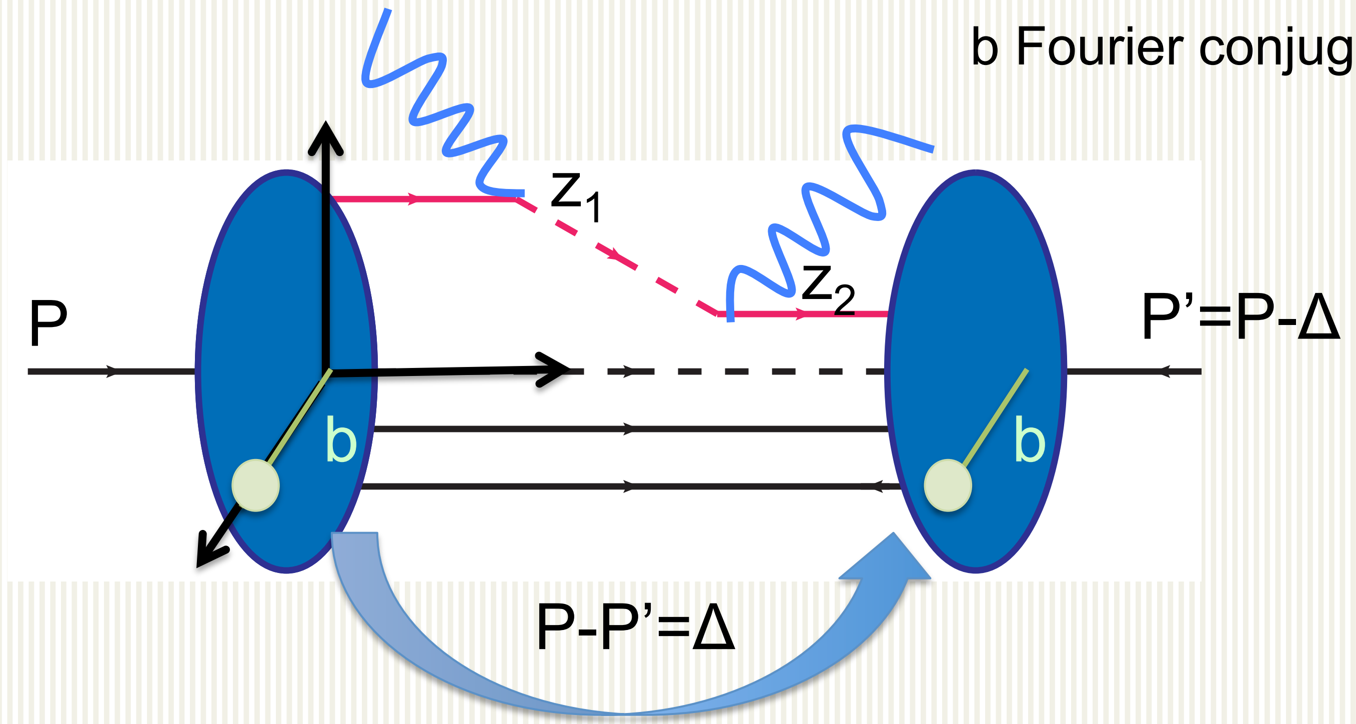
# Deeply Virtual Compton Scattering



# Space-time picture

z Fourier conjugate to $x_{Bj}$

b Fourier conjugate to Δ



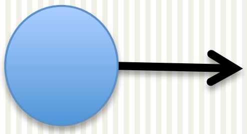

$$\mathcal{H},\mathcal{E} \longrightarrow \langle P-\Delta,\Lambda' \mid \bar{q}(0)\gamma^{+}\mathcal{U}(0,z)q(z^{-}) \mid P,\Lambda\rangle_{\mathbf{z}_T=0}$$

# GPDs and TMDs: Connecting Momentum and Spatial 3D structure

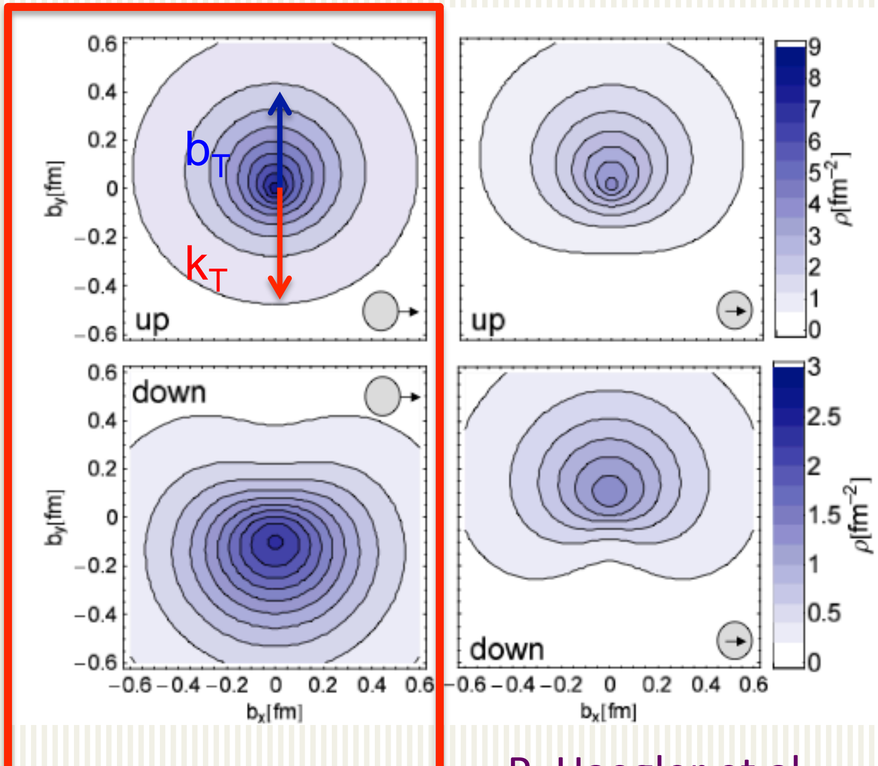
$$F^{[\gamma^+]}_{\Lambda\Lambda'} = \frac{1}{2P^+}\bar{U}(p,\Lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+j}\Delta_j}{2M}E(x,\xi,t)\right]U(p,\Lambda)$$

Ji (1997), Radyushkin (1997), Mueller et al (1994)

$S_T$



Transversely polarized proton




P. Haegler et al.

E → $\sigma^{+j}\Delta_j \Rightarrow \vec{S}_T \times \vec{\Delta}$

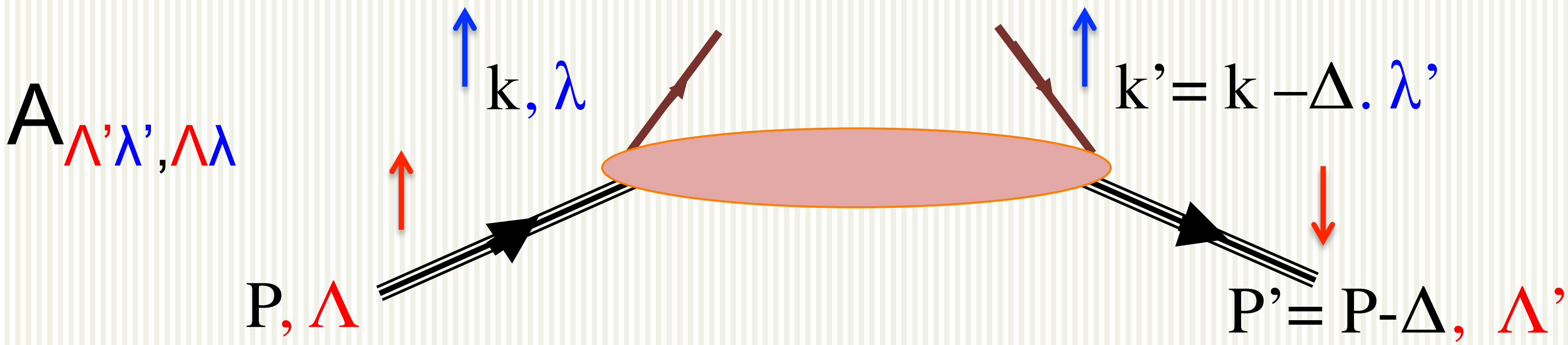
Lensing Mechanism (Burkardt)

-- $b_T$ corresponds to net $k_T$ in the opposite direction

-- attractive color force due to FSI!

$f_{1T}^{\perp}$ → $\sigma^{+j}k_j \Rightarrow \vec{S}_T \times \vec{k}$

# Helicity Structure of E




- E is the spin flip part of the amplitude
- E measures J through a <u>change</u> of one unit of L

$$S_z = -1/2 \rightarrow 1/2 \;\Rightarrow \Delta L_z = 1$$

at fixed J

$$\left(A_{++,++} + A_{+-,+-} + A_{-+,-+} + A_{--,--}\right) + \left(A_{++,-+} + A_{+-,--} - A_{--,+-} - A_{-+,++}\right)$$

Helicity basis (flip)

$$\left(A^X_{++,++} + A^X_{+-,+-} + A^X_{-+,-+} + A^X_{--,--}\right) + \left(A^X_{++,++} + A^X_{+-,+-} - A^X_{-+,-+} - A^X_{--,--}\right)$$

$$\approx H - i\Delta_2 E$$

Transversity basis (non flip)

Brodsky and Drell '80s, Belitsky, Ji and Yuan, '90's

One needs to measure both H and E similarly to how we measure both the Dirac and Pauli form factors to obtain the anomalous magnetic moment: indirect signal of the existence of OAM!

SPIN 1/2

SPIN 1

$$J_q = \frac{1}{2}\int dx\, x\, [H_q(x,0,0) + E_q(x,0,0)]\,, \quad \longrightarrow \quad J_q = \frac{1}{2}\int dx\, x\, H_2^q(x,0,0),$$

$F_1+F_2= G_M$

$G_M$

**Deuteron Angular Momentum Sum Rule**

S. Taneja, K. Kathuria, S. Liuti, G. Goldstein, PRD86 (2012)

**Ji Sum Rule relates GPDs to the total angular momentum:**

$$\frac{1}{2}\int_{-1}^{1} dx\, x\left[H_q(x,0,0)+E_q(x,0,0)\right]=J_q$$

**Helicity distribution singles out the contribution of spin:**

$$\int_{-1}^{1} dx\, g_1(x)=\frac{1}{2}\Delta\Sigma_q$$

**The orbital contribution can be obtained indirectly through:**

$$J_q = ? + \frac{1}{2}\Delta\Sigma_q$$

**However, to verify the (quark part of) the spin sum rule one needs to perform three independent measurements**

The next generation of studies of the 3D structure will:

- Verify the angular momentum sum rule
    - GPDs in the quark sector: Jlab@12 GeV
    - Open question: gluon GPDs … EIC…

Experiment:

- ➢ Transversely polarized targets
- ➢ DVCS, Timelike Compton Scattering, Recoil Polarization DVCS: universality

- Go beyond, to understand L
    - Twist 3 GPDs
    - GTMDs

Experiment:

- ➢ Longitudinally polarized targets
- ➢ DVCS, and more complex exclusive experiments

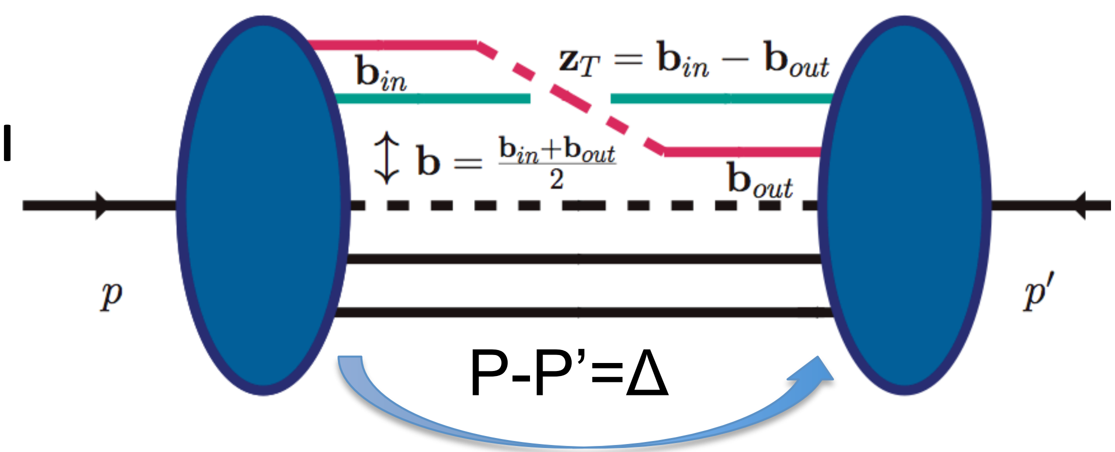
# Wigner Distribution and how to measure it

Generalized Transverse Momentum Distribution (GTMD)

$$\mathcal{W}^{\mathcal{U}}\left(x,\vec{b}_T,\vec{k}_T\right)=\int\frac{d^2\Delta_T}{(2\pi)^2}\,e^{i\Delta_T\cdot b_T}\int\frac{d^2z_T d^2z^-}{(2\pi)^3}\,e^{i(xP^+z^- - k_T\cdot z_T)}\left\langle P',\Lambda'\middle|\bar{\psi}(0)\gamma^+\mathcal{U}(0,\infty\,|\,n)\psi(z)\middle|P,\Lambda\right\rangle\Big|_{z^+=0}$$

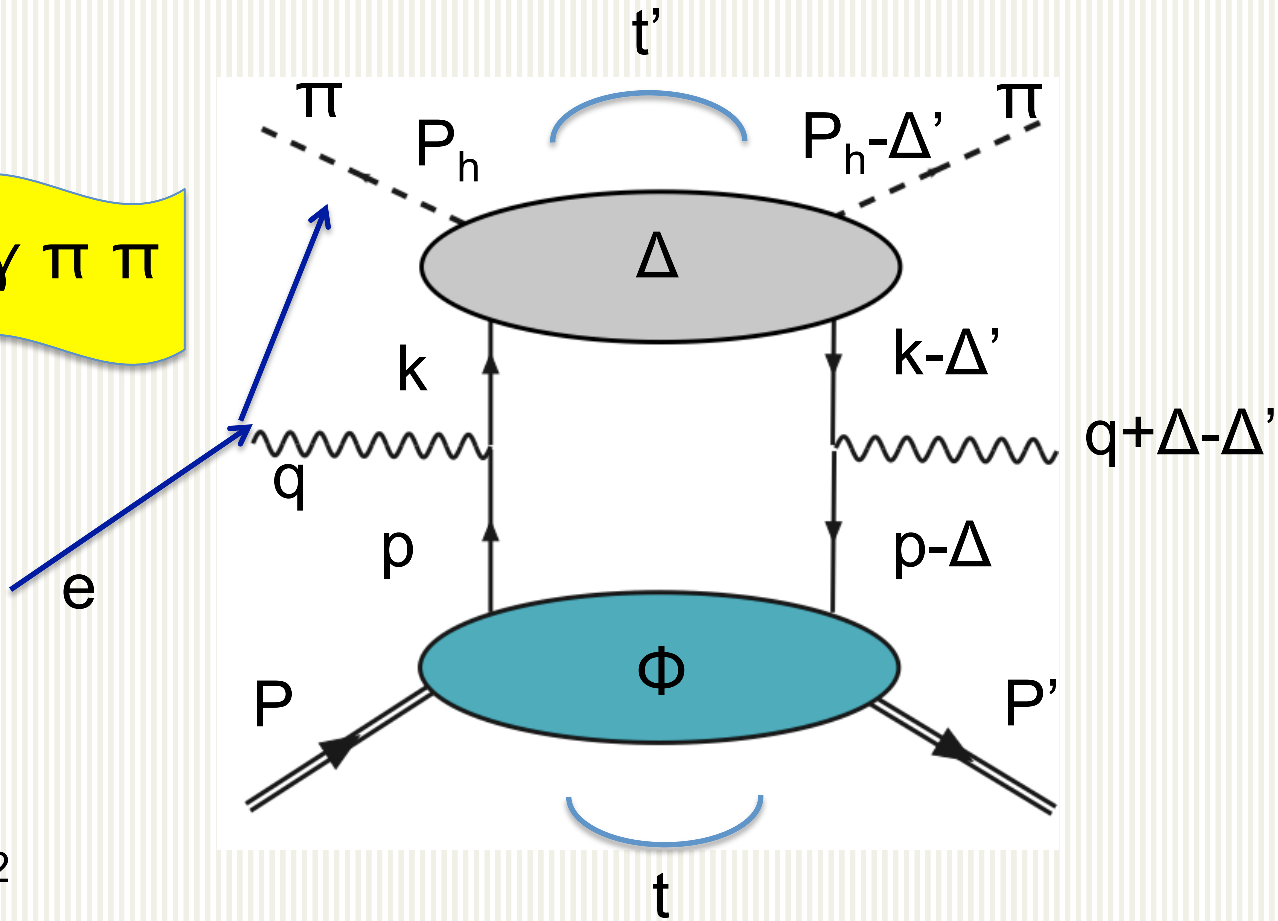
**b= transverse position of the quark inside the nucleon/nucleus**

**$z_T$=transverse distance traveled by the struck quark between the initial and final scattering**



# GTMDs from "off-forward SIDIS"

ep → e'p' γ π π




$t, t', P_h^2 < Q^2$

$$g_{\Lambda'_\gamma,\Lambda'_N,0;\Lambda_\gamma,\Lambda_N,0} = \sum_{\lambda,\lambda'} \tilde{g}^{\lambda'\lambda}_{\Lambda'_\gamma\Lambda_\gamma} \otimes A_{\Lambda'_N,\lambda',\Lambda_N,\lambda}(x,\xi,t) \otimes F^{\pi_1}_{\lambda 0}(z) F^{\pi_2}_{\lambda' 0}(v)$$

Φ Δ

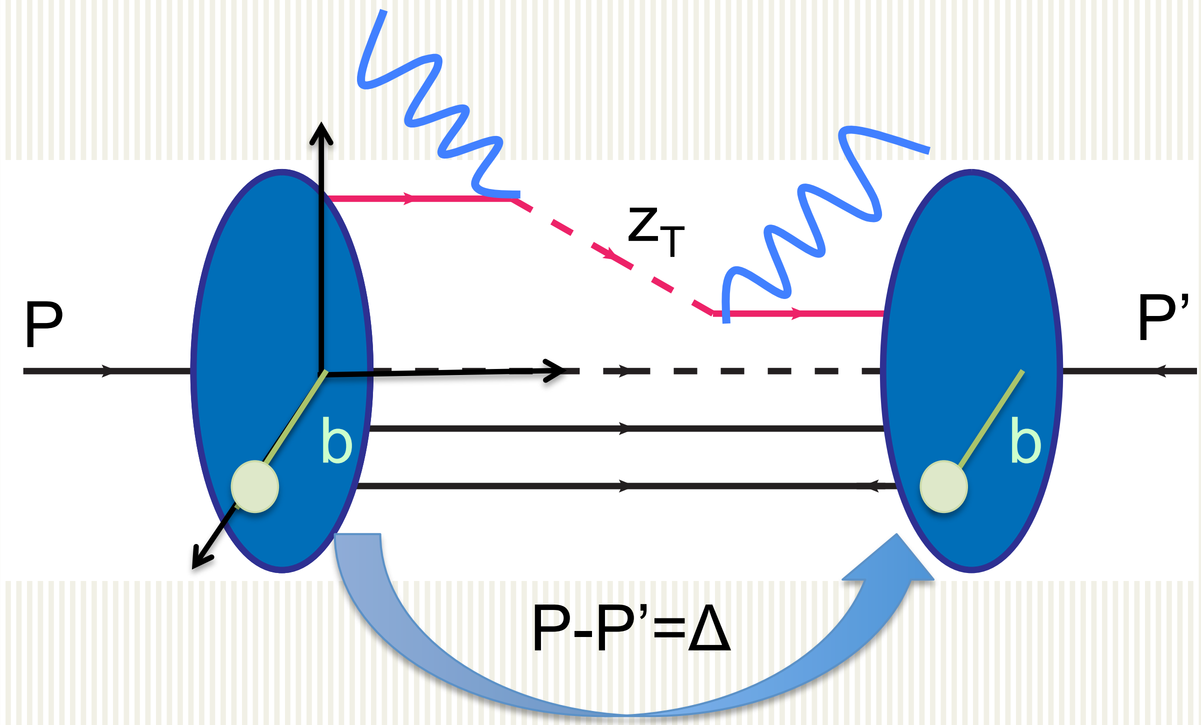
# What gives L?

## OAM is measured through a Wigner Distribution/GTMD

Lorce, Pasquini, Hatta, Meissner, Metz, Schlegel

$$\mathcal{L}_q, L_q = \int dx\, d^2b\, d^2k_T \left(\vec{b} \times \vec{k}_T\right)_3 \mathcal{W}\left(x, \vec{b}, \vec{k}_T\right)$$

$$F_{14}(x, k_T, \Delta) \to \int \frac{dz^- d^2\mathbf{z}_T}{(2\pi)^3} e^{i(xP^+ z^- - \mathbf{k}_T \cdot \mathbf{z}_T)} \langle P', \Lambda' \mid \bar{q}(0)\gamma^+ q(z^-) \mid P, \Lambda \rangle$$



## Integrated OAM is also obtained through a twist three GPD

Polyakov et al., Hatta

$$\int_0^1 dx\, xG_2 = -\frac{1}{2}\int_0^1 dx\, x(H+E) + \frac{1}{2}\int_0^1 dx\, \tilde{H}$$

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

**To understand how it is possible that the same quantity, L, is given by two different observables one needs to derive a relation at the unintegrated in $k_T$ level**

$$\frac{d}{dx}\int d^2k_T\, \frac{k_T^2}{M^2}\, F_{14} = G_2$$

A. Rajan, A. Courtoy, M. Engelhardt, S.L. arXiv:1602.00160

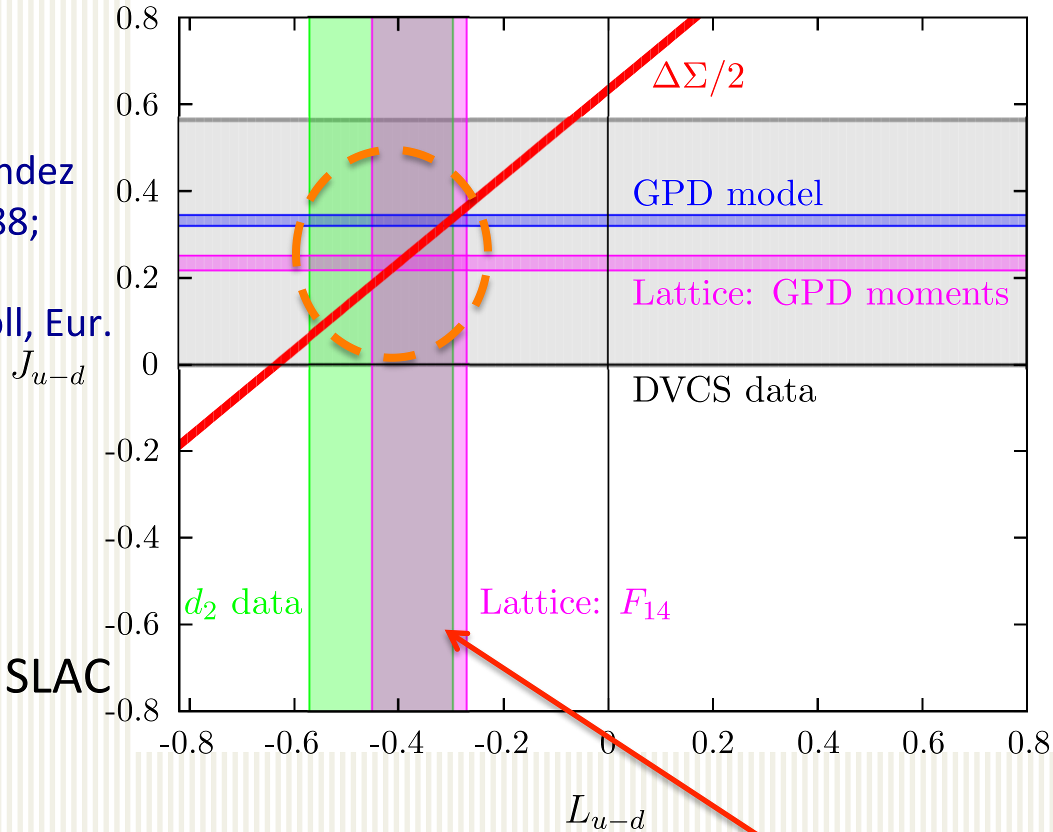
## Unintegrated relation between a GTMD and a Twist three GPD

# We now know how to access $L_q$ experimentally through twist three spin correlations in DVCS

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

A. Rajan, A. Courtoy, M. Engelhardt, S.L. arXiv:1602.00160

Model Calculation
O. Gonzalez Hernandez et al., Phys. Rev. C88; arXiv:1206.1876
M. Diehl and P. Kroll, Eur. Phys. J. C73; arXiv: 1302.4604




d$_2$ from SLAC

Mazous et al. PRL (2007)

DVCS + VGG

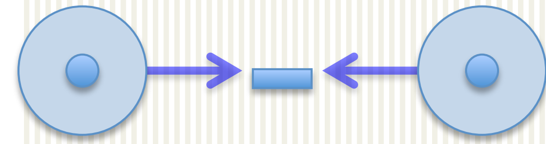
$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14} = G_2$$

# We identified the quark-proton helicity amplitudes combinations

A.Courtoy, G.Goldstein, O.Gonzalez Hernandez, S.Liuti and A.Rajan, PLB 731(2014)

$$i\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14} = \underline{A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--}}$$

$$-i\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}G_{11} = A_{++,++} - A_{+-,+-} + A_{-+,-+} - A_{--,--}$$



$A_{UL}$ correlation

In terms of quark-proton helicity amplitudes,

$$i(\mathbf{k}\times\mathbf{\Delta})_3 F_{14} = A^{tw2}_{++,++} + A^{tw2}_{+-,+-} - A^{tw2}_{-+,-+} - A^{tw2}_{--,--}$$

G$_2$

$$(k_1 - ik_2)F_{27} + (\Delta_1 - i\Delta_2)F_{28} = A^{tw3}_{+-^*,++} - A^{tw3}_{+-,++^*} - A^{tw3}_{--^*,-+} + A^{tw3}_{--,-+^*}$$

$A_{\Lambda'\lambda',\Lambda\lambda}$=quark-proton helicity ampltidue

**In order to understand how twist three GPDs can be measured one needs to develop the appropriate formalism**

**We are developing the framework for extracting OAM and the 3D Structure of the nucleon from experimental data**

# Increased Complexity compared to PDFs (restrict to GPD sector)

Theoretical Framework:

✓ Observables in terms of GPDs satisfy more complex constraints (polynomiality, positivity)

Global Fit:

✓ Increase in number of observables

unpolarized and polarized observables, together!

processes, DVCS, various DVMP, TCS, Recoil Polarization CS (RPCS)…

more kinematical variables

# Theoretical Framework: Cross section for SIDIS

$$\frac{d^4\sigma}{dx_{Bj}\,dy\,d\phi\,dt} = \Gamma\Big\{\Big[F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{2\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi} + h\sqrt{2\epsilon(1-\epsilon)}\,\sin\phi F_{LU}^{\sin\phi}$$

$$+\; S_{||}\Big[\sqrt{2\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi} + h\Big(\sqrt{1-\epsilon^2}\,F_{LL} + \sqrt{2\epsilon(1-\epsilon)}\,\cos\phi\, F_{LL}^{\cos\phi}\Big)\Big]$$

$$+\; S_{\perp}\Big[\sin(\phi-\phi_S)\Big(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\Big) + \epsilon\Big(\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)}\Big)$$

$$+\; \sqrt{2\epsilon(1+\epsilon)}\Big(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\Big)\Big]$$

$$+\; S_{\perp}h\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\Big(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big)\Big]\Big\}$$

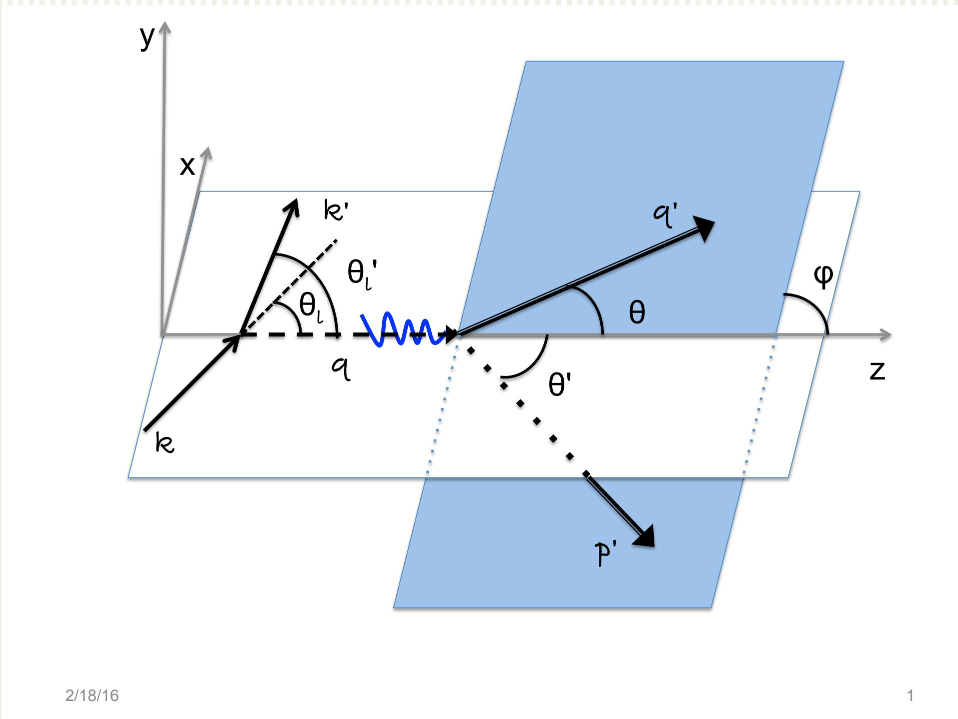
$$A_{LU} = \sqrt{\epsilon(1-\epsilon)}\,\frac{F_{LU}^{\sin\phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

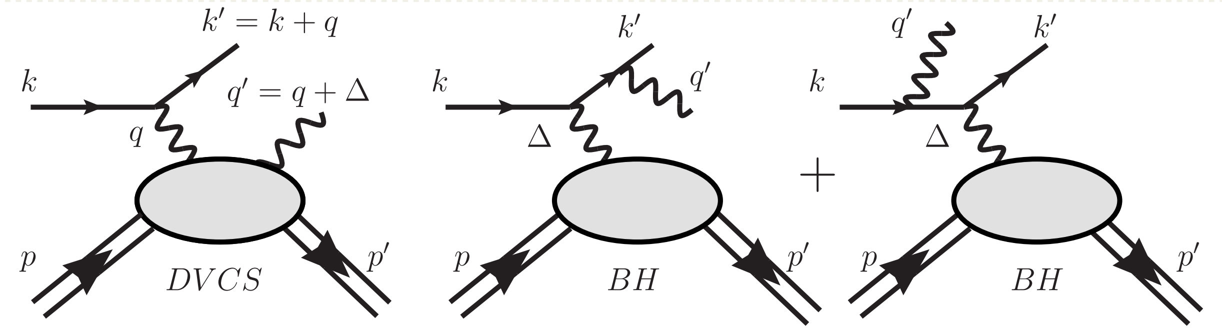
This framework was fully developed for TMDs and SIDIS: we are working out a similar, unified approach for GPDs and Exclusive processes

$$A_{UL} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{\epsilon(\epsilon+1)}\cdots}{F_{UU,T} + \epsilon\, F_{UU,L}} + \frac{\cdots}{F_{UU,T} + \epsilon\, F_{UU,L}}$$

$$A_{LL} = \frac{N^{\rightarrow}_{s_z=+} - N^{\rightarrow}_{s_z=-} + N^{\leftarrow}_{s_z=+} - N^{\leftarrow}_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{1-\epsilon^2}\;F_{LL}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\sqrt{\epsilon(1-\epsilon)}\,\cos\phi\, F_{LL}^{\cos\phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

# DVCS in Target Rest Frame






$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}\left[|T_{\mathrm{BH}}|^2 + |T_{\mathrm{DVCS}}|^2 + \mathcal{I}\right]$$

The Bethe-Heitler cross section is exactly calculable using the electromagnetic form factors at low t

The pure DVCS and the Interference term need to be separated from one another

It is important to write each component of the cross section in terms of helicity amplitudes

Example from upcoming manuscript: pure DVCS

$$\sigma^{UU} = \Gamma \left[ F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)} \cos\phi F_{UU}^{\cos\phi} \right]$$

$$= \frac{\Gamma}{2} \sum_{h,\Lambda,\Lambda',\Lambda'_\gamma} \left( T^{h\Lambda'_\gamma}_{DVCS,\Lambda\Lambda'} \right)^* T^{h\Lambda'_\gamma}_{DVCS,\Lambda\Lambda'},$$

$F_{UU,T}$

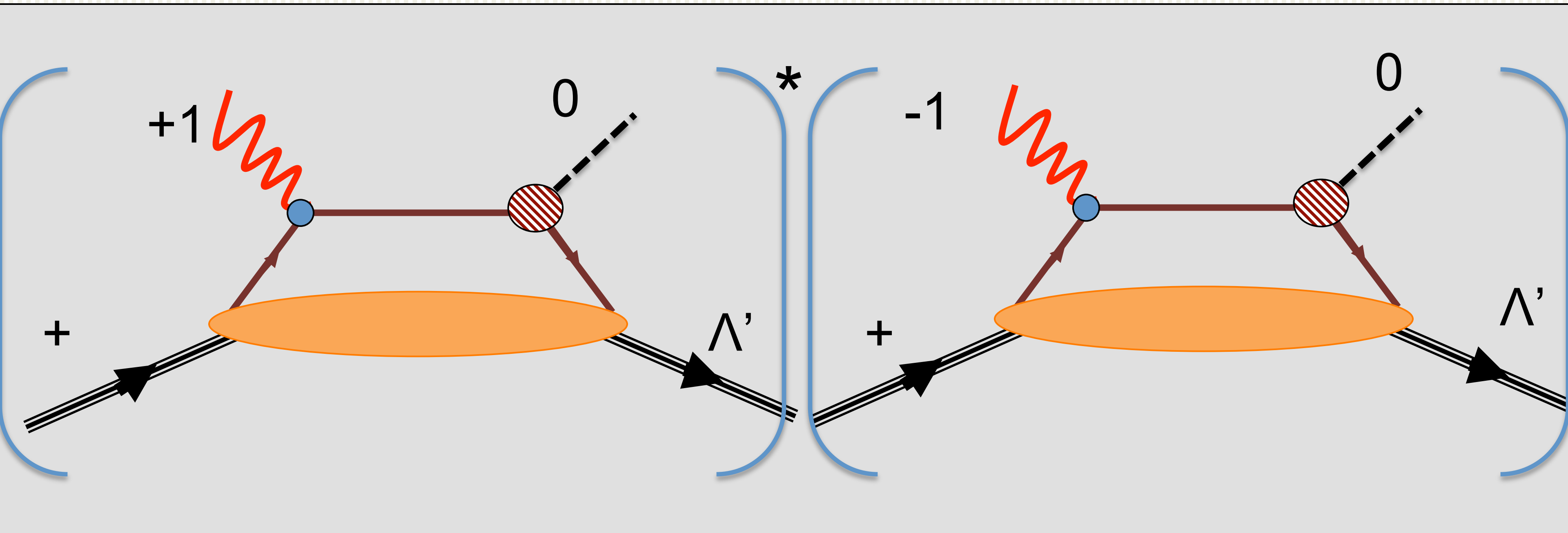
$$\sum_{\Lambda'} F^{11}_{+\Lambda'} + F^{11}_{-\Lambda'} = \left(f^{11}_{++}\right)^* f^{11}_{++} + \left(f^{11}_{+-}\right)^* f^{11}_{+-} + \left(f^{11}_{-+}\right)^* f^{11}_{-+} + \left(f^{11}_{--}\right)^* f^{11}_{--}$$

helicity amplitudes

$F_{UU}^{\cos\varphi}$

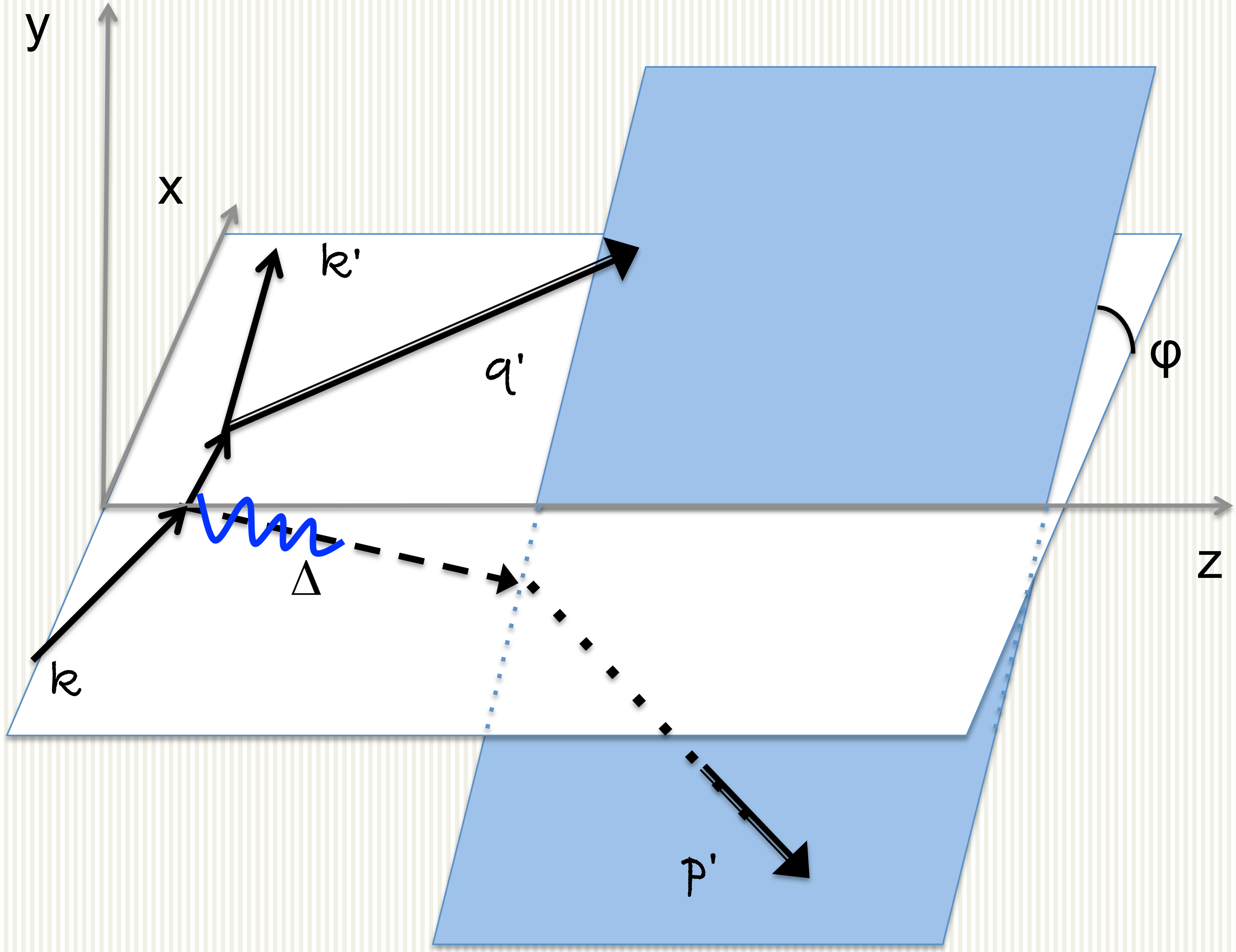
$$\cos\phi \sum_{\Lambda'} F^{01}_{+\Lambda'} - F^{01}_{-\Lambda'} = \left[ \left(f^{01}_{++}\right)^* f^{11}_{++} + \left(f^{01}_{+-}\right)^* f^{11}_{+-} - \left(f^{01}_{-+}\right)^* f^{11}_{-+} + \left(f^{01}_{--}\right)^* f^{11}_{--} \right]$$

contains twist three GPD




Example of observable:

$$F_{UU}^{\cos 2\phi} = -\Re e\, F_{1-1}^{++} = -\Re e \sum_{\Lambda'} f_{10}^{+\Lambda' *} f_{-10}^{+\Lambda'}$$

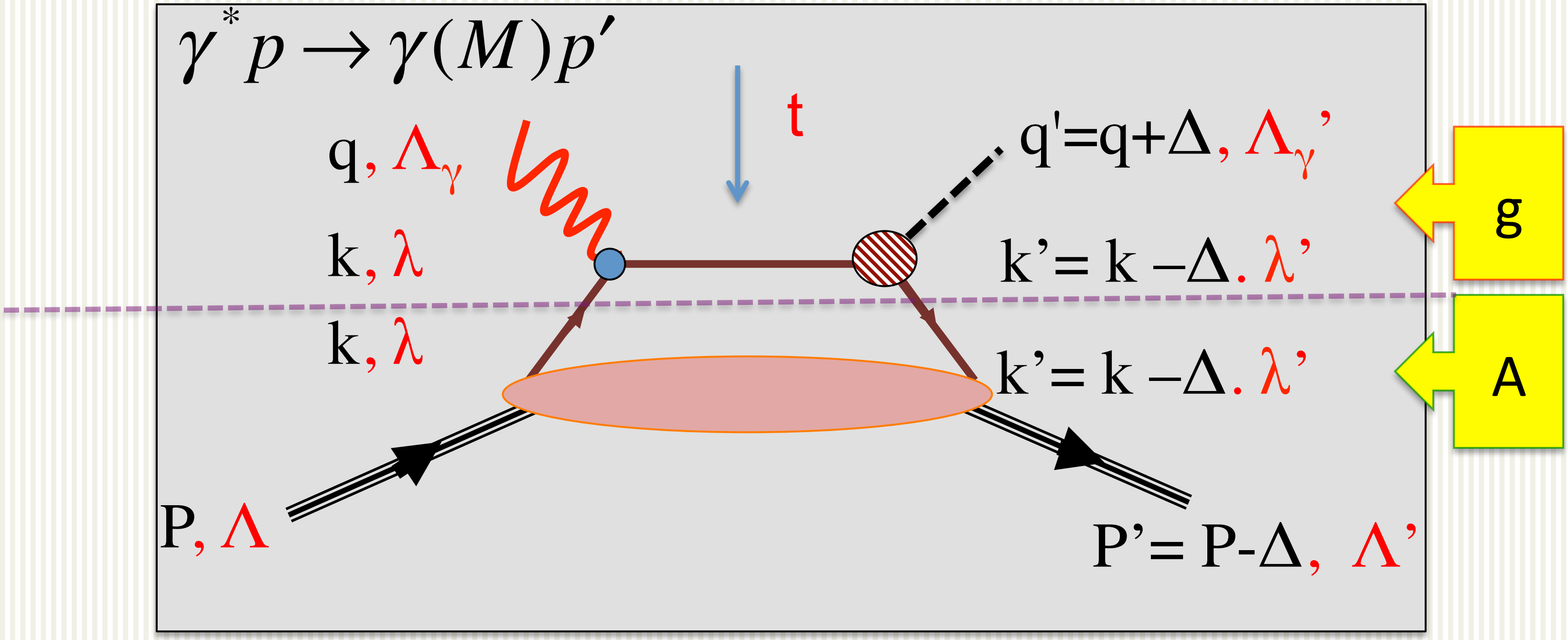



BH φ dependence is more complicated because the virtual photon is not along the z-axis

BH $\varphi$ dependence is more complicated because the virtual photon is not along the z-axis: it includes kinematical dependence besides phase dependence

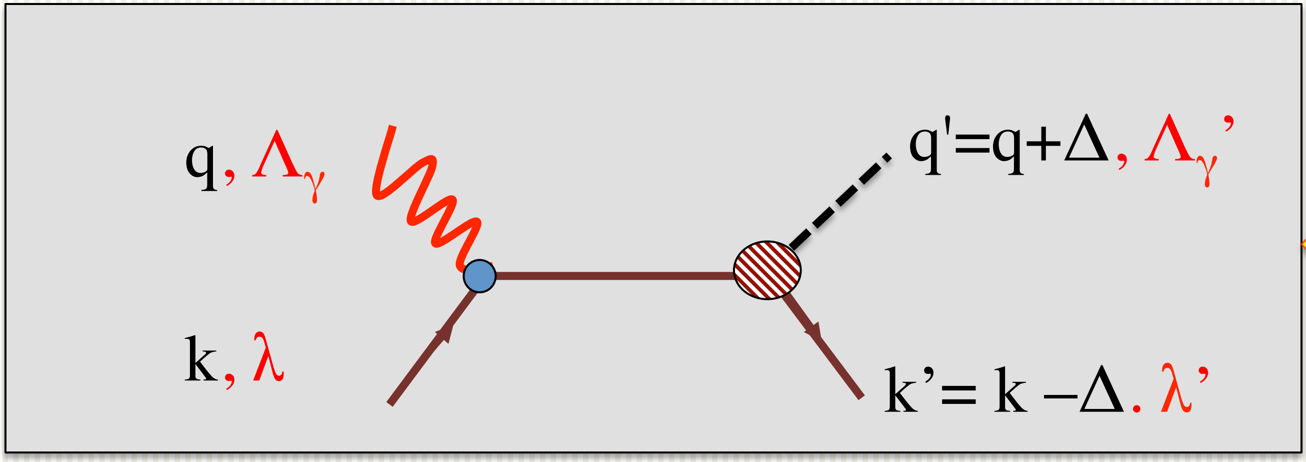
BH is treated as an exactly calculable term to correct the DVCS cross section with: QED + small t nucleon form factors

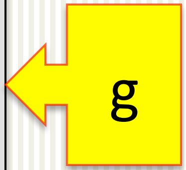
# Where are the GPDs? Factorization in exclusive processes (DVCS, DVMP…)




Convolution of "hard part" with quark-proton amplitudes

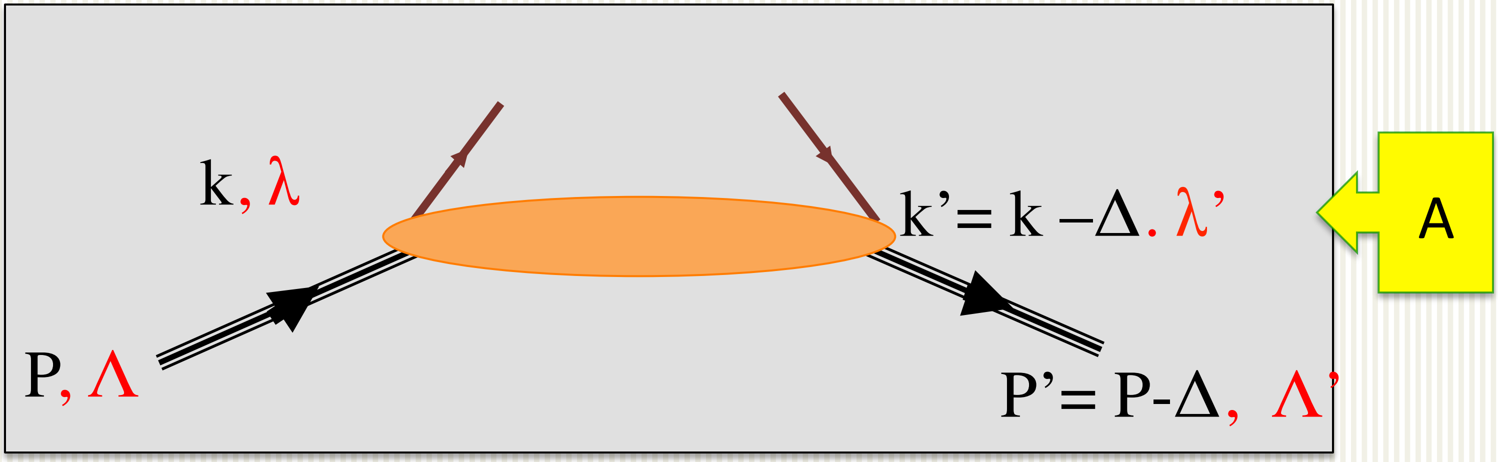
$$f_{\Lambda_\gamma,\Lambda;\Lambda'_\gamma,\Lambda'} = \sum_{\lambda,\lambda'} g_{\lambda,\lambda'}^{\Lambda_\gamma,\Lambda'_{\gamma(M)}}(x,k_T,\zeta,t;Q^2) \otimes A_{\Lambda',\lambda';\Lambda,\lambda}(x,k_T,\zeta,t),$$




$$g_{\Lambda_\gamma 0}^{\lambda\lambda\,(even)} = g_\pi^{even}(Q^2) e_q\, q^- \left[\bar{u}(k',\lambda)\gamma^\mu\gamma^+\gamma^\nu\gamma_5 u(k,\lambda)\right] \times \epsilon_\mu^{\Lambda_\gamma} q'_\nu \left(\frac{1}{\hat{s}-i\epsilon}+\frac{1}{\hat{u}-i\epsilon}\right)$$

# Quark-proton helicity amplitudes




$$
\begin{aligned}
f_{10}^{++} &= g_{10}^{+-} \otimes A_{+-,++} \\
f_{10}^{+-} &= g_{10}^{+-} \otimes A_{--,++} \\
f_{10}^{-+} &= g_{10}^{+-} \otimes A_{+-,-+} \\
f_{10}^{--} &= g_{10}^{+-} \otimes A_{++,+-} \\
f_{00}^{+-} &= g_{00}^{+-} \otimes (A_{--,++} - A_{+-,-+}) \\
f_{00}^{++} &= g_{00}^{+-} \otimes (A_{++,+-} - A_{+-,++}),
\end{aligned}
$$

$$
\begin{aligned}
f_{00}^{+-,\,even} &= \frac{\zeta}{\sqrt{1-\zeta}} \frac{1}{1-\zeta/2} \frac{\sqrt{t_o - t}}{2M} \widetilde{\mathcal{E}}, \\
f_{00}^{++,\,even} &= \frac{\sqrt{1-\zeta}}{1-\zeta/2} \widetilde{\mathcal{H}} + \frac{-\zeta^2/4}{(1-\zeta/2)\sqrt{1-\zeta}} \widetilde{\mathcal{E}},
\end{aligned}
$$

Write observables in terms of GPDs: convolution analogous to PDF sector

$$A_{\Lambda'\pm,\Lambda\pm} \Leftrightarrow H, E, \tilde{H}, \tilde{E}$$

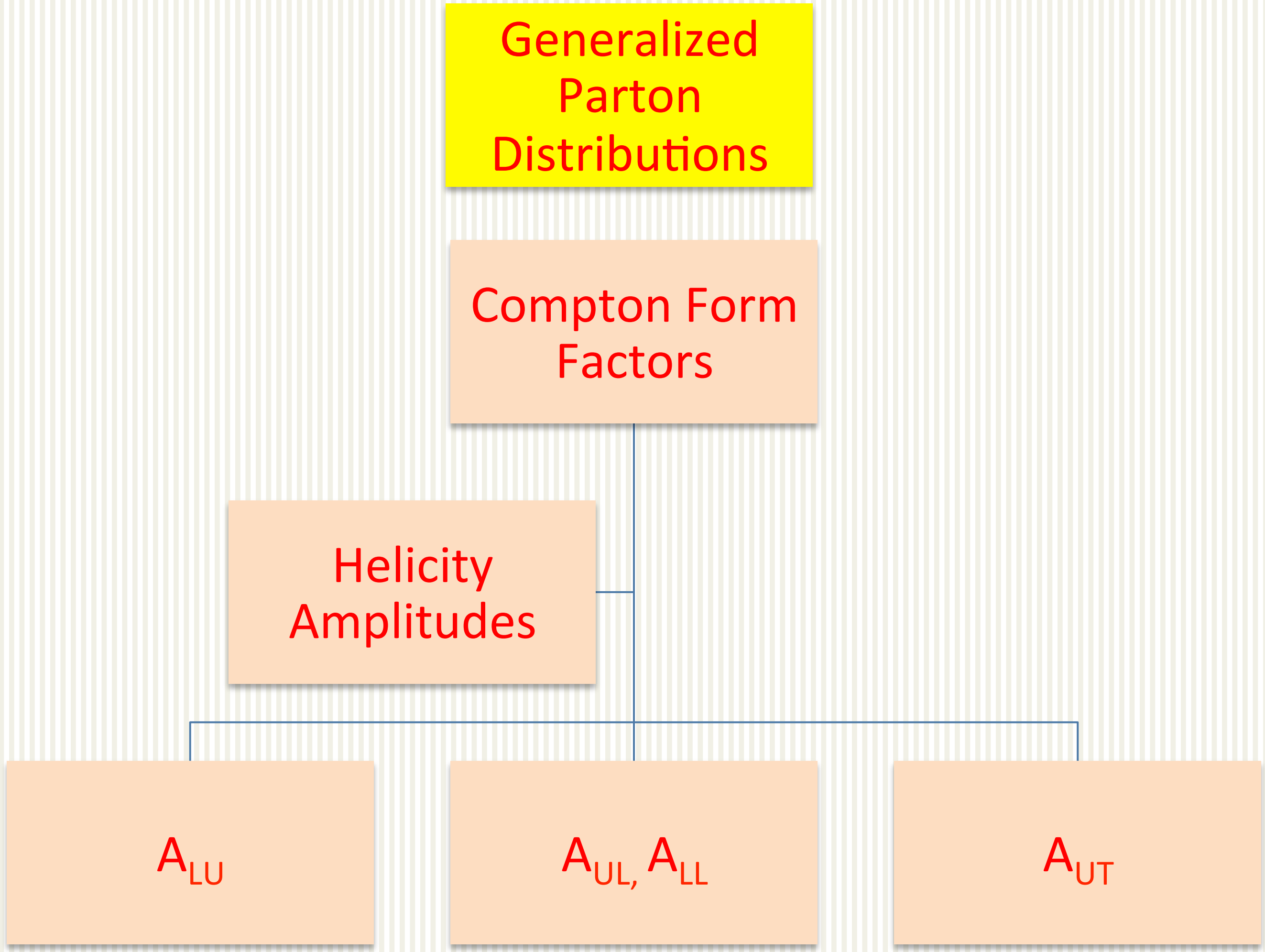
g

$$\mathcal{H}(\xi,t;Q^2) = \int dx \left[ \frac{1}{x-\xi-i\varepsilon} \mp \frac{1}{x+\xi-i\varepsilon} \right] H(x,\xi,t;Q^2)$$

$$\rightarrow \left( P.V. \int dx \frac{H(x,\xi,t;Q^2)}{x-\xi} + i\pi H(\xi,\xi,t;Q^2) \right) \mp (symm.term)$$

Compton Form Factors: both Real and Imaginary parts

## Global Fitting Procedure


Generalized Parton Distributions
Compton Form Factors
Helicity Amplitudes
$A_{LU}$
$A_{UL,}$ $A_{LL}$
$A_{UT}$

## 1st practical problem: Finding a parametric functional form given the enhanced complexity

**From DIS**

$$q(x,Q_o^2) = A_q\, x^{-\alpha_q}(1-x)^{\beta_q} F(x,c_q,d_q,\ldots)$$

**to DVCS, DVMP**

$$H_q(x,\xi,t;Q_o^2) = \qquad ?$$

GPD Parametrizations

- Double distributions (Radyushkin, Kroll, Goloskokov, VGG, Moutarde, Sabatie)
- LF Wave Function Overlap (Diehl,Kroll)
- Conformal Moments (Kumericki, Muller)
- Covariant Scattering Matrix (Brodsky, Llanes-Estrada, Sczcepaniak, Goldstein, Gonzalez Hernandez, Liuti)

State of the art in a nutshell

✓ **Double distributions**, Conformal Moments aim at getting $\xi$ dependence correctly (polynomiality and positivity can be satisfied a priori) but introduce $t$ dependence empirically

✓ **Partonic picture based** have to implement polynomiality order by order, but $t$ dependence is built in from the beginning

From DIS

$$q(x,Q_o^2) = A_q\, x^{-\alpha_q}(1-x)^{\beta_q} F(x,c_q,d_q,\ldots)$$

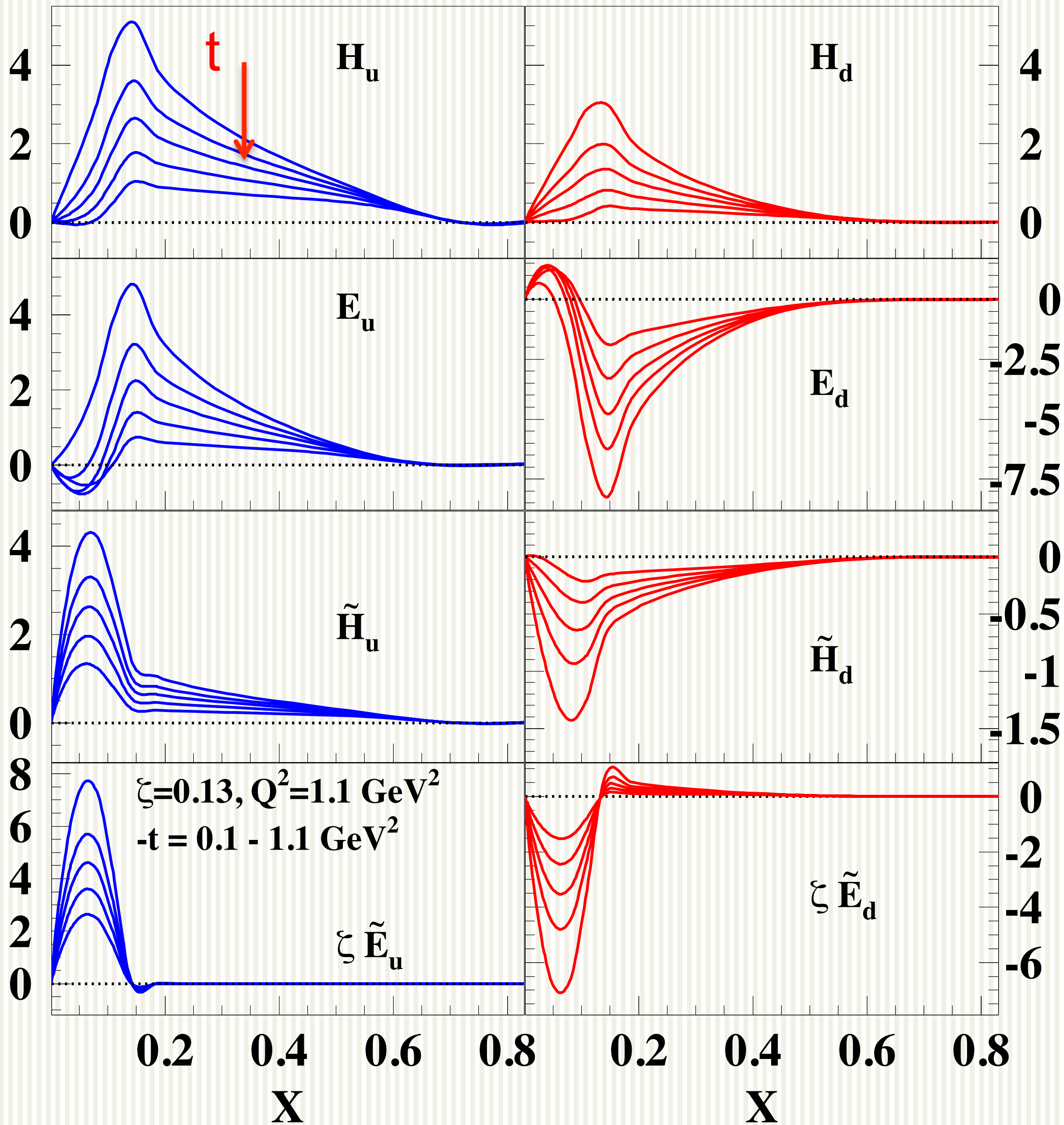
to DVCS, DVMP

## Covariant Scattering Matrix Framework

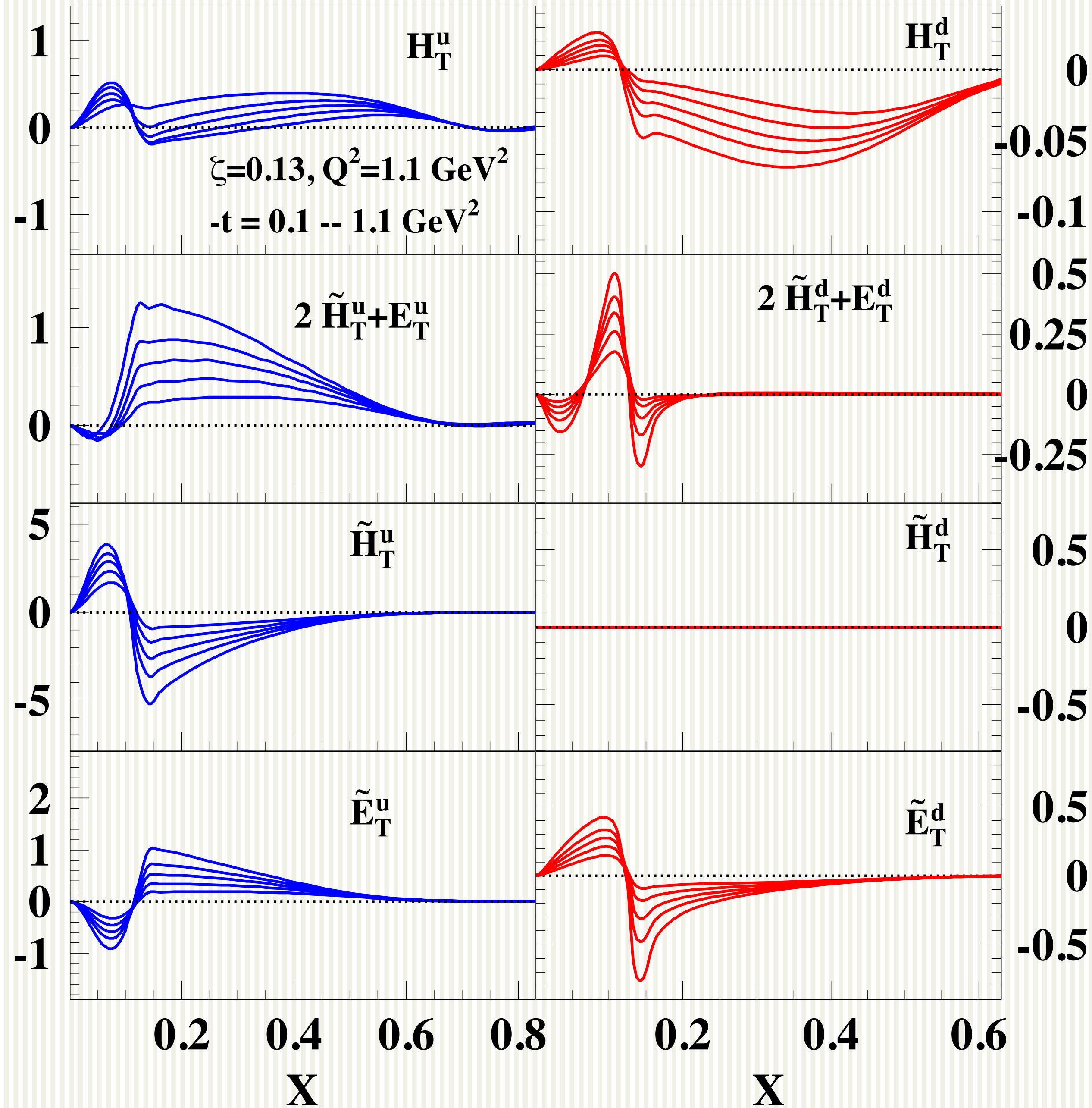
$$H_q(x,\xi,t;Q_o^2) = N_q\, x^{-\left[\alpha_q+\alpha'_q(1-x)^p t + f(\beta\xi)\right]} G(x,\xi,t;m_q,M_X^q,M_\Lambda^q)$$

Reggeized diquark model

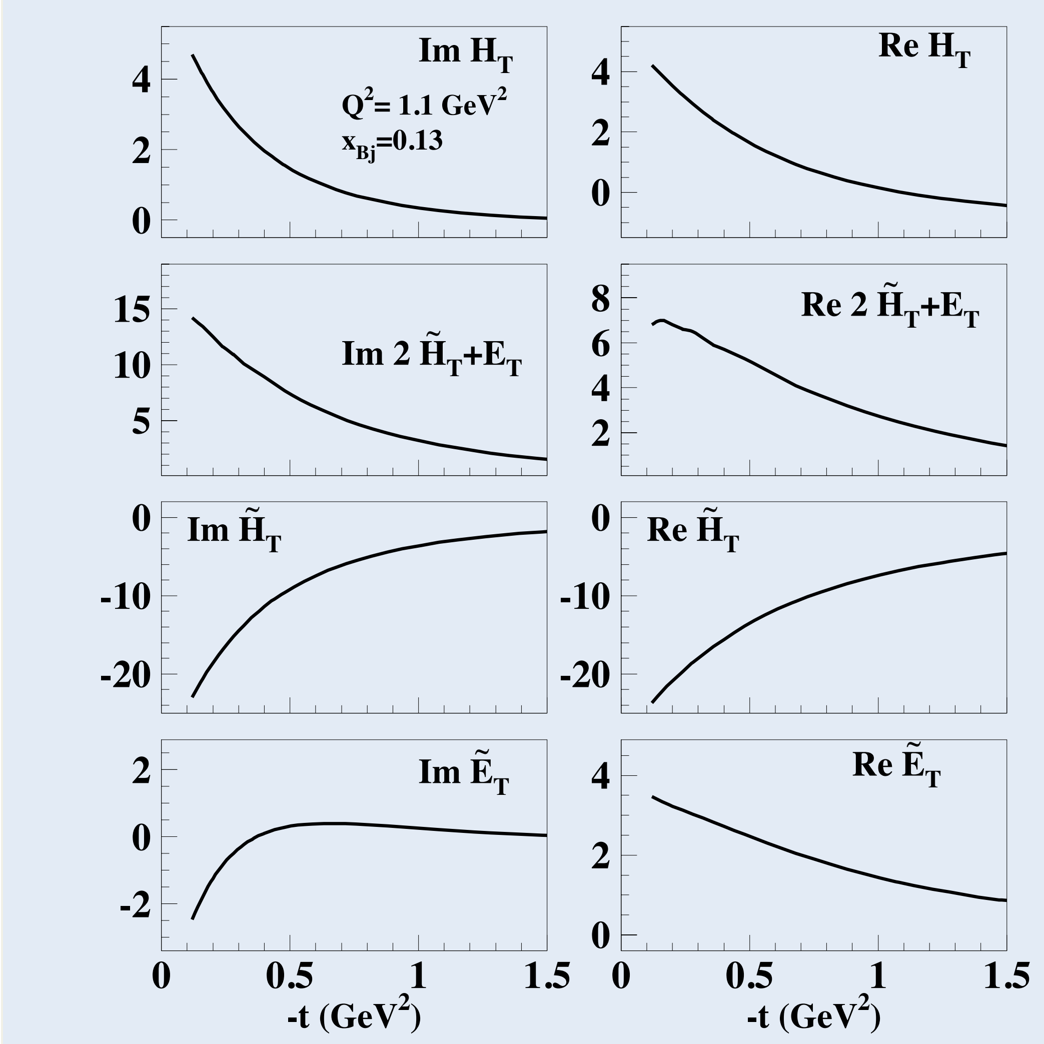
# GPDs (with error, Hessian, not shown)



# Chiral odd GPDs

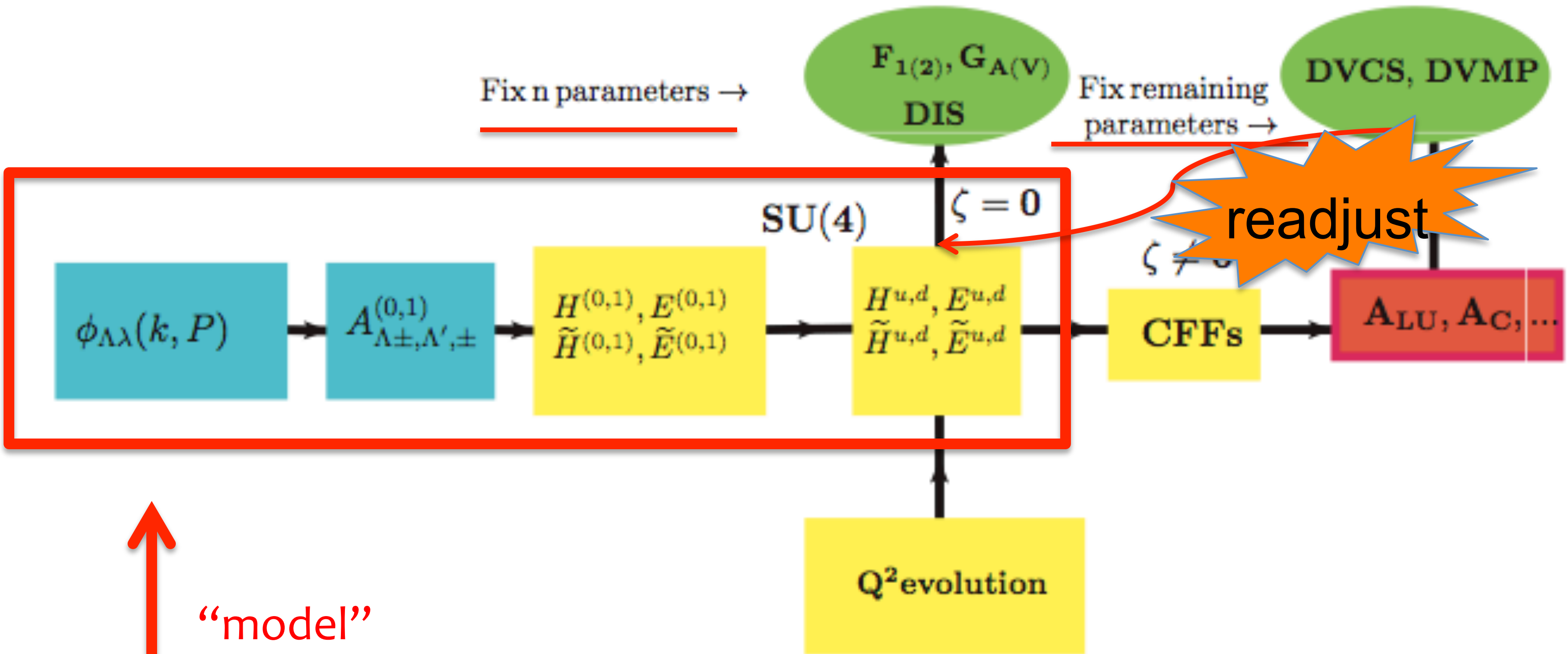


# Compton Form Factors

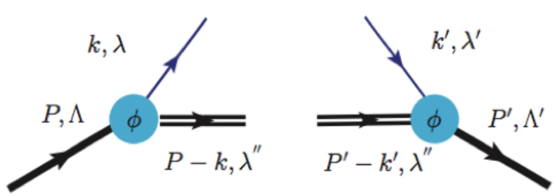


O. Gonzalez Hernandez et al., Phys. Rev. C88; arXiv:1206.1876

# Recursive Fit



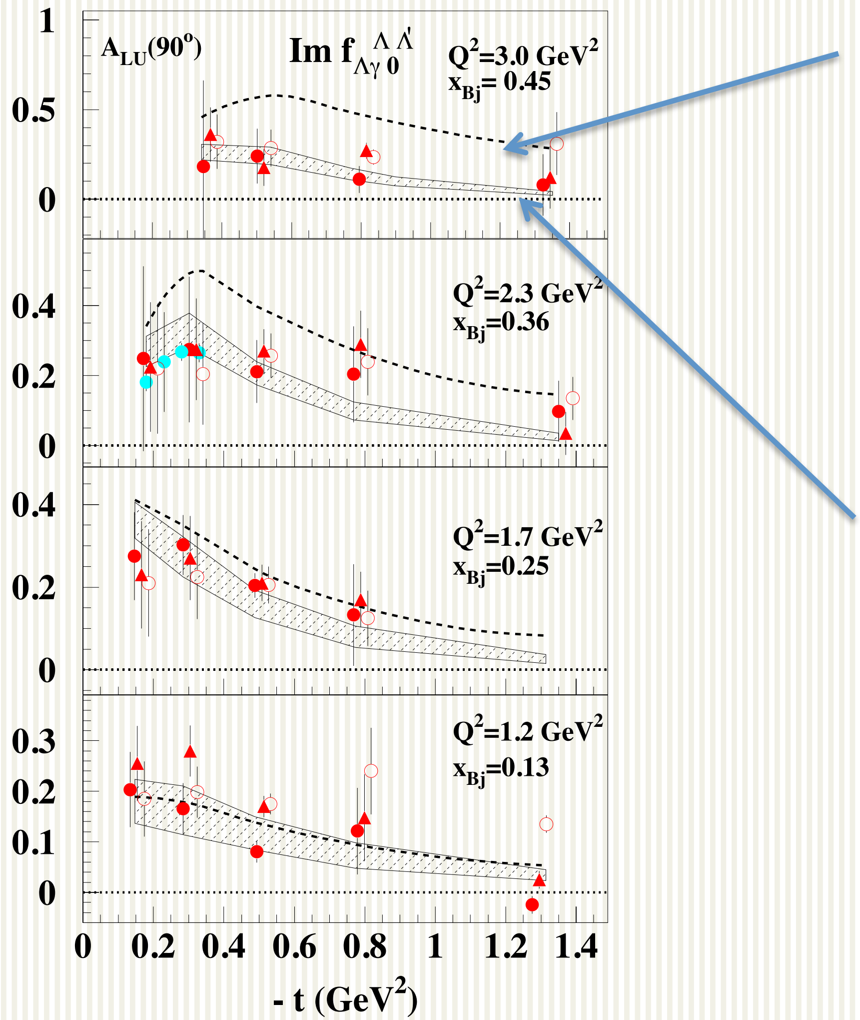

"model"




$$\phi_{\Lambda,\lambda}(k,P)=\Gamma(k)\frac{\bar{u}(k,\lambda)U(P,\Lambda)}{k^2-m^2}$$

$$\phi^*_{\Lambda'\lambda'}(k',P')=\Gamma(k')\frac{\overline{U}(P',\Lambda')u(k',\lambda')}{k'^2-m^2},$$

# Comparison with Hall B DVCS data (Girod et al.)




Before fitting additional parameters

After fit

## Conclusions

I discussed how OAM is directly obtained from longitudinal spin correlations, NOT transverse

I outlined a framework to extract 3D functions (in particular GPDs) from data.

In establishing benchmarks for an experimental extraction, attention must be paid to:

- ✓ helicity structure and twist expansion, phase structure
- ✓ kinematical dependence in both models and general formalism, in particular t-dependence and interference with the twist expansion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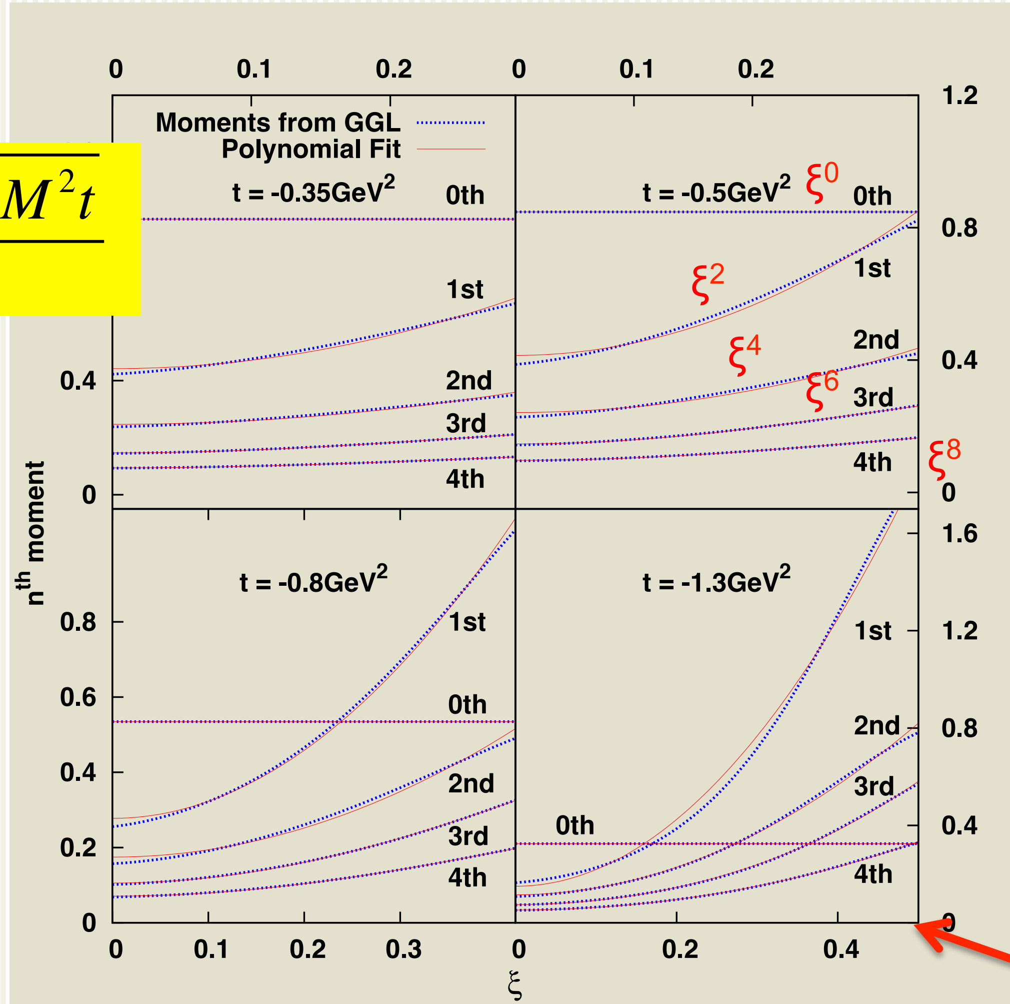
Backup

# Polynomiality in the reggeized quark-diquark approach ➔arXiv:1012.3776

$$x_{Bj}^{MAX} = \frac{-t + \sqrt{t^2 + 4M^2 t}}{2M^2}$$




$x_{Bj}=0.7$

# Compare to constraint for PDFs

The extent to which baryon number is conserved by PDF parametrizations