

Nuclear transparency of small-size configurations

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- 1 Motivation
- 2 Transparencies in a Glauber Model (Ghent group)
 - Model
 - Applications and Results
 - Density Dependence
- 3 Semi-inclusive DIS off deuteron (w M. Sargsian)
 - Model: Ingredients and approximations
 - Comparison with Deeps
 - Q^2 , W evolution of rescattering parameters
- 4 Conclusions

Motivation

- Look for phenomena predicted in QCD that introduce **deviations** from traditional nuclear physics observations
- the nuclear transparency as a function of a tunable scale parameter (t or Q^2) is a good quantity to study the crossover between the two regimes

Nuclear transparency: effect of nuclear attenuations on escaping hadrons

$$T(A, Q^2) = \frac{\text{cross section on a target nucleus}}{A \times \text{cross section on a free nucleon}}$$

- Onset of **color transparency** (Brodsky, Mueller) will show as a rise in T
- Interpretation of the transparency experiments requires the availability of reliable and advanced traditional nuclear-physics calculations to compare the data with

Building a Model



- * To interpret the data from experiments, comparison to results from up-to-date **nuclear models** is necessary to identify deviations originating from QCD effects
- * Semi-classical models are available
- * Develop a **relativistic and quantum mechanical** model

Ingredients

- **Relativistic wave functions** for beam, target and residual nucleus, outgoing particles
- **Impulse approximation**: incoming particle (leptonic or hadronic) interacts with one nucleon
- Describe the **final state interactions** of the ejected particles with **Glauber scattering theory**

NPA A728 (2003) 226



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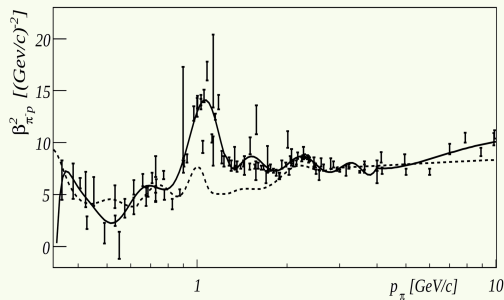
NPA **A728** (2003) 226

Glauber scattering theory

- Uses the **eikonal approximation**, originating from optics:
$$\phi_{\text{out}}(\vec{r}) = e^{i\chi(\vec{r})} \phi_{\text{in}}(\vec{r}) = (1 - \Gamma(\vec{r})) \phi_{\text{in}}(\vec{r})$$
- Works when the wavelength of the particle is a lot smaller than the range of the scattering potential → **OK** for the performed experiments!
- Particles scatter over small angles and follow a **linear trajectory**
- Second order eikonal corrections have been computed → **small**

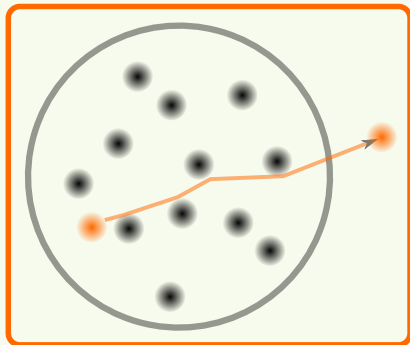
Profile function in $N - N$ and $\pi - N$ scattering

$$\Gamma_{\pi N}(\vec{b}) = \frac{\sigma_{\pi N}^{\text{tot}}(1 - i\epsilon_{\pi N})}{4\pi\beta_{\pi N}^2} \exp\left(-\frac{\vec{b}^2}{2\beta_{\pi N}^2}\right)$$



- Profile function can be related to the scattering amplitude
- Three energy-dependent parameters
 - ▶ total cross section
 - ▶ slope parameter
 - ▶ real to imaginary ratio
- Fit parameters to $N - N$ and $\pi - N$ scattering data
- range $\sqrt{2}\beta$ is of the order 0.75 fm \rightarrow short range

Relativistic Multiple-Scattering Glauber Approximation



Multiple scattering

- Frozen approximation is adopted

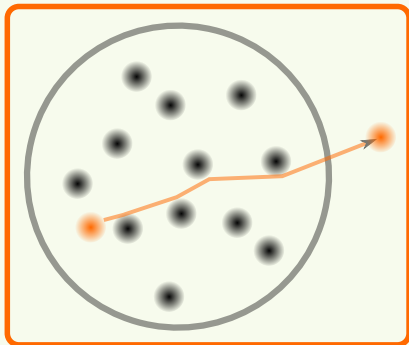
- Phase-shift additivity

$$e^{i\chi_{\text{tot}}} = \prod_i \left(1 - \Gamma_i(\vec{b}_i) \right)$$

- Profile functions are weighted with the Dirac wave function
- Only nucleons in forward path contribute

$$G(\vec{b}, z) = \prod_{\alpha_{\text{occ}} \neq \alpha} \left[1 - \int d\vec{r}' |\phi_{\alpha_{\text{occ}}}(\vec{r}')|^2 \left[\theta(z' - z) \Gamma(\vec{b}' - \vec{b}) \right] \right]$$

Relativistic Multiple-Scattering Glauber Approximation



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Implementing Short-Range Correlations

- In standard Glauber: effect of intranuclear attenuations is computed as if the density remains unaffected by the presence of a nucleon at $\vec{r} = (\vec{b}, z)$
- $\sqrt{2}\beta \sim 0.75\text{fm} \rightarrow$ attenuations will be mainly affected by the **short-range structure** of the transverse density in the residual nucleus
- Mean field does not contain repulsive short-range behavior of the $N - N$ force
- Introduce **correlated** two-body density

$$\rho_A^{[2]}(\vec{r}', \vec{r}) = \frac{A-1}{A} \gamma(\vec{r}) \rho_A^{[1]}(\vec{r}) \gamma(\vec{r}') \rho_A^{[1]}(\vec{r}') g(|\vec{r} - \vec{r}'|)$$

- $\gamma(\vec{r})$ ensures normalization

Color Transparency: Quantum diffusion parametrization

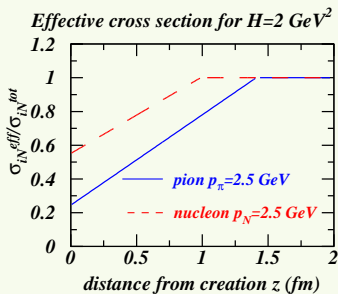
$$\sigma_{iN}^{\text{eff}}(z) = \sigma_{iN}^{\text{tot}} \left\{ \left[\frac{z}{l_h} + \frac{\langle n^2 k_t^2 \rangle}{\mathcal{H}} \left(1 - \frac{z}{l_h} \right) \theta(l_h - z) \right] + \theta(z - l_h) \right\} \quad i = \pi \text{ or } N.$$

- Replace the total cross section with an **effective** one
- Parameters are based on theoretical grounds but values are **educated guesses**
- Pion cross section is more strongly reduced and formation length is longer

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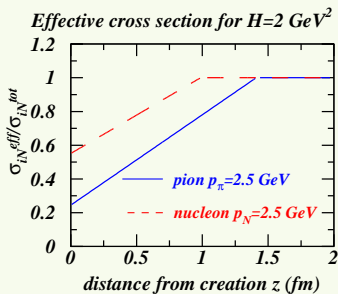
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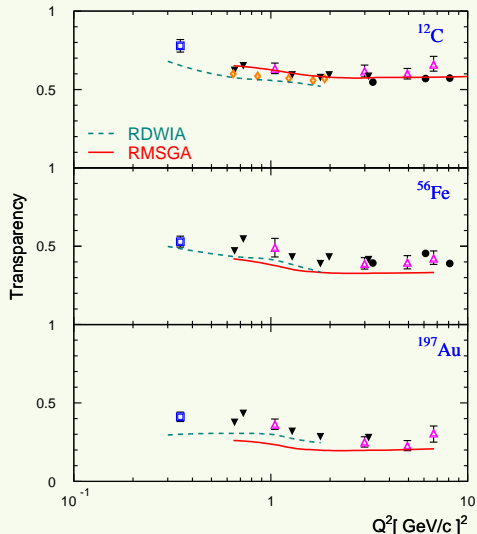
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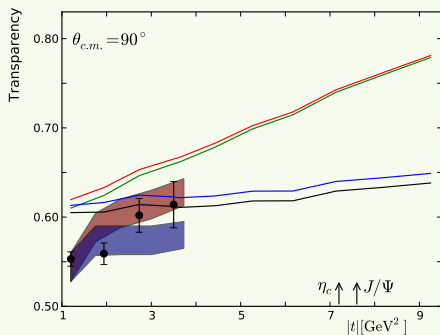
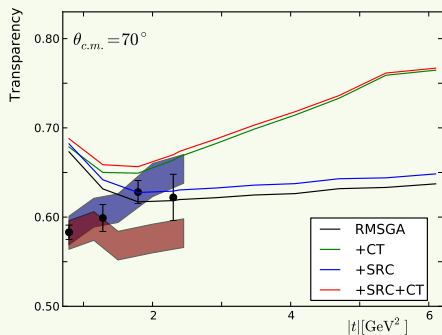
The nuclear transparency from $A(e, e'p)$



P. Lava et al. PLB595 (2004), 177-186

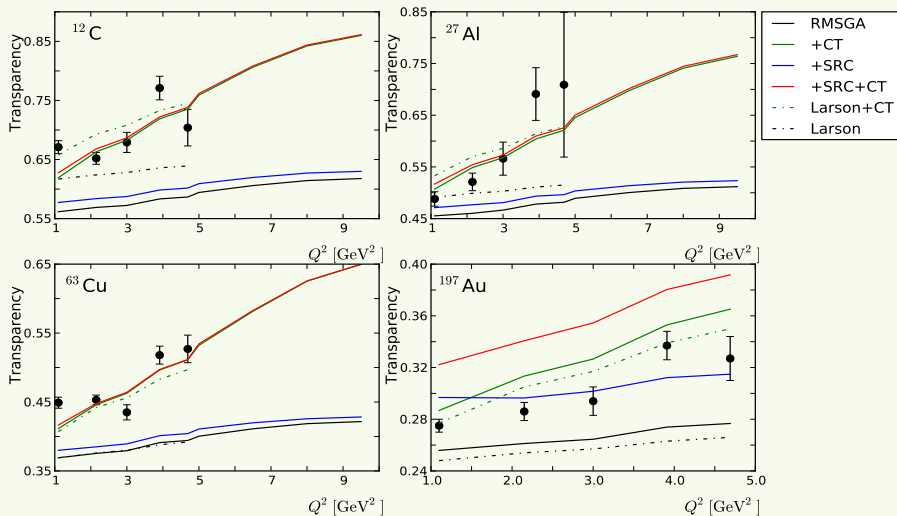
- Calculations tend to underestimate the measured proton transparencies
- In the region of overlap: RMSGA and RDWIA predictions are **not dramatically different !!**
- Data from **MIT**, **JLAB** and **SLAC**
- CT effects are very small for $Q^2 \leq 10 \text{ GeV}^2$

$^4\text{He}(\gamma, p\pi^-)$ transparencies



- **Theory:** W. Cosyn et al., PRC74 (2006) 062201
- **Data:** D. Dutta et al., PRC68 (2003) 021001
- **Semiclassical theory:** H. Gao et al., PRC54 (1996) 2779 [normalized to first data point]

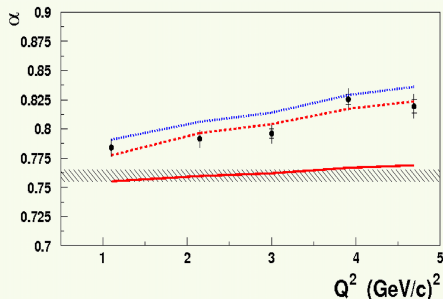
$A(e, e' \pi^+)$ transparencies: Q^2 dependence



$A(e, e' \pi^+)$ data from JLab, B. Clasie *et al.*, PRL99 (2007) 242502

Dashed lines from semi-classical calc. by A. Larson *et al.*, PRC79 (2006) 018201

$A(e, e'\pi^+)$ transparencies: A dependence



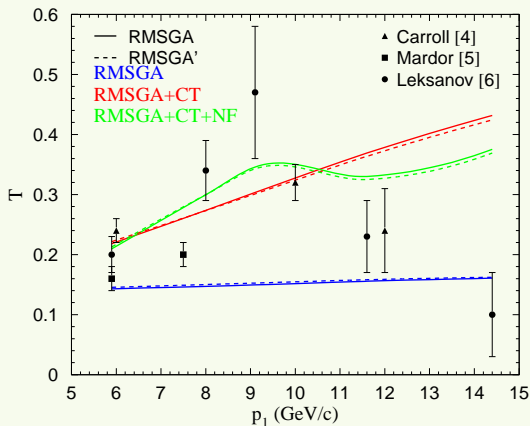
GI.+SRC+CT

Semi-classical Larson

Hatched area: value from $\pi - A$ scatt.

- Parametrize $T = A^{\alpha-1}$
- Clear Q^2 dependence, deviates from expected value
- Models in good agreement

The nuclear transparency from $^{12}\text{C}(p, 2p)$



Parameterization of the CT effects compatible with pion production results!

Density Dependence

The RMSGA model provides an excellent basis to study the density dependence of removal reactions

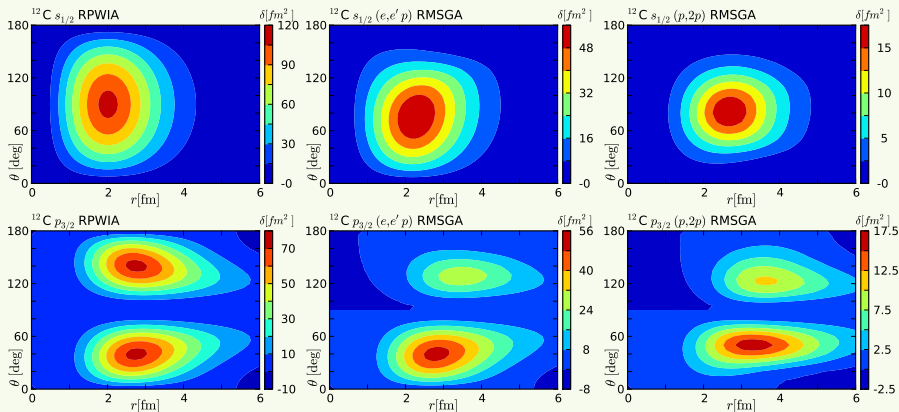
- Variety of reactions
- Scattering parameters are relatively smooth above 1 GeV \rightarrow universal statements
- Density dependence of the attenuation will determine the **effective nuclear density** which can be probed

- * Compare $A(e, e'p)$ (1 proton), $A(\gamma, pp)$ (2 protons) and $A(p, 2p)$ (3 protons) on ^{12}C and ^{56}Fe
- * Outgoing particles have 1.5 GeV kinetic energy

J. Ryckebusch & WC arXiv:1102.0905

WC & JR, PRC80:011602 (2009)

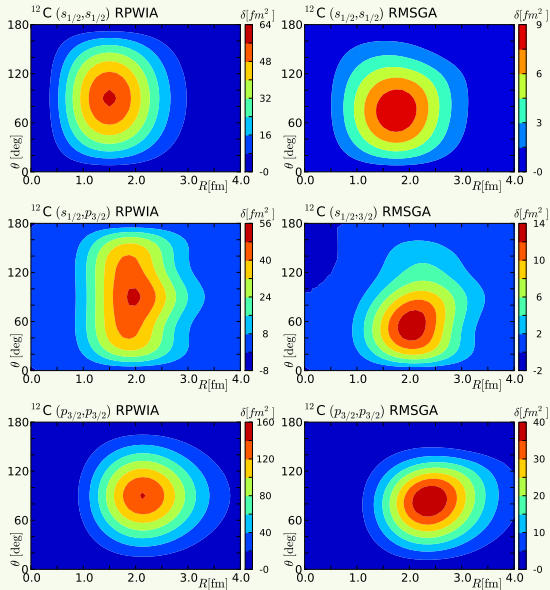
Density Dependence: $^{12}\text{C}(e, e'p)$ and $^{12}\text{C}(p, 2p)$



rms radius of $^{12}\text{C} \rightarrow 2.464 \pm 0.012$ fm

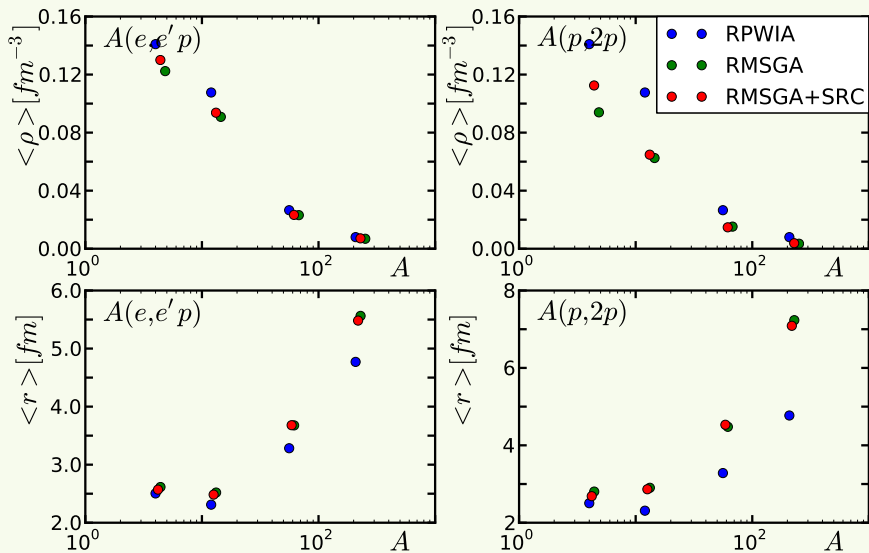
- FSI shift contributions to larger r and upper hemisphere
- Larger effect for $A(p, 2p) \rightarrow$ surface is probed

Density Dependence: $^{12}\text{C}(\gamma, pp)$

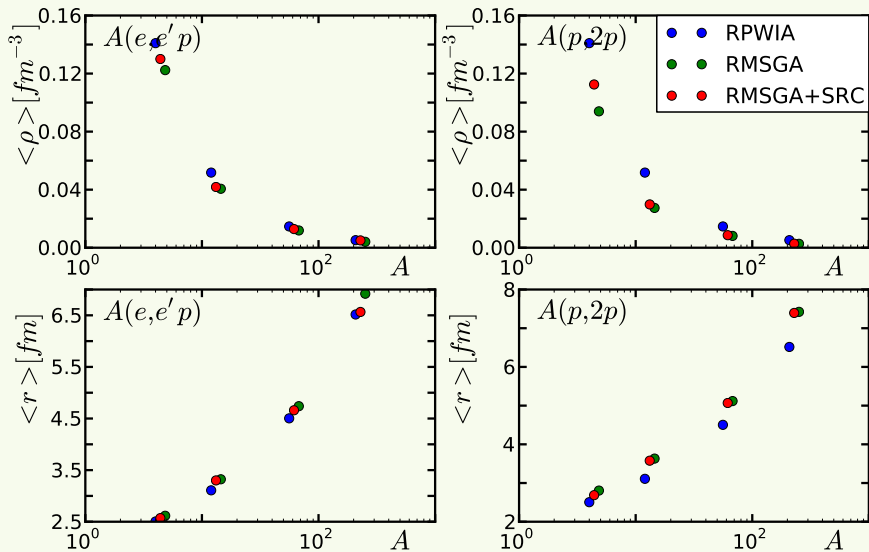


- Knockout of a correlated pair
- Strength remains in the high density regions of the nucleus

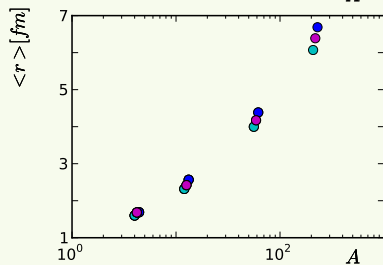
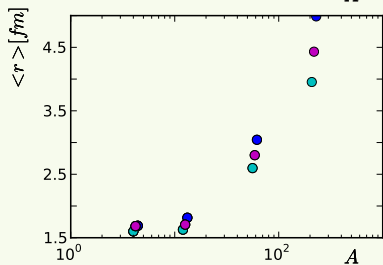
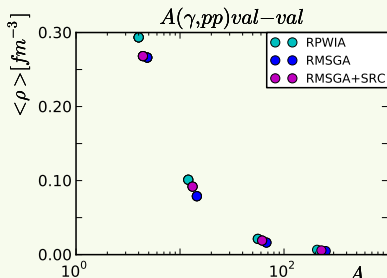
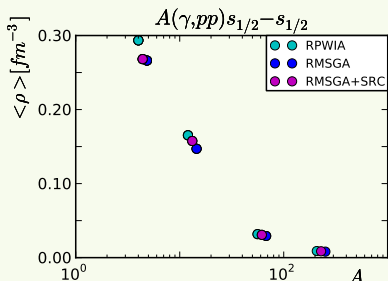
A dependence 1 nucl knockout: s-shell



A dependence 1 nucl knockout: valence shell



A dependence 2 nucl knockout

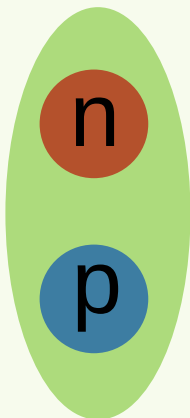


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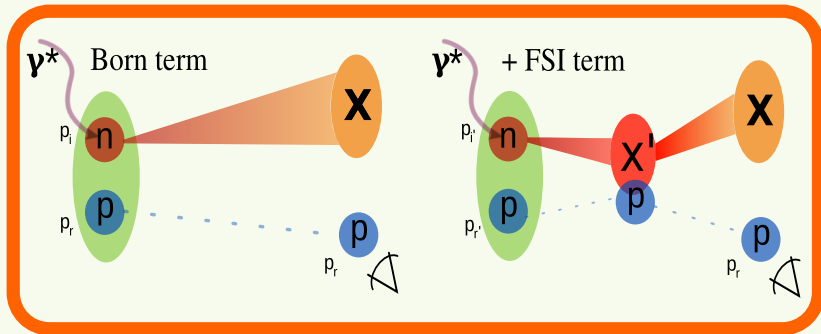
- Quantify effect of FSI
- $X?$: details about composition and space-time evolution (function of (x, Q^2)) of produced hadronic system after DIS unknown
- Use general properties of soft scattering theory, without specifying X
- Factorized approach: split photon interaction and rescattering part
In $D(e, e'N)N$: works well up to $p_s \approx 400$ MeV

Virtual Nucleon Approximation



- Consider only pn component of Deuteron
- Spectator proton is **on-shell**
- Deuteron wf normalization obeys baryon number conservation $\int \alpha |\Phi_D(p)|^2 d^3p = 1$, but violates momentum sum rule $\int \alpha^2 |\Phi_D(p)|^2 d^3p < 1$
- Neglect negative energy contribution of virtual neutron propagator
→ $p_s \leq 700$ MeV
- Photon interactions with exchanged mesons are neglected
→ $Q^2 > 1\text{GeV}^2$

Reaction diagrams



$$\frac{d\sigma}{dx dQ^2 d\phi_{e'}} \frac{d^3 p_S}{2E_S (2\pi)^3} = \frac{2\alpha_{EM}^2}{xQ^4} \left(1 - y - \frac{x^2 y^2 m_n^2}{Q^2}\right) \left(F_L^D(x, Q^2) + v_T F_T^D(x, Q^2) + v_{TL} \cos \phi F_{TL}^D(x, Q^2) + \cos 2\phi F_{TT}^D(x, Q^2) \right)$$

Factorization

- Relate Deuteron structure functions to the neutron ones for a moving nucleon at $\hat{x} = \frac{Q^2}{2p_i \cdot q} \approx \frac{x}{2-\alpha_s} \dots$

$$F_T^D(x, Q^2) = [2F_{1N}(\hat{x}, Q^2) + \frac{p_T^2}{m_i \hat{\nu}} F_{2N}(\hat{x}, Q^2)] \times S^D(p_r) (2\pi)^3 2E_r$$

- ...times a **distorted spectral function** that contains a **plane-wave** and **FSI** part

$$S^D(p_r) = \frac{1}{3} \sum_{M, S_r, S_s} \left| \overbrace{\Phi_D^M(p_i s_i, p_s s_s)}^{PW} - \int \overbrace{\frac{d^3 p_{s'}}{(2\pi)^3} \chi(p_{s'}, m_{s'}) \langle p_r X | \mathcal{F} | p_{s'} X' \rangle \frac{\Phi_D^M(p_i' s_i, p_{s'} s_s)}{(p_{s'}^z - p_s^z + \Delta')}}^{FSI} \right|^2$$

FSI: Generalized eikonal approximation

- Scattering amplitude is parametrized with the standard diffractive form

$$\langle p_r, X | \mathcal{F} | p_{r'}, X' \rangle = \sigma_{\text{tot}}(W, Q^2) (i + \epsilon(W, Q^2)) e^{\frac{\beta(W, Q^2)}{2} t} \delta_{s_r, s_{r'}} \delta_{s_X s_{X'}}$$

- Eikonal regime gives approximate conservation law $p_s^- = p_{s'}^-$ in the high q limit. This leads to $m_X^2 > m_{X'}^2$, and yields pole values in the FSI integral of

$$\Delta' = \frac{\nu + M_D}{|\vec{q}|} (E_s - m_p) + \frac{m_X^2 - m_{X'}^2 (p_{i'} = 0)}{2 |\vec{q}|} \quad \text{for } m_{X'}^2 (p_{i'} = 0) \leq m_X^2,$$
$$\Delta' = \frac{\nu + M_D}{|\vec{q}|} (E_s - m_p) \quad \text{for } m_{X'}^2 (p_{i'} = 0) > m_X^2.$$

Comparison with Deeps: approach

- Use **SLAC parametrization** for neutron structure functions (as in Deeps data analysis)
- Take $\sigma_{\text{tot}}(W, Q^2)$ (and $\beta(W, Q^2)$) as **free parameter** in the distorted spectral function. Fits are done for each W, Q^2 over the 5 measured spectator momenta (300-560 MeV).

- Deuteron wave function: $\Phi_D(p) = \Phi_D^{\text{NR}}(p) \sqrt{\frac{M_D}{2(M_D - E_s)}}$
Obeys baryon number conservation $\int \alpha |\Phi_D(p)|^2 d^3p = 1$

Parametrization of the off-shell rescattering amplitude

Three approaches:

- **no off-shell FSI**: off-shell rescattering amplitude is zero

$$f_{X'N, XN}^{\text{off}} \equiv 0$$

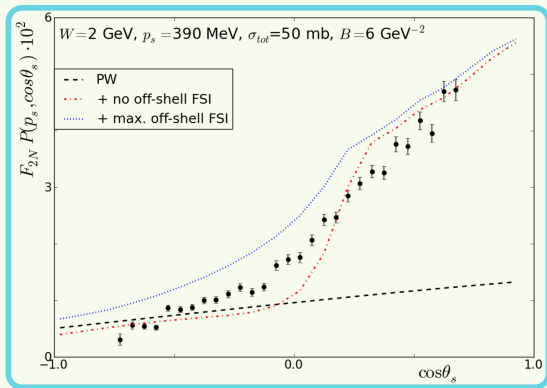
- **maximum off-shell FSI**: off-shell amplitude is taken equal to the on-shell one

$$f_{X'N, XN}^{\text{off}} = f_{X'N, XN}^{\text{on}}$$

- **fitted off-shell FSI**: off-shell amplitude is parametrized as the on-shell one with a suppression factor dependent on (x, Q^2)

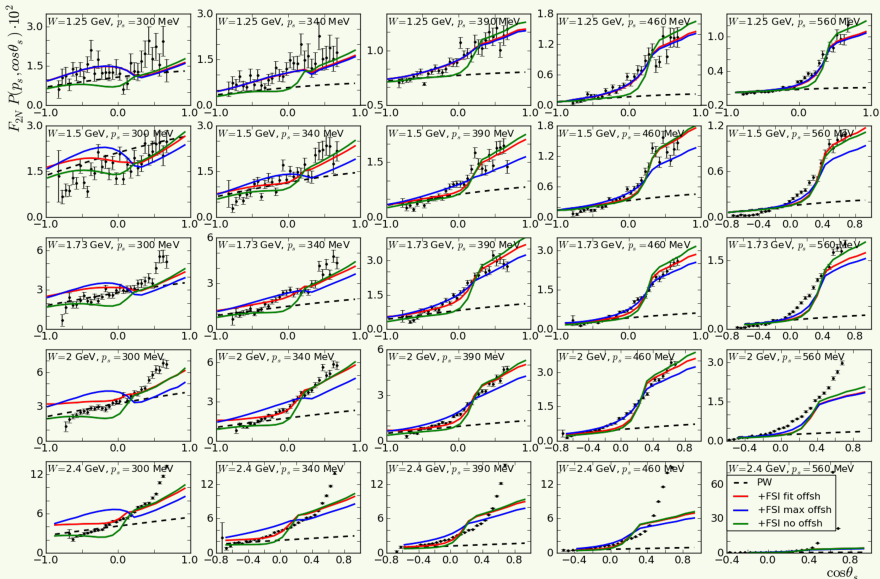
$$f_{X'N, XN}^{\text{off}} = f_{X'N, XN}^{\text{on}} e^{-\mu(x, Q^2)t}$$

Calculation without fits

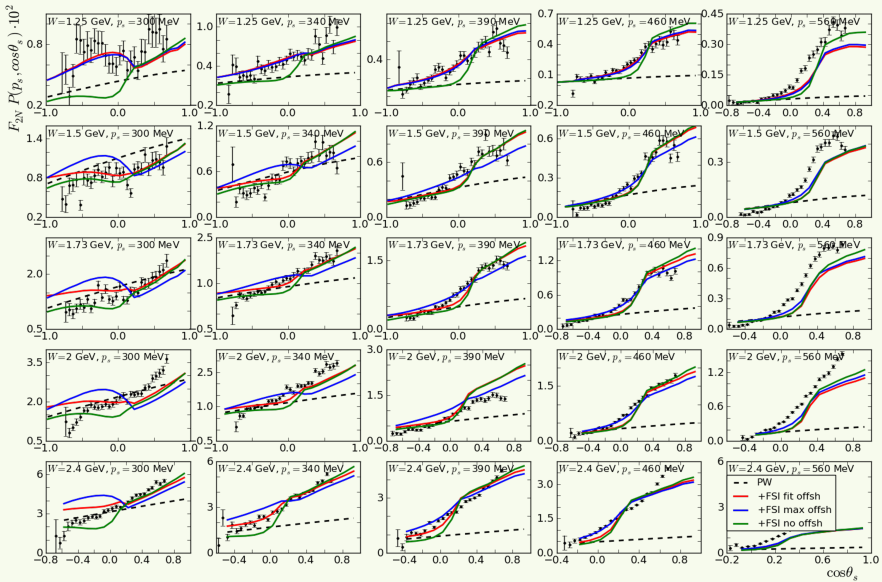


- Plain-wave calculation shows little dependence on spectator angle
- FSI effects **grow** in forward direction, different from quasi-elastic case
- **Small** contribution from off-shell amplitude

$Q^2 = 1.8\text{GeV}^2$: σ and β free, $\epsilon = -0.5$



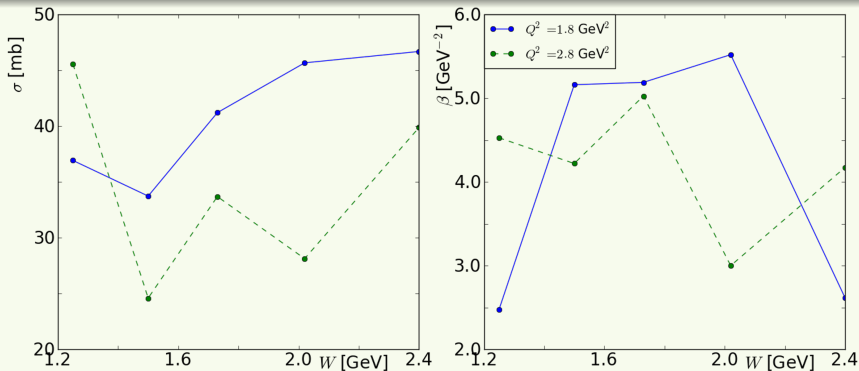
$Q^2 = 2.8\text{GeV}^2$: σ and β free, $\epsilon = -0.5$



Results discussion

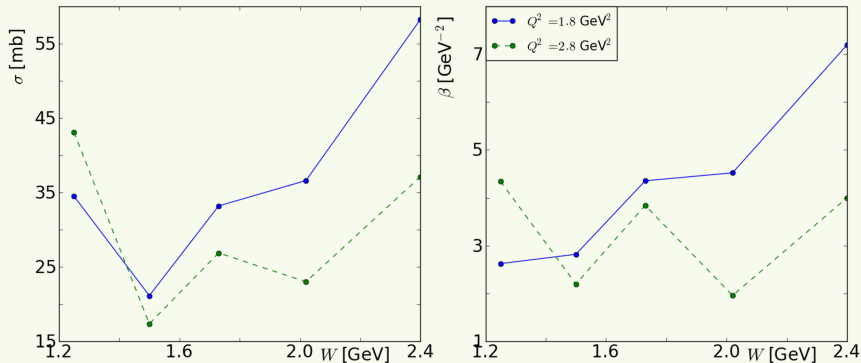
- Systematic **underestimation** of data at $p_s = 560$ MeV, breakdown of factorization
- Difference between off-shell descriptions **diminishes** with increasing p_s
- At lowest spectator momentum plain-wave and FSI amplitude **comparable in magnitude**, sensitive to small differences
- Fitted off-shell calculations correspond more with no off-shell ones, pointing to **suppressed** off-shell amplitude

No off-shell FSI



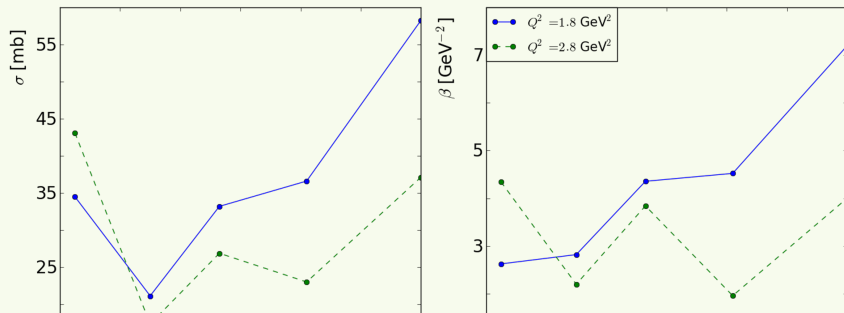
- σ rises with W , no sign of hadronization plateau
- σ drops with Q^2 , small-sized configuration?
- β largely correlated with σ

Max off-shell FSI



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Max off-shell FSI



- More measurements at higher Q^2 needed to make more definite statements
- These values can be used as input in the computation of FSI effects in inclusive DIS

Conclusions



A “flexible” eikonal framework to model the propagation of fast nucleons and pions through the nuclear medium

- Glauber approach computes full $(A - 1)$ multiple-scattering series and has no free parameters
- Provides common framework to describe a variety of nuclear reactions with electroweak and hadronic probes.
- Effect of central short-range correlations and color transparency can be implemented. The two can be clearly separated in results, due to different hard scale dependence
- Pion electroproduction data in agreement with CT calculations, Fair results for $A(p, pN)$
- Knockout of a correlated pair and s -shell knockout in $A(p, 2p)$ probe the high density regions of the nucleus

Conclusions (II)



- Model for semi-inclusive DIS on the deuteron based on general properties of soft rescattering.
- Fair description of the Deeps data
- Discrepancies at $p_s = 300$ MeV with high W . Possible breakdown of factorization at highest $p_s = 560$ MeV.
- Cross section rises with W and shows no signs of a plateau (hadronization) yet, drops with Q^2 .
- More measurements at higher Q^2 needed