TMDs in covariant approach

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(based on collaboration and discussions
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Outline

- Introduction
- Covariant approach:
  - TMDs: calculation, predictions, QCD evolution…
  - spin & OAM, role of gluons
- Summary

Introduction

Intrinsic motion in composite systems is required by QM:

electrons in atom *non-relativistic motion, OAM & spin are decoupled*

$$\boxed{d \approx 10^{-10} m, \quad p \approx 10^{-3} MeV, \quad m_e \approx 0.5 MeV, \quad \beta \approx 0.002}$$

nucleons in nucleus *Fermi motion*

$$\boxed{d \approx 10^{-15} m, \quad p \approx 10^{2} MeV, \quad m_N \approx 940 MeV, \quad \beta \approx 0.1}$$

quarks in nucleon *relativistic motion, OAM & spin cannot be decoupled*

$$\boxed{d \approx 10^{-15} m, \quad p \approx 10^{2} MeV, \quad m_e \approx 5 MeV, \quad \beta \approx 1}$$

Covariant approach

Main results:

- Sum rules: Wanzura-Wilczek (WW), Burhardt-Cottinngham (BC) and Efremov-Leader-Teryaev (ELT)
- Relations between TMDs, PDFs and TMDs (giving predictions for TMDs
- Study and prediction of the role of OAM

[1] P.Zavada, Phys. Lett. B 751, 525 (2015).
[2] A. V. Efremov, O. V. Teryaev and P. Zavada, J.Phys.Conf.Ser. 678 (2016) no.1, 012001, arXiv:1511.01164 [hep-ph].
[3] P. Zavada, Phys. Rev. D 89, 014012 (2014).
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[12] A. V. Efremov, P. Schweitzer, O. V. Teryaev and P. Zavada, Phys. Rev. D 80, 014021 (2009).
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Parton model postulates:

- DIS can be (in a leading order) described as an incoherent superposition of interactions of a probing lepton with the individual effectively free quarks (one-photon exchange).
- The kinematical degrees of freedom of the quarks are described by a set of probabilistic distribution functions. Integration of the quark tensors with the corresponding distributions gives the hadronic tensor (structure functions).

In the *conventional, collinear parton model* (1D kinematics) this picture is assumed only in the frame, where the proton is fast moving

$\Rightarrow$ $x_B \approx p/P$

In the *covariant approach*, both conditions are postulated in *any reference frame* (3D kinematics)

Paradigm of covariant approach

- **Large Q^2:** In the rest frame we have

$$\boxed{|\mathbf{q}_R|^2 = Q^2 + \nu^2 = Q^2 \left(1 + \frac{Q^2}{(2Mx)^2}\right)} \Rightarrow \boxed{|\mathbf{q}_R| \gtrsim \nu = \frac{Q^2}{2Mx} \geq \frac{Q^2}{2M}}$$

$$\Rightarrow \boxed{\Delta\lambda \lesssim \Delta\tau \approx \frac{2Mx}{Q^2}}$$

So a space-time domain of lepton -quark QED interaction is limited.

- **Effect of asymptotic freedom:** Limited extend of this domain prevent the quark from an interaction with the rest of nucleon during the lepton-quark interaction – **in any reference frame.**

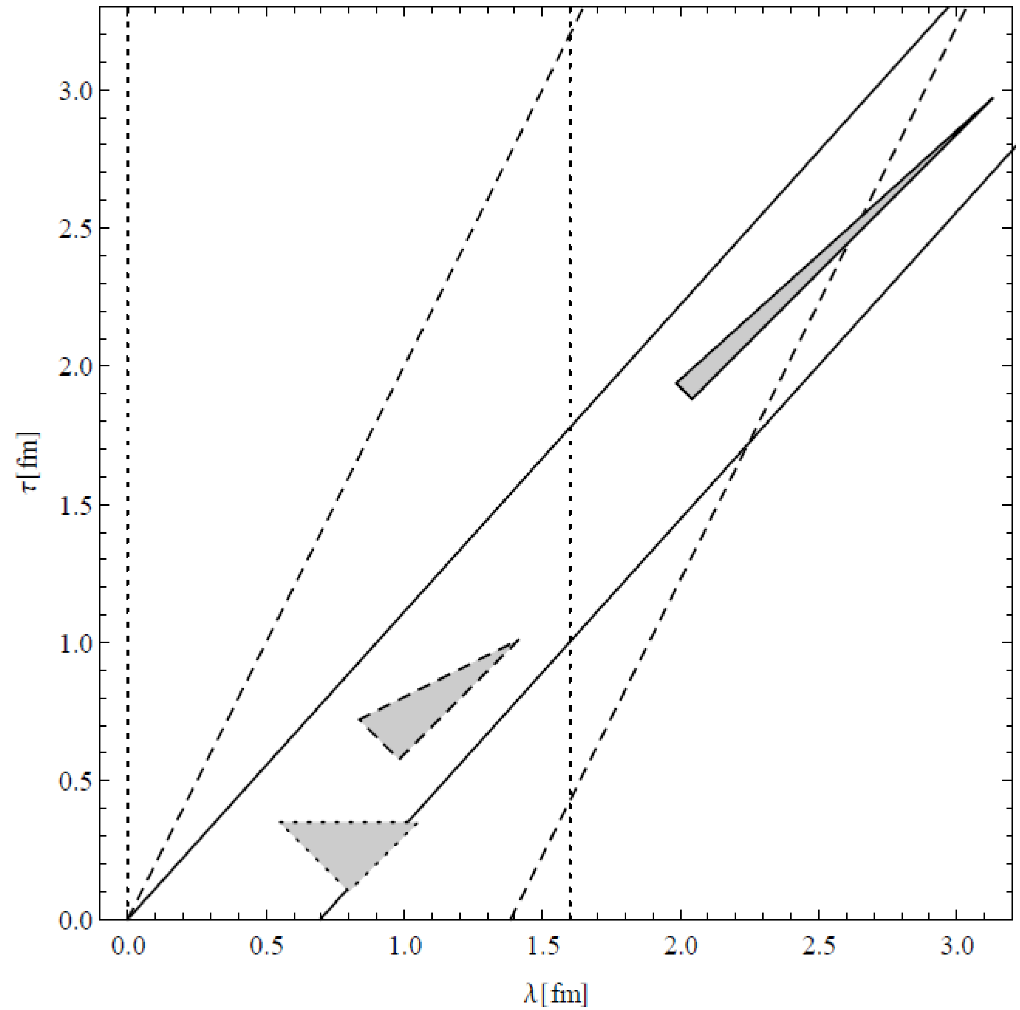


FIG. 1: The space-time domain of the photon momentum transfer to the quark in different Lorentz frames. The different styles of lines and triangles represent the proton boundary and the domain for: rest frame, $\beta = 0$ *(dotted)*, $\beta = 0.5$ *(dashed)*, $\beta = 0.9$ *(solid)*. Note that Lorentz boosts does not change the area of the domain $\Delta\lambda \times \Delta\tau$.

$$\lambda(\beta) = \frac{\lambda_0 + \beta\tau_0}{\sqrt{1-\beta^2}} \qquad \tau(\beta) = \frac{\tau_0 + \beta\lambda_0}{\sqrt{1-\beta^2}} \qquad \Delta\lambda(\beta) = \Delta\lambda_0\sqrt{1-\beta^2} \qquad \Delta\tau(\beta) = \frac{\Delta\tau_0}{\sqrt{1-\beta^2}}$$

In fact we assume characteristic time of QCD process accompanying γ absorption much greater than absorption time itself:

$$\Delta\tau \ll \Delta\tau_{QCD}$$

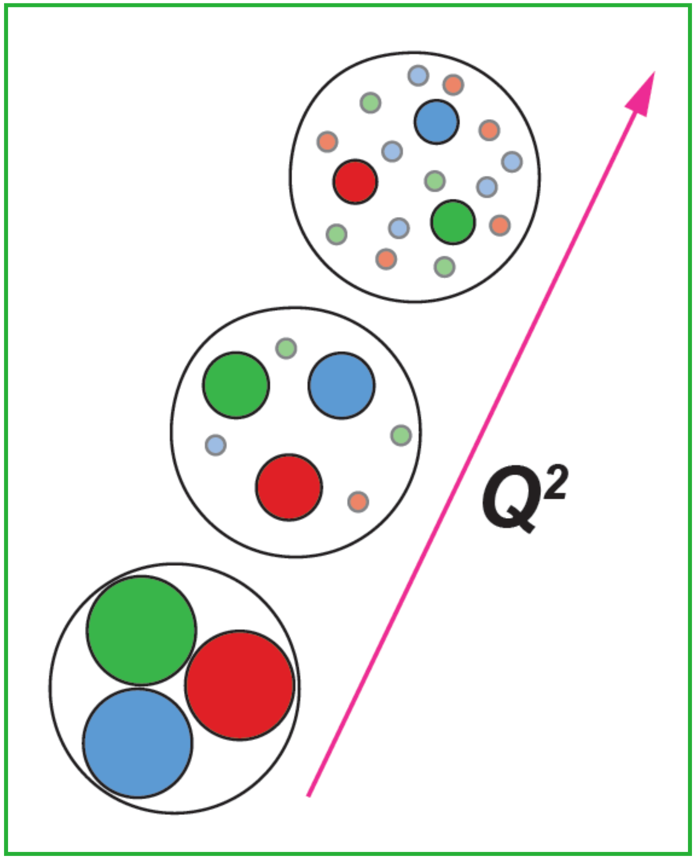
Since Lorentz time dilation is universal, the first relation holds in any reference frame. This is essence of our covariant leading order approach.

$$\Delta T(\beta) = \frac{\Delta T_0}{\sqrt{1-\beta^2}}$$

Remarks:

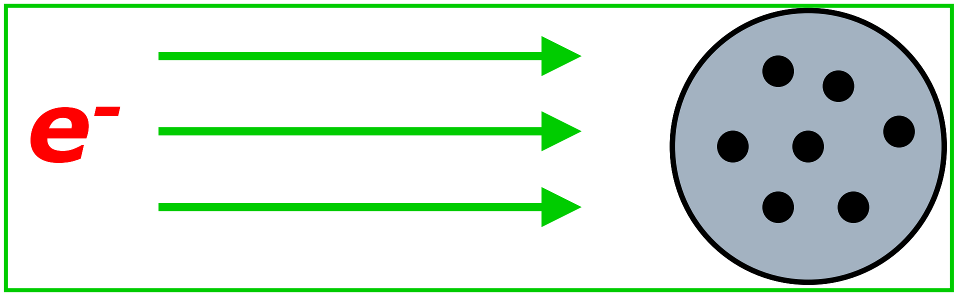
- We suppose $\Delta\tau_{QCD}$ has a good sense in any reference frame - even if we cannot transform QCD corrections…
- We do not aim to describe complete nucleon dynamic structure, but only a picture of short time interval corresponding to DIS.
- We assume the approximation of quarks by free waves in limited space-time domain is acceptable for description of DIS regardless of the reference frame.
- We assume Q^2-dependence of this

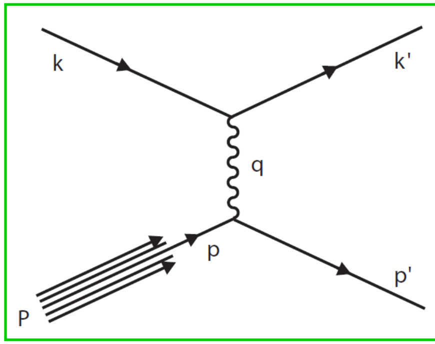
Lorentz-invariant "effective" picture: $n_q(\mathbf{p},Q^2)$.



Structure functions

General framework: $\Delta\sigma(x, Q^2) \sim |A|^2 = L_{\alpha\beta} W^{\alpha\beta}$





$W^{\alpha\beta} \Rightarrow$

$$F_1(x, Q^2)$$
$$F_2(x, Q^2)$$
$$g_1(x, Q^2)$$
$$g_2(x, Q^2)$$

The quarks are represented by the quasifree fermions, which are in the proton rest frame described by the set of distribution functions with spheric symmetry

$$G_q^{\pm}(p_0) d^3p; \qquad p_0 = \sqrt{m^2 + \mathbf{p}^2},$$

which are expected to depend effectively on Q^2. These distributions measure the probability to find a quark in the state

$$u(p, \lambda\mathbf{n}) = \frac{1}{\sqrt{N}} \begin{pmatrix} \phi_{\lambda\mathbf{n}} \\ \frac{\mathbf{p}\sigma}{p_0+m} \phi_{\lambda\mathbf{n}} \end{pmatrix}; \qquad \frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}} = \lambda\phi_{\lambda\mathbf{n}},$$

where m and p are the quark mass and momentum, $\lambda = \pm 1/2$ and $\mathbf{n}$ coincides with the direction of target polarization $\mathbf{J}$.

Input distributions

- **Rest frame:**

The distributions allow to define the generic functions G and ΔG:

$$G(p_0) = \sum_q e_q^2 G_q(p_0), \quad G_q(p_0) \equiv G_q^+(p_0) + G_q^-(p_0),$$

$$\Delta G(p_0) = \sum_q e_q^2 \Delta G_q(p_0), \quad \Delta G_q(p_0) \equiv G_q^+(p_0) - G_q^-(p_0)$$

from which the structure functions can be obtained.

- **Arbitrary frame:** $p_0 \rightarrow pP/M$

<u>Rotational symmetry (rest frame) & Lorentz invariance</u>

F_1, F_2 – exact and manifestly covariant form:

$$F_1(x) = \frac{M}{2}\left(\frac{B}{\gamma} - A\right), \qquad F_2(x) = \frac{Pq}{2M\gamma}\left(\frac{3B}{\gamma} - A\right),$$

where

$$A = \frac{1}{Pq}\int G\left(\frac{Pp}{M}\right)[m^2 - pq]\,\delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0},$$

$$B = \frac{1}{Pq}\int G\left(\frac{pP}{M}\right)\left[\left(\frac{Pp}{M}\right)^2 + \frac{(Pp)(Pq)}{M^2} - \frac{pq}{2}\right]\delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0},$$

$$\gamma = 1 - \left(\frac{Pq}{Mq}\right)^2.$$

conventional collinear approach: $p_\mu \rightarrow xP_\mu$

... similarly for g_1, g_2 :

$$g_1 = Pq\left(G_S - \frac{Pq}{qS}G_P\right), \qquad g_2 = \frac{(Pq)^2}{qS}G_P,$$

where

$$G_P = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[\frac{pS}{pP+mM}1 + \frac{1}{mM}\left(pP - \frac{pu}{qu}Pq\right)\right]$$
$$\times\,\delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0},$$

$$G_S = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[1 + \frac{pS}{pP+mM}\frac{M}{m}\left(pS - \frac{pu}{qu}qS\right)\right]$$
$$\times\,\delta\left(\frac{pq}{Pq} - x_B\right)\frac{d^3p}{p_0};$$

$$u = q + (qS)S - \frac{(Pq)}{M^2}P.$$

Rest frame representation

If one assumes $Q^2 \gg 4M^2x^2$, then:

$$F_2(x) = Mx^2 \int G(p_0)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}$$

$$g_1(x) = \frac{1}{2}\int \Delta G(p_0)\left(m + p_1 + \frac{p_1^2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0},$$

$$g_2(x) = -\frac{1}{2}\int \Delta G(p_0)\left(p_1 + \frac{p_1^2 - p_T^2/2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}$$

- ❑ integrals can be inverted
- ❑ study and prediction OAM

… or in terms of conventional distributions:

$$\begin{aligned}
f_1^a(x) &= Mx\int G^a(p_0)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}, \\
g_1^a(x) &= \int \Delta G^a(p_0)\left(m + p_1 + \frac{p_1^2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}, \\
g_2^a(x) &= -\int \Delta G^a(p_0)\left(p_1 + \frac{p_1^2 - p_T^2/2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}
\end{aligned}$$

- ❑ *G,ΔG* are not known, but integrals imply relations between distributions: WW relation, sum rules WW, BC, ELT; helicity↔transversity, transversity ↔ pretzelosity; unpolarized+SU(6) → polarized
- ❑ partial integration (only over p_1) defines p_T – dependent distributions: $f(x) \rightarrow f(x, p_T)$
- ❑ relations between TMDs, but also TMDs↔PDFs

TMDs

$\phi(x, \mathbf{p}_T)_{ij}$ light-front correlators

$\frac{1}{2}\,\mathrm{tr}[\gamma^+\ \phi(x,\mathbf{p}_T)] = f_1(x,\mathbf{p}_T) - \frac{\varepsilon^{jk} p_T^j S_T^k}{M} f_{1T}^{\perp}(x,\mathbf{p}_T)$
$\frac{1}{2}\,\mathrm{tr}[\gamma^+\gamma_5\ \phi(x,\mathbf{p}_T)] = S_L\, g_1(x,\mathbf{p}_T) + \frac{\mathbf{p}_T \cdot \mathbf{S}}{M} g_{1T}^{\perp}(x,\mathbf{p}_T)$
$\frac{1}{2}\,\mathrm{tr}[i\sigma^{j+}\gamma_5\ \phi(x,\mathbf{p}_T)] = S_T^j\, h_1(x,\mathbf{p}_T) + S_L\, \frac{p_T^j}{M} h_{1L}^{\perp}(x,\mathbf{p}_T)$
$+ \frac{(p_T^j p_T^k - \frac{1}{2}\mathbf{p}_T^2 \delta^{jk}) S_T^k}{M^2} h_{1T}^{\perp}(x,\mathbf{p}_T) + \frac{\varepsilon^{jk} p_T^k}{M} h_1^{\perp}(x,\mathbf{p}_T)$

LI & **RS** generate relations also between some TMDs !

for details see A.Efremov, P.Schweitzer, O.Teryaev and P. Z., Phys. Rev. D 80,014021 (2009).

PDF-TMD relations

1. UNPOLARIZED

$$f_1^a(x,\mathbf{p}_T) = -\frac{1}{\pi M^2}\,\frac{d}{dy}\left[\frac{f_1^a(y)}{y}\right]_{y=\xi(x,\mathbf{p}_T^2)}$$

$$\xi(x,\mathbf{p}_T^2) = x\left(1+\frac{\mathbf{p}_T^2}{x^2M^2}\right)$$

For details see:
P.Z. Phys.Rev.D **83**, 014022 (2011), **arXiv:0908.2316 [hep-ph]**
A.Efremov, P.Schweitzer, O.Teryaev and P.Z. Phys.Rev.D **83**, 054025(2011)
arXiv:0912.3380 [hep-ph], arXiv:1012.5296 [hep-ph]

The same relation was shortly afterwards obtained independently:
U. D'Alesio, E. Leader and F. Murgia, Phys.Rev. D **81**, 036010 (2010),
arXiv:0909.5650 [hep-ph]

we assume $m \to 0$ (if not stated otherwise)

PDF-TMD relations

2. POLARIZED

$$g_1^a(x,\mathbf{p}_T) = \frac{2x-\xi}{2} K^a(x,\mathbf{p}_T) \ ,$$
$$h_1^a(x,\mathbf{p}_T) = \frac{x}{2} K^a(x,\mathbf{p}_T) \ ,$$
$$g_{1T}^{\perp a}(x,\mathbf{p}_T) = K^a(x,\mathbf{p}_T) \ ,$$
$$h_{1L}^{\perp a}(x,\mathbf{p}_T) = -K^a(x,\mathbf{p}_T) \ ,$$
$$h_{1T}^{\perp a}(x,\mathbf{p}_T) = -\frac{1}{x} K^a(x,\mathbf{p}_T) \ .$$

Known $f_1(x)$, $g_1(x)$ allow us to predict some unknown TMDs

$$K^a(x,\mathbf{p}_T) = \frac{2}{\pi \xi^3 M^2} \left(2 \int_\xi^1 \frac{dy}{y} \, g_1^a(y) + 3\, g_1^a(\xi) - x \, \frac{dg_1^a(\xi)}{d\xi} \right)$$

$$\xi(x,\mathbf{p}_T^2) = x \left(1 + \frac{\mathbf{p}_T^2}{x^2 M^2} \right)$$

Numerical results:
(unpolarized)

Another model approaches to TMDs give compatible results:

1. U. D'Alesio, E. Leader and F. Murgia, Phys.Rev. D 81, 036010 (2010)

2. C.Bourrely, F.Buccellla, J.Soffer, Phys.Rev. D 83, 074008 (2011); Int.J.Mod.Phys. A28 (2013) 1350026

Input for $f_1(x)$
MRST LO at *4 GeV²*

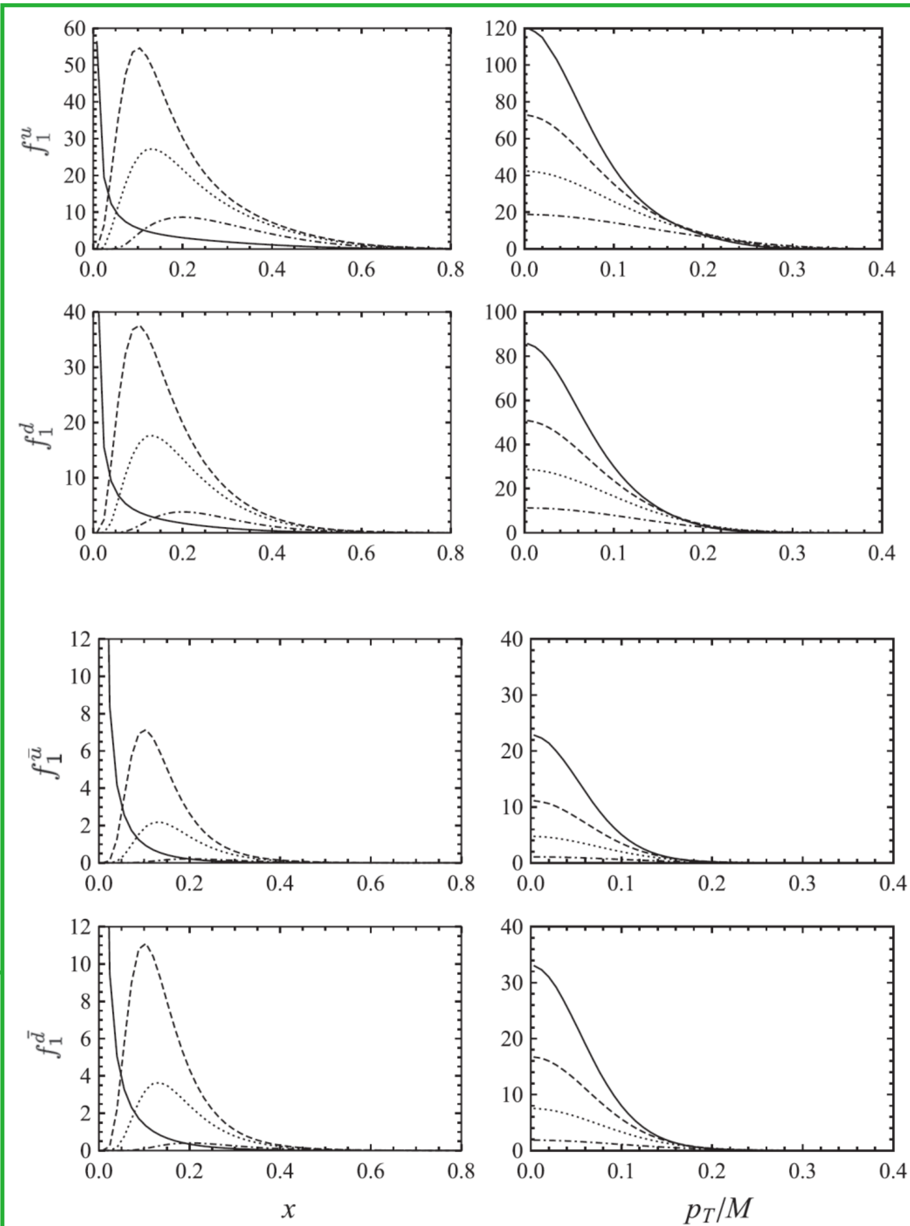


FIG. 1. The TMDs $f_1^a(x, \mathbf{p}_T)$ for u, d (*upper part*) and $\bar{u}$, $\bar{d}$-quarks (*lower part*). *Left panel*: $f_1^a(x, \mathbf{p}_T)$ as a function of x for $p_T/M = 0.10$ (dashed line), 0.13 (dotted line), 0.20 (dash-dotted line). The solid line corresponds to the input distribution $f_1^a(x)$. *Right panel*: $f_1^a(x, \mathbf{p}_T)$ as a function of p_T/M for $x = 0.15$ (solid line), 0.18 (dashed line), 0.22 (dotted), 0.30 (dash-dotted line).

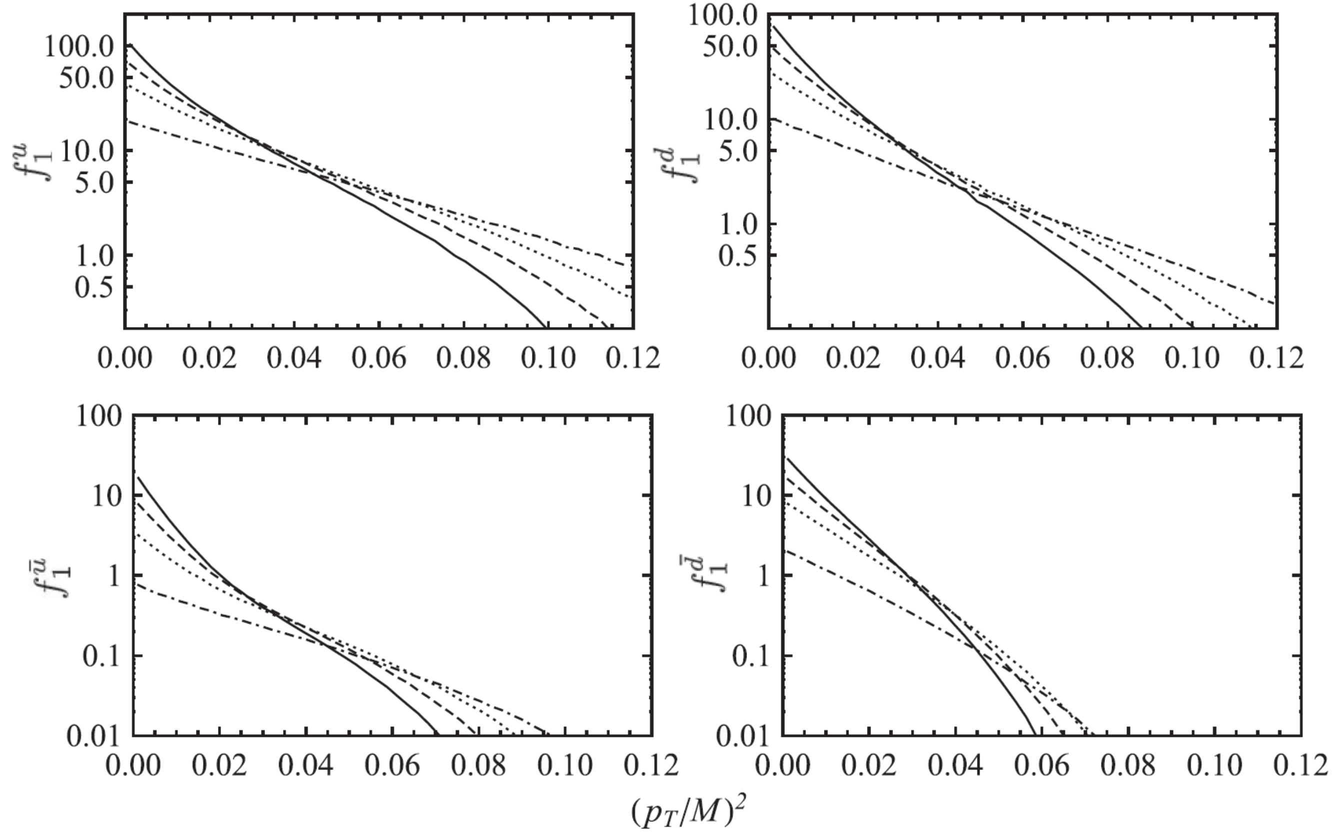


FIG. 2. $f_1^a(x, \mathbf{p}_T)$ as a function of $(p_T/M)^2$ for $x = 0.15$ (solid), 0.18 (dashed), 0.22 (dotted), 0.30 (dash-dotted line).

- Gaussian shape – is supported by phenomenology
- $<p_T^2>$ depends on x , is smaller for sea quarks
- $<p_T> < 0.1\text{GeV}$, $p_T/M < 0.5$

Numerical results:
(polarized)

Input for g_1 : LSS LO at *4 GeV*^{2}

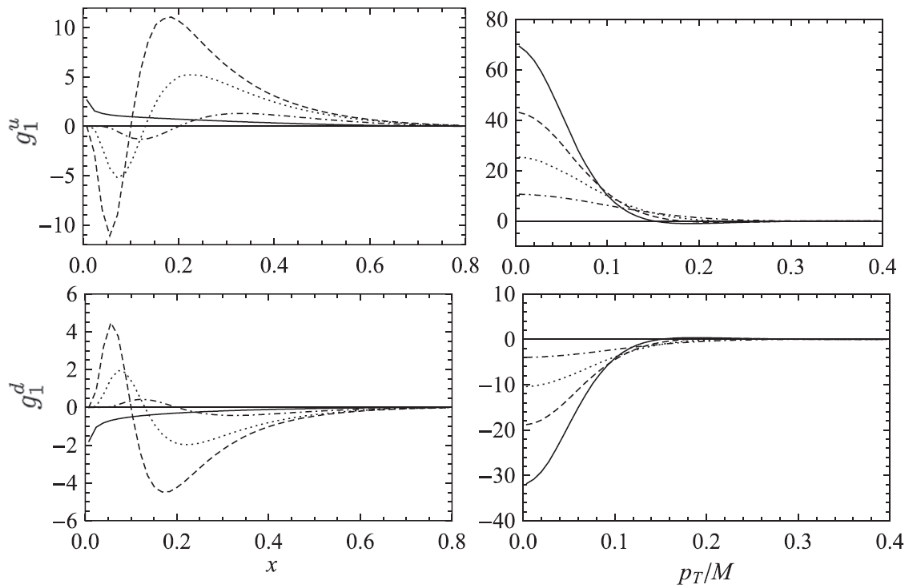
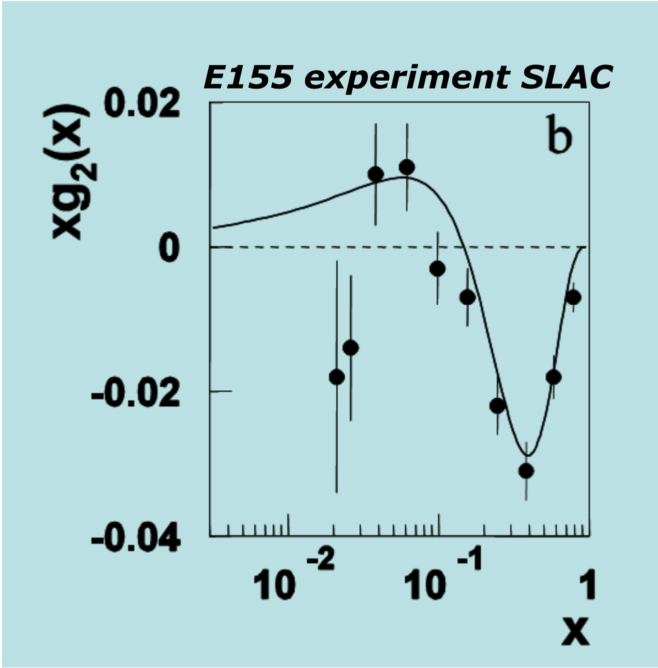


FIG. 3. $g_1^q(x, \mathbf{p_T})$ for u- (*upper panel*) and d-quarks (*lower panel*). *Left panel*: $g_1^q(x, \mathbf{p_T})$ as a function of x for $p_T/M =$ 0.10 (dashed line), 0.13 (dotted line), 0.20 (dash-dotted line). The solid line corresponds to the input distribution $g_1^q(x)$. *Right panel*: $g_1^q(x, \mathbf{p_T})$ as a function of p_T/M for $x = 0.15$ (solid line), 0.18 (dashed line), 0.22 (dotted line), 0.30 (dash-dotted line).

… can be compared to g_2 *(x)*: In both cases the sign is correlated with the sign of p_L in the rest frame



P.Z. Phys.Rev.D **67**, 014019 (2003)

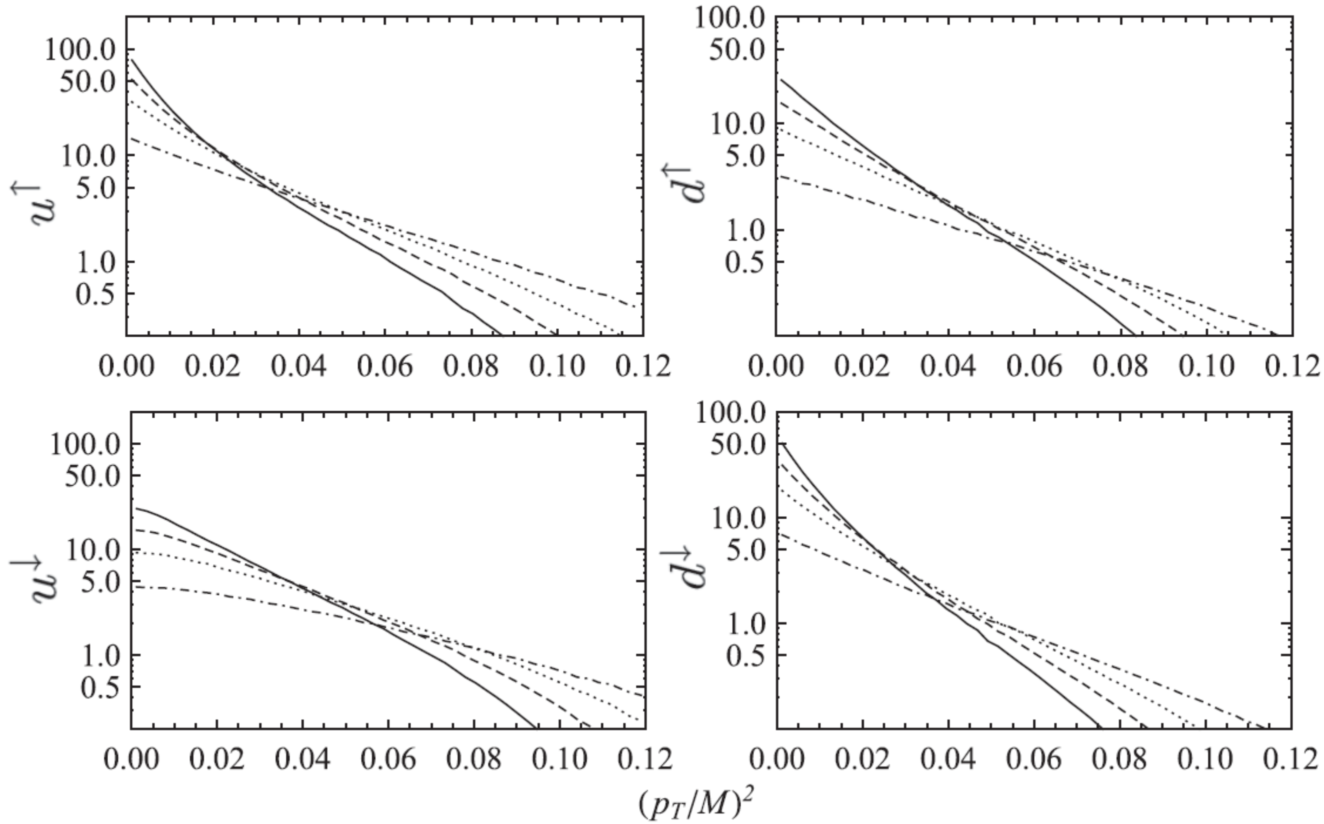


FIG. 4. The TMDs $q^{\uparrow}(x, \mathbf{p_T}) = \frac{1}{2}(f_1^q + g_1^q)(x, \mathbf{p_T})$ (*upper panel*) and $q^{\downarrow}(x, \mathbf{p_T}) = \frac{1}{2}(f_1^q - g_1^q)(x, \mathbf{p_T})$ (*lower panel*) as functions of p_T^2/M^2 for fixed values of $x = 0.15$ (solid line), 0.18 (dashed line), 0.22 (dotted line), 0.30 (dash-dotted lines). *Left panel*: u-flavor. *Right panel*: d-flavor.

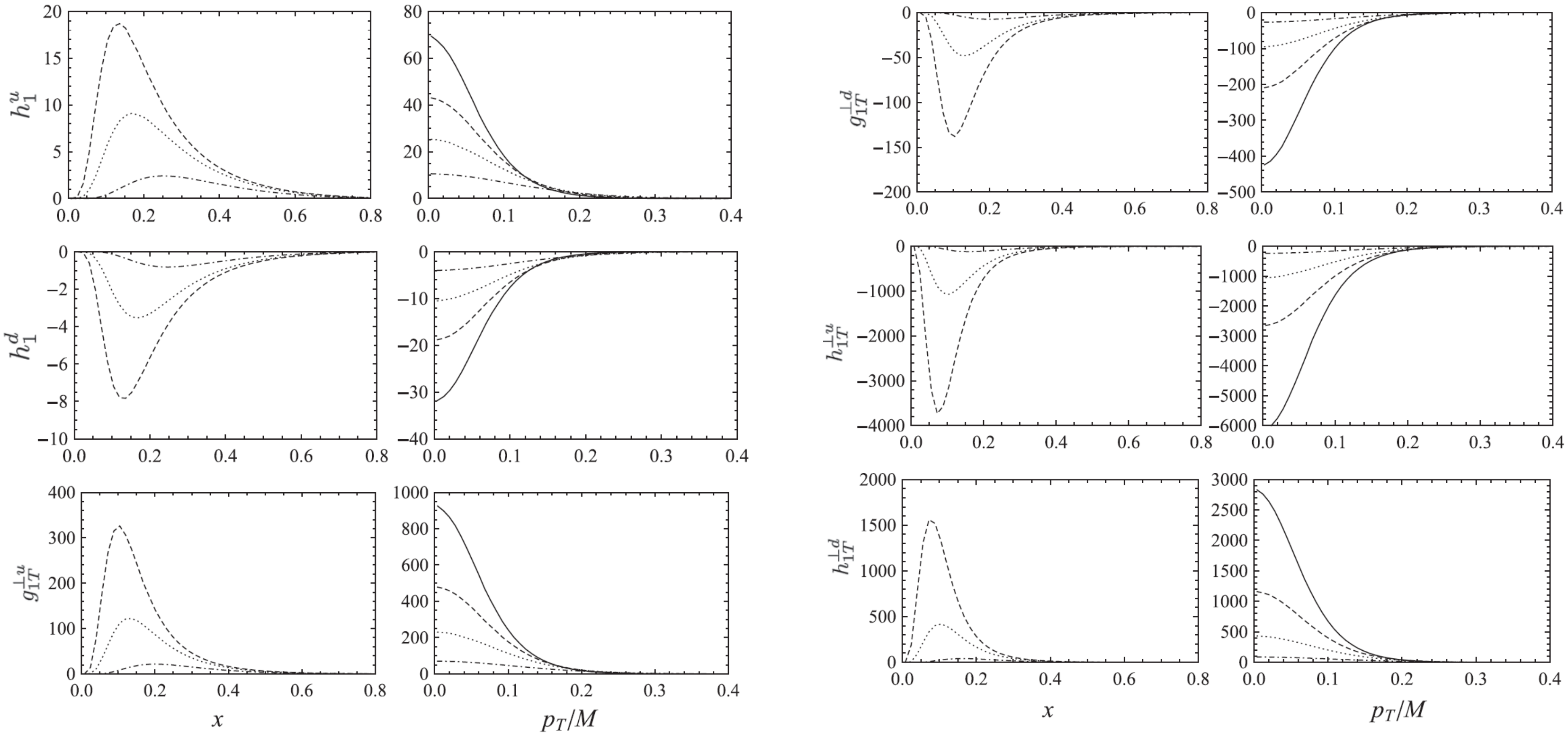


FIG. 5. The TMDs $h_1^q(x, \mathbf{p_T})$, $g_{1T}^{\perp q}(x, \mathbf{p_T})$, $h_{1T}^{\perp q}(x, \mathbf{p_T})$ for u- and d-quarks. *Left panel*: The TMDs as functions of x for $p_T/M = 0.10$ (dashed line), 0.13 (dotted line), 0.20 (dash-dotted lines). *Right panel*: The TMDs as functions of p_T/M for $x = 0.15$ (solid line), 0.18 (dashed line), 0.22 (dotted line), 0.30 (dash-dotted lines).

QCD evolution of TMDs

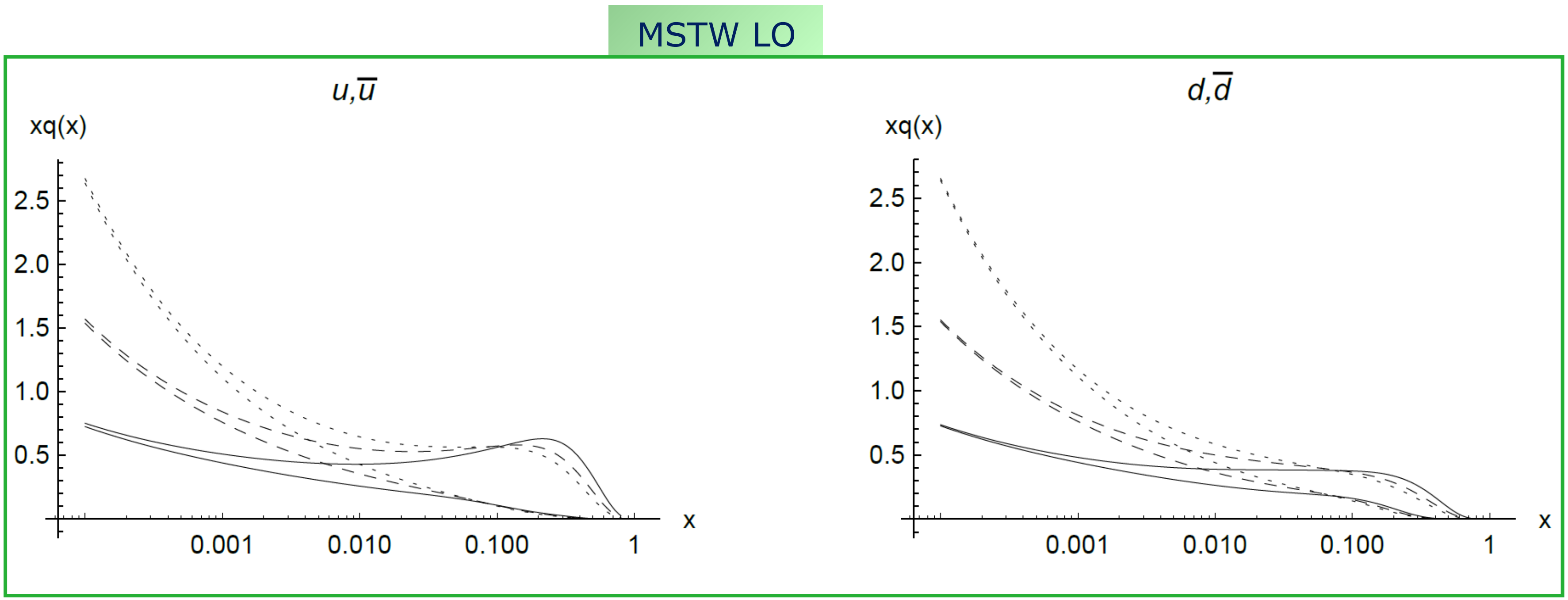
LI & **RS** generate the relations **TMDs↔PDFs:**

$$\boxed{f_1^a(x, \mathbf{p}_T) = -\frac{1}{\pi M^2}\frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{q(y)}{y}\right]_{y=\xi}; \qquad \xi = x\left(1 + \frac{\mathbf{p}_T^2}{x^2M^2}\right)}$$

The most direct way to introduce evolution is via ***q(x, Q²)*** :

$$\boxed{f_1^a(x, \mathbf{p}_T, Q^2) = -\frac{1}{\pi M^2}\frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{q(y, Q^2)}{y}\right]_{y=\xi}; \qquad \xi = x\left(1 + \frac{\mathbf{p}_T^2}{x^2M^2}\right)}$$

for details see A. Efremov, O. Teryaev and P.Z., J.Phys.Conf.Ser. 678 (2016), no.1, 012001, arXiv:1511.01164 [hep-ph]. (in progress)



Input PDF of $u, \bar{u}$ (left) and d, d (right) quarks at different scales: $Q^2 = 4, 40, 400 GeV$ (solid, dashed, dotted curves)

TMDs - numerical results:

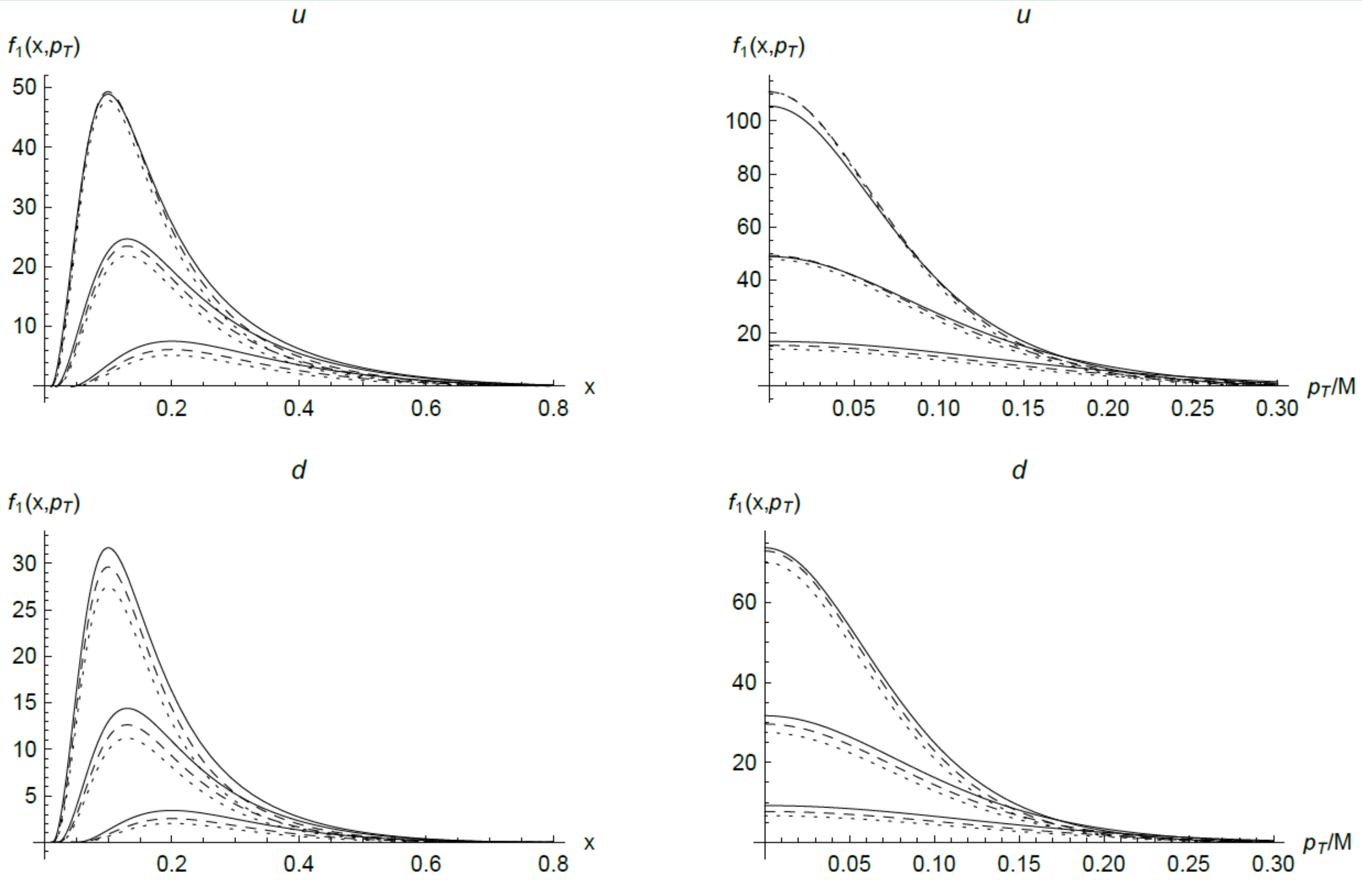


FIG. 3: TMD at different scales: $Q^2 = 4, 40, 400 GeV$ (solid, dashed, dotted curves) for u and d quarks. Sets of curves in left panels (from top) correspond to fixed $p_T/M = 0.1, 0.13, 0.20$. The curves in left panels (from top) correspod to fixed $x = 0.18, 0.22, 0.30$.

Remark

For $Q^2 \gg 4M^2 x_B^2$ **and if we assume LI & RS, then:**

$$\boxed{0 \leq x_B \leq 1} \quad \Rightarrow \quad \boxed{p_T \leq M/2}$$

$$x_B = \frac{pq}{Pq} = \frac{Q^2}{2Pq}$$

quark on-mass-shell is not necessary!
P. Z. Phys. Rev. D 85, 037501 (2012)

or equivalently:

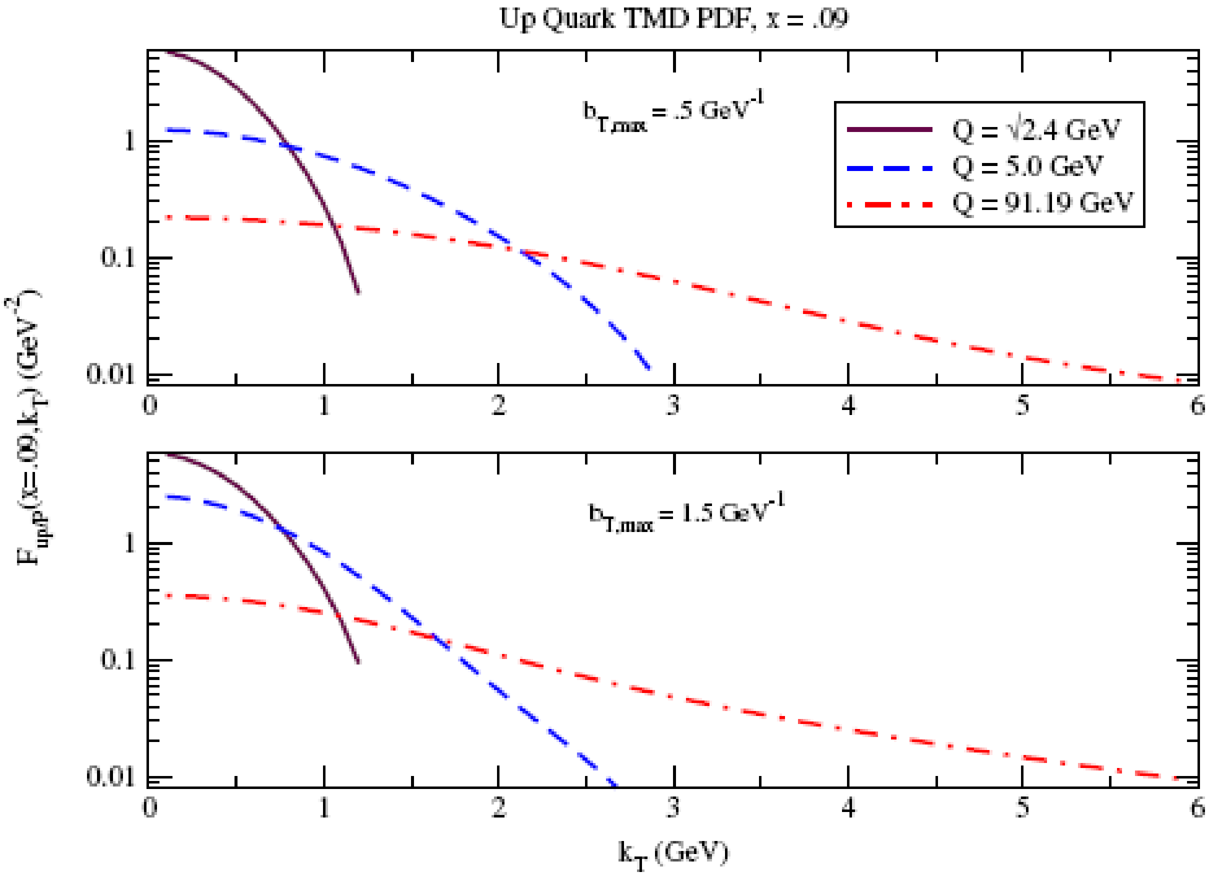
$\boxed{p_T > M/2} \Rightarrow$ **LI** or **RS** or $\boxed{0 \leq x_B \leq 1}$

… something is not satisfied

PHYSICAL REVIEW D **83**, 114042 (2011)

Transverse momentum dependent parton distribution and fragmentation functions with QCD evolution

S. Mert Aybat[1,2,*] and Ted C. Rogers[2,†]



k_T>M/2 ?

Why the results of the calculations differ so much?

We compare results on TMDs:

- pQCD evolution:
  correct dynamics (QCD) + reduced kinematic
  (no covariance, no rest frame sphericity…)

- Covariant approach:
  simplistic model + correct 3D kinematics
  (constrained by LI & RS)

- <u>Correct answer:</u>
  will come from JLab experiments!

Spin & OAM

Eigenstates of angular momentum

Usual plane-wave spinors are replaced by spinor spherical harmonics (both in momentum representation):

$$u(\mathbf{p},\lambda\mathbf{n}) = \frac{1}{\sqrt{N}}\begin{pmatrix} \phi_{\lambda\mathbf{n}} \\ \frac{\mathbf{p}\sigma}{p_0+m}\phi_{\lambda\mathbf{n}} \end{pmatrix}$$

$$\frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}} = \lambda\phi_{\lambda\mathbf{n}}\,, \qquad N = \frac{2p_0}{p_0+m}$$

$\Rightarrow$

$$|j,j_z\rangle = \Phi_{jl_pj_z}(\omega) = \frac{1}{\sqrt{2\epsilon}}\begin{pmatrix} \sqrt{\epsilon+m}\Omega_{jl_pj_z}(\omega) \\ -\sqrt{\epsilon-m}\Omega_{j\lambda_pj_z}(\omega) \end{pmatrix}$$

$$\Omega_{jl_pj_z}(\omega) = \begin{pmatrix} \sqrt{\frac{j+j_z}{2j}}Y_{l_p,j_z-1/2}(\omega) \\ \sqrt{\frac{j-j_z}{2j}}Y_{l_p,j_z+1/2}(\omega) \end{pmatrix}; \quad l_p = j-\frac{1}{2},$$

$$\Omega_{jl_pj_z}(\omega) = \begin{pmatrix} -\sqrt{\frac{j-j_z+1}{2j+2}}Y_{l_p,j_z-1/2}(\omega) \\ \sqrt{\frac{j+j_z+1}{2j+2}}Y_{l_p,j_z+1/2}(\omega) \end{pmatrix}; \quad l_p = j+\frac{1}{2}$$

where ω represents the polar and azimuthal angles (θ,φ) of the momentum p with respect to the quantization axis, $l_p = j \pm 1/2$ and $\lambda_p = 2j - l_p$ (l_p defines parity).

New representation is convenient for general discussion about role of OAM. The rest frame of the composite system is a starting reference frame.

P. Z. Phys. Rev. D 89, 014012 (2014)

Spinor spherical harmonics $|j, j_z\rangle$

- SSH represent solutions of the free Dirac equation, which reflects the known QM rule:

In relativistic case spin and OAM are not decoupled (separately conserved), but only sums j and $j_z = s_z + l_z$ are conserved.

- However, one can always calculate the mean values of corresponding operators:

$$\boxed{s_z = \frac{1}{2}\begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}, \qquad l_z = -i\left(p_x \frac{\partial}{\partial p_y} - p_y \frac{\partial}{\partial p_x}\right)}$$

result:

$$\boxed{\langle s_z \rangle_{j,j_z} = \frac{1 + (2j+1)\,\mu}{4j\,(j+1)} j_z, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1 + (2j+1)\,\mu}{4j\,(j+1)}\right) j_z,}$$

where $\mu = m/\varepsilon$.

$j \geq 1/2$

$l_p = j\text{-}1/2$

Non-relativistic limit (*μ=1*):

$$\langle s_z \rangle_{j,j_z} = \frac{j_z}{2j}, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1}{2j}\right) j_z$$

Relativistic case (*μ → 0*):

$$\langle s_z \rangle_{j,j_z} = \frac{j_z}{4j\,(j+1)}, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1}{4j\,(j+1)}\right) j_z$$

$$\Downarrow$$

$$\left|\langle s_z \rangle_{j,j_z}\right| \leq \frac{1}{4\,(j+1)} \leq \frac{1}{6}, \qquad \frac{\left|\langle s_z \rangle_{j,j_z}\right|}{\left|\langle l_z \rangle_{j,j_z}\right|} \leq \frac{1}{4j^2 + 4j - 1} \leq \frac{1}{2}$$

… and for *j=1/2* :

$$\left|\langle s_z \rangle_{j,j_z}\right| = \frac{1}{6} \qquad \frac{\langle s_z \rangle_{j,j_z}}{\langle l_z \rangle_{j,j_z}} = \frac{1}{2}$$

Remark:
The ratio *μ=m/ε* plays a crucial role, since it controls a "contraction" of the spin component which is compensated by the OAM*.* It is an **QM effect of relativistic kinematics.**

In other words, lower component can play an important role!
cf. Bo-Qiang Ma, DSPIN2015 talk

Many-fermion states

Composition of one-particle states (SSH) representing composed particle with spin ***J=J_z=1/2:***

$$\boxed{\left|(j_1, j_2, ...j_n)_c J, J_z\right\rangle = \sum_{j_{z1}=-j_1}^{j_1} \sum_{j_{z2}=-j_2}^{j_2} ... \sum_{j_{zn}=-j_n}^{j_n} c_j \left|j_1, j_{z1}\right\rangle \left|j_2, j_{z2}\right\rangle ... \left|j_n, j_{zn}\right\rangle}$$

where c_j's consist of Clebsch-Gordan coefficients:

$$\boxed{c_j = \left\langle j_1, j_{z1}, j_2, j_{z2} \left| J_3, J_{3z} \right.\right\rangle \left\langle J_3, J_{z3}, j_3, j_{z3} \left| J_4, J_{z4} \right.\right\rangle ... \left\langle J_n, J_{zn}, j_n, j_{zn} \left| J, J_z \right.\right\rangle}$$

$$\langle \mathbb{S}_z \rangle_{c,1/2,1/2} = \langle s_{z1} + s_{z2} + ... + s_{zn} \rangle_c\,, \qquad \langle \mathbb{L}_z \rangle_{c,1/2,1/2} = \langle l_{z1} + l_{z2} + ... + l_{zn} \rangle_c$$

$$\langle \mathbb{S}_z \rangle_{c,1/2,1/2} + \langle \mathbb{L}_z \rangle_{c,1/2,1/2} = \frac{1}{2},$$

$$\boxed{|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6},} \qquad \boxed{\frac{|\langle \mathbb{S}_z \rangle|}{|\langle \mathbb{L}_z \rangle|} \leq \frac{1}{2}} \qquad \boxed{J_z = \langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}}$$

for ***μ → 0***

Spin structure functions: explicit form

For $Q^2 \gg 4M^2x^2$ we get (in terms of rest frame variables)

$x = Q^2/2Pq$

$$\boxed{\begin{aligned} g_1(x) &= \frac{1}{2}\int\left(\mathrm{u}(\epsilon)\left(p_1+m+\frac{p_1^2}{\epsilon+m}\right)+\mathrm{v}(\epsilon)\left(p_1-m+\frac{p_1^2}{\epsilon-m}\right)\right)\delta\left(\frac{\epsilon+p_1}{M}-x\right)\frac{d^3p}{\epsilon}, \\ g_2(x) &= -\frac{1}{2}\int\left(\mathrm{u}(\epsilon)\left(p_1+\frac{p_1^2-p_T^2/2}{\epsilon+m}\right)+\mathrm{v}(\epsilon)\left(p_1+\frac{p_1^2-p_T^2/2}{\epsilon-m}\right)\right)\delta\left(\frac{\epsilon+p_1}{M}-x\right)\frac{d^3p}{\epsilon}. \end{aligned}}$$

where u, v are functions related to the polarization tenzor, which is defined by the initial state $\Psi_{1/2}$

This result is exact for SFs generated by (free) many-fermion state **J=1/2** represented by the spin spherical harmonics.

For given state $\Psi_{1/2}$ we have checked calculation:

$$\boxed{\begin{aligned} \langle \mathbb{S}_z \rangle &= \left\langle \Psi_{1/2} \left| \mathbb{S}_z \right| \Psi_{1/2} \right\rangle = \left\langle s_{z1} + s_{z2} + \ldots + s_{zn} \right\rangle \\ \Gamma_1 &= \int_0^1 g_1(x)\, dx \end{aligned}}$$

give equivalent results! ✓

Proton spin structure

The SSH formalism can be used for proton description in conditions of DIS. We assume:

- The proton state can be at each Q^2 represented by a superposition of Fock states:

$$\boxed{\Psi = \sum_{q,g} a_{qg} \left|\varphi_1, \ldots \varphi_{n_q}\right\rangle \left|\psi_1, \ldots \psi_{n_g}\right\rangle}$$

- In a first step we ignore possible contribution of gluons, then:

$$\boxed{\Psi = \sum_{q} a_{q} \left|\varphi_1, \ldots \varphi_{n_q}\right\rangle}$$

where the quark states $\left|\varphi_1, \ldots \varphi_{n_q}\right\rangle$ are represented by eigenstates:

$$\boxed{J = J_z = \langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}}$$

Proton spin content

We have shown the system **J=1/2** composed of (quasi) free fermions $\mu \to 0$ satisfies:

$$\boxed{|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6},}$$

(or the same in terms of Γ_1)

Reduced spin is compensated by OAM

$$\boxed{\langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}}$$

and equality takes place for a simplest configuration:

$$\boxed{\jmath_1 = \jmath_2 = \jmath_3 = \ldots = \jmath_{n_q} = \frac{1}{2}}$$

If we change notation

$$\boxed{|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6},} \quad \Rightarrow \quad \boxed{\Delta\Sigma \lesssim 1/3}$$

this result is well compatible with the data
(cf. experiments [30-32]):

$$\boxed{\Delta\Sigma = 0.32 \pm 0.03(stat.)}$$

[30] M. G. Alekseev et al. [COMPASS Collaboration], Phys. Lett. B 693, 227 (2010)].
[31] V. Y. Alexakhin et al. [COMPASS Collaboration], Phys. Lett. B 647, 8 (2007) .
[32] A. Airapetian et al. [HERMES Collaboration], Phys. Rev. D 75, 012007 (2007).
[33] C. Adolph et al. [COMPASS Collaboration], Phys. Lett. B 718, 922 (2013) .
[34] A. Airapetian et al. [HERMES Collaboration], JHEP 1008, 130 (2010) .

Role of gluons in proton spin

- Until now we assumed the simplest scenario: $\mu = m/\varepsilon \to 0$ and $\mathbf{J_g}=0$, which gave $\Delta\Sigma \approx 1/3$. This complies with the data very well, for both, quarks and gluons.
- However, the recent data from RHIC can suggest $\mathbf{J_g}>0$. Such value does not contradict our approach. If one admits also $\mu = m/\varepsilon > 0$, then instead of

$$\boxed{|\langle \mathbb{S}_z^q \rangle| = \frac{1}{6} \qquad \frac{\langle \mathbb{S}_z^q \rangle}{\langle \mathbb{L}_z^q \rangle} = \frac{1}{2}}$$

we have

$$\boxed{|\langle \mathbb{S}_z^q \rangle| = \frac{1+2\tilde{\mu}}{6} \qquad \frac{\langle \mathbb{S}_z^q \rangle}{\langle \mathbb{L}_z^q \rangle} = \frac{1+2\tilde{\mu}}{2-2\tilde{\mu}} \qquad J^q = \langle \mathbb{S}_z^q \rangle + \langle \mathbb{L}_z^q \rangle \qquad \tilde{\mu} = \left\langle \frac{m}{\epsilon} \right\rangle}$$

At the same time:

$$\boxed{\frac{1}{2} = J^q + J^g} \quad \Rightarrow \quad \boxed{\Delta\Sigma = \frac{1}{3}\left(1 - 2J^g\right)\left(1 + 2\tilde{\mu}\right)}$$

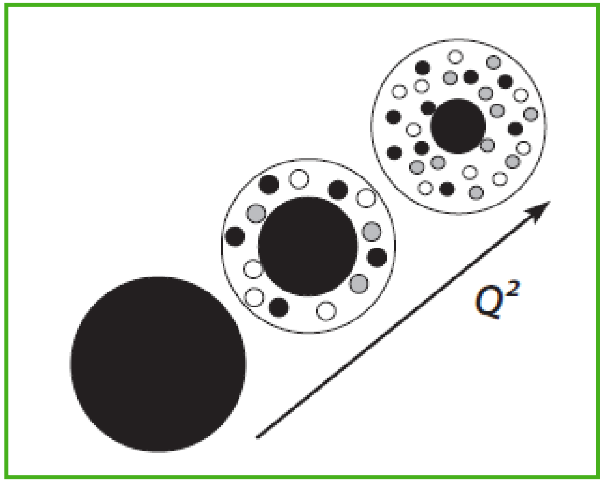
for details see P.Z. Phys. Lett. B 751, 525 (2015).

SPIN OF THE PARTICLE IN ITS SCALE DEPENDENT PICTURE

Two questions:

- How much do the virtual particles surrounding bare particle contribute to the spin of corresponding real, dressed particle?

- How much do the virtual particles mediating binding of the constituents of a composite particle contribute to its spin?



The **electron**, as a Dirac particle, in its rest frame has AM defined by its spin, $s = 1/2$. This value is the same for the dressed electron (as proved experimentally) and for the bare one (as defined by the QED Lagrangian).

So, can the AM contribution of virtual cloud $J^{\gamma}(Q^2)$ differ from zero and how much?

For similarly motivated studies see:
Bo-Qiang Ma; talk for DSPIN-15
Tianbo Liu, Bo-Qiang Ma; Phys.Rev. D91 (2015) 017501
S. J. Brodsky, Dae Sung Hwang, Bo-Qiang Ma, I. Schmidt ; Nucl. Phys. B 593 (2001) 311–335
Matthias Burkardt and Hikmat BC; Phys.Rev. D79 (2009) 071501(R)
Xinyu Zhang, Bo-Qiang Ma; Phys.Rev. D85 (2012) 114048

Semiclassical calculation of stationary electromagnetic field in the frame defined by spinor spherical harmonic:

$$\Phi_{jl_pj_z}(\mathbf{r}) = \frac{1}{\sqrt{2\epsilon}} \begin{pmatrix} \sqrt{\epsilon+m}R_{kl_p}\Omega_{jl_pj_z}(\omega) \\ -\sqrt{\epsilon-m}R_{k\lambda_p}\Omega_{j\lambda_pj_z}(\omega) \end{pmatrix}$$

Our reference frame is the rest frame of the composite system of these states.

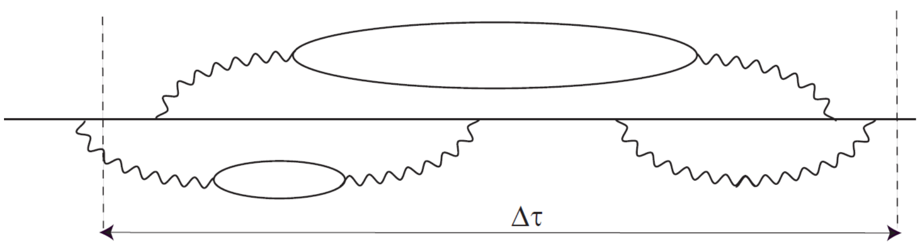
$$I_\mu = (I_0, \mathbf{I}) = \Phi^\dagger_{jl_pj_z}(\mathbf{r})\gamma^0\gamma_\mu\Phi_{jl_pj_z}(\mathbf{r})$$

$$\mathbf{E}(\mathbf{r}) = \int I_0(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^{3/2}}d^3\mathbf{r}'$$

$$\mathbf{H}(\mathbf{r}) = \int \mathbf{I}(\mathbf{r}')\times\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^{3/2}}d^3\mathbf{r}'$$

$$\mathbf{J}^\gamma = \int \mathbf{r}\times(\mathbf{E}\times\mathbf{H})d^3\mathbf{r}, \quad \Rightarrow \quad \boxed{\mathbf{J}^\gamma = 0}$$

This result represents a mean value, which is not influenced by the fluctuations generated by single γ.



Can we do a similar calculation for the color field ?

Summary

Covariant approach:

- **Constrains on LI & RS are crucial!**
- **TMDs: relations, calculation, predictions, QCD evolution…**
- **Interplay of spin & OAM, role of gluons…**
- **Agreement with the data, particularly as for $\Delta\Sigma$, is a strong argument for this approach**

Thank you for your attention!