Separating Structure functions in SIDIS

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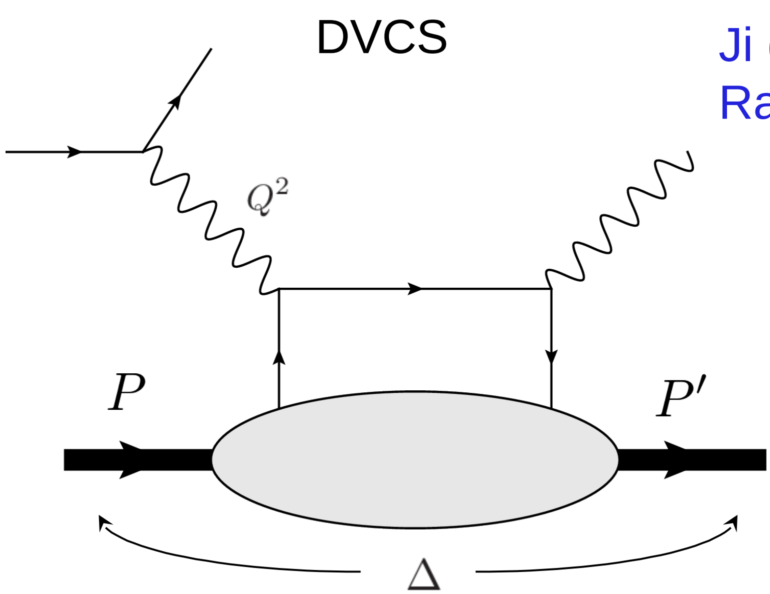
GPDs



Ji (1997)
Radyushkin (1997)

Q^2 ensures hard scale, pointlike interaction

$\Delta = P' - P$ momentum transfer can be varied independently

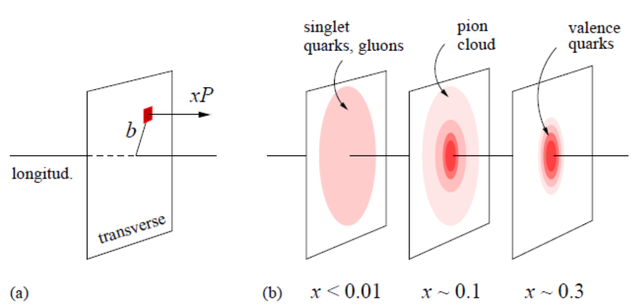
Connection to 3D structure

Burkardt (2000)
Burkardt (2003)

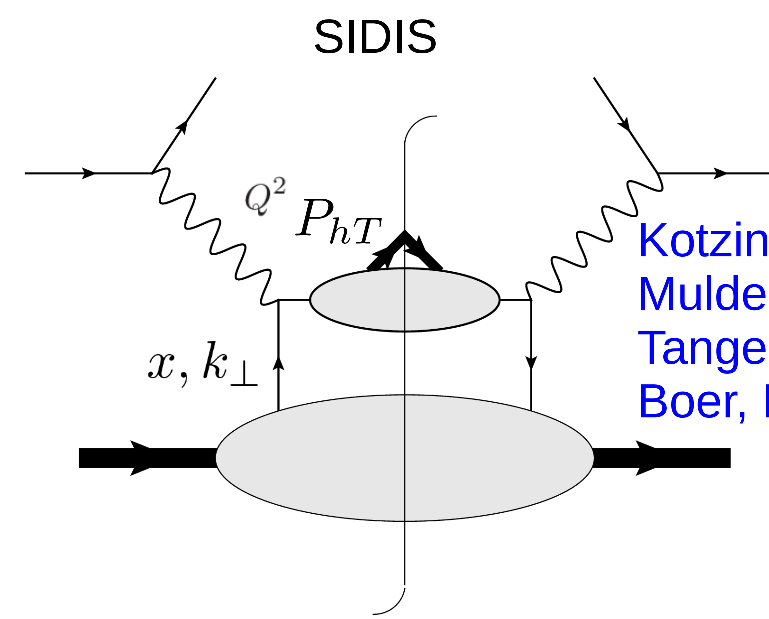
$$\rho(x, \vec{r}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} H_q(x, \xi = 0, t = -\vec{\Delta}_\perp^2)$$

Drell-Yan frame $\Delta^+ = 0$

Weiss (2009)



TMDs



Kotzinian (1995),
Mulders,
Tangerman (1995),
Boer, Mulders (1998)

Q^2 ensures hard scale, pointlike interaction

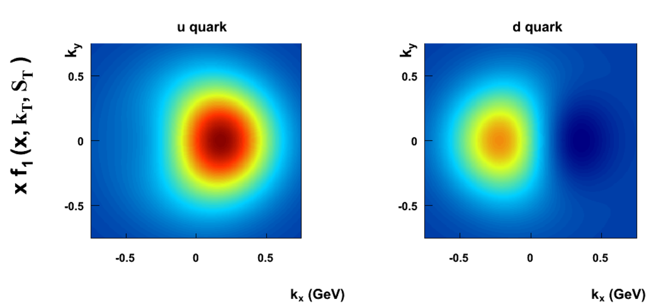
P_{hT} final hadron transverse momentum can be varied independently

Connection to 3D structure

Ji, Ma, Yuan (2004)
Collins (2011)

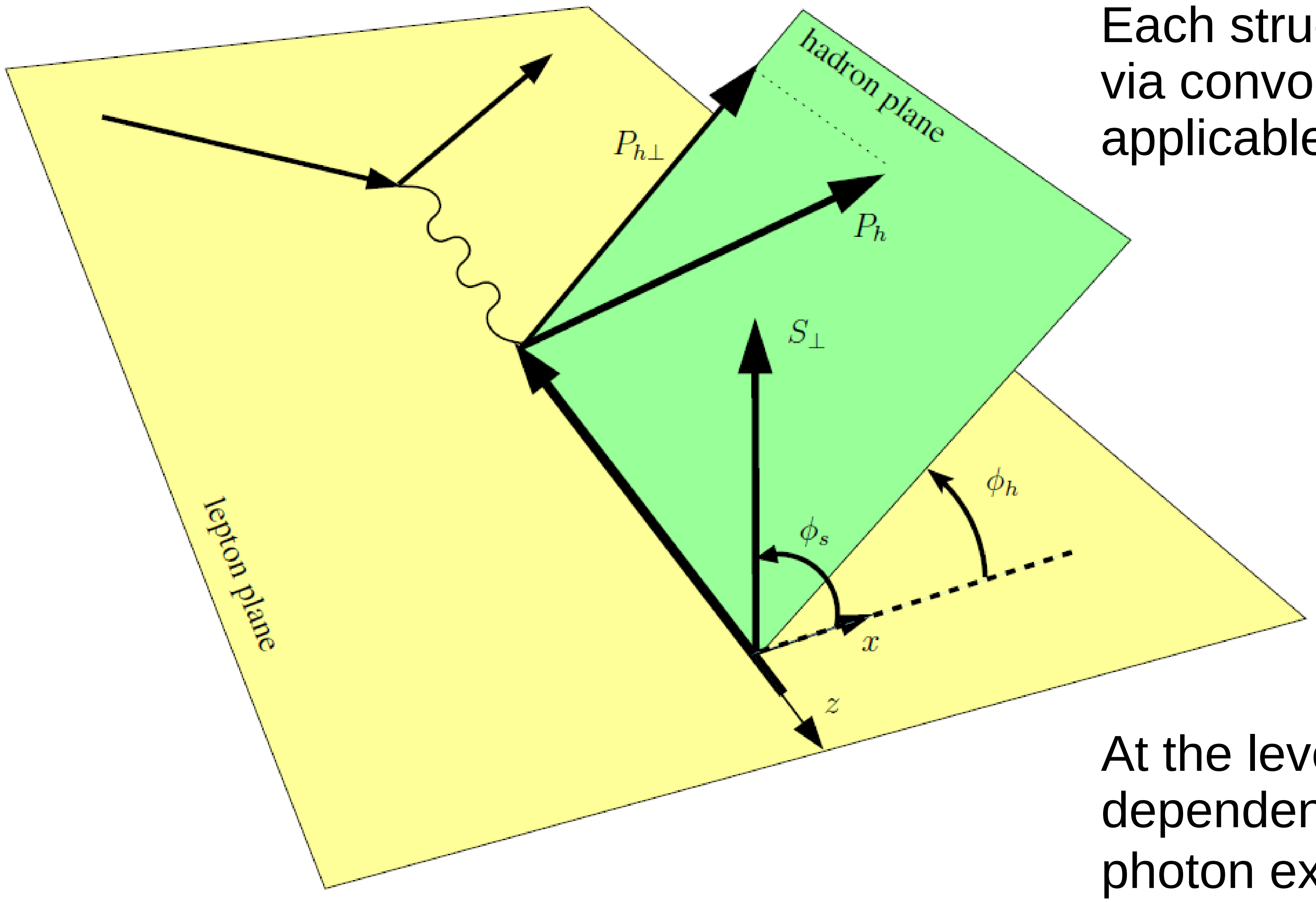
$$\tilde{f}(x, \vec{b}_T) = \int d^2k_\perp e^{i\vec{b}_T \cdot \vec{k}_\perp} f(x, \vec{k}_\perp)$$

$\vec{b}_T$ is the transverse separation of parton fields in configuration space



AP (2012)

$$\ell P \to \ell' \pi X$$



One can rewrite the cross-section in terms of **18** structure functions

Each structure function encodes parton dynamics via convolutions of TMDs when factorization is applicable

Kotzinian (1995),
Mulders, Tangerman (1995),
Boer, Mulders (1998)
Bacchetta et al (2007)

At the level of structure functions **no model** dependence – the only assumption is one photon exchange

Structure functions are convolutions of unobserved partonic momenta:

$$F \sim \int d^2\vec{k}_\perp d^2\vec{p}_\perp \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{hT})\omega f(x, \vec{k}_\perp) D(z, \vec{p}_\perp)$$

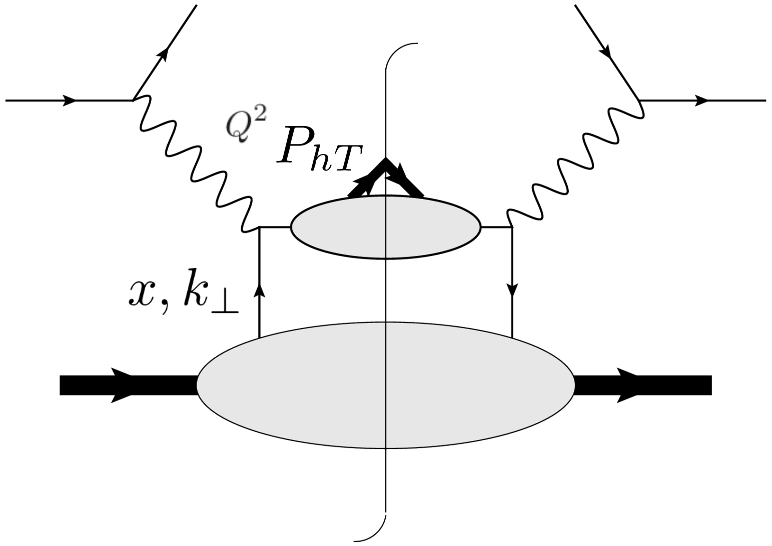
Related to the observed hadron momentum

Convolution notation:

$$F \sim f \otimes D \equiv \sum_q e_q^2 \; f_q \otimes D_q$$

The TMD approximation is valid in the region

$$P_{hT}/z \ll Q$$



TMD distributions

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

N \ q	U	L	T
U	D_1		$H_1^{\perp}$
L		G_{1L}	$H_{1L}^{\perp}$
T	$H_{1T}^{\perp}$	G_{1T}	H_1 $H_{1T}^{\perp}$

8 functions describing fragmentation of a quark into spin ½ hadron

Mulders, Tangerman (1995), Meissner, Metz, Pitonyak (2010)

TMD Fragmentation Functions

N \ q	U	L	T
U	D_1		$H_1^{\perp}$
L		G_{1L}	$H_{1L}^{\perp}$
T	$H_{1T}^{\perp}$	G_{1T}	H_1 $H_{1T}^{\perp}$

8 functions describing fragmentation of a quark into spin ½ hadron

Pion production

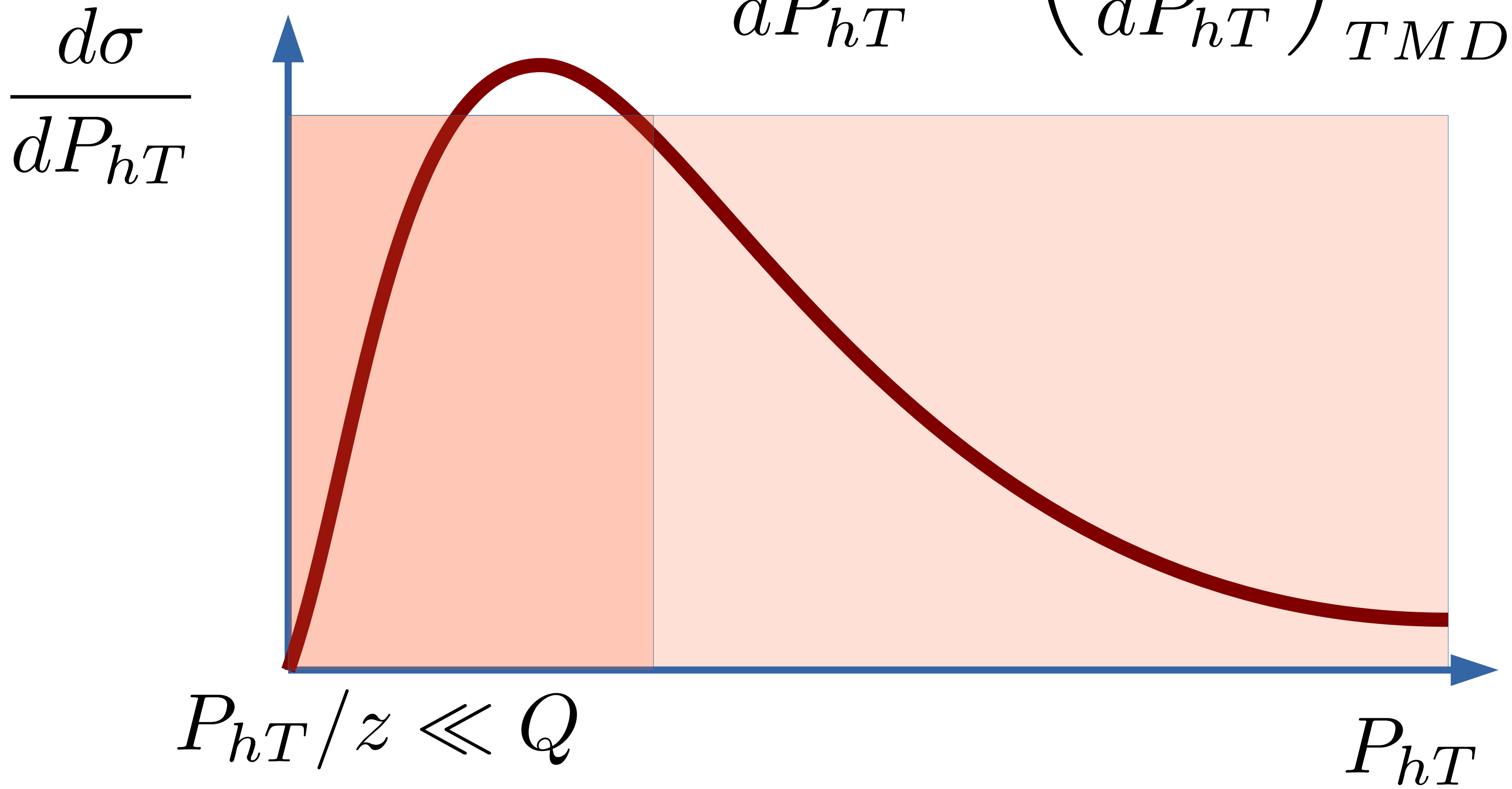
Mulders, Tangerman (1995), Meissner, Metz, Pitonyak (2010)

TMD

TMD factorization has a validity region $P_{hT}/z \ll Q$
(two scale problem)

In order to describe cross section in a wide region of transverse momentum one needs to add a Y term

$$\frac{d\sigma}{dP_{hT}} = \left(\frac{d\sigma}{dP_{hT}}\right)_{TMD} + Y$$



Improved approach that aims to describe low-Q region:

Collins, Gamberg, AP, Rogers, Sato, Wang arXiv:1605.00671

Cross section of SIDIS

Cross-section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\cdot$$

$$\Big(1+\cos\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UU}^{\cos\phi_h}+\cos 2\phi_h\,\varepsilon\,A_{UU}^{\cos 2\phi_h}$$
$$+\lambda\,\sin\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LU}^{\sin\phi_h}$$
$$+S_L\left[\sin\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UL}^{\sin\phi_h}+\sin 2\phi_h\,\varepsilon\,A_{UL}^{\sin 2\phi_h}\right]$$
$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\,A_{LL}+\cos\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LL}^{\cos\phi_h}\right]$$
$$+S_T\,\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)}$$
$$+S_T\,\sin(\phi_h+\phi_S)\,\varepsilon\,A_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+S_T\,\sin(3\phi_h-\phi_S)\,\varepsilon\,A_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+S_T\,\sin\phi_S\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin\phi_S}$$
$$+S_T\,\sin(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin(2\phi_h-\phi_S)}$$
$$+S_T\lambda\,\cos(\phi_h-\phi_S)\,\sqrt{1-\varepsilon^2}\,A_{LT}^{\cos(\phi_h-\phi_S)}$$
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virtual photon polarization ($F_{UU,T}$)

λ beam polarization

Target polarization

S_L longitudinal

S_T transverse

Longitudinal to transverse photon flux ratio

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2}$$

$$\gamma = \frac{2Mx}{Q}$$

Cross-section

Bacchetta et al (2007)

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\cdot$$

$$\Big(1 + \cos\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UU}^{\cos\phi_h} + \cos 2\phi_h\ \varepsilon\ A_{UU}^{\cos 2\phi_h}$$

$$+\lambda\ \sin\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LU}^{\sin\phi_h}$$

$$+S_L\left[\sin\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UL}^{\sin\phi_h} + \sin 2\phi_h\ \varepsilon\ A_{UL}^{\sin 2\phi_h}\right]$$

$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\ A_{LL} + \cos\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LL}^{\cos\phi_h}\right]$$

$$+S_T\ \sin(\phi_h-\phi_S)\ A_{UT}^{\sin(\phi_h-\phi_S)}$$

$$+S_T\ \sin(\phi_h+\phi_S)\ \varepsilon\ A_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+S_T\ \sin(3\phi_h-\phi_S)\ \varepsilon\ A_{UT}^{\sin(3\phi_h-\phi_S)}$$

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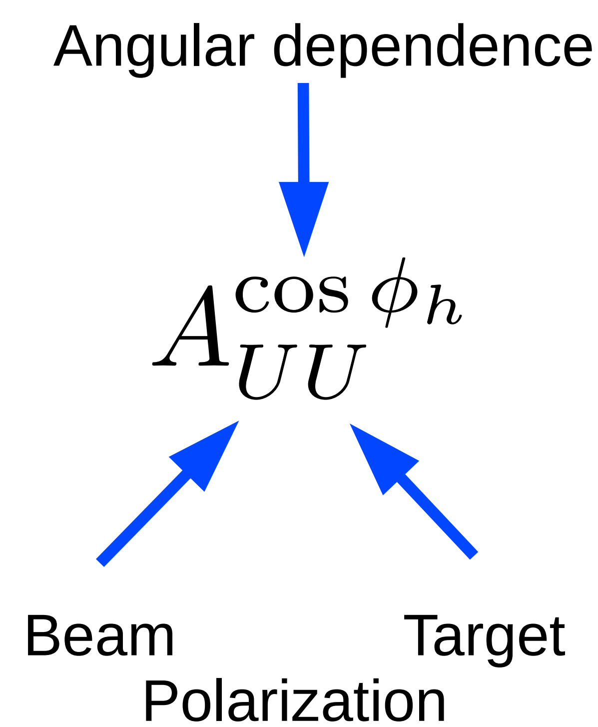
Asymmetry



Bacchetta et al (2007)

Cross-section

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$$+S_T\lambda\ \cos(\phi_h-\phi_S)\ \sqrt{1-\varepsilon^2}\ A_{LT}^{\cos(\phi_h-\phi_S)}$$

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Single Spin Asymmetry

Cross-section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\cdot$$

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Single Spin Asymmetry

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Unpolarized modulations

Single Spin Asymmetry

Double Spin Asymmetry

Cross-section

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Unpolarized cross section

Unpolarized modulations

Single Spin Asymmetry

Double Spin Asymmetry

Partonic interpretation of structure functions in SIDIS

✓ – twist 2 ✗ – twist 3

Cross-section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

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Single Spin Asymmetry

$$A_{LU}^{\sin(\phi_h)} \sim \frac{M}{Q}(xe\otimes H_1^\perp+\ldots)$$ ✗

$$A_{UL}^{\sin(\phi_h)} \sim \frac{M}{Q}(h_{1L}^\perp\otimes\tilde{H}+\ldots)$$ ✗

$$A_{UL}^{\sin(2\phi_h)} \sim h_{1L}^\perp\otimes H_1^\perp$$ ✓

$$A_{UT}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^\perp\otimes D_1$$ ✓

$$A_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1\otimes H_1^\perp$$ ✓

$$A_{UT}^{\sin(3\phi_h-\phi_S)} \sim h_{1T}^\perp\otimes H_1^\perp$$ ✓

$$A_{UT}^{\sin(\phi_h)} \sim \frac{M}{Q}(xf_T\otimes D_1+\ldots)$$ ✗

$$A_{UT}^{\sin(2\phi_h-\phi_S)} \sim \frac{M}{Q}(xf_T^\perp\otimes D_1+\ldots)$$ ✗

Bacchetta et al (2007)   ✓ – twist 2   ✗ – twist 3

Cross-section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$
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$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\,A_{LL}+\cos\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LL}^{\cos\phi_h}\right]$$
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Double Spin Asymmetry

$$A_{LL}\sim g_1\otimes D_1$$ ✓

$$A_{LL}^{\cos\phi_h}\sim\frac{M}{Q}(g_1\otimes\tilde{D}_1+\ldots)$$ ✗

$$A_{LT}^{\cos(\phi_h-\phi_S)}\sim g_{1T}\otimes D_1$$ ✓

$$A_{LT}^{\cos\phi_h}\sim\frac{M}{Q}(g_{1T}\otimes\tilde{D}_1+\ldots)$$ ✗

$$A_{LT}^{\cos(2\phi_h-\phi_S)}\sim\frac{M}{Q}(g_{1T}\otimes\tilde{D}_1+\ldots)$$ ✗

✓ – twist 2 ✗ – twist 3

Cross-section

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$$+S_L\left[\sin\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UL}^{\sin\phi_h} + \sin 2\phi_h\ \varepsilon\ A_{UL}^{\sin 2\phi_h}\right]$$

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$$+S_T\ \sin(3\phi_h-\phi_S)\ \varepsilon\ A_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+S_T\ \sin\phi_S\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin\phi_S}$$

$$+S_T\ \sin(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin(2\phi_h-\phi_S)}$$

$$+S_T\lambda\ \cos(\phi_h-\phi_S)\ \sqrt{1-\varepsilon^2}\ A_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+S_T\lambda\ \cos\phi_S\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos\phi_S}$$

$$+S_T\lambda\ \cos(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

Unpolarized modulations
Single Spin Asymmetry

$$A_{UU}^{\cos\phi_h} \sim \frac{M}{Q}(f_1\otimes\tilde{D}_1+\ldots)$$ ✗

$$A_{UU}^{\cos 2\phi_h} \sim h_1^\perp \otimes H_1^\perp$$ ✓

Bacchetta et al (2007) ✓ – twist 2 ✗ – twist 3

Cross-section

Unpolarized cross section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

$$\Big(1+\cos\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UU}^{\cos\phi_h}+\cos 2\phi_h\ \varepsilon\ A_{UU}^{\cos 2\phi_h}$$
$$+\lambda\ \sin\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LU}^{\sin\phi_h}$$
$$+S_L\left[\sin\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UL}^{\sin\phi_h}+\sin 2\phi_h\ \varepsilon\ A_{UL}^{\sin 2\phi_h}\right]$$
$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\ A_{LL}+\cos\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LL}^{\cos\phi_h}\right]$$
$$+S_T\ \sin(\phi_h-\phi_S)\ A_{UT}^{\sin(\phi_h-\phi_S)}$$
$$+S_T\ \sin(\phi_h+\phi_S)\ \varepsilon\ A_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+S_T\ \sin(3\phi_h-\phi_S)\ \varepsilon\ A_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+S_T\ \sin\phi_S\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin\phi_S}$$
$$+S_T\ \sin(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin(2\phi_h-\phi_S)}$$
$$+S_T\lambda\ \cos(\phi_h-\phi_S)\ \sqrt{1-\varepsilon^2}\ A_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+S_T\lambda\ \cos\phi_S\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos\phi_S}$$
$$+S_T\lambda\ \cos(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

$$F_{UU,T}\sim f_1\otimes D_1$$ ✓

PennState Berks

What do we know about structure functions in SIDIS?

Cross-section

Unpolarized cross section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

$$\Big(1+\cos\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UU}^{\cos\phi_h}+\cos 2\phi_h\,\varepsilon\,A_{UU}^{\cos 2\phi_h}$$

$$+\lambda\,\sin\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LU}^{\sin\phi_h}$$

$$+S_L\left[\sin\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UL}^{\sin\phi_h}+\sin 2\phi_h\,\varepsilon\,A_{UL}^{\sin 2\phi_h}\right]$$

$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\,A_{LL}+\cos\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LL}^{\cos\phi_h}\right]$$

$$+S_T\,\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)}$$

$$+S_T\,\sin(\phi_h+\phi_S)\,\varepsilon\,A_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+S_T\,\sin(3\phi_h-\phi_S)\,\varepsilon\,A_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+S_T\,\sin\phi_S\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin\phi_S}$$

$$+S_T\,\sin(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin(2\phi_h-\phi_S)}$$

$$+S_T\lambda\,\cos(\phi_h-\phi_S)\,\sqrt{1-\varepsilon^2}\,A_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+S_T\lambda\,\cos\phi_S\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos\phi_S}$$

$$+S_T\lambda\,\cos(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

$$F_{UU,T} \sim f_1 \otimes D_1$$ ✓

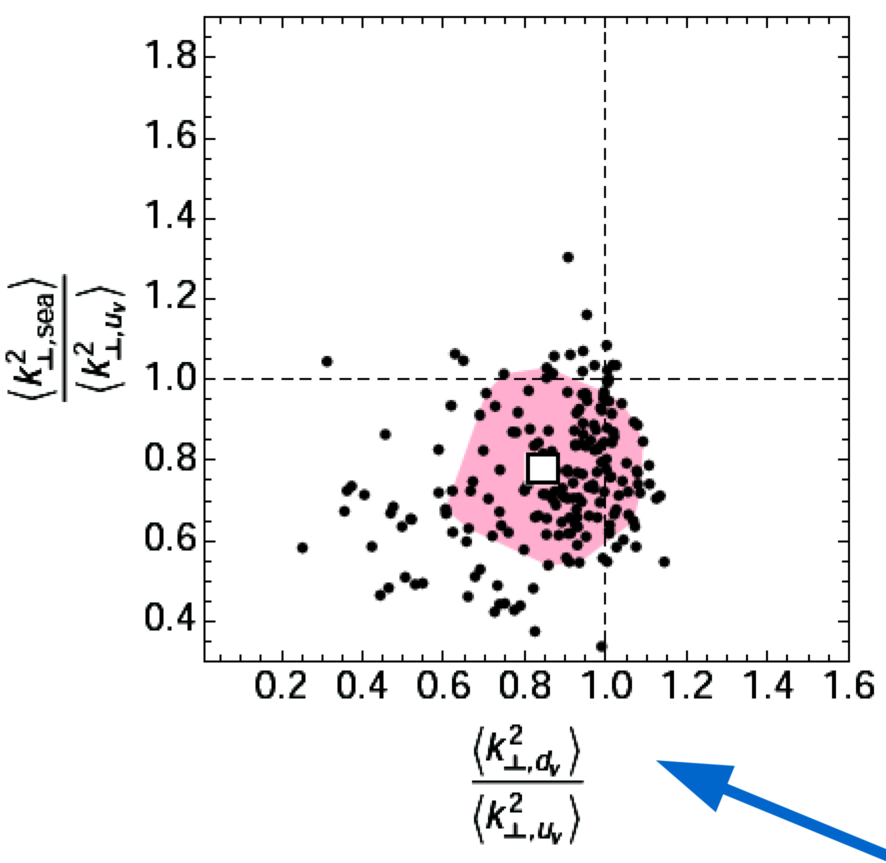
$F_{UU,T} \sim f_1 \otimes D_1$

Signori, Bacchetta, Radici, Schnell 2014
Anselmino, Boglione, Gonzalez, Melis, AP 2014

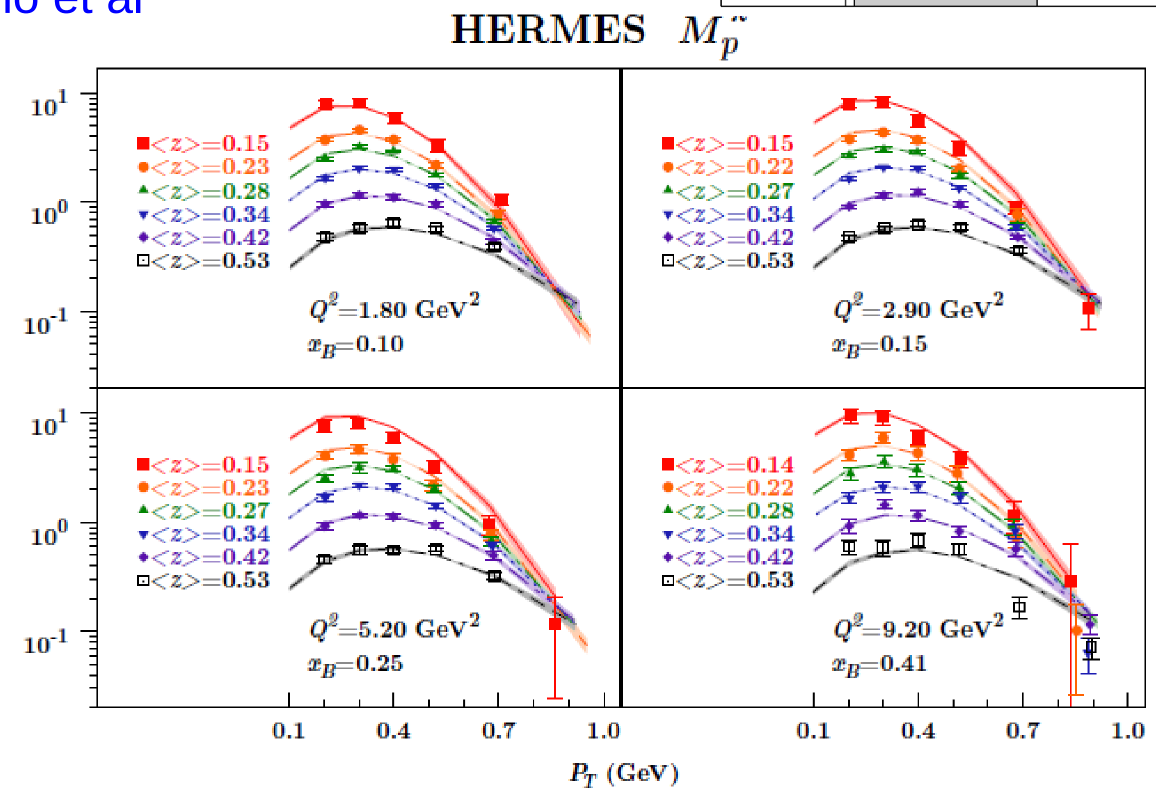
N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Flavor dependence of the widths of unpolarized valence distributions and sea distributions

Signori et al



Anselmino et al



Flavor dependence seen in the data or not?

Perhaps studies that include TMD evolution will give a more definite answer.

✅ – twist 2 ❌ – twist 3

Cross-section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

$$\Big(1+\cos\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UU}^{\cos\phi_h}+\cos 2\phi_h\ \varepsilon\ A_{UU}^{\cos 2\phi_h}$$
$$+\lambda\ \sin\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LU}^{\sin\phi_h}$$
$$+S_L\left[\sin\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UL}^{\sin\phi_h}+\sin 2\phi_h\ \varepsilon\ A_{UL}^{\sin 2\phi_h}\right]$$
$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\ A_{LL}+\cos\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LL}^{\cos\phi_h}\right]$$
$$+S_T\ \sin(\phi_h-\phi_S)\ A_{UT}^{\sin(\phi_h-\phi_S)}$$
$$+S_T\ \sin(\phi_h+\phi_S)\ \varepsilon\ A_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+S_T\ \sin(3\phi_h-\phi_S)\ \varepsilon\ A_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+S_T\ \sin\phi_S\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin\phi_S}$$
$$+S_T\ \sin(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin(2\phi_h-\phi_S)}$$
$$+S_T\lambda\ \cos(\phi_h-\phi_S)\ \sqrt{1-\varepsilon^2}\ A_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+S_T\lambda\ \cos\phi_S\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos\phi_S}$$
$$+S_T\lambda\ \cos(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

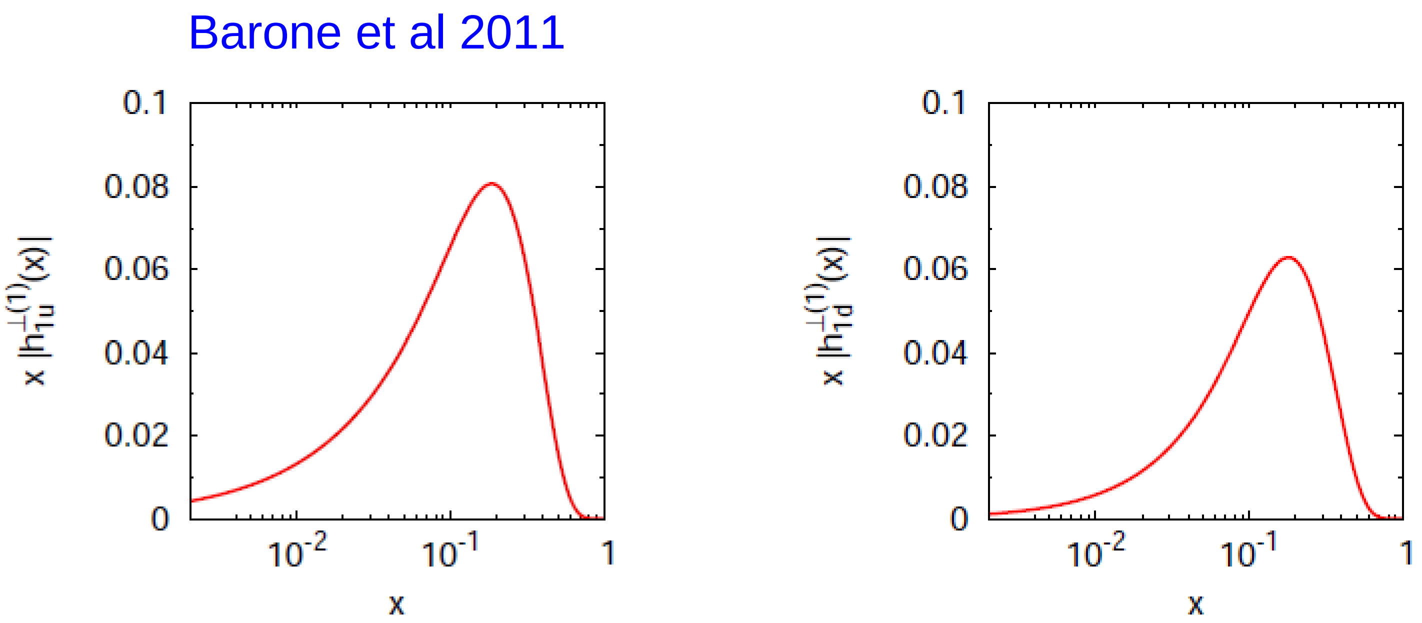
Unpolarized modulations
Single Spin Asymmetry

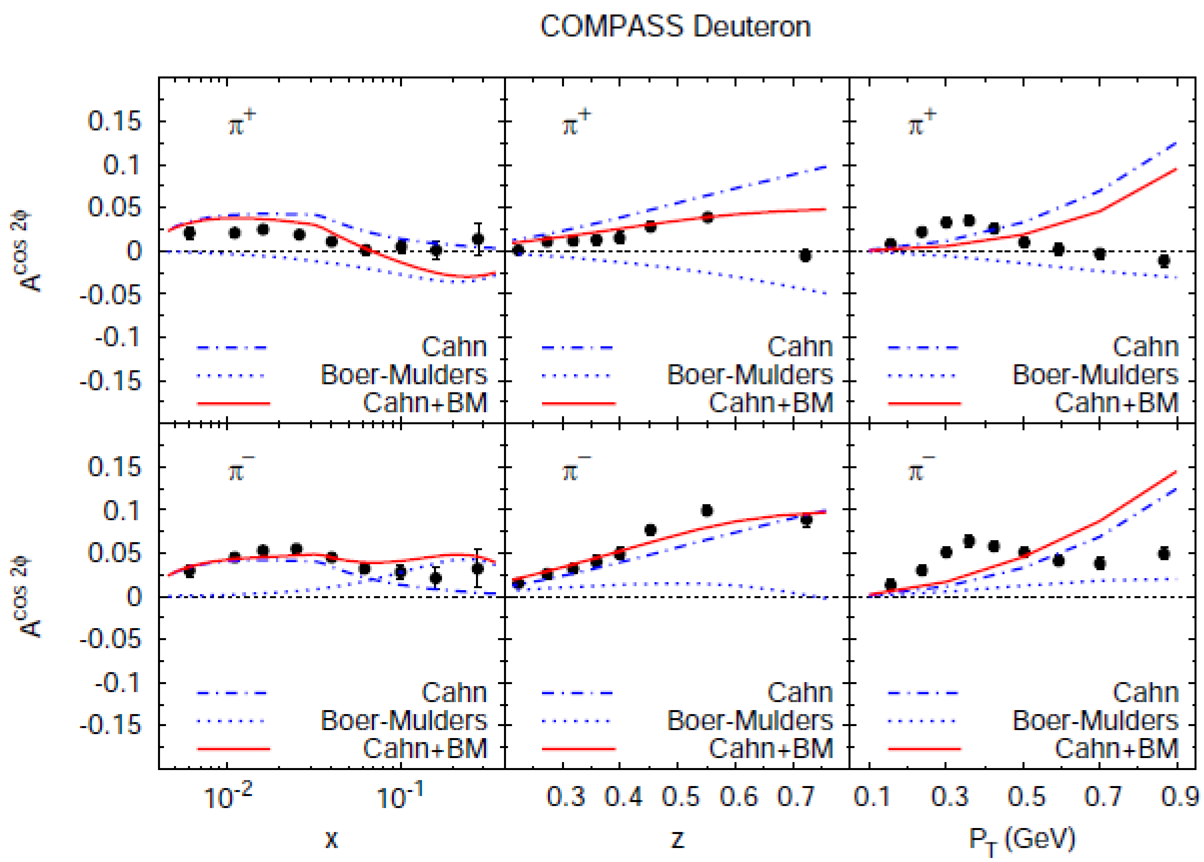
$$A_{UU}^{\cos\phi_h}\sim\frac{M}{Q}(f_1\otimes\tilde{D}_1+\ldots)$$ ❌

$$A_{UU}^{\cos 2\phi_h}\sim h_1^\perp\otimes H_1^\perp$$ ✅

$A_{UU}^{\cos 2\phi_h} \sim h_1^\perp \otimes H_1^\perp$

q \ N	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$





Boer-Mulders function

Negative u-quark
Negative d-quark

Compatible with models

Naive T-odd, should change sign in DY with respect to SIDIS

✓ – twist 2 ✗ – twist 3

Cross-section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

$$\Big(1+\cos\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UU}^{\cos\phi_h}+\cos 2\phi_h\,\varepsilon\,A_{UU}^{\cos 2\phi_h}$$

$$+\lambda\,\sin\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LU}^{\sin\phi_h}$$

$$+S_L\left[\sin\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UL}^{\sin\phi_h}+\sin 2\phi_h\,\varepsilon\,A_{UL}^{\sin 2\phi_h}\right]$$

$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\,A_{LL}+\cos\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LL}^{\cos\phi_h}\right]$$

$$+S_T\,\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)}$$

$$+S_T\,\sin(\phi_h+\phi_S)\,\varepsilon\,A_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+S_T\,\sin(3\phi_h-\phi_S)\,\varepsilon\,A_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+S_T\,\sin\phi_S\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin\phi_S}$$

$$+S_T\,\sin(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin(2\phi_h-\phi_S)}$$

$$+S_T\lambda\,\cos(\phi_h-\phi_S)\,\sqrt{1-\varepsilon^2}\,A_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+S_T\lambda\,\cos\phi_S\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos\phi_S}$$

$$+S_T\lambda\,\cos(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

Single Spin Asymmetry

$$A_{UL}^{\sin(2\phi_h)} \sim h_{1L}^{\perp}\otimes H_1^{\perp}$$ ✓

$$A_{UT}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^{\perp}\otimes D_1$$ ✓

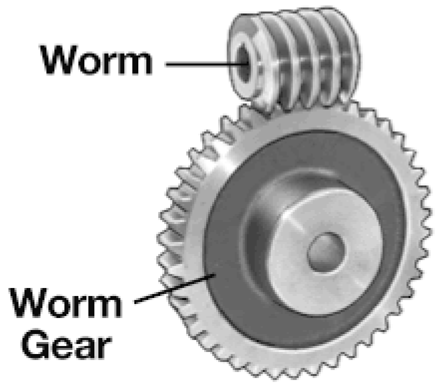
$$A_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1\otimes H_1^{\perp}$$ ✓

$$A_{UT}^{\sin(3\phi_h-\phi_S)} \sim h_{1T}^{\perp}\otimes H_1^{\perp}$$ ✓

Single Spin Asymmetries at twist-2

$$A_{UL}^{\sin(2\phi_h)} \sim h_{1L}^{\perp} \otimes H_1^{\perp}$$

Worm-gear



$$A_{UT}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^{\perp} \otimes D_1$$

Sivers asymmetry

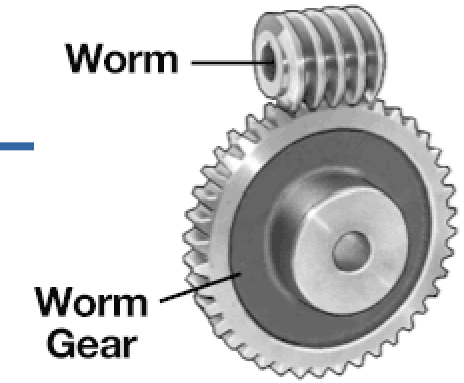
$$A_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1 \otimes H_1^{\perp}$$

Collins asymmetry

$$A_{UT}^{\sin(3\phi_h-\phi_S)} \sim h_{1T}^{\perp} \otimes H_1^{\perp}$$

Pretzelosity

$A_{UL}^{\sin(2\phi_h)} \sim h_{1L}^{\perp} \otimes H_1^{\perp}$ Worm-gear



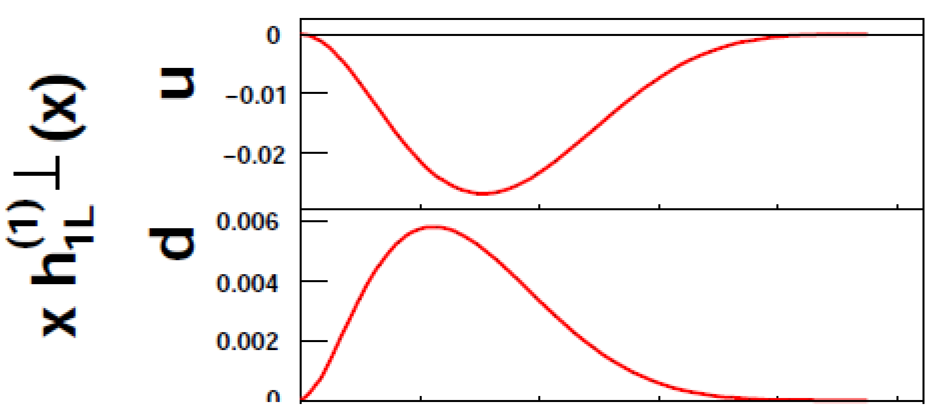
Teckentrup, Metz, Schweitzer 2009
Avakian, Bastami, Efremov, Kotzinian, Parsamyan, AP, Schlegel, Schweitzer in preparation

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

"Wandzura-Wilczek" approximation

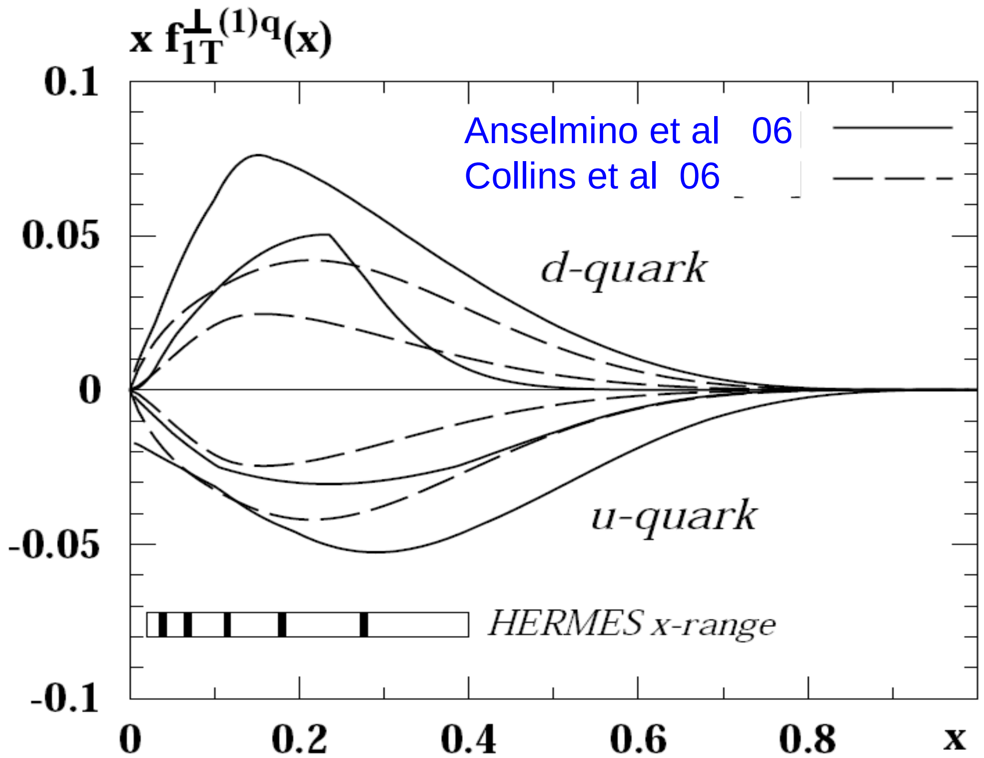
$$h_{1L}^{\perp(1)a}(x) \overset{\text{WW-type}}{\approx} -x^2 \int_x^1 \frac{dy}{y^2}\, h_1^a(y)$$

Connection to transversity



Talk by Bastami

$A_{UT}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^{\perp} \otimes D_1$ Sivers asymmetry



Negative u-quark
Positive d-quark

Big uncertainty

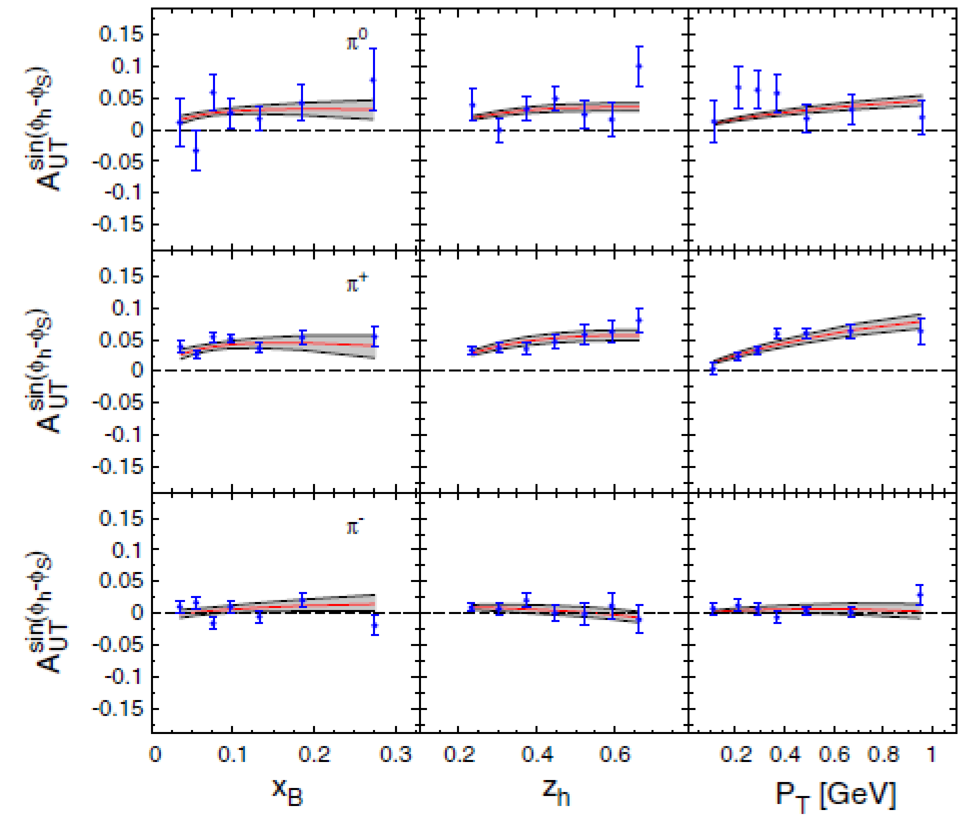
Precise data are needed

Sivers function is related to OAM

Vogelsang, Yuan 2005
Anselmino et al 2006, 2009,2013
Collins et al 2006
Bacchetta, Radici 2011
Echevaria et al 2014
Sun, Yuan 2014

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

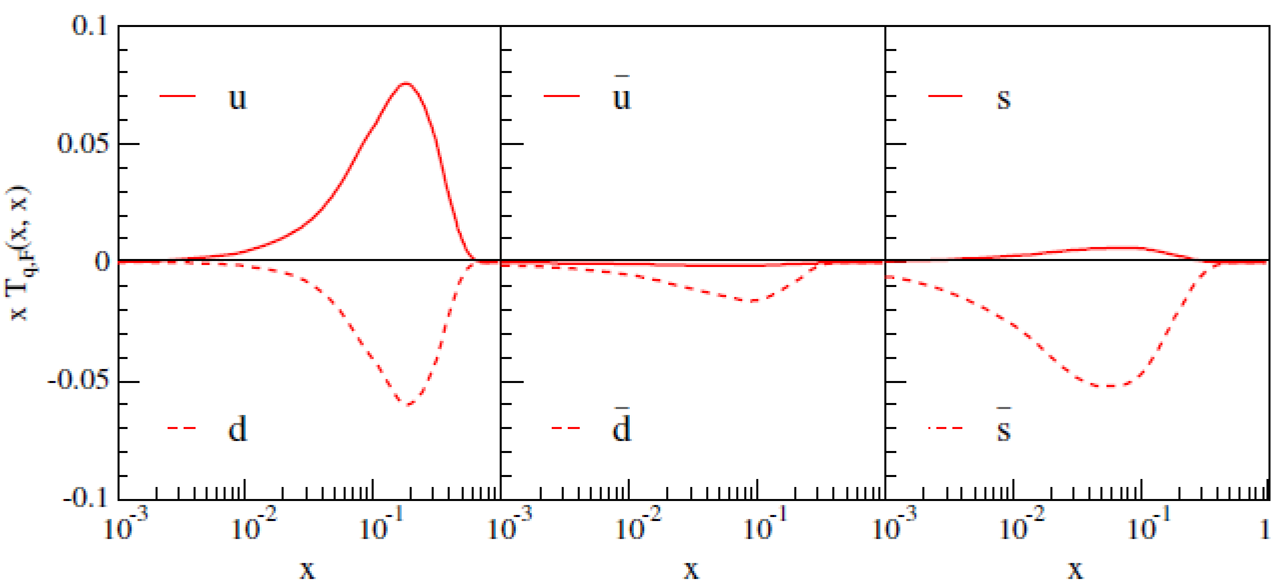
Anselmino et al 2012



TMD evolution included

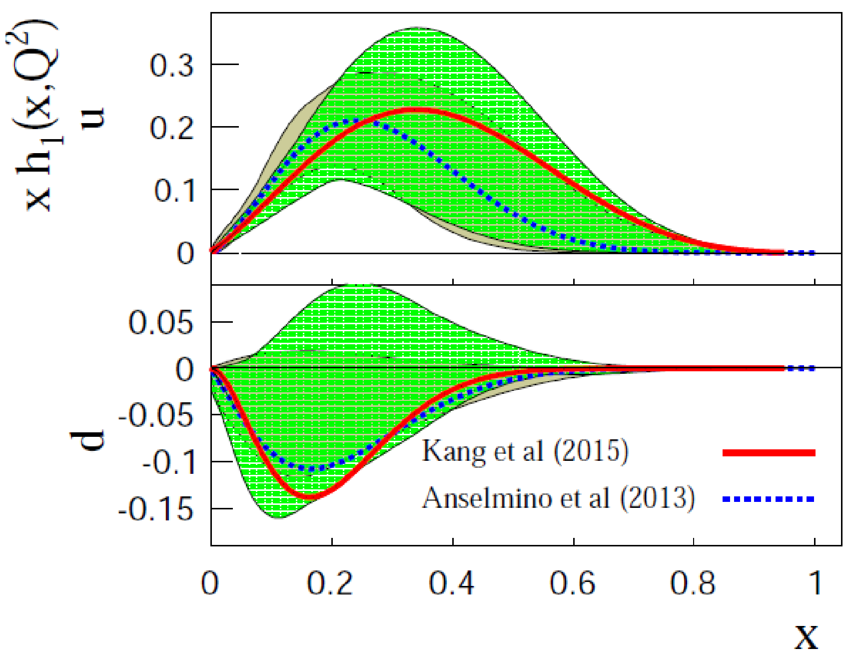
Sun, Yuan 2014
Echevarria et al 2014

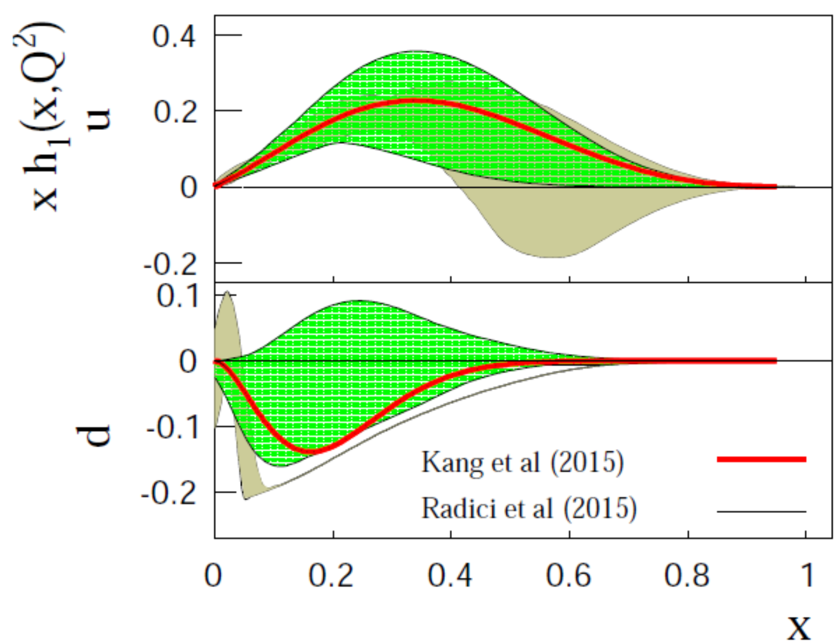
Echevarria et al 2014



$$A_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1 \otimes H_1^{\perp}$$

Collins asymmetry





Transversity:

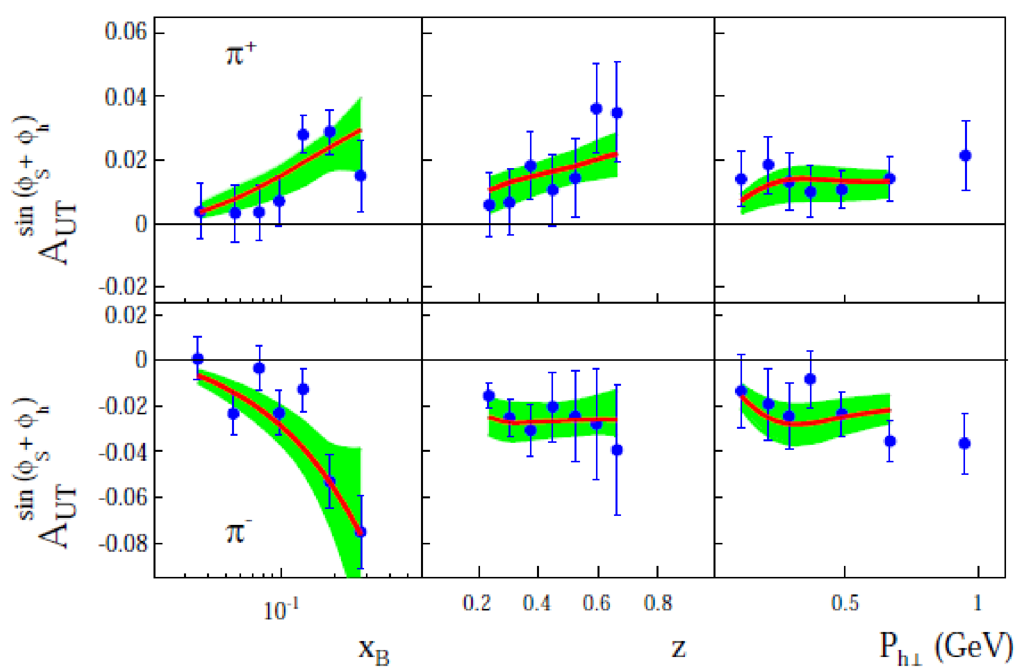
Positive u-quark
Negative d-quark

Access to tensor charge

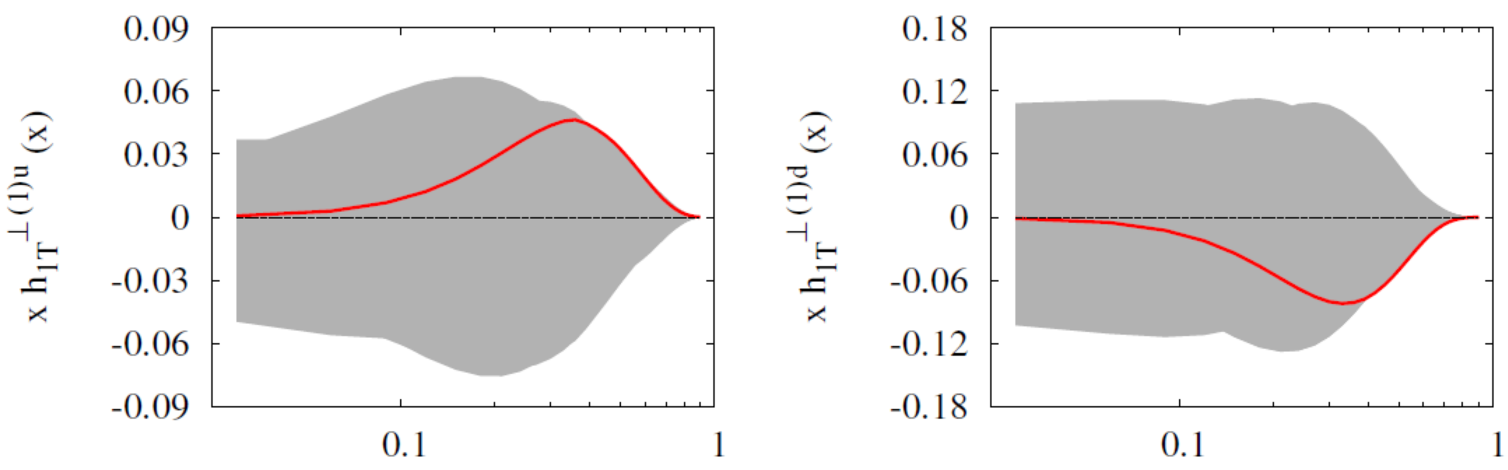
Anselmino et al 2009, 2013
Kang et al 2014
Radici et al 2014
...

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

HERMES



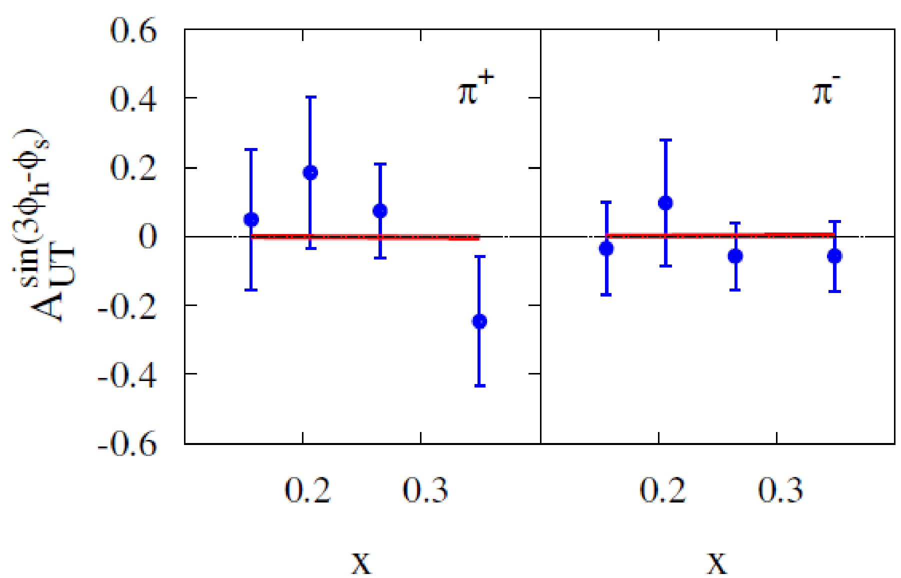
$A_{UT}^{\sin(3\phi_h-\phi_S)} \sim h_{1T}^{\perp} \otimes H_1^{\perp}$ Pretzelosity



Lefky, AP PRD 91 (2015)

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

Zhang et al, JLAB HALL A Coll. (2013)



Positive u-quark
Negative d-quark

Huge 100% uncertainty

Precise data are needed

Pretzelosity is related in models to OAM

✓ – twist 2 ✗ – twist 3

Cross-section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

$$\Big(1+\cos\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UU}^{\cos\phi_h}+\cos 2\phi_h\,\varepsilon\,A_{UU}^{\cos 2\phi_h}$$
$$+\lambda\,\sin\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LU}^{\sin\phi_h}$$
$$+S_L\left[\sin\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UL}^{\sin\phi_h}+\sin 2\phi_h\,\varepsilon\,A_{UL}^{\sin 2\phi_h}\right]$$
$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\,A_{LL}+\cos\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LL}^{\cos\phi_h}\right]$$
$$+S_T\,\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)}$$
$$+S_T\,\sin(\phi_h+\phi_S)\,\varepsilon\,A_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+S_T\,\sin(3\phi_h-\phi_S)\,\varepsilon\,A_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+S_T\,\sin\phi_S\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin\phi_S}$$
$$+S_T\,\sin(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin(2\phi_h-\phi_S)}$$
$$+S_T\lambda\,\cos(\phi_h-\phi_S)\,\sqrt{1-\varepsilon^2}\,A_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+S_T\lambda\,\cos\phi_S\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos\phi_S}$$
$$+S_T\lambda\,\cos(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

Double Spin Asymmetry

$$A_{LL}\sim g_1\otimes D_1$$ ✓

$$A_{LT}^{\cos(\phi_h-\phi_S)}\sim g_{1T}\otimes D_1$$ ✓

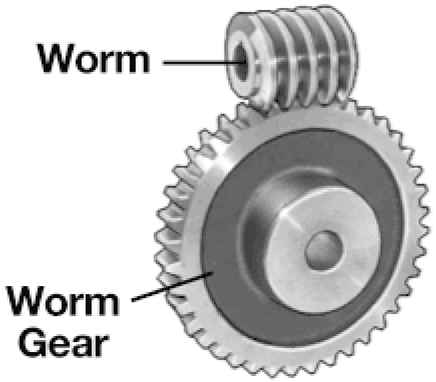
Double Spin Asymmetries at twist-2

$$A_{LL} \sim g_1 \otimes D_1$$

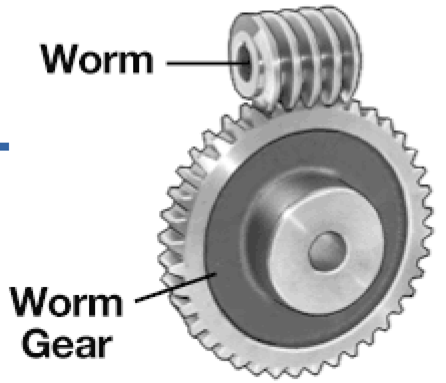
Helicity distributions

$$A_{LT}^{\cos(\phi_h - \phi_S)} \sim g_{1T} \otimes D_1$$

Worm-gear



$A_{LT}^{\cos(\phi_h-\phi_S)} \sim g_{1T} \otimes D_1$ Worm-gear



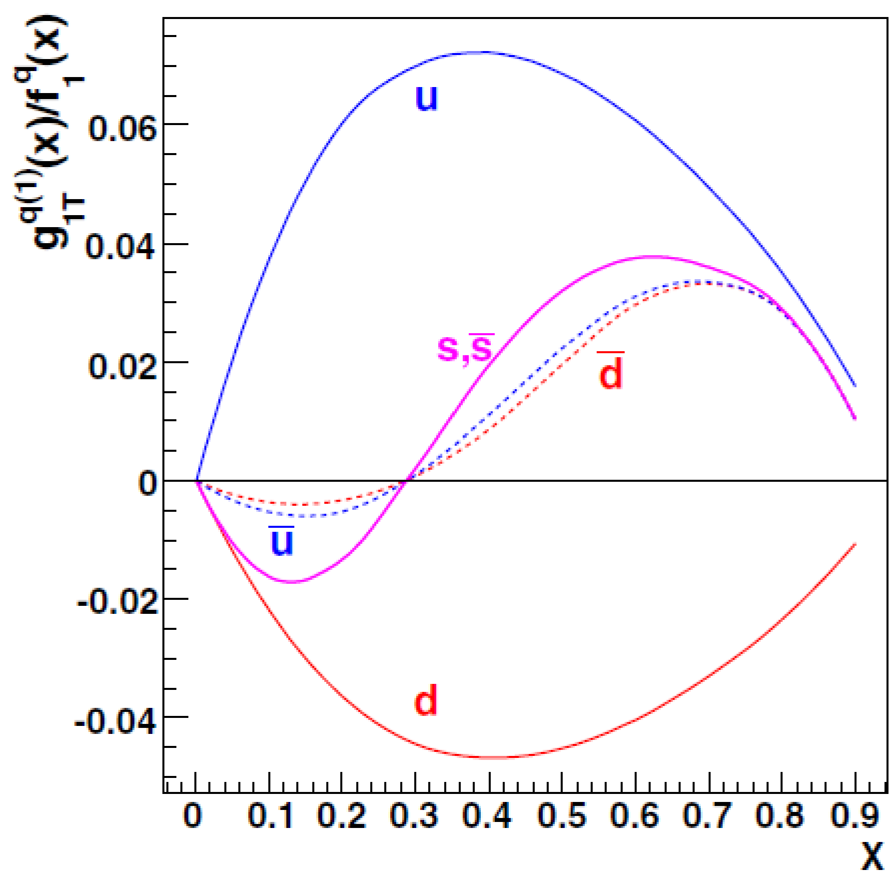
Kotzinian, Parsamyan, AP 2006
Anselmino, Efremov, Kotzinian, Pasamyan 2006
Avakian, Bastami, Efremov, Kotzinian, Parsamyan, AP, Schlegel, Schweitzer in preparation

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

"Wandzura-Wilczek" approximation

$$g_{1T}^{(1)q}(x) \approx x \int_x^1 \frac{dy}{y} g_1^q(y)$$

Connection to helicity distributions



Positive u-quark
Negative d-quark

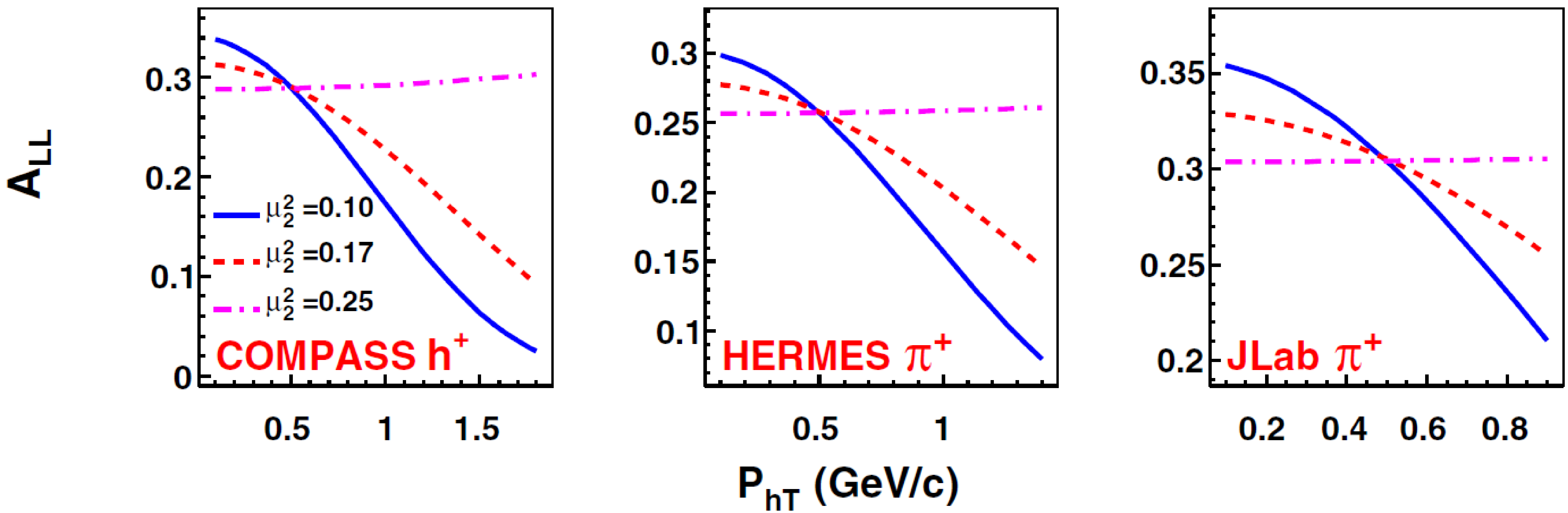
$A_{LL} \sim g_1 \otimes D_1$

helicity

Anselmino, Efremov, Kotzinian, Pasamyan 2006
Avakian, Bastami, Efremov, Kotzinian, Parsamyan, AP, Schlegel, Schweitzer in preparation

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Anselmino, Efremov, Kotzinian, Pasamyan 2006



What are the widths of helicity TMD distributions?

Cross-section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

$$\Big(1+\cos\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UU}^{\cos\phi_h}+\cos 2\phi_h\ \varepsilon\ A_{UU}^{\cos 2\phi_h}$$

$$+\lambda\ \sin\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LU}^{\sin\phi_h}$$

$$+S_L\left[\sin\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UL}^{\sin\phi_h}+\sin 2\phi_h\ \varepsilon\ A_{UL}^{\sin 2\phi_h}\right]$$

$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\ A_{LL}+\cos\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LL}^{\cos\phi_h}\right]$$

$$+S_T\ \sin(\phi_h-\phi_S)\ A_{UT}^{\sin(\phi_h-\phi_S)}$$

$$+S_T\ \sin(\phi_h+\phi_S)\ \varepsilon\ A_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+S_T\ \sin(3\phi_h-\phi_S)\ \varepsilon\ A_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+S_T\ \sin\phi_S\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin\phi_S}$$

$$+S_T\ \sin(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin(2\phi_h-\phi_S)}$$

$$+S_T\lambda\ \cos(\phi_h-\phi_S)\ \sqrt{1-\varepsilon^2}\ A_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+S_T\lambda\ \cos\phi_S\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos\phi_S}$$

$$+S_T\lambda\ \cos(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

What if you need all modulations for a Monte Carlo etc?

What if you need all modulations for a Monte Carlo etc?

Use parameterizations from extractions

Use model calculations

Use lattice QCD calculations

Use various approximations and relations

If everything else fails, assume to be zero