Wide-angle Compton scattering and partonic structure

(theory overview and prospects)

A.V. Radyushkin

ODU & JLab

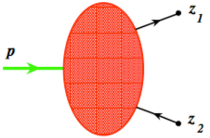
Hadrons in Terms of Quarks and Gluons

How to relate hadronic states $|p, s\rangle$

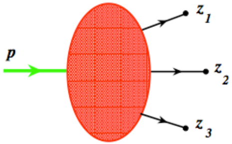
to quark and gluon fields $q(z_1)\ ,\ q(z_2)\ ,\ \ldots$?

Standard way: use matrix elements

$$\langle\, 0\,|\, \bar{q}_\alpha(z_1)\, q_\beta(z_2)\,|\, M(p), s\,\rangle\ \ ,\ \ \langle\, 0\,|\, q_\alpha(z_1)\, q_\beta(z_2)\, q_\gamma(z_3)|\, B(p), s\,\rangle$$



Meson–quark matrix element



Baryon–quark matrix element

- Can be interpreted as hadronic wave functions

Light-cone formalism

- Describe hadron by Fock components in infinite-momentum frame

For nucleon

$$|P\rangle = |q(x_1P,k_{1\perp})\,q(x_2P,k_{2\perp})\,q(x_3P,k_{3\perp})\rangle + |qqqG\rangle + |qqq\bar{q}q\rangle + |qqqGG\rangle + \ldots$$

- x_i : momentum fractions

$$\sum_i x_i = 1$$

- $k_{i\perp}$: transverse momenta

$$\sum_i k_{i\perp} = 0$$

Problems of LC Formalism

- In principle: Solving bound-state equation

$$H|P\rangle = E|P\rangle$$

  one gets $|P\rangle$ which gives complete information about hadron structure
- In practice: Equation (involving infinite number of Fock components) has not been solved and is unlikely to be solved in near future
- Experimentally: LC wave functions are not directly accessible
- Way out: Description of hadron structure in terms of phenomenological functions

Phenomenological Functions

“Old” functions:

- Form Factors
- Usual Parton Densities
- Distribution Amplitudes

“New” functions:

Generalized Parton Distributions (GPDs)

GPDs = Hybrids of

Form Factors, Parton Densities and Distribution Amplitudes

“Old” functions

are limiting cases of “new” functions

Form Factors

Form factors are defined through matrix elements

of electromagnetic and weak currents between hadronic states

Nucleon EM form factors:

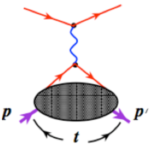
$$\langle p',\, s' \,|\, J^\mu(0) \,|\, p,\, s \,\rangle = \bar{u}(p', s') \left[\gamma^\mu F_1(t) + \frac{\Delta^\nu \sigma^{\mu\nu}}{2m_N} F_2(t) \right] u(p, s)$$

$(\Delta = p - p', t = \Delta^2)$

- Electromagnetic current $J^\mu(z) = \sum_f e_f \bar{\psi}_f(z) \gamma^\mu \psi_f(z)$
- Helicity non-flip form factor $F_1(t) = \sum_f e_f F_{1f}(t)$
- Helicity flip form factor $F_2(t) = \sum_f e_f F_{2f}(t)$

Form Factors are measurable

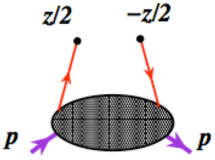
through elastic eN scattering



Usual Parton Densities

Parton Densities are defined through forward matrix elements

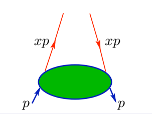
of quark/gluon fields separated by lightlike distances



Unpolarized quarks case:

$$\langle p \,|\, \bar{\psi}_a(-z/2)\gamma^\mu \psi_a(z/2)\,|\, p\,\rangle\big|_{z^2=0}$$
$$= 2p^\mu \int_0^1 \left[e^{-ix(pz)} f_a(x) - e^{ix(pz)} f_{\bar{a}}(x)\right] dx$$

Momentum space interpretation



$f_{a(\bar{a})}(x)$ is probability

to find a $(\bar{a})$ quark with momentum xp

Local limit $z = 0$

$\Rightarrow$ sum rule

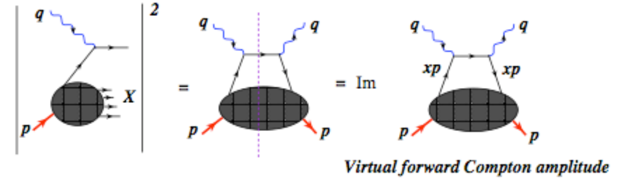
$\int_0^1 [f_a(x) - f_{\bar{a}}(x)]\, dx = N_a$

for valence quark numbers

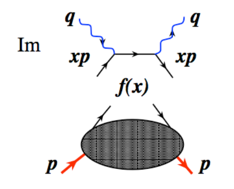
Deep Inelastic Scattering

Classic process to access usual parton densities:

deep inelastic scattering $\gamma^* N \to X$



Spacelike momentum transfer $q^2 \equiv -Q^2$



$$\text{Im}\ \frac{1}{(q+xp)^2} \approx \frac{\pi}{2(pq)}\delta(x - x_{Bj})$$

Bjorken variable: $x_{Bj} = \frac{Q^2}{2(pq)}$

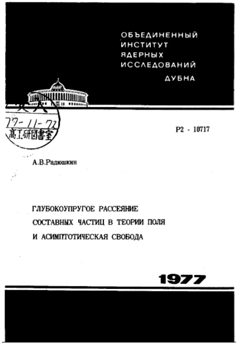
DIS measures $f(x_{Bj})$

Comparing to form factors:

point vertex instead of quark propagator and $p \neq p'$

Small-size Configurations: Perturbative QCD for Pion EM Form Factor

- First application of pQCD to exclusive processes


ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ ДУБНА

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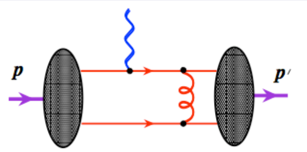
А.В.Радюшкин

ГЛУБОКОУПРУГОЕ РАССЕЯНИЕ СОСТАВНЫХ ЧАСТИЦ В ТЕОРИИ ПОЛЯ И АСИМПТОТИЧЕСКАЯ СВОБОДА

1977

Hard gluon exchange diagram



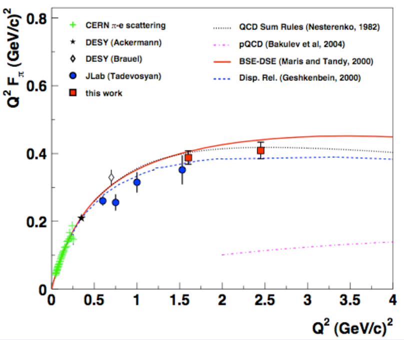
$$F_\pi^{\rm as}(Q^2) = \frac{8\pi f_\pi^2 \alpha_s(Q^2)}{Q^2}$$

- Hard scenario:
  small-size configurations dominate large-Q^2 behavior

Pion Form Factor Data and pQCD

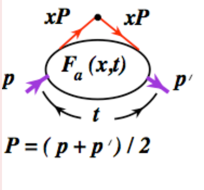
- With $\varphi_\pi^{as}(\alpha)$, hard pQCD contribution to $F_\pi(Q^2)$ is $(2\alpha_s/\pi)(0.67\,\mathrm{GeV}^2)/Q^2$: less than 1/3 of experimental value
- Competing nonperturbative soft mechanism dominates for available Q^2

Experimental Data on Pion Form Factor



Nonforward Parton Densities (aka GPDs at zero skewness)

Combine form factors with parton densities



$$F_1(t) = \sum_a F_{1a}(t)$$

$$F_{1a}(t) = \int_0^1 \mathcal{F}_{1a}(x,t)\,dx$$

Flavor components of form factors

$$\mathcal{F}_{1a}(x,t) \equiv e_a[\mathcal{F}_a(x,t) - \mathcal{F}_{\bar{a}}(x,t)]$$

Forward limit $t = 0$

$$\mathcal{F}_{a(\bar{a})}(x, t=0) = f_{a(\bar{a})}(x)$$

Interplay between x and t dependencies

Simplest factorized ansatz

$$\mathcal{F}_a(x,t) = f_a(x) F_1(t)$$

satisfies both forward and local constraints

Forward constraint

$$\mathcal{F}_a(x, t=0) = f_a(x)$$

Local constraint

$$\int_0^1 [\mathcal{F}_a(x,t) - \mathcal{F}_{\bar{a}}(x,t)]dx = F_{1a}(t)$$

Reality is more complicated:

LC wave function with Gaussian $k_\perp$ dependence

$$\Psi(x_i, k_{i\perp}) \sim \exp\left[-\frac{1}{\lambda^2}\sum_i \frac{k_{i\perp}^2}{x_i}\right]$$

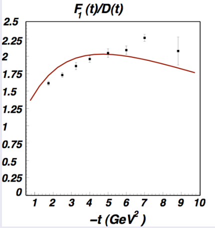
suggests

$$\mathcal{F}_a(x,t) = f_a(x) e^{\bar{x}t/2x\lambda^2}$$

$f_a(x)$=experimental densities

Adjusting λ^2 to provide

$$\langle k_\perp^2 \rangle \approx (300\text{MeV})^2$$



Feynman mechanism

Drell-Yan formula

$$F(\Delta^2) = \int_0^1 dx \int \Psi^*(x, k_\perp + (1-x)\Delta_\perp)\,\Psi(x, k_\perp)\,d^2k_\perp$$
$$\equiv \int_0^1 \mathcal{F}_a(x, t = -\Delta_\perp^2)\,dx$$

Form Factor in Gaussian $k_\perp$ model

$$F_a(t) = \int_0^1 \mathcal{F}_a(x,t)\,dx = \int_0^1 f_a(x)\,e^{(1-x)t/2x\lambda^2}\,dx$$

Large-t behavior is dominated by $x \to 1$ region

$F_a(t) \sim 1/t^{n_a+1}$ if $f_a(x) \sim (1-x)^{n_a}$

Regge-type models for NPDs

“Regge” improvement:

$$f(x) \sim x^{-\alpha(0)}$$
$$\Rightarrow \mathcal{F}(x,t) \sim x^{-\alpha(t)}$$
$$\Rightarrow \mathcal{F}(x,t) = f(x) x^{-\alpha' t}$$

Accomodating “canonical” quark counting rules:

$$\mathcal{F}(x,t) = f(x) x^{-\alpha' t(1-x)}|_{x\to 1}$$
$$\sim f(x) e^{\alpha'(1-x)^2 t}$$

Does not change small-x behavior but provides

$f(x)|_{x\to 1}$ vs. $F(t)|_{t\to\infty}$ interplay:

$$f(x) \sim (1-x)^n \Rightarrow F_1(t) \sim t^{-(n+1)/2}$$

Note: no pQCD involved in these counting rules!

Extra $1/t$ for $F_2(t)$

can be produced by taking

$$\mathcal{E}_a(x,t) \sim (1-x)^2 \mathcal{F}_a(x,t)$$

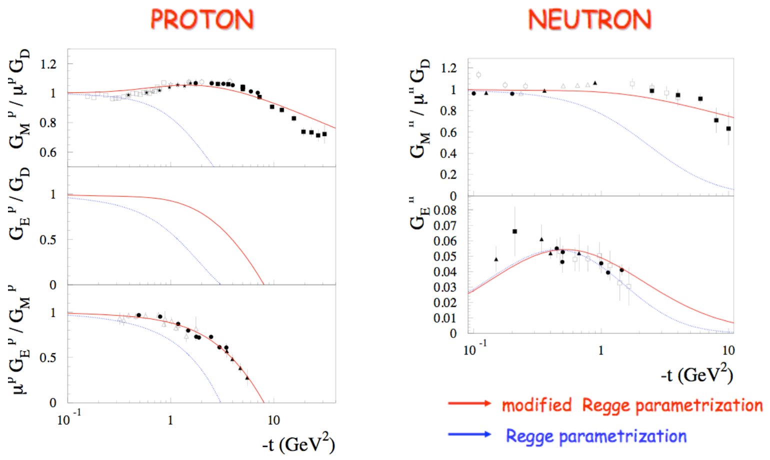
for “magnetic” NPDs

More general:

$$\mathcal{E}_a(x,t) \sim (1-x)^{\eta_a} \mathcal{F}_a(x,t)$$

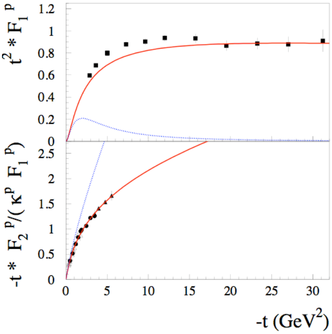
Fit : $\eta_u = 1.6$, $\eta_d = 1$

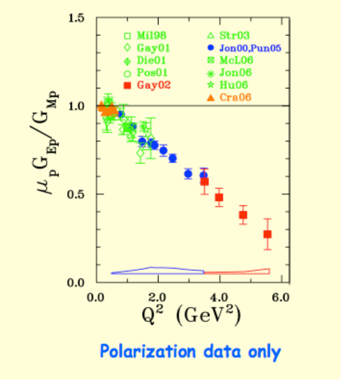
Fit to All Four Nucleon $G_{E,M}$ Form Factors



Fit to $F_{1,2}$ Proton Form Factors

Modified Regge parametrization describes JLab polarization transfer data on G_E^p/G_M^p and F_2^p/F_1^p





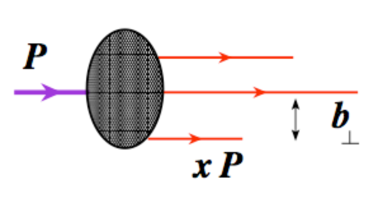
Impact Parameter Distributions

NPDs can be treated as Fourier transforms of impact parameter $b_\perp$ distributions $f_a(x, b_\perp)$

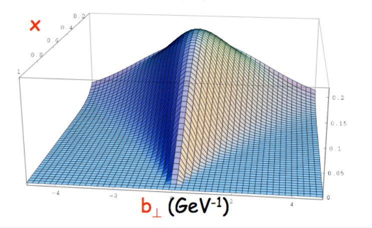
$$\mathcal{F}_a(x, t = -\Delta_\perp^2) = \int f_a(x, b_\perp) e^{i(\Delta_\perp b_\perp)} d^2 b_\perp$$

$b_\perp$ = $\perp$ distance to center of momentum

IPDs describe nucleon structure in transverse plane

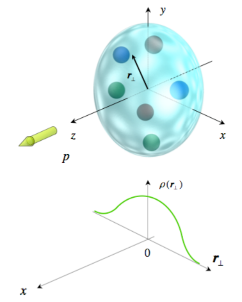


Distribution $f_u^p(x, b_\perp)$

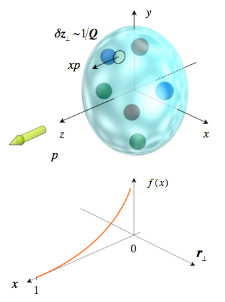


Structure of IPDs

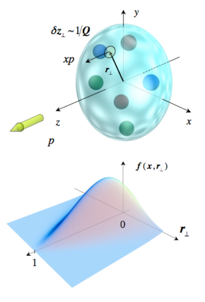
Distribution in $r_\perp$ plane



Distribution in z momentum



Combined $(x, r_\perp)$ distribution



Defining "center" in $\perp$ plane:

Geometric center: $r_\perp = \sum_i r_{i\perp}$
Center of momentum: $R_\perp = \sum_i x_i r_{i\perp}$

Impact parameter: $b_\perp = r_\perp^{\text{active}} - R_\perp$

Shape of $(x, b_\perp)$ distribution:

Shrinks when $x \to 1$: leading parton determines center of momentum

Purely kinematical effect!

Impact Parameter and Hadron Size

Compare impact parameter $b_\perp$ representation

$$\mathcal{F}_a(x, t = -\Delta_\perp^2) = \int f_a(x, b_\perp) e^{i(\Delta_\perp b_\perp)} d^2 b_\perp$$

with Drell-Yan formula in $r_\perp$ representation

$$\mathcal{F}_a(x, t = -\Delta_\perp^2) = \int |\Psi_a(x, r_\perp)|^2 e^{i(1-x)(\Delta_\perp r_\perp)} d^2 r_\perp$$

$r_\perp$ = $\perp$ distance between active quark and center of momentum of spectators

$$b_\perp = (1 - x) r_\perp$$

Feynman mechanism:

Form factors at large t are dominated by $x \to 1$, i.e. small $b_\perp$.

But this does not mean that small size configurations dominate at large t!

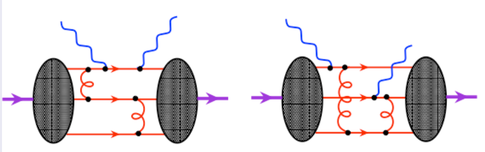
Logical conclusion:

$b_\perp$ is an artificial and confusing variable!

Size of configuration is determined by $r_\perp$!

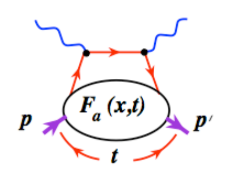
Application to Wide-Angle Compton Scattering

Perturbative QCD hard scattering mechanism



Small-size configurations

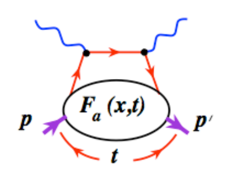
Handbag (soft) term



Feynman mechanism

Handbag model for Wide-Angle Compton Scattering

Handbag (soft) term



Approximately given by

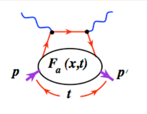
$$\left[\sum_a e_a^2 R_V^a(t)\right]^2 \frac{d\sigma}{dt}\Big|_{KN}$$

New form factor

$$R_V^a(t) = \int_0^1 \frac{\mathcal{F}^a(x,t)}{x}\,dx$$

NB: $R_V^a(t)$ is obtained from the same NPD as for FF

P. Kroll et al. model for WACS



More detailed formula

$$\frac{d\sigma}{dt} = \frac{d\hat{\sigma}}{dt}\left\{\frac{1}{2}\left[R_V^2(t) + \frac{-t}{4m_p^2}R_T^2(t) + R_A^2(t)\right] - \frac{us}{s^2+u^2}\left[R_V^2(t) + \frac{-t}{4m_p^2}R_T^2(t) - R_A^2(t)\right]\right\}$$

$$R_V(t) \simeq \sum_{q=u,d} e_q^2 \int_0^1 \frac{dx}{x} H_v^q(x,t)\,,$$

$$R_A(t) \simeq \sum_{q=u,d} e_q^2 \int_0^1 \frac{dx}{x} \widetilde{H}_v^q(x,t)\,,$$

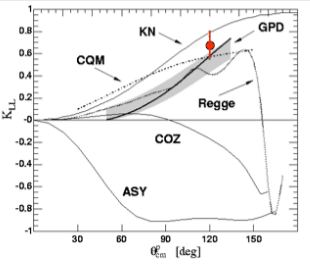
$$R_T(t) \simeq \sum_{q=u,d} e_q^2 \int_0^1 \frac{dx}{x} E_v^q(x,t)$$

Asymmetry test of WACS mechanism

Soft and hard mechanisms give drastically different predictions for polarization asymmetry

$$K_{LL} \approx A_{LL} = \frac{d\sigma(\uparrow, \lambda = 1)/dt - d\sigma(\uparrow, \lambda = -1)/dt}{d\sigma(\uparrow, \lambda = 1)/dt + d\sigma(\uparrow, \lambda = -1)/dt}$$

JLab result for K_{LL}



Lessons:

- Soft mechanism dominates
- Struck quark carries proton spin

Scaling test of WACS mechanism

pQCD prediction:

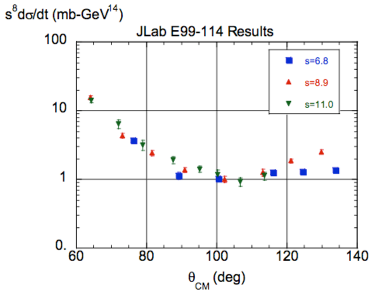
$$\frac{d\sigma}{dt} = \frac{1}{s^6} f(\theta_{CM})$$

Handbag prediction:

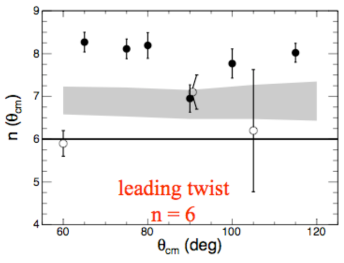
$$\frac{d\sigma}{dt} = \frac{1}{s^{N(\theta_{CM})}} f(\theta_{CM})$$

with $N(\theta_{CM}) \approx 8$

JLab result for scaling test

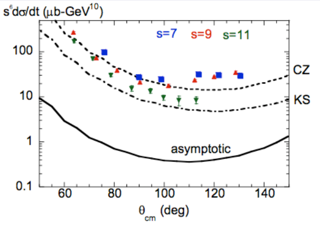


JLab result for effective power $N(\theta_{CM})$

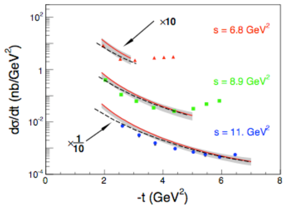


Absolute value of cross section

Comparison with pQCD



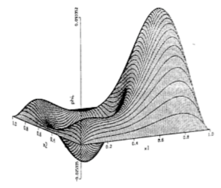
Comparison with soft models



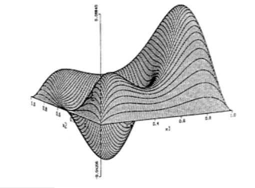
Dashed: CQM (J. Miller)

Bands: GPD (P. Kroll)

CZ wave function



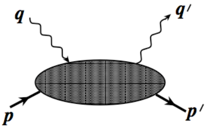
KS wave function



Soft models

describe magnitude of cross section without exotic assumptions

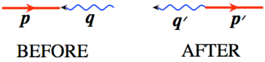
GPDs & Deeply Virtual Compton Scattering



Kinematics

Total CM energy $s = (q+p)^2 = (q'+p')^2$
LARGE: Above resonance region
Initial photon virtuality $Q^2 = -q^2$
LARGE $(> 1\,\text{GeV}^2)$
Invariant momentum transfer $t = \Delta^2 = (p-p')^2$
SMALL $(\ll 1\text{GeV}^2)$

- Picture in $\gamma^* N$ CM frame

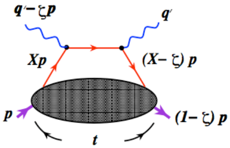


- Virtual photon momentum $q = q' - x_{Bj}p$ has component $-x_{Bj}p$ canceled by momentum transfer Δ
- $\Rightarrow$ Momentum transfer Δ has longitudinal component

$$\Delta^+ = x_{Bj}p^+ \quad , \quad x_{Bj} = \frac{Q^2}{2(pq)}$$

- "Skewed" Kinematics: $\Delta^+ = \zeta p^+$, with $\zeta = x_{Bj}$ for DVCS

Parton Picture for DVCS



Nonforward parton distribution

$\mathcal{F}_\zeta(X;t)$ depends on
X : fraction of p^+
ζ : skeweness
t : momentum transfer

- In forward $\Delta = 0$ limit

$$\mathcal{F}^a_{\zeta=0}(X, t=0) = f_a(X)$$

- Note: $\mathcal{F}^a_{\zeta=0}(X, t=0)$ comes from Exclusive DVCS Amplitude, while $f_a(X)$ comes from Inclusive DIS Cross Section
- Zero skeweness $\zeta = 0$ limit for nonzero t corresponds to nonforward parton densities

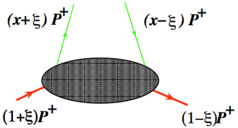
$$\mathcal{F}^a_{\zeta=0}(X, t) = \mathcal{F}^a(X, t)$$

- Local limit: relation to form factors

$$\int_0^1 \mathcal{F}^a_\zeta(X, t)\, dX/(1-\zeta/2) = F_1^a(t)$$

Off-forward Parton Distributions

Momentum fractions taken wrt average momentum $P = (p + p')/2$



4 functions of x, ξ, t:

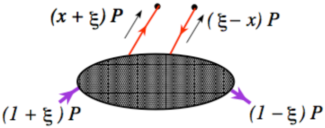
$H, E, \widetilde{H}, \widetilde{E}$

wrt hadron/parton helicity flip

$+/+, -/+, +/-, -/-$

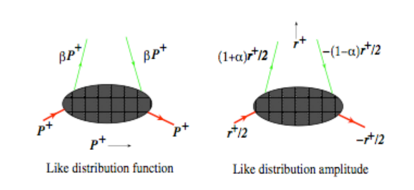
- Skeweness $\xi \equiv \Delta^+/2P^+$ is $\xi = x_{Bj}/(2 - x_{Bj})$
- 3 regions:

$\xi < x < 1$ $\sim$ quark distribution
$-1 < x < -\xi$ $\sim$ antiquark distribution
$-\xi < x < \xi$ $\sim$ distribution amplitude for $N \to \bar{q}qN'$

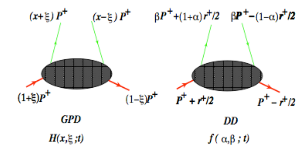


Double Distributions

"Superposition" of P^+ and r^+ momentum fluxes



Connection with OFPDs



Basic relation between fractions

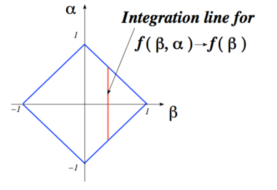
$$x = \beta + \xi\alpha$$

- Zero skewness limit $\xi = 0$ gives nonforward parton densities

$$\int_{-1+|\beta|}^{1-|\beta|} f_a(\beta, \alpha; t)\, d\alpha = \mathcal{F}_a(\beta, t)$$

Getting GPDs from DDs

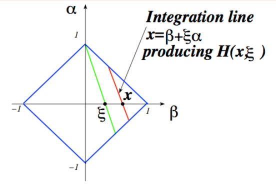
DDs live on rhombus $|\alpha| + |\beta| \le 1$



"Munich" symmetry:

$$f_a(\beta, \alpha; t) = f_a(\beta, -\alpha; t)$$

Converting DDs into GPDs



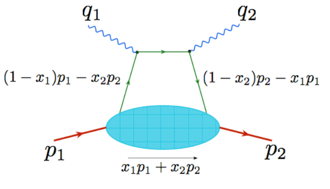
GPDs $H(x, \xi)$ are obtained from DDs $f(\beta, \alpha)$

by scanning DDs
at ξ-dependent angles

$\Rightarrow$ DD-tomography

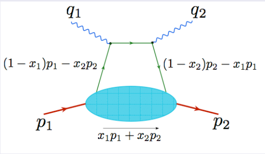
WACS in terms of DDs

Wide-Angle Compton Scattering



- Active quarks carry the fractions of initial and final hadron momenta
- Requiring small spectator mass gives $x_1x_2|t| \lesssim M^2$
- Requiring further small virtualities for active quarks gives $x_1, x_2 \lesssim M^2/|t|$
- Dominant region: small x_1, x_2

Hard subprocess amplitude



$$T(s,t) = e^2 \int_0^1 \int_0^1 \frac{F(x_1, x_2; t)\, \theta(x_1 + x_2 \leq 1)\, dx_1\, dx_2}{(1 - x_1 - x_2)s - x_1 x_2 t}$$

$$\approx e^2 \int_0^1 \frac{\mathcal{F}(\beta; t)}{\beta s}\, d\beta$$

$$x_1 = (1 - \beta + \alpha)/2 \ , \ x_2 = (1 - \beta - \alpha)/2$$

DD formalism gives formulas of A.R. and Kroll et al. with new form factors

Conclusions

- In nonrelativistic quantum mechanics, large-Q^2 asymptotics of form factors is determined by $r \sim 1/Q \to 0$ behavior of wave functions $\Psi(r)$
- In relativistic case, an extra possibility exists: Feynman mechanism, when $x \sim 1 - M/Q \to 1$ region of $\Psi(x, k_\perp)$ dominates, not requiring the system to be in small-size configuration
- Strong experimental evidence exists that, in QCD, form factors and WACS amplitude are dominated by Feynman/Drell-Yan mechanism
- For JLab: important to measure angular dependence of the WACS cross section