“Time-reversal odd” (T-odd) effects in semi-inclusive deep-inelastic lepton-nucleon scattering

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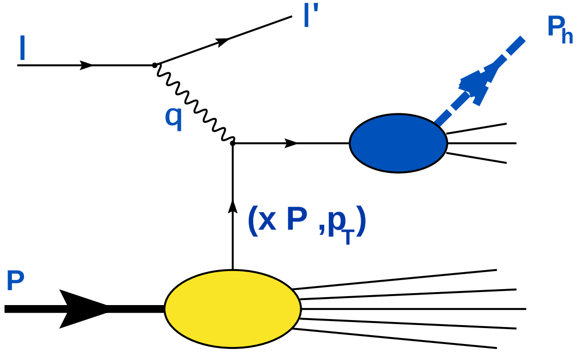
Partonic picture of semi-inclusive deep-inelastic scattering (SIDIS)

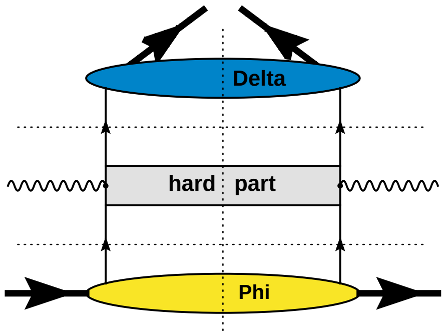


kinematic variables:

$$P^+ \text{ large} \qquad ; \qquad -q^2 = Q^2 \text{ large}$$

$$x_B = \frac{Q^2}{2P\cdot q}; \quad z_h = \frac{P\cdot P_h}{P\cdot q}; \quad |\vec{P}_{h\perp}| \ll Q$$

The SIDIS cross section



$$\sigma_{\text{SIDIS}} = \Phi \otimes (\text{hard}) \otimes \Delta$$

quark-quark correlators:

$$\Phi_{ij}(x,\vec{p}_T) = \int \frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3} \, e^{ixP^+\xi^- - i\vec{p}_T\cdot\vec{\xi}_T} \langle P|\, \bar{\psi}_j(0)\, \psi_i(\xi)\, |P\rangle \Big|_{\xi^+=0}$$

$$\Delta_{ij}(z,\vec{k}_T) = \mathcal{FT} \sum_X \langle 0|\, \psi_i(\xi)\, |P_h;X\rangle\langle P_h;X|\, \bar{\psi}_j(0)\, |0\rangle \Big|_{\xi^-=0}$$

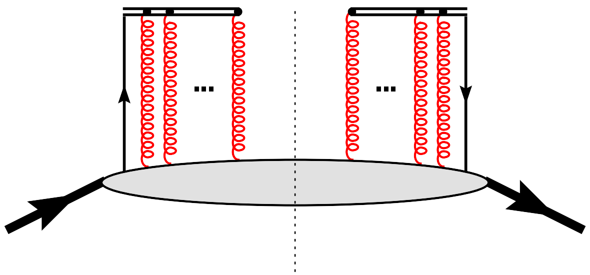
- Intrinsic dependence on the transv. parton momentum $\vec{p}_T$.
- $\Phi \longrightarrow p_T$-dependent parton distributions.
- $\Delta \longrightarrow k_T$-dependent fragmentation functions.

Color gauge invariance

mathematical definition of the gauge link:

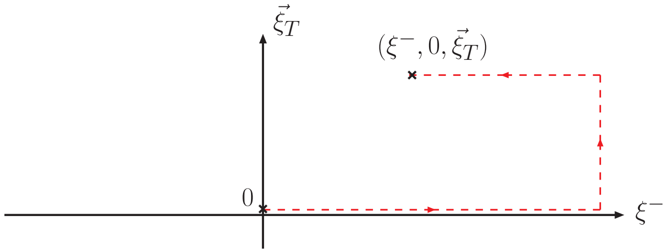
$$\mathcal{W}(a, b|\text{path}) = \mathscr{P} \exp\left(-ig \int_a^b ds^\mu \; A_\mu(s)\right)$$

physical picture:



$$\Phi(x, \vec{p}_T) = \mathcal{FT}\left[\langle P|\, \bar{\psi}_j(0)\, \mathcal{W}(0,\xi)\, \psi_i(\xi)\, |P\rangle\big|_{\xi^+=0}\right]$$

Wilson line in SIDIS:



$$\mathcal{W}(0,\xi|n_-) = \left[0\,;\, \infty, 0, \vec{0}_T\right] \times \left[\infty, 0, \vec{0}_T\,;\, \infty, 0, \vec{\xi}_T\right] \times \left[\infty, 0, \vec{\xi}_T\,;\, \xi^-, 0, \vec{\xi}_T\right]$$

- Consequences of the Gauge link: T-odd parton distributions, contributions to single-spin asymmetries.

Parameterization of qq-correlators

1.) Φ_{ij}, Δ_{ij} matrices in Dirac-space $\longrightarrow$ decomposition into $\mathbb{1}$, γ_5, γ^μ, $\gamma^\mu\gamma_5$, $i\sigma^{\mu\nu}\gamma_5$

2.) Coefficients $\Longrightarrow$ parameterization of traces in PDFs and FFs, e.g.

$$\frac{1}{2}\mathrm{Tr}\left[\Phi(x,\vec{p}_T)\gamma^+\right] = f_1(x,\vec{p}_T^{\,2}) - \frac{\vec{p}_T \times \vec{S}_T}{M} f_{1T}^{\perp}(x,\vec{p}_T^{\,2})$$

3.) Parameterization restricted by constraints due to parity (P) and time-reversal (T)-transformations.

4.) T-constraint only valid if the Wilson line is neglected $\Longrightarrow$ Wilson line enables T-odd PDFs.

5.) Choice of Wilson line $\Longrightarrow$ dependence of Φ and Δ on lightcone vectors n_- and n_+,

$$\Phi = \Phi_{ij}(x,\vec{p}_T;S\,|\,n_-) \quad ; \quad \Delta = \Delta_{ij}(z,\vec{k}_T;S_h\,|\,n_+)$$

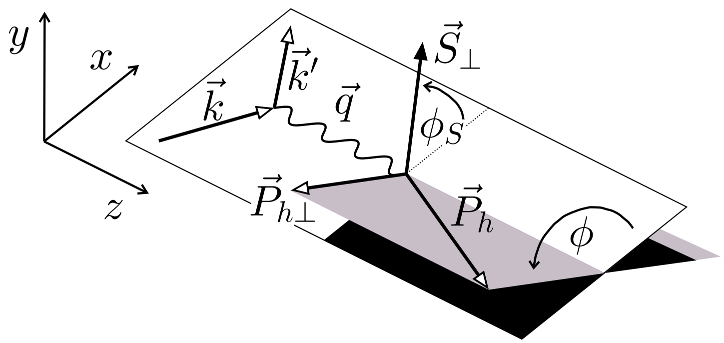
$\Longrightarrow$ induces additional T-odd twist-3 PDFs and FFs $g^\perp$, $e_T^\perp$, f_T', $G^\perp$.

6.) *Complete parameterization.*

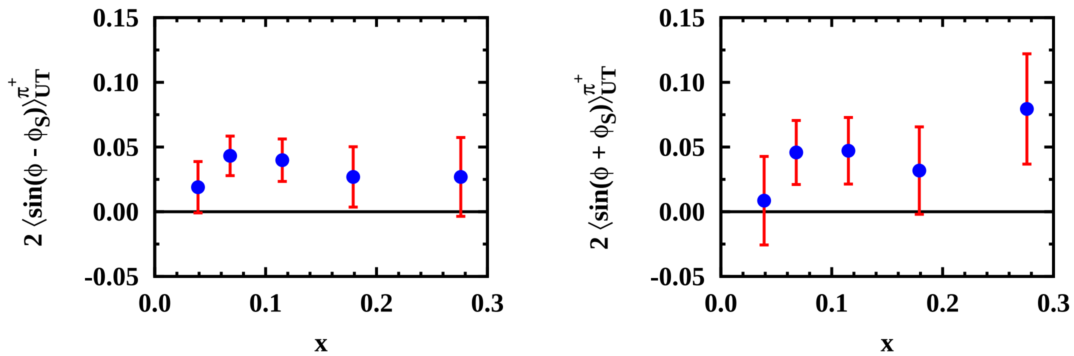
Single-Spin asymmetries $A = \frac{\sigma(\uparrow)-\sigma(\downarrow)}{\sigma(\uparrow)+\sigma(\downarrow)}$ in SIDIS

Twist-2 SSA for transversely pol. target:

$$A_{UT}^{\text{Sivers}} \propto \sin(\phi_h - \phi_s) f_{1T}^{\perp} D_1$$
$$A_{UT}^{\text{Collins}} \propto \sin(\phi_h + \phi_s) h_1 H_1^{\perp},$$

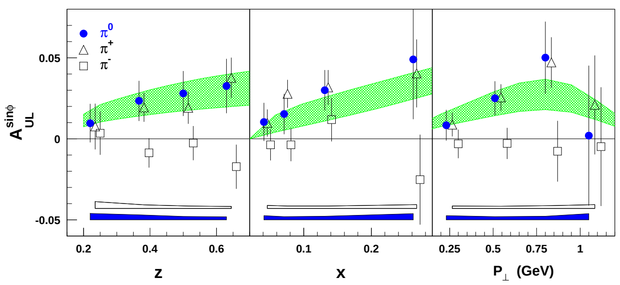


Experiments by HERMES (also by COMPASS):

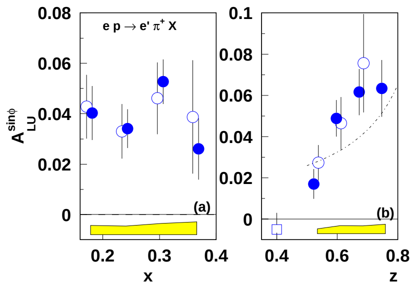


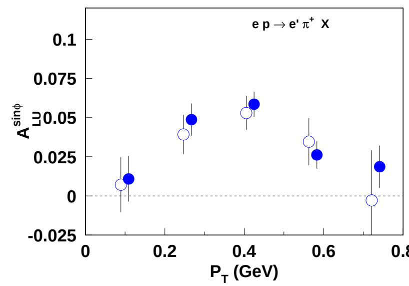
Twist-3 SSAs for longitudinally pol. target and beam:

$A_{UL}^{\sin\phi}$ measured by HERMES (2001):



$A_{LU}^{\sin\phi}$ measured by CLAS (2003):





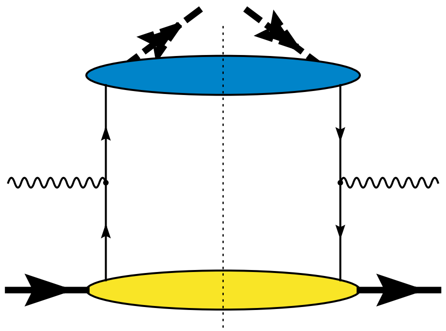
$\Longrightarrow$ Twist-3 observables theoretically more complicated!

“Tree-level” formalism of twist-3 observables

(Mulders, Tangerman, 1996; Boer, Mulders, Pijlman, 2003)

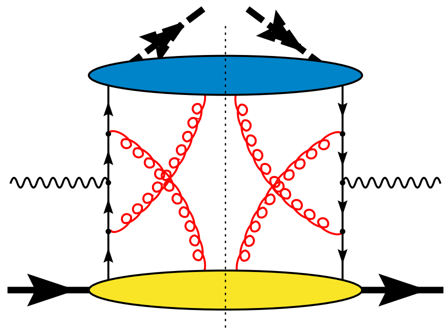
Contributions to twist-3 observables:

1. Twist-3 correlation functions:



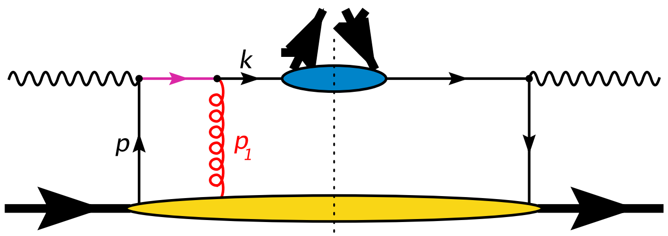
Contribution contains twist-3 PDFs/FFs

2. Genuine twist-3:



Quark-Gluon-Quark correlators

Treatment of qgq-Correlators:



$$\propto \int dp_1\, \mathcal{FT}\left(\langle P,S|\, \bar{\psi}(0)\gamma^\mu \Delta \gamma_\rho \frac{\not k - \not p_1 + m}{(k-p_1)^2 - m^2 + i0} g A^\rho(\eta) \gamma^\nu \psi(\xi)\, |P,S\rangle \right)$$

$1/Q$-expansion of the propagator:

- A^+ ($A_T^i(x^- = \infty)$)-gluons $\longrightarrow$ Gauge Link
- Transverse $A_T^i(x)$-gluons $\longrightarrow$ Twist-3 $\longrightarrow$ color g.i. qgq-corr. Φ_A^i

- Parameterization of qgq-correlator $\Phi_A^i \propto \langle P|\,\bar{\psi}\,(\int \mathcal{W}\,F^{+i}\,\mathcal{W})\,\psi\,|P\rangle$ in terms of *tilde*-functions, e.g.

$$\mathrm{Tr}\left[\Phi_{A,i}\, i\sigma^{i+}\gamma_5\right] = 2Mx\left((\tilde{h}_L + i\tilde{e}_L) + \frac{\vec{p}_T\cdot\vec{S}_T}{M}(\tilde{h}_T + i\tilde{e}_T)\right), \ldots$$

- Connection between *tilde*-functions and PDFs/FFs via QCD-equation of motion, e.g.

$$\tilde{h}_L = h_L + \frac{\vec{p}_T^{\,2}}{2M^2}\frac{h_{1L}^\perp}{x} - \frac{m}{M}\frac{g_{1L}}{x} \quad ; \quad \tilde{g}_T = g_T - \frac{\vec{p}_T^{\,2}}{2M^2}\frac{g_{1T}}{x} - \frac{m}{M}\frac{h_1}{x}, \ldots$$

$$\tilde{e}_T = e_T \quad ; \quad \tilde{e}_T^\perp = e_T^\perp - \frac{m}{M}\frac{f_{1T}^\perp}{x}, \ldots$$

- Decomposition of SIDIS-cross section into structure functions $F(x_B, z_h, Q^2, \vec{P}_{h\perp}^2)$:

Twist-2: F_{UU}, $F_{UU}^{\cos 2\phi_h}$, $F_{UL}^{\sin 2\phi_h}$, F_{LL}, $F_{UT}^{\sin(\phi_h-\phi_S)}$, $F_{UT}^{\sin(\phi_h+\phi_S)}$, $F_{UT}^{\sin(3\phi_h-\phi_S)}$, $F_{LT}^{\cos(\phi_h-\phi_S)}$

Twist-3: $F_{UU}^{\cos\phi_h}$, $F_{LU}^{\sin\phi_h}$, $F_{UL}^{\sin\phi_h}$, $F_{LL}^{\cos\phi_h}$, $F_{UT}^{\sin\phi_S}$, $F_{UT}^{\sin(2\phi_h-\phi_S)}$, $F_{LT}^{\cos\phi_h}$, $F_{LT}^{\cos(2\phi_h-\phi_S)}$

- Structure functions in terms of PDFs and FFs, e.g.
(Mulders, Tangerman, 1996; Boer, Mulders, 1998; Bacchetta, Mulders, Pijlman, 2004;
Bacchetta, Diehl, Goeke, Metz, Mulders, M.S., 2006)

$$F_{LT}^{\cos(\phi_h-\phi_S)} \propto I\left[\alpha g_{1T} D_1\right],$$

$$F_{LT}^{\cos\phi_S} \propto \frac{M}{Q}\, I\left[\alpha_1 g_T D_1 + \alpha_2 h_1 \tilde{E} + \alpha_3 e_T H_1^\perp + \alpha_4 g_{1T}\tilde{D}^\perp + \alpha_5 e_T^\perp H_1^\perp + \alpha_6 f_{1T}^\perp \tilde{G}^\perp\right],$$

$$F_{LU}^{\sin\phi_h} \propto \frac{M}{Q}\, I\left[\beta_1 e H_1^\perp + \beta_2 f_1 \tilde{G}^\perp + \beta_3 g^\perp D_1 + \beta_4 h_1^\perp \tilde{E}\right], \ldots$$

- Simplifications:

1.) Wandzura-Wilczek-approximation: neglect *tilde*-functions

$$F_{LT}^{\cos\phi_S} \longrightarrow \frac{M}{Q}\, I\left[\beta g_{1T} D_1\right]$$

2.) Semi-inclusive jet production: $D_1 \longrightarrow \delta(1-z)\delta^{(2)}(\vec{k}_T),\ H_1^{\perp} = \ldots = 0$

jet asymmetries

$$\boxed{A_{LU,\,\text{jet}}^{\sin\phi_h} \propto \frac{M}{Q}\frac{g^{\perp(1)}(x)}{f_1(x)}} \quad ; \quad \boxed{A_{UL,\,\text{jet}}^{\sin\phi_h} \propto \frac{M}{Q}\frac{f_L^{\perp(1)}(x)}{f_1(x)}}$$

- Results valid when applying “tree level” formalism for twist-3 observables.

Check of tree-level formalism?

↓

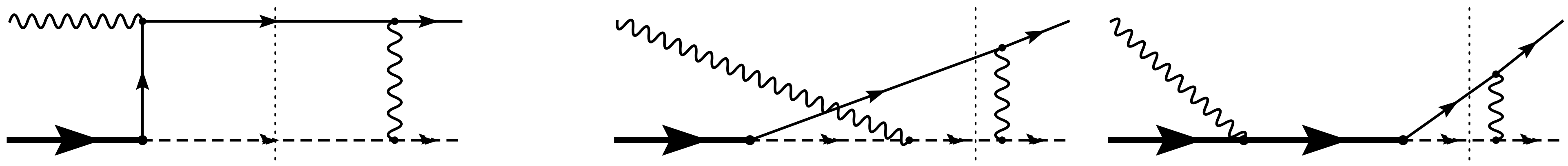
Model calculations

Longitudinal jet-SSAs in the diquark-spectator-model:

(Afanasev, Carlson, 2003, 2006; Metz, M.S., 2004)

Diagrams with imaginary parts necessary for SSAs:

Rescattering effect (RE): non-compatible (NC) diagrams:



Non-compatible diagrams needed to restore electromagnetic gauge invariance ($M = \varepsilon_\mu(q) J^\mu$):

$$q_\mu(\mathrm{Im}\,(J^\mu_{RE}) + \mathrm{Im}\left(J^\mu_{NC,D}\right) + \mathrm{Im}\left(J^\mu_{NC,N}\right)) = 0$$

Difference to twist-2 observables:

1) A_{UT}: NC-diagrams ($J^\perp_{NC}$) are suppressed by $\mathcal{O}(1/Q^2)$.

2) A_{UL}, A_{LU}: NC-diagrams ($J^\pm_{NC}$) are of the same order as RE-diagram ($J^\pm_{RE}$).

$\Longrightarrow$ Result: Non-vanishing asymmetries $\boxed{A_{UL} \neq 0}$, $\boxed{A_{LU} \neq 0}$.

Model calculation of $g^{\perp}$

(Gamberg, Hwang, Metz, M.S., 2006)

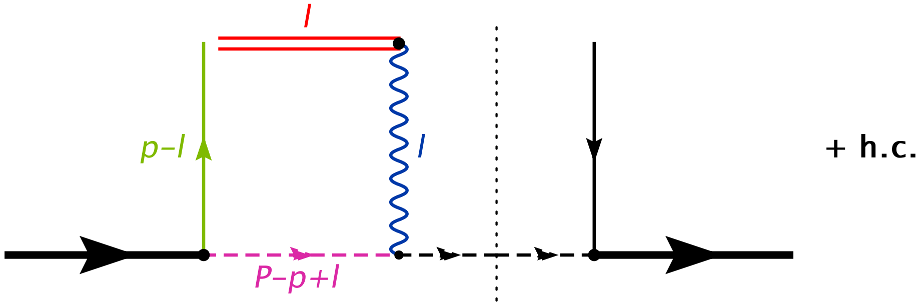
Beam-spin asymmetry: $A_{LU,\,\mathrm{jet}}^{\sin\phi} \propto g^{\perp(1)}(x)$ $\longrightarrow$ $g^{\perp}$ in the diquark-spectator-model

- Starting "from scratch":

$$\Phi_{ij}(x,\vec{p}_T;P,S|n) = \mathcal{FT}\Big(\langle P,S|\,\bar{\psi}_j(0)\,\mathcal{W}^{[-]}(0;\infty^-|n)\quad |\mathrm{dq}\rangle\langle\mathrm{dq}|\quad W^{[-]}((\infty^-,\vec{\xi}_T);\xi|n)\,\psi_i(\xi)\,|P,S\rangle\Big)$$

$$g^{\perp} \propto \mathrm{Tr}\left[\Phi(x,\vec{p}_T)\gamma^i\gamma_5\right]\Big|_{\mathrm{unpol}}$$

- Leading non-trivial order:



$$g^{\perp} \propto \sum_{\pm}\int \frac{d^4l}{(2\pi)^4}\,\frac{v\cdot(2P-2p+l)\left[\epsilon_T^{ij}p_T^j\left(P^+l^- - P^-l^+\right)+\epsilon_T^{ij}l_T^j\left(P^-p^+ - P^+p^-\right)\right]}{[(l\cdot n)\pm i0]\,[l^2\mp i0]\,[(P-p+l)^2-m_s^2\mp i0]\left[(p-l)^2-m_q^2\mp i0\right]}$$

For $n=[1^-,0^+,\vec{0}_T]$ on the light-cone $\longrightarrow$ *Divergence*!

- Regularization: "non lightlike" Wilson lines: $n \longrightarrow v = [v^-, v^+, \vec{0}_T], \left|\frac{v^+}{v^-}\right| \ll 1$
- Result:

$$g^\perp(x, \vec{p}_T^{\,2}, v) \propto \frac{1-x}{x} \ln\left(\frac{v^2 \tilde{m}^2}{2(v \cdot P)^2}\right) + \text{finite} + \mathcal{O}\left(\left|\frac{v^+}{v^-}\right|\right)$$

- Shows LC-divergence explicitly $\longrightarrow$ same divergence for *all* T-odd twist-3 PDFs, also in quark-target-model.

- Finite Box-graph contributions for twist-2 PDFs, especially $f_{1T}^\perp, h_1^\perp$.

- Regularization procedure: "Tree-level" predictions $A_{LU} \propto g^\perp$ must be modified.

- Factorization theorem for twist-2 observables: (Ji, Ma, Yuan, 2004)

$$\frac{d\sigma_{UU}}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} \propto \int d^2p_T\, d^2k_T \left(\int d^2l_T\; S(\vec{l}_T)\delta^{(2)}(\vec{p}_T - \frac{\vec{P}_{h\perp}}{z_h} - \vec{k}_T + \vec{l}_T)\right) f_1(x_B, \vec{p}_T^{\,2})\, D_1(z_h, \vec{k}_T^{\,2}) + \ldots$$

Soft factor $S(\vec{l}_T)$ due to soft gluon radiation $\longrightarrow$ modifies δ-function.

PDFs/FFs: "non light-like" Wilson lines $\longrightarrow$ "non-light-likeness" parameter $\zeta = \sqrt{\frac{2(P\cdot v)^2}{v^2}}$

- Generalization of "all-order factorization" for twist-3 observables possible?

Summary & Conclusions

- ✘ **Semi-inclusive DIS:**
  Transverse parton momentum has to be taken into account
  $\longrightarrow$ p_T-dependent parton distributions and fragmentation functions
  $\longrightarrow$ Gauge link ensures color gauge invariance and enables T-odd parton distributions.
  $\longrightarrow$ Single-spin asymmetries.

- ✘ **Analysis of the SIDIS-cross section:**
  Quark-Quark Correlators $\Phi_{ij}(x, \vec{p}_T)$ und $\Delta_{ij}(z, \vec{k}_T)$.
  $\longrightarrow$ complete determination of the twist-3 contribution to spin-observables in SIDIS in the parton-model using "tree-level" formalism.

- ✘ **Testing "tree-level" approximation:**
  Calculation of longitudinal asymmetries A_{UL} and A_{LU} in a diquark-spectator-model.
  $\longrightarrow$(IR and collinear) finite results.
  Calculation of T-odd twist-3 parton distributions
  $\longrightarrow$ Light-cone divergences.
  $\longrightarrow$ Modification of existing twist-3 predictions necessary.