

High t form factors & Compton Scattering - quark based models

Gerald A. Miller
University of Washington



Basic Philosophy- model wave function Ψ

- Given Ψ compute form factors, densities, Compton scattering
- Make guess at how QCD works, improve guess, rule out simple scenarios
- Non-relativistic quark model
- 3 quarks
- 0 orbital angular momentum
- proton is round
- What can Compton scattering say?

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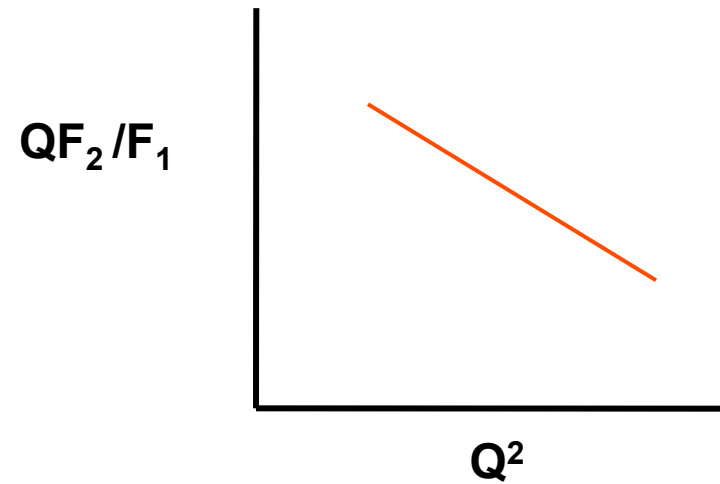
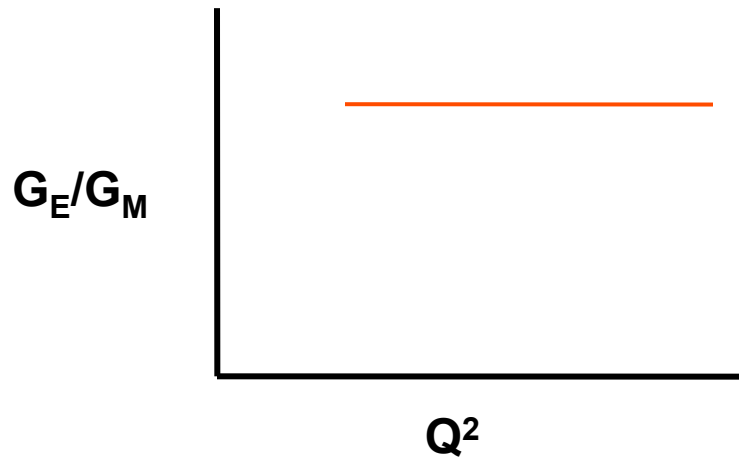
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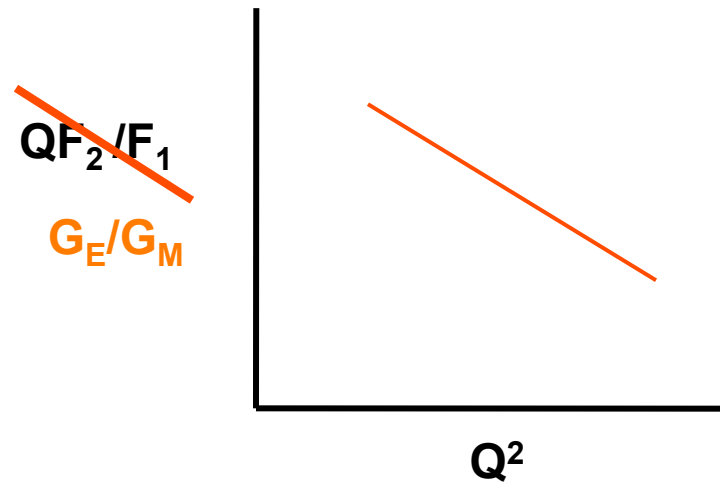
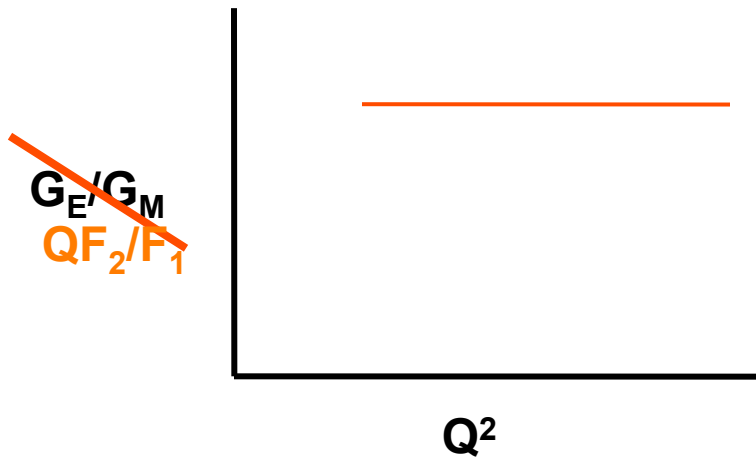
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Expectations- Pre Jlab



$\frac{G_E}{G_M}$ constant : non – relativistic quark model

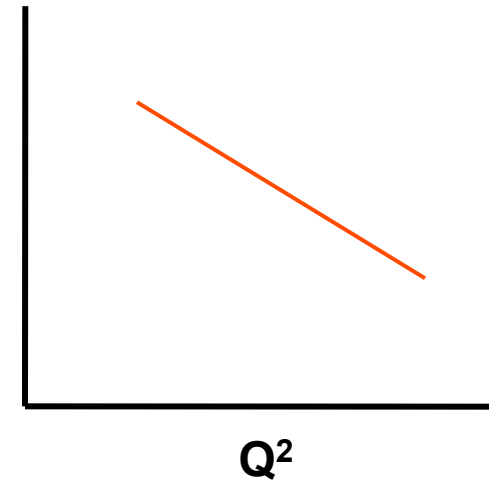
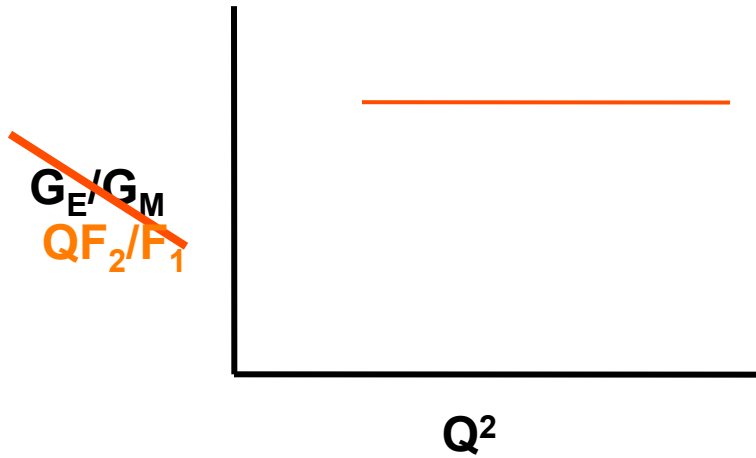
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$\frac{G_E}{G_M}$ constant : non – relativistic quark model

Form Factor

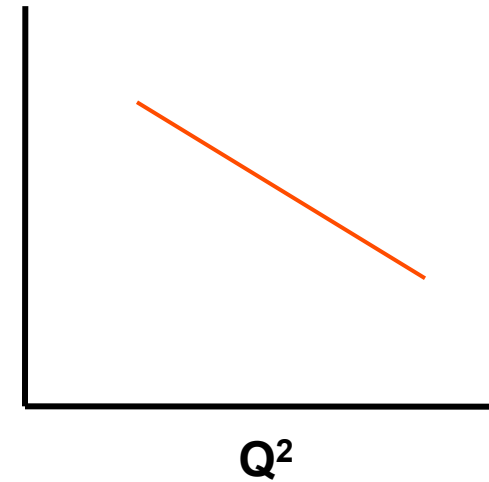
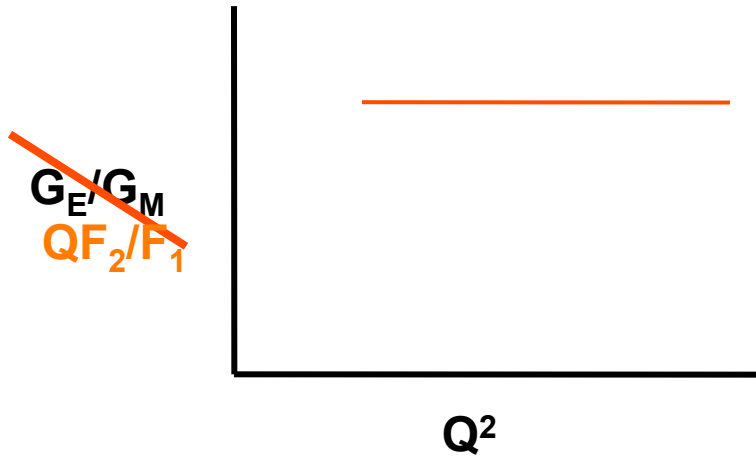
Jlab



$\frac{G_E}{G_M}$ constant : non – relativistic quark model

Form Factor

Jlab



$\frac{G_E}{G_M}$ constant : non – relativistic quark model

Relativistic model needed- light front coordinates

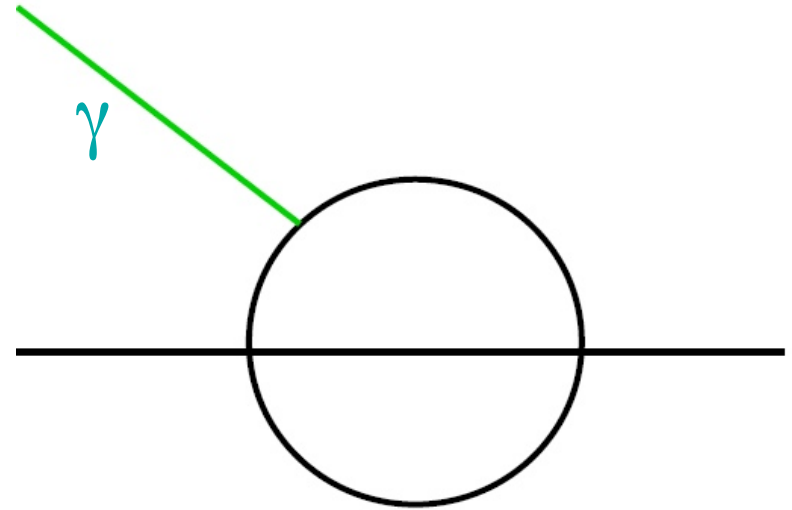
Understand phenomena-model

Model proton wave function: 3 quarks

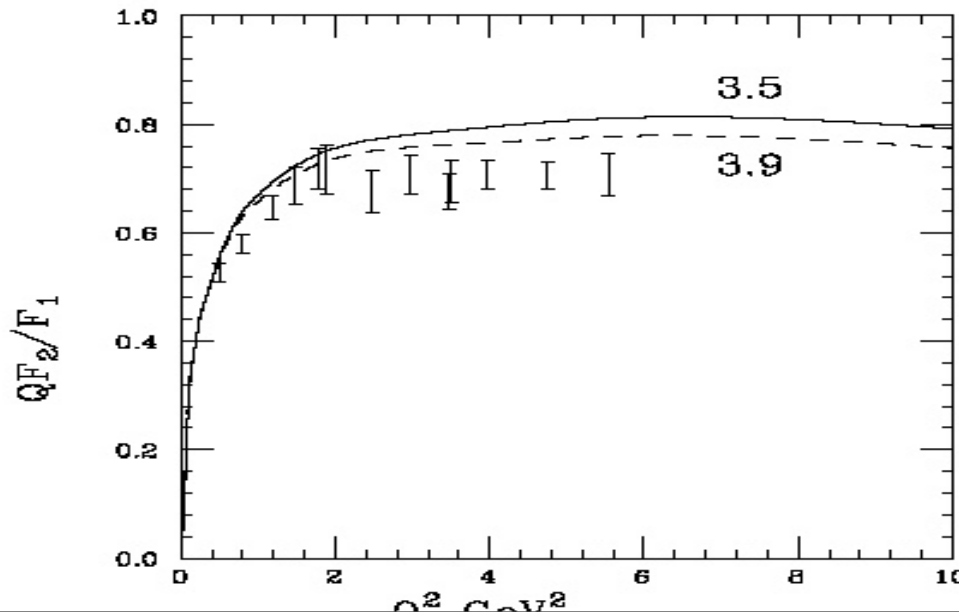
Lorentz and rotationally invariant

Light front variables

Dirac spinors-orbital angular momentum



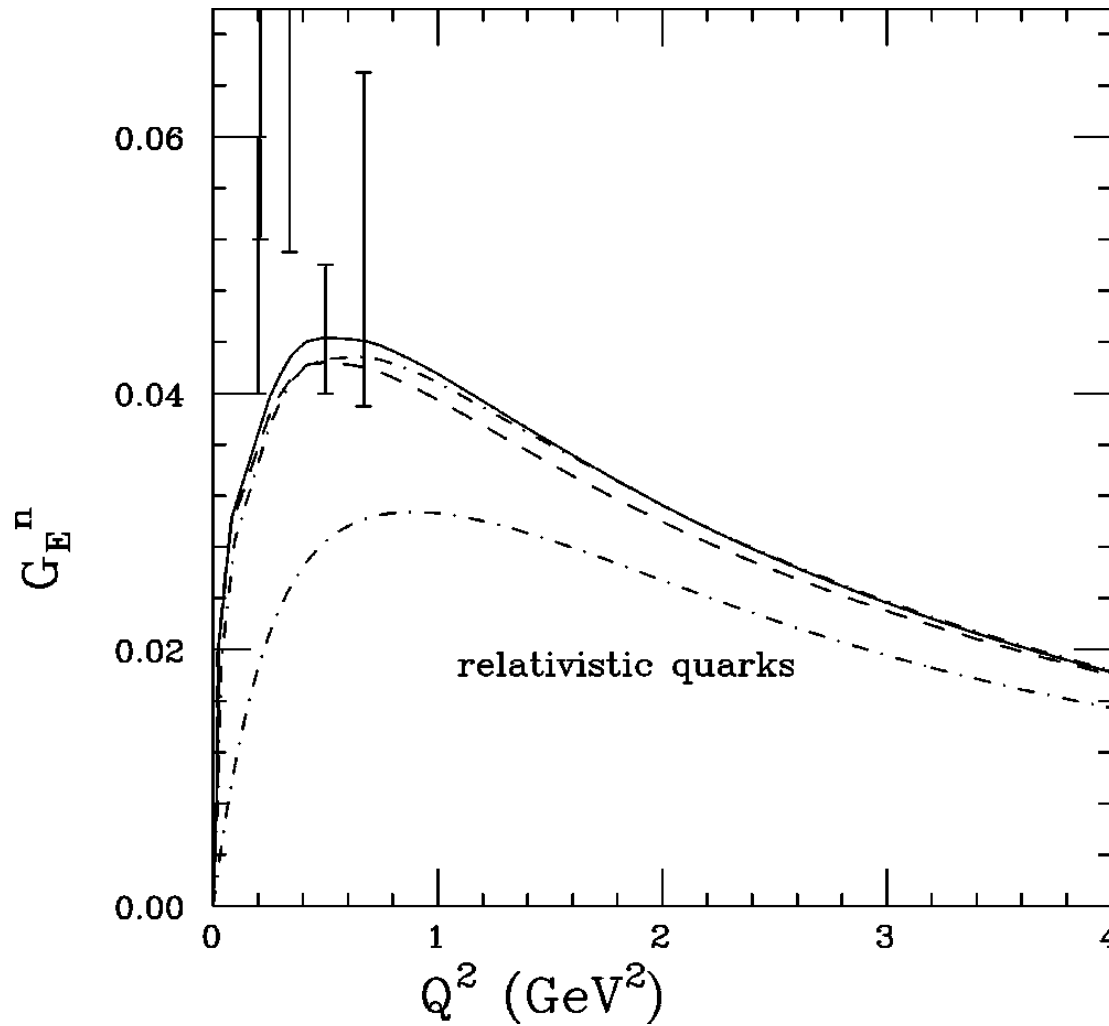
M. R. Frank, , [B.K. Jennings](#) , [G.A. Miller](#) ,. Phys.Rev.C54:920-935,1996.



Theory 1995 Data 2000

Quark spin is 75 %
of proton total
angular momentum

Neutron- requires pion cloud



[Gerald A. Miller](#), Phys.Rev.C66:032201,2002⁵

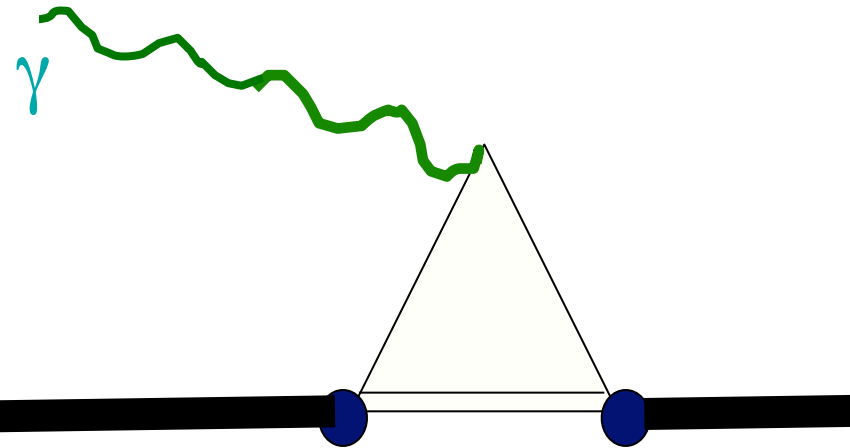
Improved model-Cloet & Miller '11 20

Model proton wave function: quark-diquark

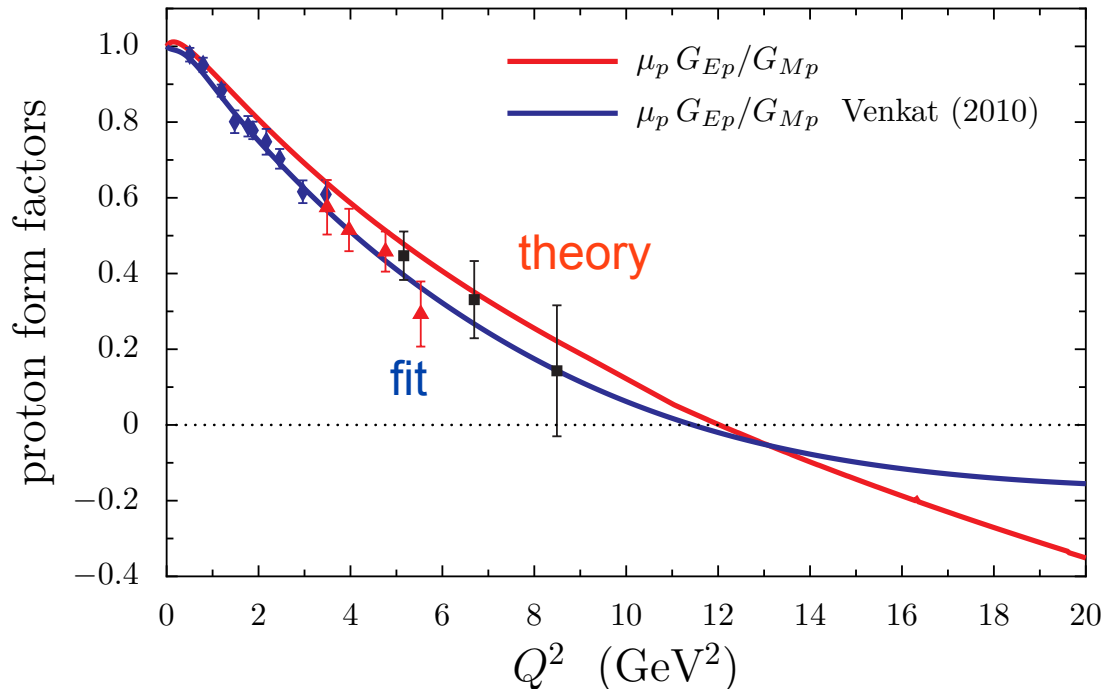
Lorentz and rotationally invariant-different forms!

Light front variables

Dirac spinors-orbital angular momentum

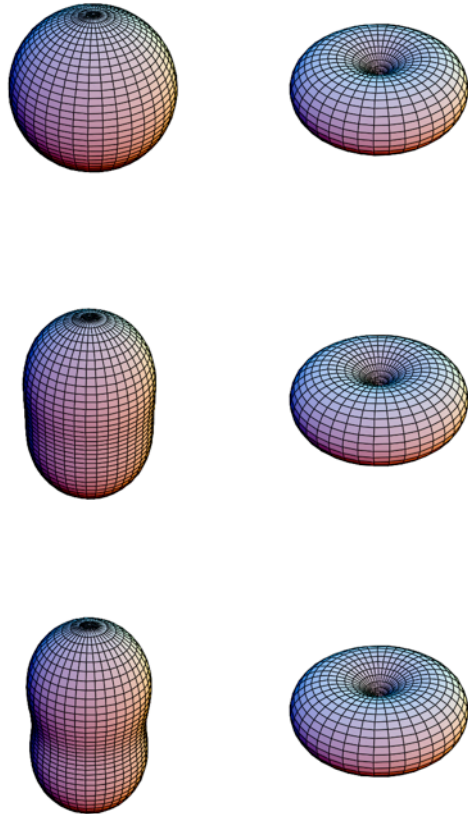


Cloet and Miller 2011



Quark spin is 35 % of proton total angular momentum

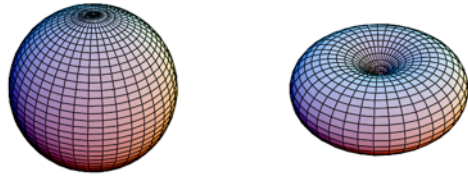
Shapes of the proton- momentum space spin-dependent-densities



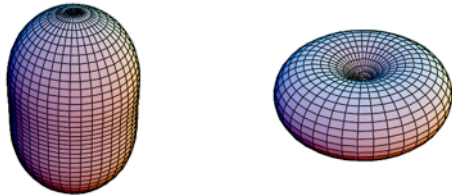
three vectors \mathbf{n} , \mathbf{K} , \mathbf{S}

Phys.Rev. C68 (2003) 022201

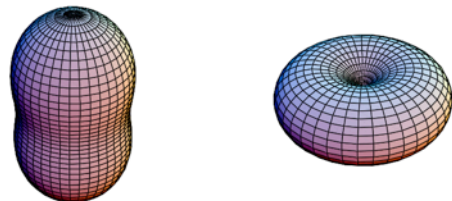
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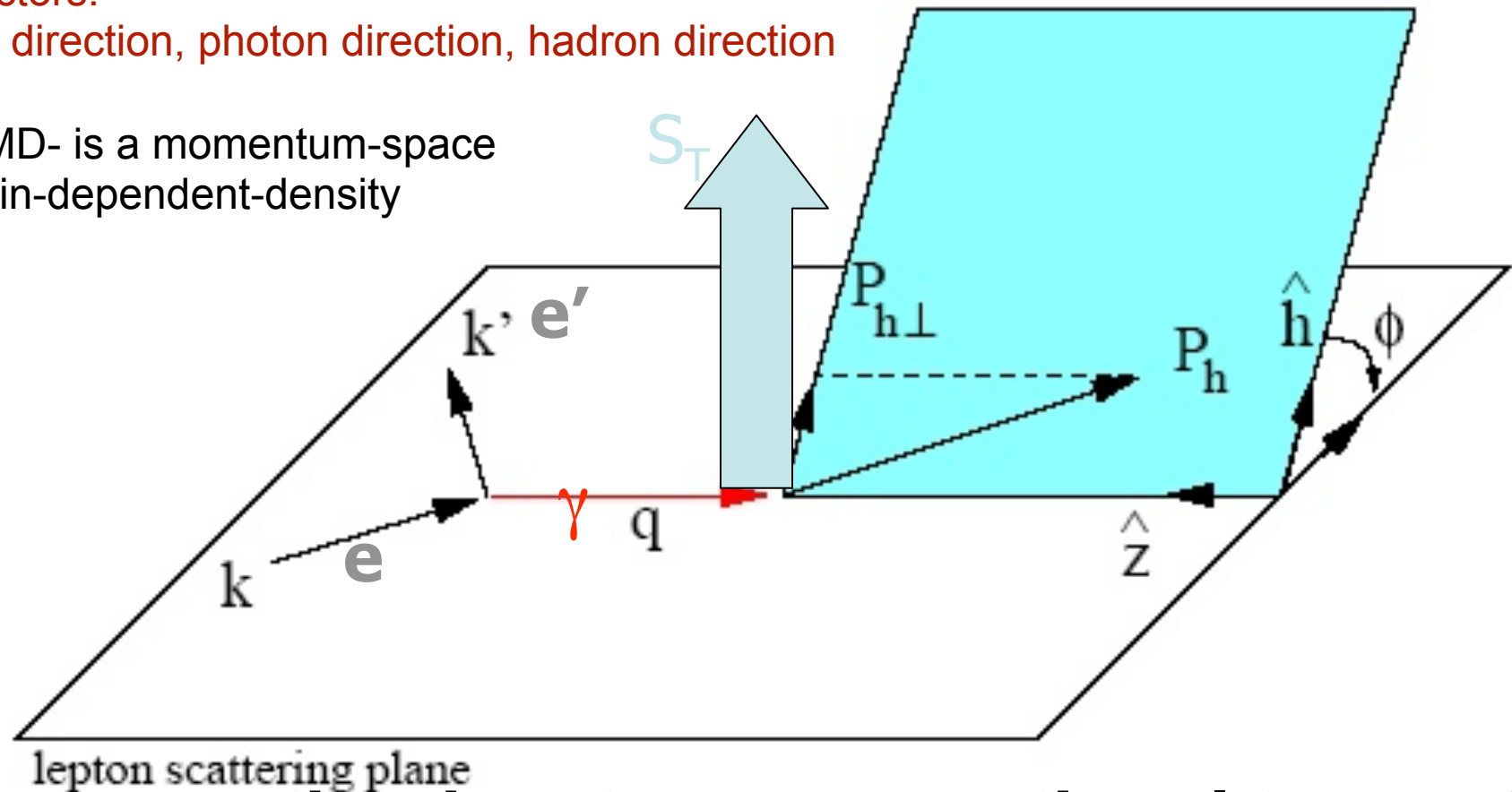


MODEL , HOW TO MEASURE? How to compute fundamentally?

Measure $h_{1T}^\perp : e + p(\uparrow) \rightarrow e' \pi X$

3 vectors:
Spin direction, photon direction, hadron direction

TMD- is a momentum-space
spin-dependent-density



Cross section has term proportional to $\cos 3\phi$

Boer Mulders '98

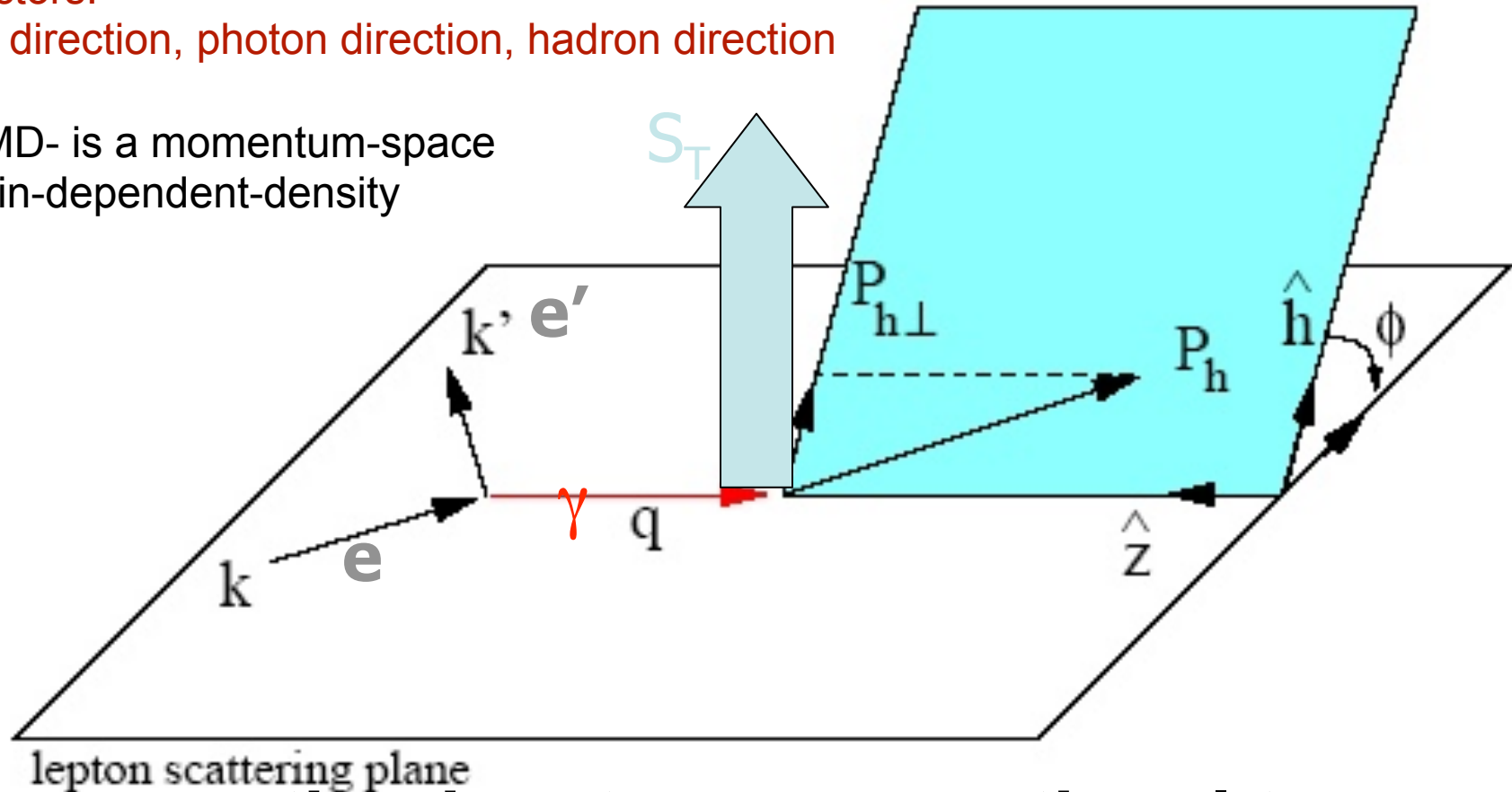
GAM Phys.Rev.C76:065209,2007

Measure $h_{1T}^\perp : e + p(\uparrow) \rightarrow e' \pi X$

H. Avakian, *et al.* "Transverse Polarization Effects in Hard Scattering at CLAS12 Jefferson Laboratory", LOI12-06-108, and H. Avakian private communication.

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GAM Phys.Rev.C76:065209,2007

Generalized Coordinate Space Densities

$$\rho^\Gamma(\mathbf{b}) = \sum e_q \int dx^- q_+(x^-, \mathbf{b}) \gamma^+ \Gamma q_+(x^-, \mathbf{b})$$
$$\Gamma = \frac{1}{2} (1 + \mathbf{n} \cdot \boldsymbol{\gamma} \gamma_5) \text{ gives spin - dependent density}$$

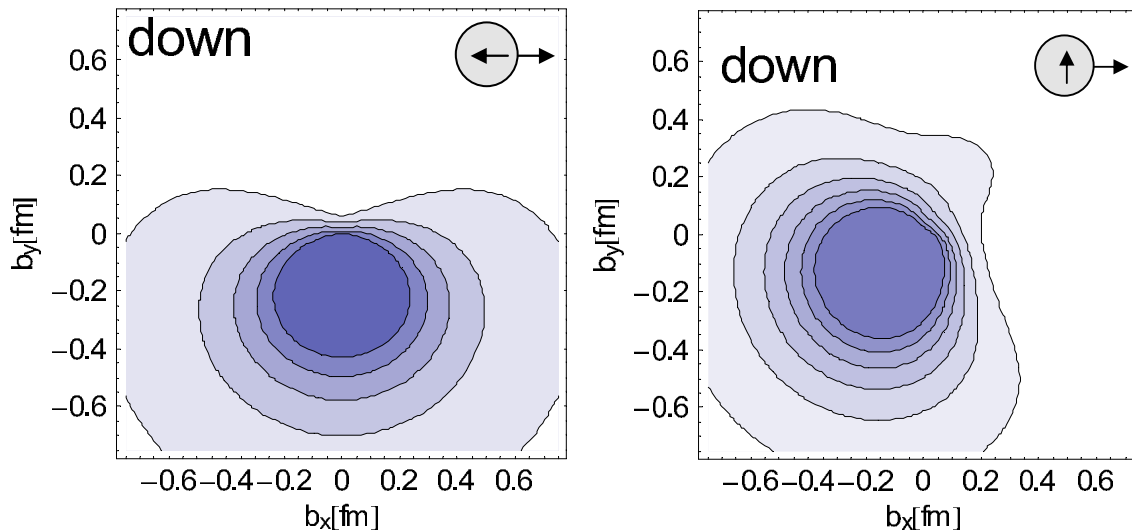
PRL 98, 222001 (2007)

PHYSICAL REVIEW LETTERS

week ending
1 JUNE 2007

Transverse Spin Structure of the Nucleon from Lattice-QCD Simulations

M. Gökeler,¹ Ph. Hägler,^{2,*} R. Horsley,³ Y. Nakamura,⁴ D. Pleiter,⁴ P. E. L. Rakow,⁵ A. Schäfer,¹ G. Schierholz,^{6,4}
H. Stüben,⁷ and J. M. Zanotti³



spin-dependent density
-depends on direction
of \mathbf{b} : proton is not round

Compton scattering

RAPID COMMUNICATIONS

PHYSICAL REVIEW C **69**, 052201(R) (2004)

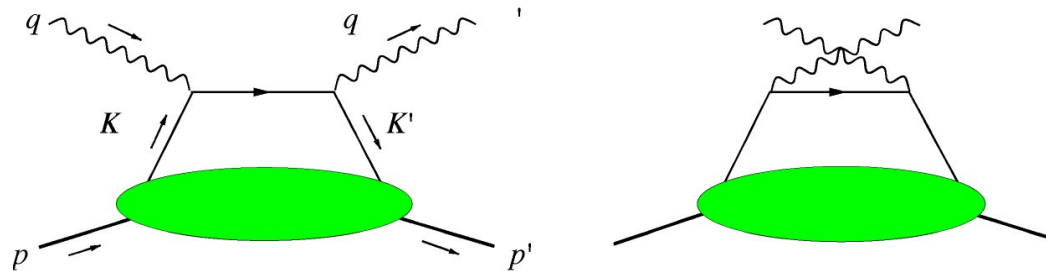
Handling the handbag diagram in Compton scattering on the proton

Gerald A. Miller

Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA

(Received 1 March 2004; published 25 May 2004)

Poincaré invariance, gauge invariance, conservation of parity, and time reversal invariance are respected in an impulse approximation evaluation of the handbag diagram. Proton wave functions, previously constrained by comparison with measured form factors, that incorporate the influence of quark transverse and orbital angular momentum (and the corresponding violation of proton helicity conservation) are used. Computed cross sections are found to be in reasonably good agreement with early measurements. The helicity correlation between the incident photon and outgoing proton, K_{LL} , is both large and positive at back angles. For photon laboratory energies of ≤ 6 GeV, we find that $K_{LL} \neq A_{LL}$, and $D_{LL} \neq 1$.



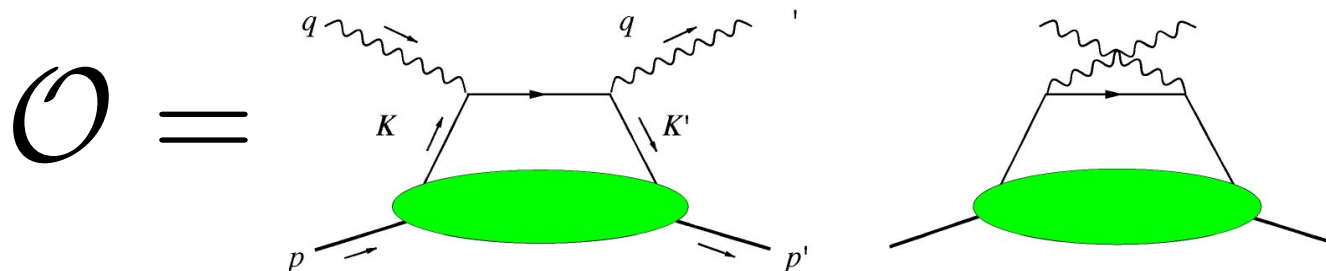
**Wave function supplies amplitudes for
on-mass shell quarks, CC respected¹⁰**

Technical aspects

- Transverse momentum of quarks included
- Photon momenta are transverse, no boosts
- No energy transfer

- $$\mathcal{M}_{S',S}(\epsilon', \epsilon) = \rho \otimes \mathcal{O}$$

$$\rho_{S',s';S,s}(\eta, K'_\perp, K_\perp) = \int d\xi d^2k_\perp \Psi_{S',s'}^\dagger(\xi, k_\perp, \eta, K'_\perp) \Psi_{S,s}(\xi, k_\perp, \eta, K_\perp)$$



$$K'_\perp = k_\perp + (1 - \eta)(q'_\perp - q_\perp) \quad 11$$

Technical II

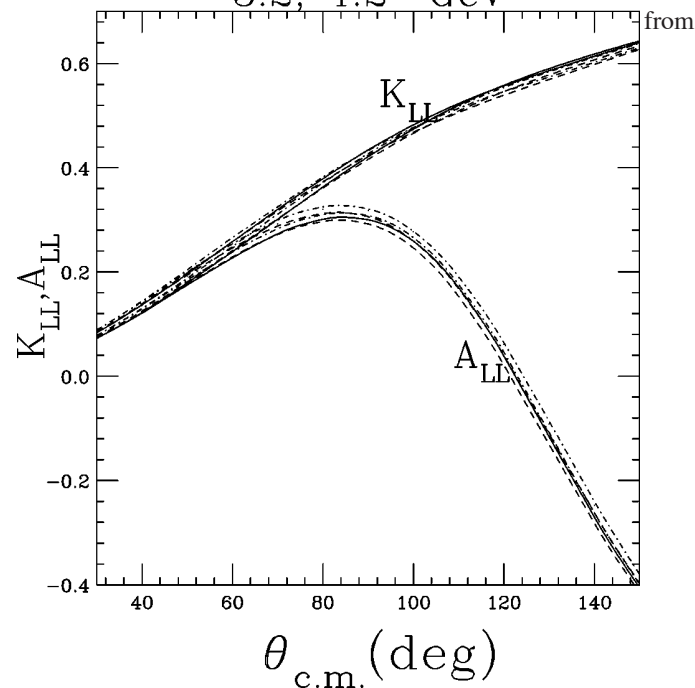
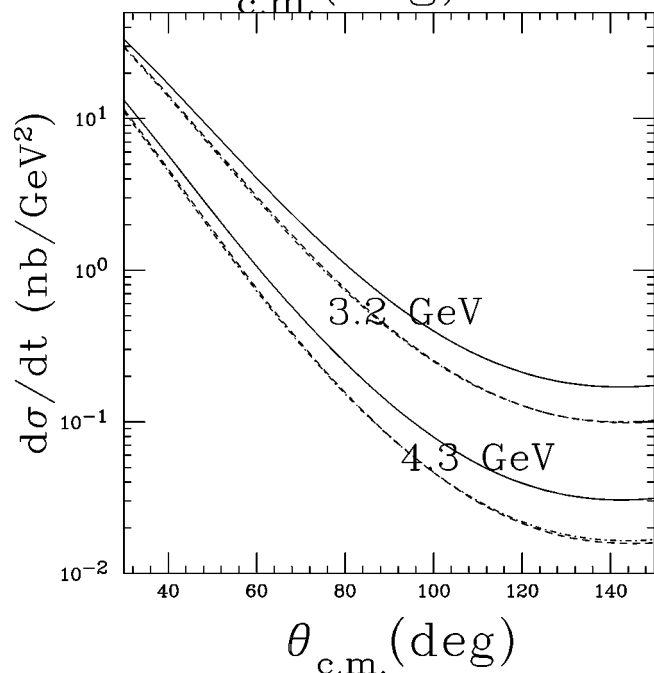
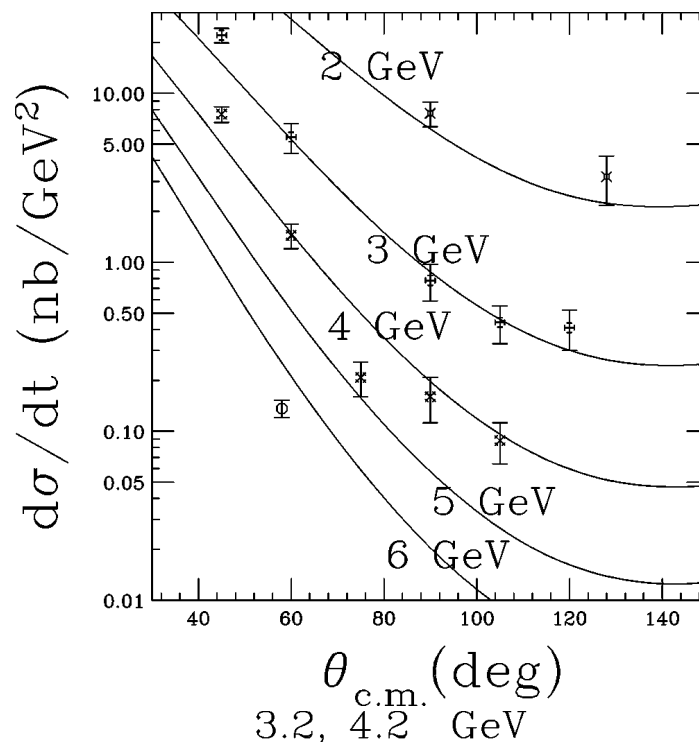
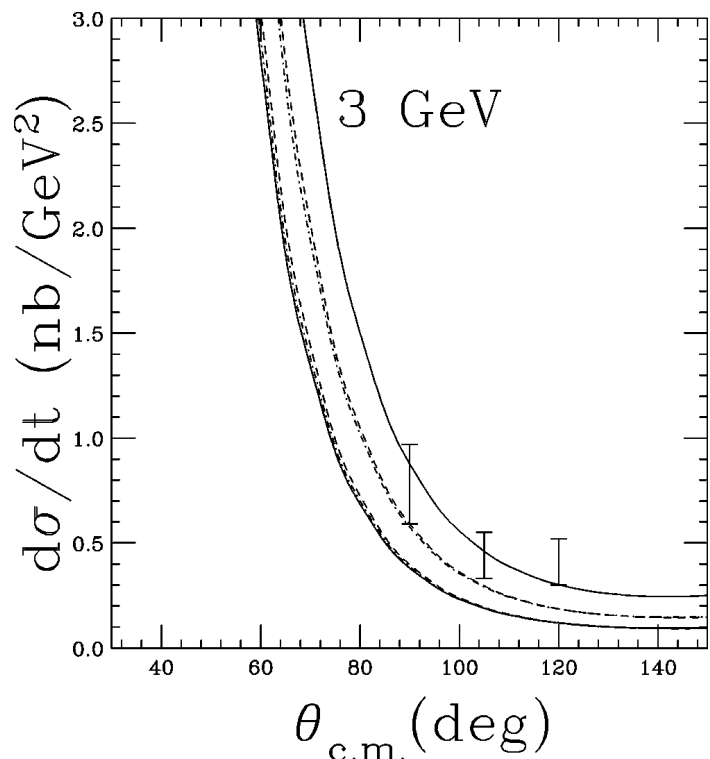
- $S', S, \varepsilon, \varepsilon'$, 16 amplitudes
- 6 independent, challenge to calc'n
- transform to helicity basis,
- λ nucleon helicity, μ photon helicity

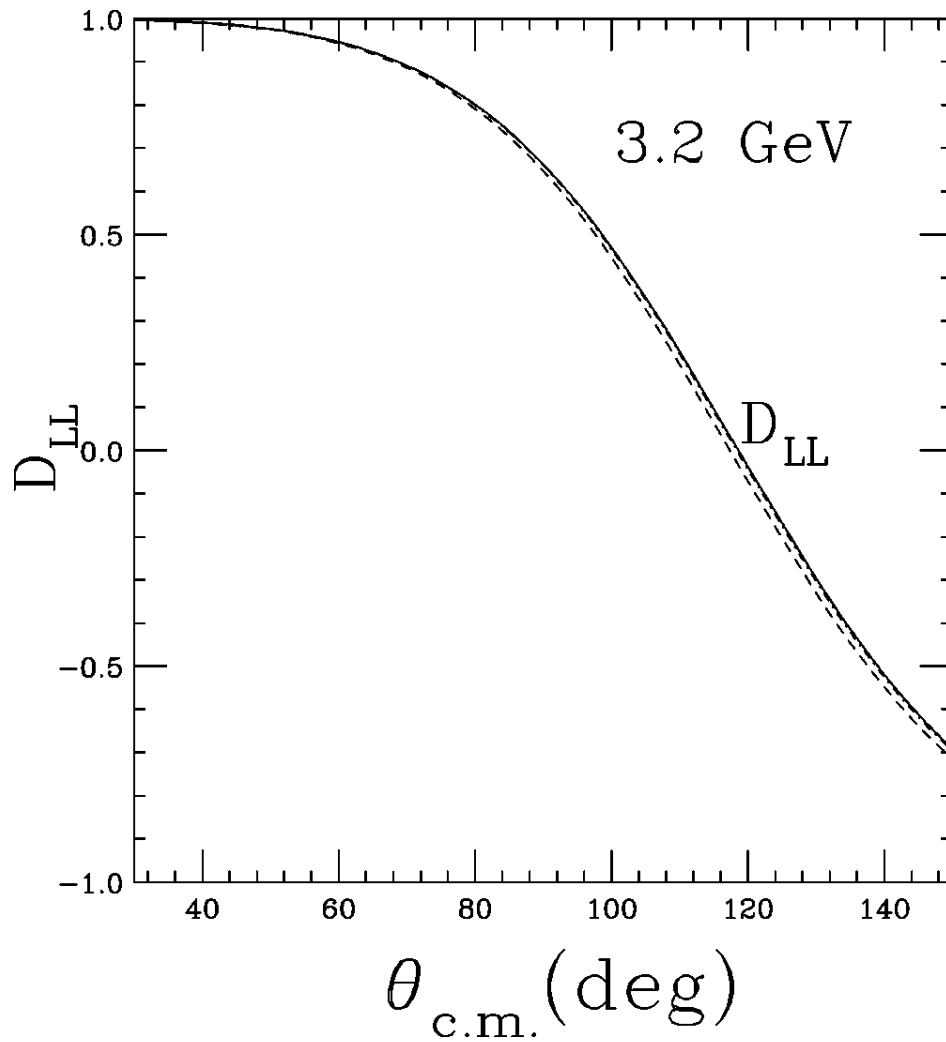
$$\frac{d\sigma}{dt} = \frac{1}{64\pi(s-m^2)^2} \sum_{\mu, \mu', \lambda, \lambda'} |\Phi_{\mu', \lambda', \mu\lambda}|^2.$$

$$A_{LL} \frac{d\sigma}{dt} = \frac{1}{2} \left[\frac{d\sigma(\mu = +, \lambda = +)}{dt} - \frac{d\sigma(\mu = +, \lambda = -)}{dt} \right].$$

$$K_{LL} \frac{d\sigma}{dt} = \frac{d\sigma(\mu = +, \lambda' = +)}{dt} - \frac{d\sigma(\mu = +, \lambda' = -)}{dt},$$

$$A_{LL} \neq K_{LL}$$





$$D_{LL} \frac{d\sigma}{dt} = \frac{d\sigma(\mu = +, \mu' = +)}{dt} - \frac{d\sigma(\mu = +, \mu' = -)}{dt}$$

Let us summarize. Poincaré invariance, gauge invariance, conservation of parity, and time reversal invariance are respected in our impulse approximation evaluation of the handbag diagrams. Proton wave functions, previously constrained by comparison with measured form factors, that incorporate the influence of quark orbital angular momentum (and the corresponding violation of proton helicity conservation) are used. Computed cross sections are in reasonably good agreement with early measurements. The value of K_{LL} is large and positive for scattering at large angles. In contrast with earlier work, we find that $K_{LL} \neq A_{LL}$, and $D_{LL} \neq 1$ at large scattering angles.

Summary

- Form factors, **GPDs**, **TMDs**, understood from unified light-front formulation, **GPD-coordinate space density**, **TMD momentum space density**
- **Potential of Compton scattering unrealized-more data needed**
- **Proton is not round- lattice QCD spin-dependent-density is not zero**
- **Experiment can whether or not proton is round by measuring h_{1T}^\perp**



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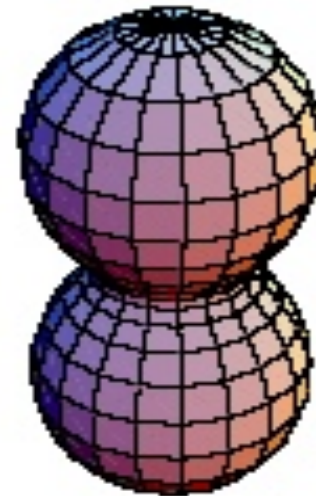


The Proton

Spares follow

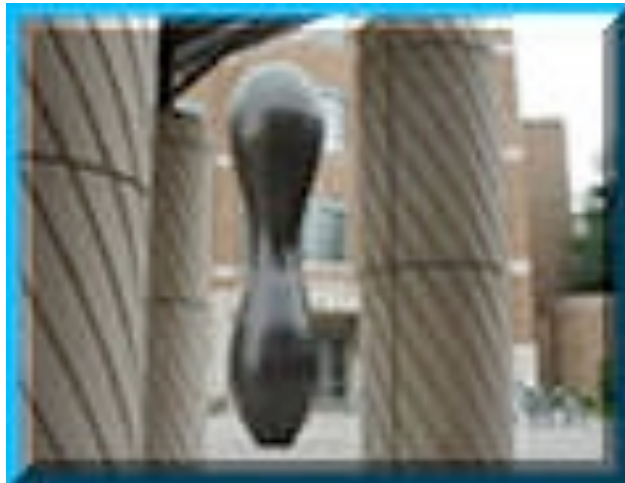
Summary of SDD

- SDD are closely related to TMD's
- If $h_{1T}^?$ is not 0, proton is not round. **Experiment can show proton ain't round.**

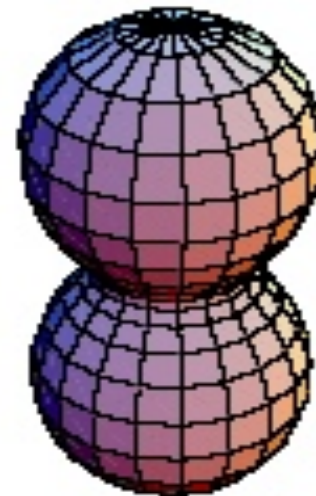


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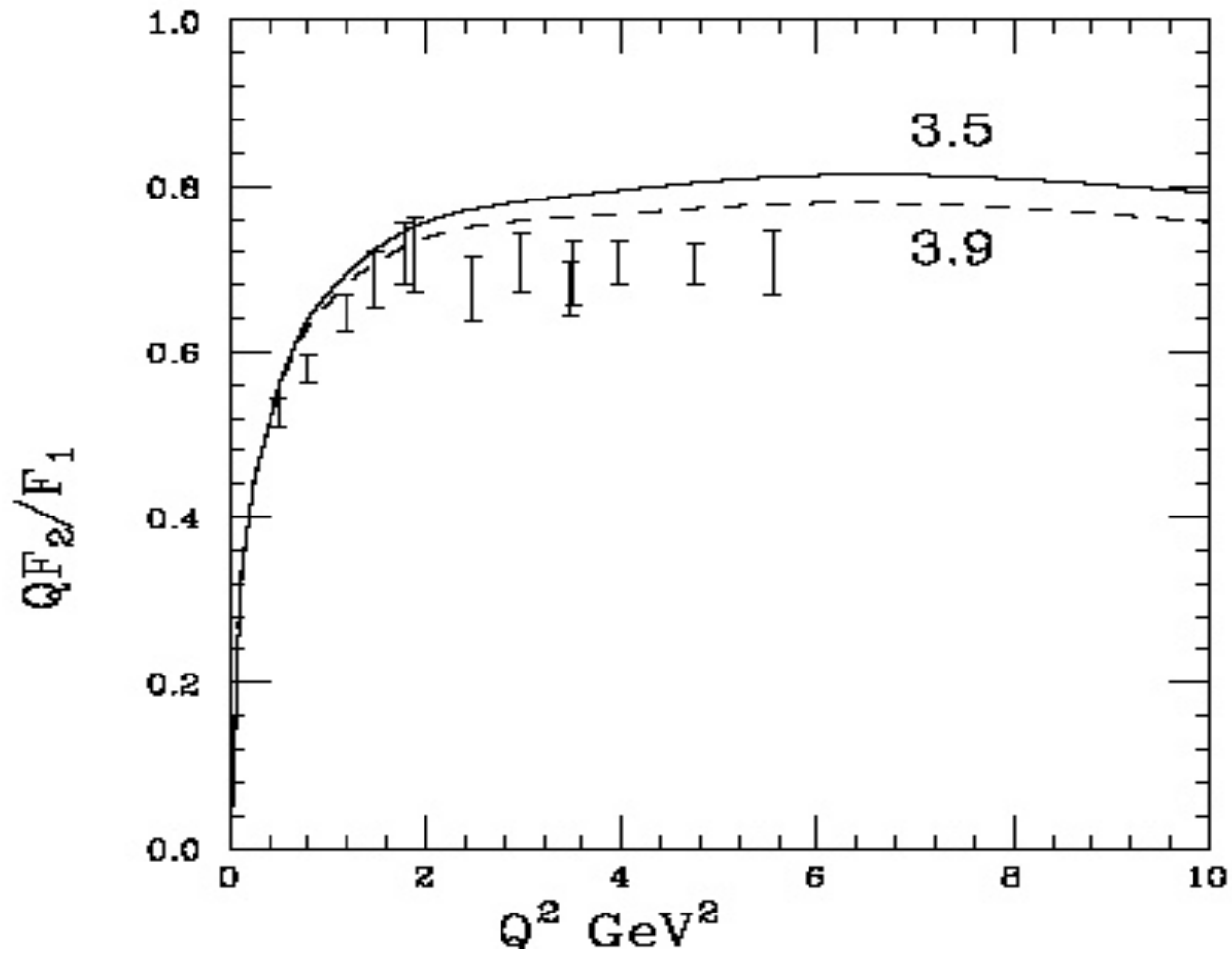
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The Proton



Ratio of Pauli to Dirac Form Factors 1995 theory, data 2000



How to study the proton?

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- EXPERIMENTS

How to study the proton?

- **EXPERIMENTS**
- Theory –numerical simulations lattice

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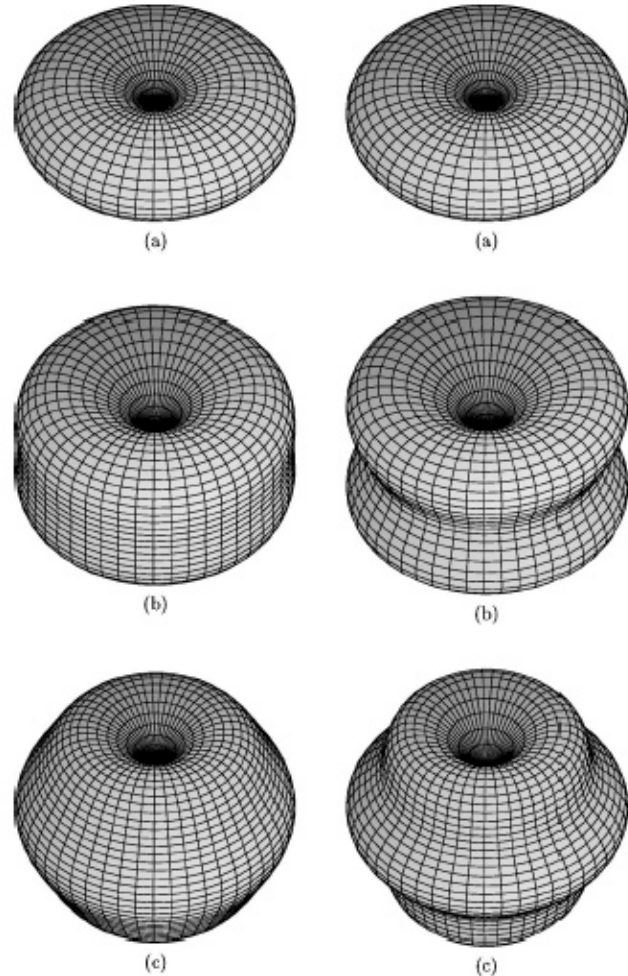
what the lattice will find

Spin density operator: $\delta(\mathbf{r}-\mathbf{r}_p) \boldsymbol{\sigma} \cdot \mathbf{n}$

Canted ferromagnetic structure of UNiGe high magnetic fields

- Neutron magnetic scattering
- Neutron, B, crystal

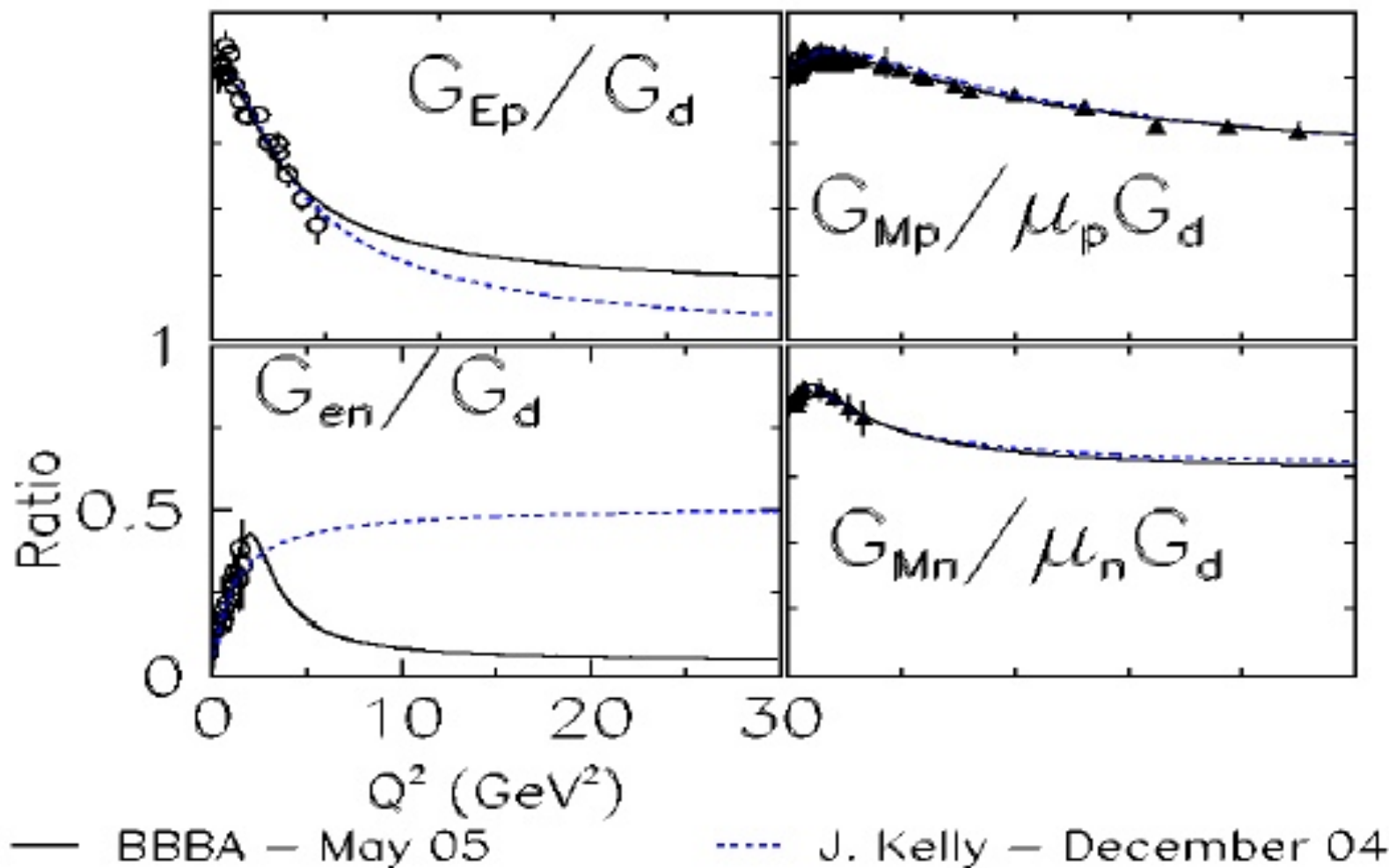
PRB65, 144429



A New Parameterization of the Nucleon Elastic Form Factors

R. Bradford,^a A. Bodek,^a H. Budd,^a and J. Arrington^b

hep-ex/0602017

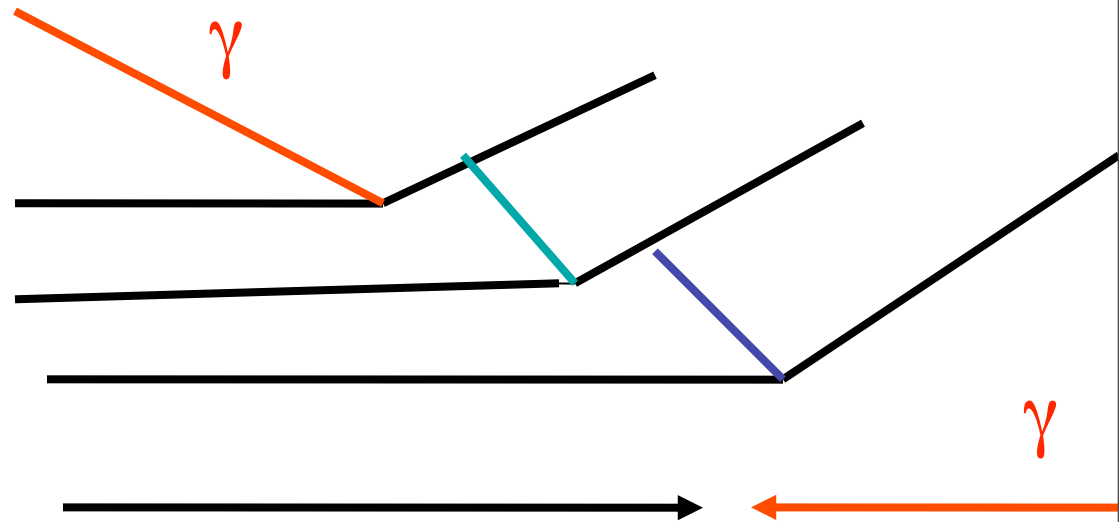


How proton holds together-high Q^2

- pQCD

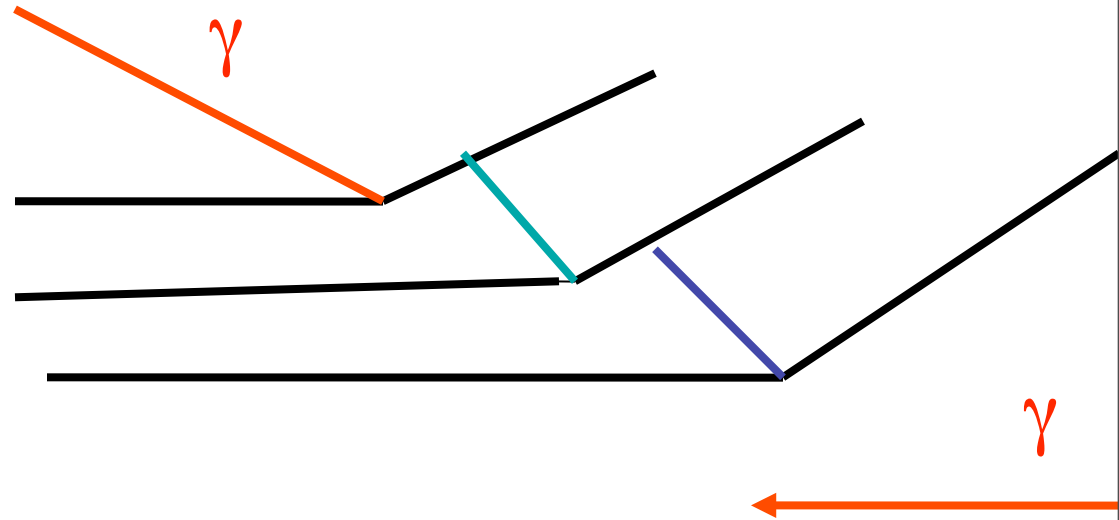
Feynman

Non perturbative
 ∞ gluon exch



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Feynman

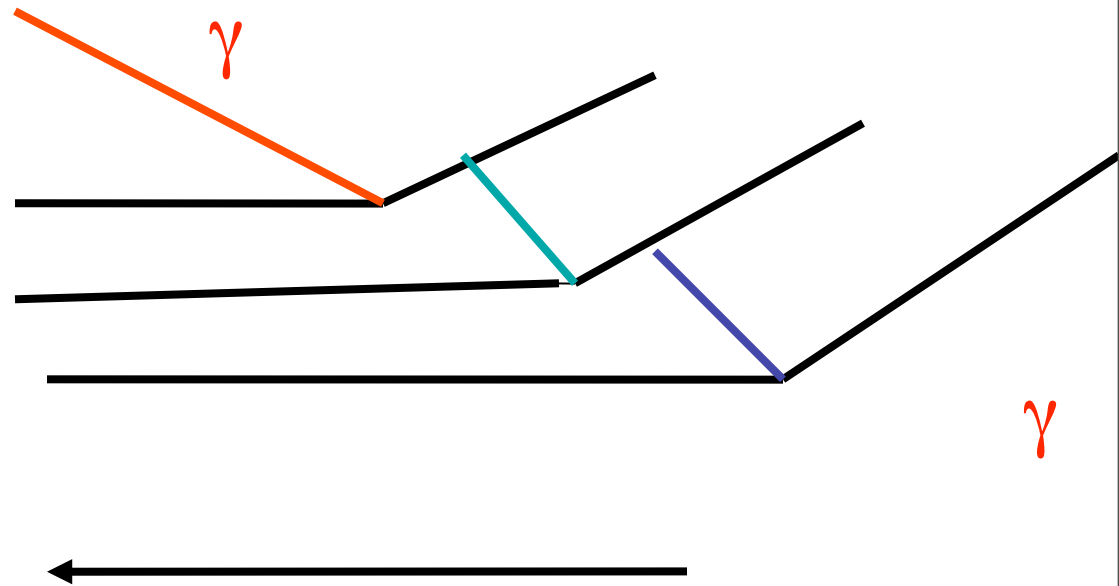
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Results

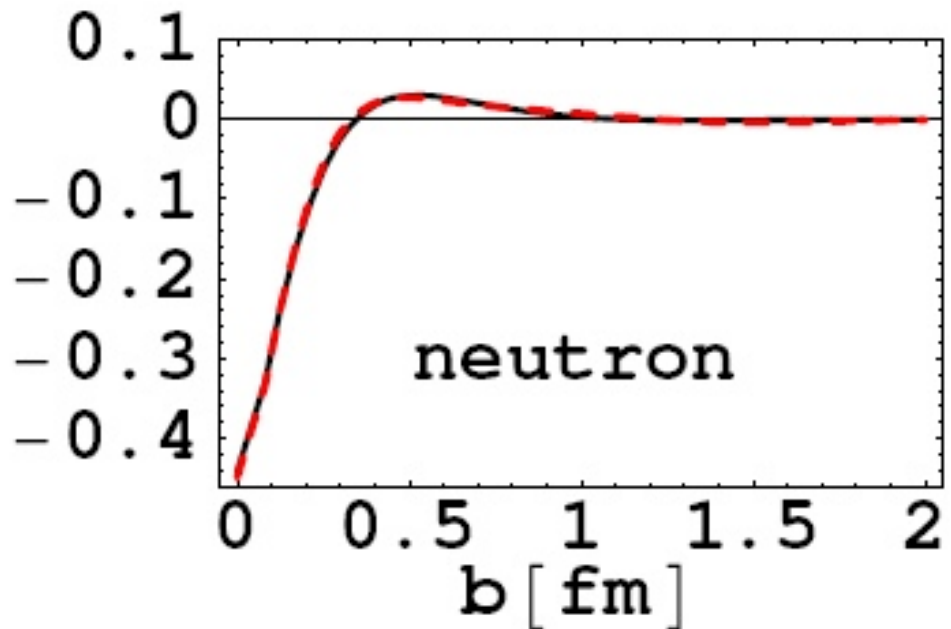
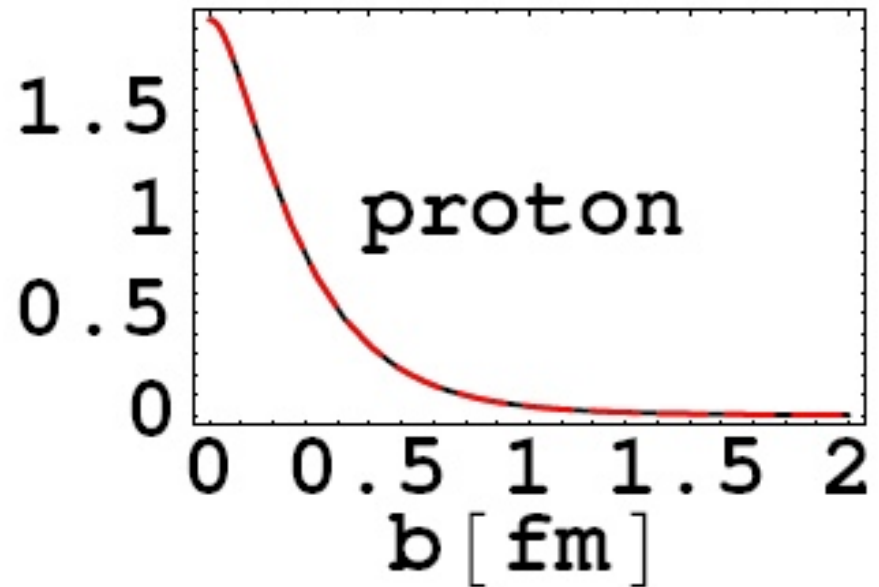
$\rho(\mathbf{b})$ [fm⁻²]

BBBA

Kelly

$\rho(\mathbf{b})$ [fm⁻²]

Negative



Results

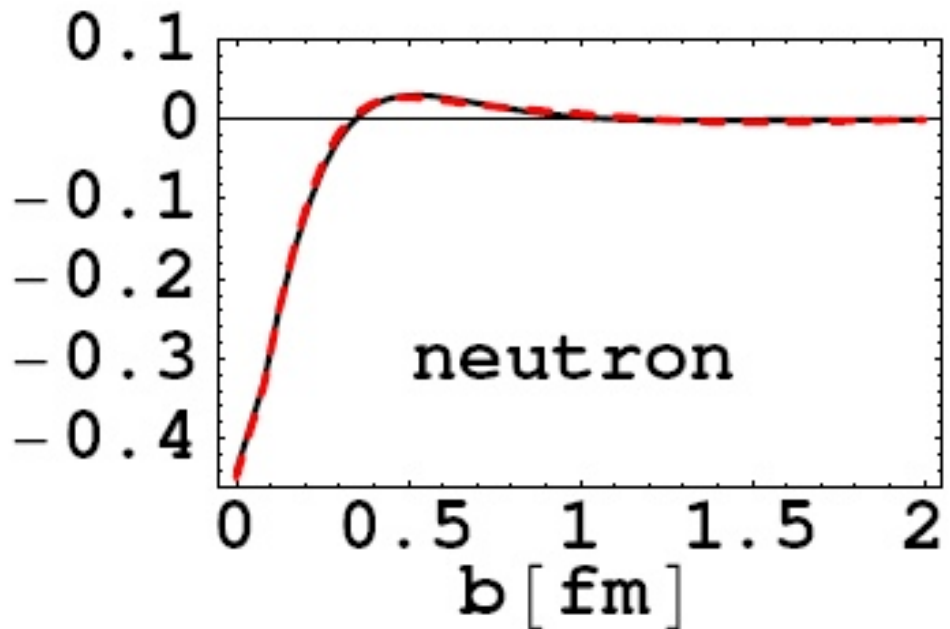
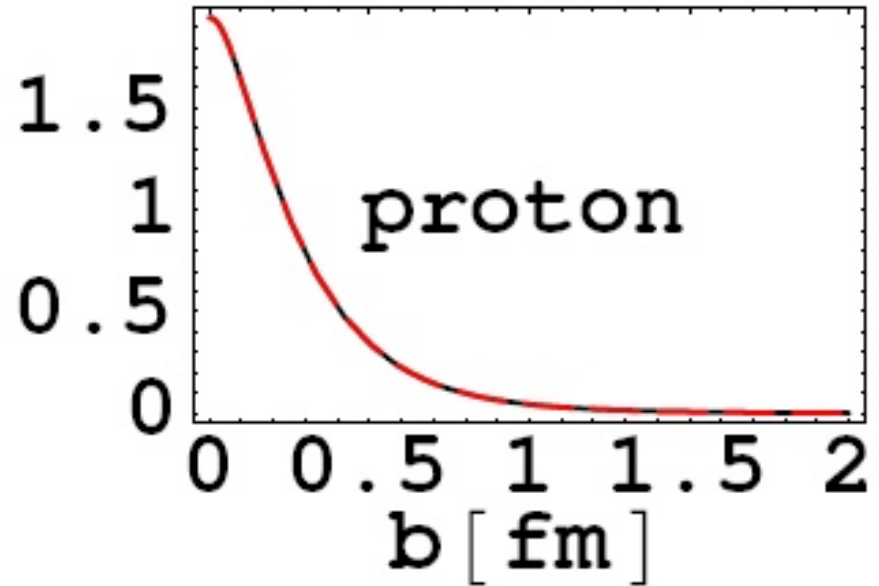
$\rho(\mathbf{b})$ [fm⁻²]

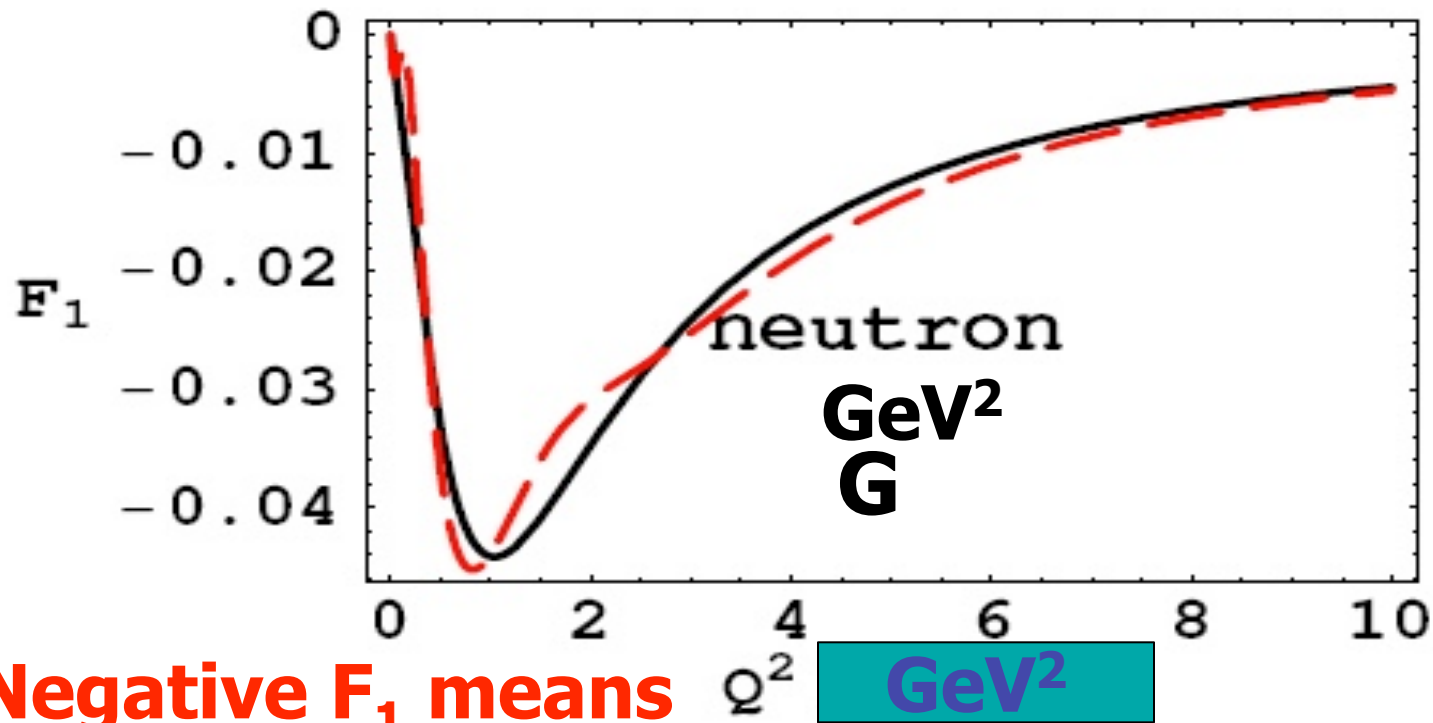
BBBA

Kelly

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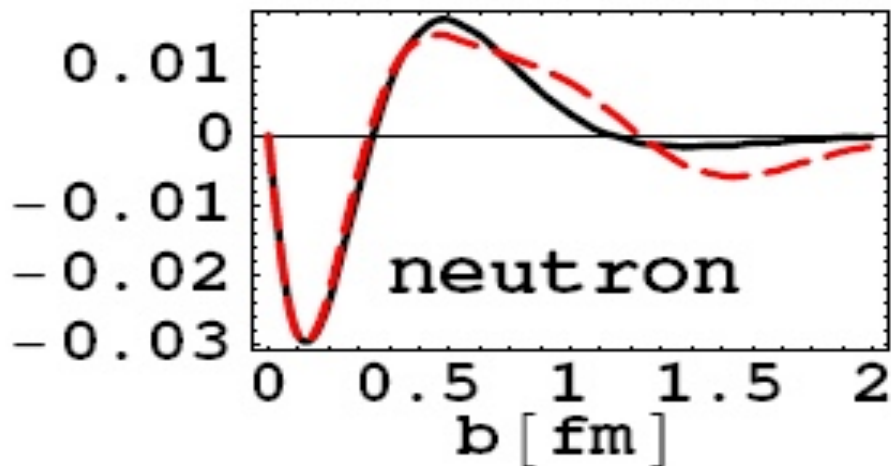
Negative



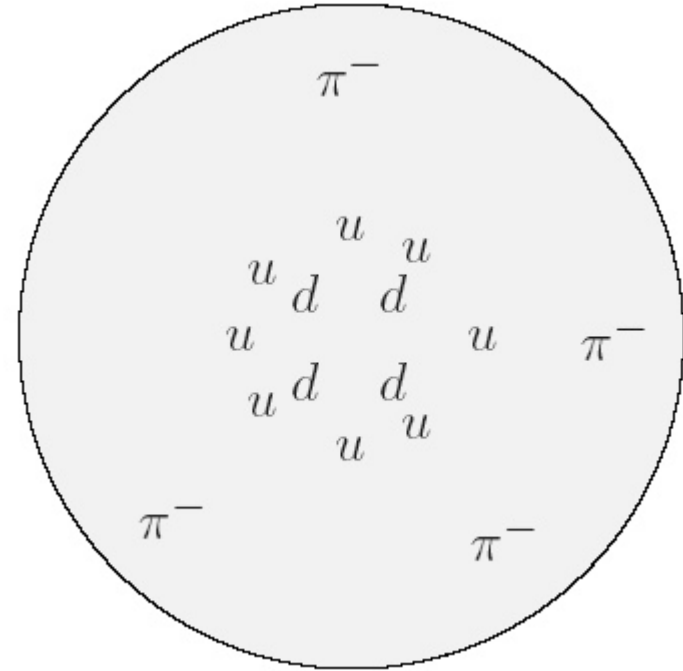
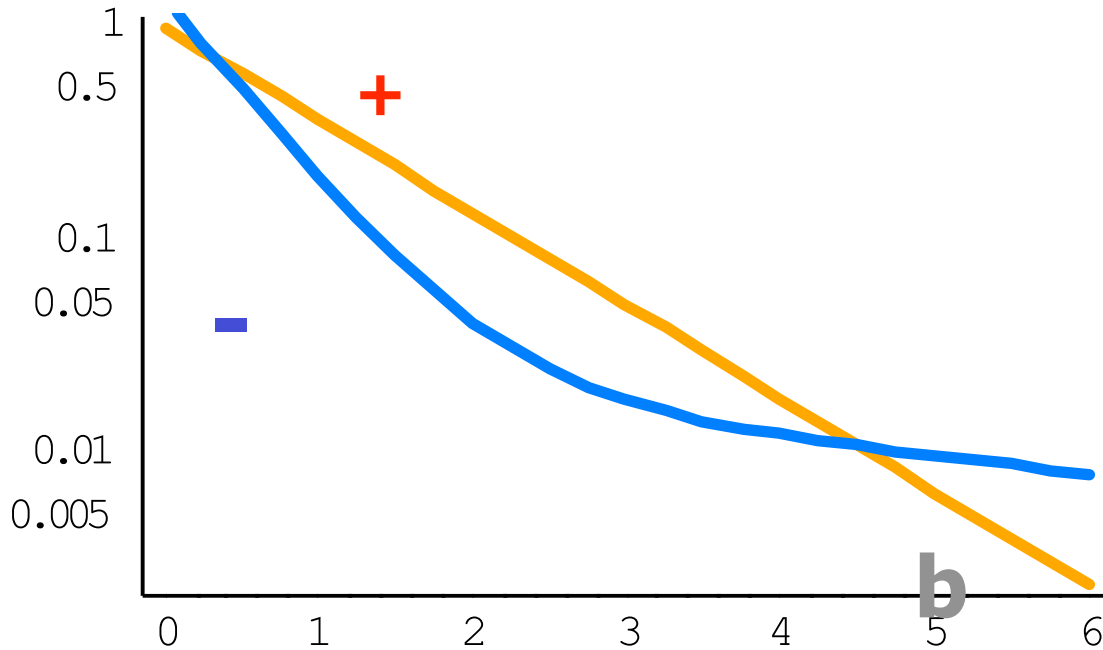


Negative F_1 means
central density
negative

b $\rho(b) [\text{fm}^{-1}]$



Neutron Interpretation



? π^- at short distance ?

**Central quark density reduced
by orbital ang. momentum OAM?**

Summary of density

- Model independent information on charge density

$$\rho(b) \equiv \sum_a e_q \int dx q(x, \mathbf{b}) = \int d^2q F_1(Q^2 = \mathbf{q}^2) e^{i \mathbf{q} \cdot \mathbf{b}}$$

- Central charge density of neutron is negative
- Pion cloud at large b

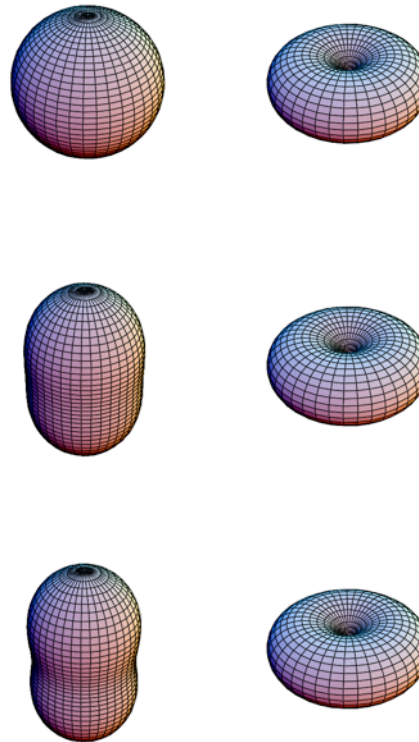
Field theoretic SDD

$$\hat{\rho}_{\text{REL}}(\mathbf{K}, \mathbf{n}) = \int \frac{d^3\xi}{(2\pi)^3} e^{-i\mathbf{K}\cdot\xi} \bar{\psi}(0)\gamma^0(1 + \boldsymbol{\gamma}\cdot\mathbf{n}\gamma_5)\mathcal{L}(0, \xi; \text{path})\psi(\xi)\Big|_{t=\xi^0=0}$$

- **Probability to have momentum \mathbf{K} , and spin direction \mathbf{n}**

**Matrix
elements
depend on
three vectors**

$\mathbf{n}, \mathbf{K}, \mathbf{S}$



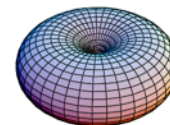
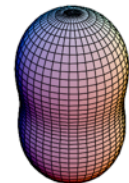
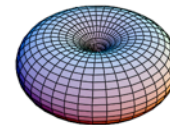
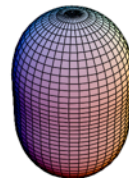
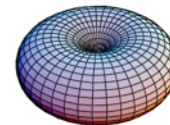
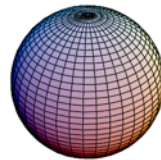
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- **Probability to have momentum \mathbf{K} , and spin direction \mathbf{n}**

Matrix elements depend on three vectors

$\mathbf{n}, \mathbf{K}, \mathbf{S}$



Equal time correlation function

Relate SDD to TMD

- SDD depend on K_x, K_y, K_z & equal time correlation function
- TMD depend on x, K_x, K_y & $\xi^+=0 = t+z$ correlation function
- Integrate SDD over $K_z \rightarrow t=0, z=0$
- Integrate TMD over $x ! \xi^s=0, t=0, z=0$

Result : non-spherical nature of proton related to h_{1T} ?