Theoretical and experimental challenges of SIDIS

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Hard scattering processes to access partons (quarks and gluons): high energy end of duality regime

- Measure longitudinal (lightcone) momentum fraction $x$ of struck quark and quark densities $f_{1}^{H\rightarrow q}(x)$

- Proton and neutron (possibly nuclei) for (limited) flavor sensitivity ($q = u, d$)

- Polarization to access longitudinal polarization $g_{1}^{q}(x) = \Delta q(x)$

- In principle unlimited access to higher twist contributions
Hard scattering processes to access partons (quarks and gluons): high energy end of duality regime

Measure longitudinal (lightcone) momentum fraction $x$ of struck quark and quark densities $f_{1 H \rightarrow q}(x)$ multiplied with decay densities $D_{1 q \rightarrow h}(z)$

Proton and neutron (possibly nuclei) for (limited) flavor sensitivity and selection of specific hadrons ($\pi$, K, ...) for further flavor selectivity.

Polarization to access longitudinal and transverse polarization, polarimetry for $\rho$ or $\Lambda$-production

Access to transverse momenta of partons: $p = xP + p_T$

Limited access to higher twist contributions!
Kinematical flexibility
Kinematical flexibility
Kinematical flexibility
(calculation of) cross section in DIS

Full calculation

OPTICAL THEOREM FOR DIS

LEADING (in 1/Q)
(calculation of) cross section in SIDIS

Full calculation

LEADING (in 1/Q)
Soft part in (SI)DIS

- In limit of large $Q^2$ the result of ‘handbag diagram’ survives
- $\ldots +$ contributions from $A^+$ gluons ensuring color gauge invariance

**SOFT PARTS IN DIS**

$$\Phi_{ij}(x) = \int dp^- d^2p_T \, \Phi(p, P, S)$$

$$= \int d\xi^- \frac{e^{ip \cdot \xi}}{2\pi} \langle P, S | \bar{\psi}_j(0) U(0, \xi) \psi_i(\xi) | P, S \rangle \bigg|_{\xi^+ = \xi_T = 0}$$

Ellis, Furmanski, Petronzio
Efremov, Radyushkin

$A^+$ gluons $\rightarrow$ gauge link
Parametrization of lightcone correlator

\[ \Phi(x) = \frac{1}{2} \left\{ f_1(x) \gamma_5 \psi_+ + S_L g_1(x) \gamma_5 \psi_+ + h_1(x) \frac{\gamma_T \gamma_5}{2} \right\} + \frac{M}{2P^+} \left\{ e(x) + g_T(x) \gamma_5 \gamma_T + S_L h_L(x) \frac{\gamma_5}{2} \right\} - \frac{M}{2P^+} \left\{ f_T(x) \gamma_T + i S_L e_L(x) \gamma_5 + h(x) \frac{\gamma_5}{2} \right\} \]

- \( M/P^+ \) parts appear as \( M/Q \) terms in cross section
- \( T \)-reversal applies to \( \Phi(x) \rightarrow \) no \( T \)-odd functions

\[ Jaffe \ & Ji \ NP \ B \ 375 \ (1992) \ 527 \]
\[ Jaffe \ & Ji \ PRL \ 71 \ (1993) \ 2547 \]
Basis of partons

- Good part of Dirac space is 2-dimensional
- Interpretation of DF’s

Two ‘Spin’ States for (Good) Quark Fields

Chiral eigenstates:

$$\psi_{R/L} = \frac{1}{2} (1 \pm \gamma_5) \psi : |R\rangle \quad \text{and} \quad |L\rangle$$

Or

Transverse spin eigenstates:

$$\psi_{\uparrow/\downarrow} = \frac{1}{2} (1 \pm \gamma^\alpha \gamma_5) \psi : |\uparrow\rangle \quad \text{and} \quad |\downarrow\rangle$$

Note: \([P_{R/L}, P_+] = [P_{\uparrow/\downarrow}, P_] = 0\)

Distribution Functions in Pictures

Unpolarized quark distribution \(q(x)\)

Helicity or chirality distribution \(\Delta q(x)\)

Transverse spin distribution or transversity \(\delta q(x)\)
Off-diagonal elements (RL or LR) are chiral-odd functions.

Chiral-odd soft parts must appear with partner in e.g. SIDIS, DY.
Theory

- Acces to transverse momenta of partons: $p = xP + p_T$
- Limited access to higher twist contributions!
Measuring transverse structure

- In a hard process one probes partons (quarks and gluons)
- Momenta fixed by kinematics (external momenta)
  - DIS \( x = x_B = Q^2/2P\cdot q \)
  - SIDIS \( z = z_h = P\cdot K_h/P\cdot q \)
- Also possible for transverse momenta
  - SIDIS \( q_T = k_T - p_T \)
    - \( = q + x_B P - K_h/z_h \approx -K_{h\perp}/z_h \)
  - 2-particle inclusive hadron-hadron scattering
    - \( q_T = p_{1T} + p_{2T} - k_{1T} - k_{2T} \)
    - \( = K_1/z_1 + K_2/z_2 - x_1 P_1 - x_2 P_2 \approx K_{1\perp}/z_1 + K_{2\perp}/z_2 \)
- Sensitivity for transverse momenta requires \( \geq 3 \) momenta
  - SIDIS: \( \gamma^* + H \to h + X \)
  - DY: \( H_1 + H_2 \to \gamma^* + X \)
  - \( e^+e^-: \gamma^* \to h_1 + h_2 + X \)
  - hadronproduction: \( H_1 + H_2 \to h + X \to h_1 + h_2 + X \)
Parametrization of $\Phi(x, p_T)$

- Additional TMD distribution functions, terms $\sim p_T$
- Link dependence allows also T-odd distribution functions since $T U[0,\infty] T^\dagger = U[0,-\infty]$
- Functions $h_{1T}^\perp$ and $f_{1T}^\perp$ (Sivers) nonzero! They come from gauge link (i.e. involve gluon fields)
- Similar functions (of course) exist as fragmentation functions (no T-constraints) $H_{1T}^\perp$ (Collins) and $D_{1T}^\perp$
- For spin 0 and spin $\frac{1}{2}$ T-odd effects require $p_T$

**DISTRIBUTION FUNCTIONS**

Parameterization of $p_T$-dependent soft part at leading order and including T-odd parts for polarized hadrons:

$$
\Phi_0(x, p_T) = \left\{ f_1(x, p_T^2) + i h_{1T}^\perp(x, p_T^2) \frac{p_T^\perp}{M} \right\} \psi_+
$$

$$
\Phi_L(x, p_T) = \left\{ S_L g_{1L}(x, p_T^2) \gamma_5 + S_L h_{1L}^\perp(x, p_T^2) \gamma_5 \frac{p_T^\perp}{M} \right\} \psi_+
$$

$$
\Phi_T(x, p_T) = \left\{ g_{1T}(x, p_T^2) \frac{p_T \cdot S_T}{M} \gamma_5 + f_{1T}^\perp(x, p_T^2) \frac{\epsilon_T \rho \sigma p_T^2 S_T^g}{M} \right. \\
+ h_{1T}(x, p_T^2) \gamma_5 S_T + h_{1T}^\perp(x, p_T^2) \frac{p_T \cdot S_T}{M} \gamma_5 \frac{p_T^\perp}{M} \right\} \psi_+
$$

$$
\Phi_{LL}(x, p_T) = \ldots
$$
Interpretation

unpolarized quark distribution

need $p_T$

T-odd

helicity or chirality distribution

need $p_T$

T-odd

transverse spin distr. or transversity

need $p_T$

need $p_T$

DISTRIBUTION FUNCTIONS IN PICTURES

\[ f_1(x, p_T^2) = \begin{array}{c}
\bullet
\end{array} = \begin{array}{c}
\text{R} + \text{L}
\end{array} \]

\[ \frac{p_T \times S_T}{M} f_{1T}(x, p_T^2) = \begin{array}{c}
\bullet
\end{array} - \begin{array}{c}
\text{L}
\end{array} \]

\[ S_L g_{1L}(x, p_T^2) = \begin{array}{c}
\text{R}
\rightarrow - \text{L}
\end{array} \]

\[ \frac{p_T \cdot S_T}{M} g_{1T}(x, p_T^2) = \begin{array}{c}
\text{R}
\rightarrow - \text{L}
\end{array} \]

\[ S_T^\alpha h_{1T}(x, p_T^2) = \begin{array}{c}
\bullet
\end{array} - \begin{array}{c}
\text{L}
\end{array} \]

\[ \frac{i}{M} \frac{p_T^\alpha}{p_T} h_{1T}^\perp(x, p_T^2) = \begin{array}{c}
\bullet
\end{array} - \begin{array}{c}
\text{L}
\end{array} \]

\[ S_L \frac{p_T^\alpha}{p_T} h_{1L}(x, p_T^2) = \begin{array}{c}
\bullet
\end{array} - \begin{array}{c}
\text{L}
\end{array} \]

\[ \frac{p_T \cdot S_T}{M} \frac{p_T^\alpha}{p_T} h_{1T}^\perp(x, p_T^2) = \begin{array}{c}
\bullet
\end{array} - \begin{array}{c}
\text{L}
\end{array} \]

unpolarized hadrons
Matrix representation for $M = [\Phi^{[\pm]}(x,p_T)\gamma^+]^T$

- $p_T$-dependent functions

**Matrix Representation for Spin 1/2**

$p_T$-dependent quark distributions:

\[
\begin{pmatrix}
    f_1 + g_{1L} & \frac{|p_T|}{M} e^{i\phi} g_{1T} & \frac{|p_T|}{M} e^{-i\phi} h_{1L} & 2h_1 \\
    \frac{|p_T|}{M} e^{-i\phi} g_{1T} & f_1 - g_{1L} & \frac{|p_T|^2}{M^2} e^{-2i\phi} h_{1T} & -\frac{|p_T|}{M} e^{-i\phi} h_{1L} \\
    \frac{|p_T|}{M} e^{i\phi} h_{1L} & \frac{|p_T|^2}{M^2} e^{2i\phi} h_{1T} & f_1 - g_{1L} & -\frac{|p_T|}{M} e^{i\phi} g_{1T} \\
    2h_1 & -\frac{|p_T|}{M} e^{i\phi} h_{1L} & -\frac{|p_T|}{M} e^{-i\phi} g_{1T} & f_1 + g_{1L}
\end{pmatrix}
\]

$T$-odd: $g_{1T} \rightarrow g_{1T} - i f_{1T} \quad \text{and} \quad h_{1L} \rightarrow h_{1L} + i h_{1L}$ (imaginary parts)

Bacchetta, Boglione, Henneman & Mulders
PRL 85 (2000) 712
Possibilities in leptoproduction of pions

Asymmetries are (theoretically) most clean in terms of transverse moments ($p_T$-weighted functions).

**Matrix Representation for Spin 0**

$p_T$-dependent quark fragmentation functions:

$$M^{(\text{dec})} = \begin{pmatrix}
D_1 & \frac{i |k_T| e^{-i\phi}}{M_h} H^\perp_1 \\
-\frac{i |k_T| e^{i\phi}}{M_h} H^\perp_1 & D_1
\end{pmatrix}$$

**SIDIS:** $\ell + H^\uparrow \rightarrow \ell + h + X$

\[
\left\langle \frac{Q_T}{M} \sin(\phi^\ell_h - \phi^\ell_S) \right\rangle_{\text{OTO}} = \frac{2\pi\alpha^2}{Q^4} |S_T| \left(1 - y + \frac{1}{2} y^2\right) \sum_{a,a} e^\ell_a x_B f_{1T}^{\perp(1)\alpha}(x_B) D_1^\alpha(z_h)
\]

\[
\left\langle \frac{Q_T}{M_h} \sin(\phi^\ell_h + \phi^\ell_S) \right\rangle_{\text{OTO}} = \frac{2\pi\alpha^2}{Q^4} |S_T| 2(1 - y) \sum_{a,a} e^\ell_a x_B h_1^{\alpha}(x_B) H_1^{\perp(1)\alpha}(z_h)
\]

$H_1^\perp$ is T-odd and chiral-odd.
Factorization studies in SIDIS

- Measurements of TMD distribution (and fragmentation functions)
- Investigate the $p_T$-dependence
- Important as experimental input on factorization behavior
- Normal situation

$$\Phi_2(x, p_T) \rightarrow \frac{\alpha(p_T^2)}{p_T^2} K \otimes \Phi_2(x)$$

- TMD functions

$$\Phi_3(x, p_T) \rightarrow \frac{\alpha(p_T^2)}{|p_T|} K \otimes \Phi_2(x)$$

After integration: subleading in $1/Q \rightarrow$ NLO (in $\alpha_s$)

- Where: look for $<\cos \phi_h>$, $<\sin \phi_h>$ azimuthal asymmetries.
Transverse momenta and higher twist

- Link between weighted TMD functions (transverse moments) and higher twist functions (QCD eom)

- $g_{1T}^{(1)}(x, p_T^2)$ from SIDIS $<q_T \cos(\phi_s-\phi_n)>$

- $g_2$ from DIS at 1/Q $<\cos(\phi_s)>$

- Role of gauge link needs to be (further) investigated.

**ESTIMATE OF $g_{1T}$**

- datapoints: SLAC $g_2$-data: $g_{1T}^{(1)}(x) = -\int_x^1 dy \ g_2(y)$ (including E155, preliminary)

- line: above relation with $g_2(x) = g_{WW}^{WW}(x)$
Single spin asymmetries
Color gauge invariance

- Nonlocal combinations of colored fields must be joined by a gauge link:
  \[
  \bar{\psi}(0)\psi(\xi) \to \bar{\psi}(0)U(0, \xi)\psi(\xi) \to U(0, \xi) = \mathcal{P} \exp \left( -ig \int_0^\xi ds^\mu A_\mu \right)
  \]

- Gauge link structure is calculated from collinear A.n gluons exchanged between soft and hard part

\[\text{DIS} \to \Phi^{[U]}\]

\[\text{SIDIS} \to \Phi^{[U^+]} = \Phi^{[+]}\]

\[\text{DY} \to \Phi^{[U^-]} = \Phi^{[-]}\]

- Link structure for TMD functions depends on the hard process!
  (Pijlman, Bomhof, PM unifies Brodsky, Schmidt, Ji, Yuan, …)
Summary

- Access to the (transverse) partonic structure of hadrons is still in a very preliminary stage (azimuthal asymmetries by themselves or combined with single or double spin asymmetries)
- Theoretical interesting parts are contained in $\alpha_s$, $p_T$ and $1/Q$ structure of cross section putting demands on experimental detection capabilities
- Like the spin puzzle, interplay of theory and experiment is essential