High t form factors & Compton Scattering - quark based models

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Basic Philosophy - model wave function

• Given compute form factors, densities, Compton scattering ....
• Make guess at how QCD works, improve guess, rule out simple scenarios
• Non-relativistic quark model
• 3 quarks
• 0 orbital angular momentum
• proton is round
• What can Compton scattering say?
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Expectations- Pre Jlab

\[
\frac{G_E}{G_M} \text{ constant : non - relativistic quark model}
\]
Expectations- Pre Jlab

\[ \frac{G_E}{G_M} \text{ constant: non-relativistic quark model} \]
\( \frac{G_E}{G_M} \) constant: non-relativistic quark model

\[ \frac{QF_2}{F_1} \]

\[ \frac{G_E}{G_M} \]

\( Q^2 \)
\[ \frac{G_E}{G_M} \text{ constant} : \text{non-relativistic quark model} \]

Relativistic model needed - light front coordinates
Understand phenomena-model

Model proton wave function: 3 quarks

Lorentz and rotationally invariant

Light front variables

Dirac spinors - orbital angular momentum


Theory 1995 Data 2000

Quark spin is 75% of proton total angular momentum
Neutron- requires pion cloud

Improved model - Cloet & Miller ’11 20

Model proton wave function: quark-diquark

Lorentz and rotationally invariant-different forms!

Light front variables

Dirac spinors-orbital angular momentum

Quark spin is 35% of proton total angular momentum

Venkat (2010)

Cloet and Miller 2011

Wednesday, March 23, 2011
Shapes of the proton- momentum space spin-dependent-densities

three vectors $n, K, S$

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MODEL, HOW TO MEASURE? How to compute fundamentally?

Measure $h_{1T}^\perp : e + p(\uparrow) \rightarrow e'\pi X$

3 vectors:
Spin direction, photon direction, hadron direction

TMD- is a momentum-space spin-dependent-density

Cross section has term proportional to $\cos 3\phi$

Boer Mulders '98

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Boer Mulders ’98


Wednesday, March 23, 2011
Nucleon and quarks both polarized

\[ \rho^\Gamma(b) = \sum e_q \int dx^- q_+(x^-, b) \gamma^+ \Gamma q_+(x^-, b) \]

\[ \Gamma = \frac{1}{2} \left(1 + n \cdot \gamma \gamma_5 \right) \text{ gives spin-dependent density} \]

Transverse Spin Structure of the Nucleon from Lattice-QCD Simulations


spin-dependent density - depends on direction of \( b \): proton is not round
Handling the handbag diagram in Compton scattering on the proton

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Poincaré invariance, gauge invariance, conservation of parity, and time reversal invariance are respected in an impulse approximation evaluation of the handbag diagram. Proton wave functions, previously constrained by comparison with measured form factors, that incorporate the influence of quark transverse and orbital angular momentum (and the corresponding violation of proton helicity conservation) are used. Computed cross sections are found to be in reasonably good agreement with early measurements. The helicity correlation between the incident photon and outgoing proton, $K_{LL}$, is both large and positive at back angles. For photon laboratory energies of $\approx 6$ GeV, we find that $K_{LL} \neq A_{LL}$, and $D_{LL} \neq 1$.

Wave function supplies amplitudes for on-mass shell quarks, CC respected.
Technical aspects

- Transverse momentum of quarks included
- Photon momenta are transverse, no boosts
- No energy transfer

\[ \mathcal{M}_{s',s}(\epsilon', \epsilon) = \rho \otimes \mathcal{O} \]

\[
\rho_{s',s';s,s}(\eta, K'_\perp, K_\perp) = \int d\xi \; d^2k_\perp \; \Psi^\dagger_{s',s'}(\xi, k_\perp, \eta, K'_\perp) \Psi_{s,s}(\xi, k_\perp, \eta, K_\perp) 
\]

\[ \mathcal{O} = \]

\[ K'_\perp = k_\perp + (1 - \eta)(q'_\perp - q_\perp) \]
Technical II

- $S', S, \varepsilon, \varepsilon'$, 16 amplitudes
- 6 independent, challenge to calc'n
- transform to helicity basis,
- $\lambda$ nucleon helicity, $\mu$ photon helicity

\[
\frac{d\sigma}{dt} = \frac{1}{64\pi(s - m^2)^2} \Sigma_{\mu, \mu', \lambda, \lambda'} |\Phi_{\mu', \lambda', \mu\lambda}|^2.
\]

\[
A_{LL} \frac{d\sigma}{dt} = \frac{1}{2} \left[ \frac{d\sigma(\mu = +, \lambda = +)}{dt} - \frac{d\sigma(\mu = +, \lambda = -)}{dt} \right].
\]

\[
K_{LL} \frac{d\sigma}{dt} = \frac{d\sigma(\mu = +, \lambda' = +)}{dt} - \frac{d\sigma(\mu = +, \lambda' = -)}{dt},
\]

$A_{LL} \neq K_{LL}$
One set involves both photon and proton helicities. The correlation, where

\[ \frac{d\sigma}{dt} \]

obtained using the parameter sets of Fig. 2.

We find that the spin-flip matrix elements

\[ M_{LL} \]

vanish, or if their sum vanishes.

In our model, the role of orbital angular momentum and nonconservation of the proton helicity is the crucial aspect in the equality

\[ dt \]

raises and lowers spins with exact strength, so that

\[ A_{LL}, A_{LL} \]

represents a handbag diagram. Proton wave functions, previously suspected in our impulse approximation evaluation of the one for the sideways polarization of the outgoing proton is obtained by increasing the quark mass.

\[ \theta_{c.m.} (\text{deg}) \]

shows that the spin-flip matrix elements

\[ K_{LL} \]

are used. Computed cross sections are in reasonably good agreement with early measurements. The value of\[ \theta_{c.m.} (\text{deg}) \]

is large and positive for scattering at large angles. In contrast to aproach, we find that

\[ K_{LL}, A_{LL} \]

do not vanish, and that the terms

\[ \text{with earlier work, we find that} \]

leaves the left-hand side of the equality

\[ \text{A} \]

invariant...
The equality of \( H_{9021}^2 \) and \( H_{9021}^6 \) can only occur if each of the proton spin flip matrix elements \( M^+ \), \( M^- \) vanish, or if their sum vanishes. Inspection of Eq. (3) shows that the spin-flip matrix elements do not vanish, and that the terms \( H_{9253} \cdot H_{20849}^q \cdot H_{20850}^q = H_{9253}^0 \cdot H_{20849}^2 \cdot H_{20850}^6 \) lead to operators \( H_{20849}^1 \), \( H_{9268} \), \( x \), \( H_{20850}^1 \) evaluated between Pauli spinors. The operator \( H_{9268} \), \( x \) raises and lowers spins with exact strength, so that \( M^+ \), \( M^- \) does not vanish, \( H_{9021}^2 \), \( H_{11005} \), and \( A_{LL} \), \( K_{LL} \).

Another way to understand this inequality is to examine the numerical effect of reducing the quark mass towards 0. We find that this causes \( A_{LL} \) to approach \( K_{LL} \).

There are other polarization variables \([10]\). The helicity transfer from the incoming to the outgoing photon is given by:

\[
D_{LL} \frac{d\sigma}{dt} = \frac{d\sigma(\mu = +, \mu' = +)}{dt} - \frac{d\sigma(\mu = +, \mu' = -)}{dt}^{14}
\]

Reference \([10]\) finds that \( D_{LL} \) is large and positive for scattering at large angles. Now consider sideways proton spin directions. The correlation between the helicity of the incoming photon and the sideways \( S \), \( S \) polarization of the incoming proton, parallel or antiparallel to the \( S \)-direction is defined \([10]\) as \( A_{LS} \), and the one for the sideways polarization of the outgoing proton is \( K_{LS} \). We find \( K_{LS} = 0 \) and \( A_{LS} = 0 \), and that the incoming photon asymmetry \([10]\) vanishes.

Let us summarize. Poincaré invariance, gauge invariance, conservation of parity, and time reversal invariance are respected in our impulse approximation evaluation of the handbag diagrams. Proton wave functions, previously constrained by comparison with measured form factors, that incorporate the influence of quark orbital angular momentum (and the corresponding violation of proton helicity conservation) are used. Computed cross sections are in reasonably good agreement with early measurements. The value of \( K_{LL} \) is large and positive for scattering at large angles. In contrast with earlier work, we find that \( K_{LL} \), \( A_{LL} \) are nearly independent of energy and quark mass.
Let us summarize. Poincaré invariance, gauge invariance, conservation of parity, and time reversal invariance are respected in our impulse approximation evaluation of the handbag diagrams. Proton wave functions, previously constrained by comparison with measured form factors, that incorporate the influence of quark orbital angular momentum (and the corresponding violation of proton helicity conservation) are used. Computed cross sections are in reasonably good agreement with early measurements. The value of $K_{LL}$ is large and positive for scattering at large angles. In contrast with earlier work, we find that $K_{LL} \neq A_{LL}$, and $D_{LL} \neq 1$ at large scattering angles.
Summary

• Form factors, GPDs, TMDs, understood from unified light-front formulation, GPD-coordinate space density, TMD momentum space density

• Potential of Compton scattering unrealized—more data needed

• Proton is not round—lattice QCD spin-dependent density is not zero

• Experiment can whether or not proton is round by measuring $h_{1T}$
Summary

• Form factors, GPDs, TMDs, understood from unified light-front formulation, GPD-coordinate space density, TMD momentum space density

• Potential of Compton scattering unrealized—more data needed

• Proton is not round—lattice QCD spin-dependent—density is not zero

• Experiment can whether or not proton is round by measuring $h_{1T}$

The Proton
Spares follow
Summary of SDD

- SDD are closely related to TMD’s.
- If $h_{1T}^?$ is not 0, proton is not round. Experiment can show proton ain’t round.
Summary of SDD

- SDD are closely related to TMD’s
- If $h_{1T}$ is not 0, proton is not round. Experiment can show proton ain’t round.

The Proton

Wednesday, March 23, 2011
Ratio of Pauli to Dirac Form Factors
1995 theory, data 2000

\[ \frac{Q_F}{F_1} \]

\[ Q^2 \, \text{GeV}^2 \]

Wednesday, March 23, 2011
How to study the proton?
How to study the proton?

- EXPERIMENTS
How to study the proton?

• EXPERIMENTS
• Theory – numerical simulations lattice
How to study the proton?

- EXPERIMENTS
- Theory – numerical simulations lattice masses + ... low $Q^2$
How to study the proton?

**EXPERIMENTS**

- Theory – numerical simulations lattice masses + … low $Q^2$
- eventually exact
How to study the proton?

- EXPERIMENTS
- Theory – numerical simulations lattice masses + … low $Q^2$
  eventually exact
- Phenomenology– symmetries, dynamical guesses, high $Q^2$
How to study the proton?

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  - Theory – numerical simulations lattice masses + … low $Q^2$
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- Phenomenology- symmetries, dynamical guesses, high $Q^2$
- Model independent techniques
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what the lattice will find
Spin density operator: $\delta(r-r_p) \sigma \cdot n$.

Canted ferromagnetic structure of UNiGe high magnetic fields

- Neutron magnetic scattering

- Neutron, B, crystal
A New Parameterization of the Nucleon Elastic Form Factors

R. Bradford, A. Bodek, H. Budd, and J. Arrington

hep-ex/0602017
How proton holds together-high $Q^2$

- pQCD

Feynman

Non perturbative
∞ gluon exch
How proton holds together-high $Q^2$

- pQCD

Feynman

Non perturbative

$\propto$ gluon exch
How proton holds together-high $Q^2$

- pQCD

Feynman

Non perturbative

\[\infty\] gluon exch
Results

\[ \rho(b) \text{ [fm}^{-2}\text{]} \]

**Proton**

\[ 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \]

**Neutron**

\[ 0 \quad 0.1 \quad 0 \quad -0.1 \quad -0.2 \quad -0.3 \quad -0.4 \]

Kelly

**Negative**
**Results**

\[ \varrho(b) \, [\text{fm}^{-2}] \]

- **BBBA**

- **Kelly**

- **Negative**
Negative $F_1$ means central density negative.
Neutron Interpretation

? $\pi^-$ at short distance?

Central quark density reduced by orbital ang. momentum $\text{OAM}$?
Summary of density

- Model independent information on charge density

\[ \rho(b) \equiv \sum_q e_q \int dx \, q(x, b) = \int d^2 q F_1(Q^2 = q^2) e^{i \cdot q \cdot b}. \]

- Central charge density of neutron is negative

- Pion cloud at large b
Field theoretic SDD

\[ \hat{\rho}_{REL}(K, n) = \int \frac{d^3 \xi}{(2\pi)^3} e^{-iK \cdot \xi} \bar{\psi}(0) \gamma^0 (1 + \gamma \cdot n \gamma_5) \mathcal{L}(0, \xi; \text{path}) \psi(\xi) \Big|_{t=\xi^0=0} \]

- Probability to have momentum $K$, and spin direction $n$

Matrix elements depend on three vectors $n, K, S$
Field theoretic SDD

\[ \hat{\rho}_{\text{REL}}(K, n) = \int \frac{d^3 \xi}{(2\pi)^3} e^{-iK \cdot \xi} \bar{\psi}(0) \gamma^0 (1 + \gamma \cdot n \gamma_5) \mathcal{L}(0, \xi; \text{path}) \psi(\xi) \bigg|_{t=\xi^0=0} \]

- Probability to have momentum K, and spin direction n

Matrix elements depend on three vectors \( n, K, S \)

Equal time correlation function

Wednesday, March 23, 2011
Relate SDD to TMD

- SDD depend on $K_x$, $K_y$, $K_z$ & equal time correlation function
- TMD depend on $x$, $K_x$, $K_y$ & $\xi^+=0 = t + z$ correlation function
- Integrate SDD over $K_z$ --> $t=0, z=0$
- Integrate TMD over $x$! $\xi=0$, $t=0, z=0$

Result: non-spherical nature of proton related to $h_{1T}$?