

Nucleon EM Form Factors in Dispersion Theory

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Agenda

- Introduction
- Dispersion Analysis Results
- Two-Photon-Echange Corrections
- Conclusions & Outlook

Why Nucleon Form Factors?

- Nucleons are basic constituents of matter
- Detailed understanding of nucleon structure
 - nucleon radii, vector meson coupling constants,...
 - **perturbative and nonperturbative aspects of QCD**
 - input for wide variety of experiments, e.g.
 - * **Lamb shift measurements** in atomic hydrogen
 - * Sea quark dynamics in **strange form factors**
- Theoretical tools
 - Effective Field Theory → **ChPT** (low momentum transfer)
 - **Dispersion theory** (all momentum transfers, space- & time-like)
 - **Lattice QCD** (this workshop)

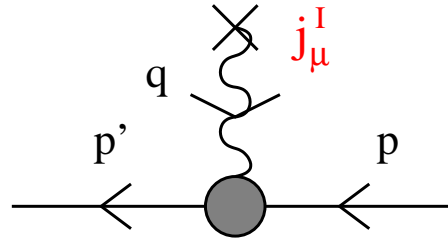
Why Dispersion Theory?

- Based on fundamental principles: unitarity, analyticity
→ (Largely) model independent
- Connects data over full range of momentum transfers:
→ time-like and space-like data
- Connects to data from different processes (πN scattering, ...)
- Simultaneous analysis of all four form factors
- Spectral functions encode perturbative and non-perturbative physics:
vector meson couplings, multi-meson continua, pion cloud,...
- Constraints from ChPT, pQCD
- Extraction of nucleon radii

Electromagnetic Form Factors

- Definition:

$$t \equiv q^2 = (p' - p)^2 = -Q^2$$



$$\langle N(p') | j_\mu^I | N(p) \rangle = \bar{u}(p') \left[F_1^I(t) \gamma_\mu + i \frac{F_2^I(t)}{2m} \sigma_{\mu\nu} q^\nu \right] u(p), \quad I = S, V$$

- Normalization: $F_1^S(0) = F_1^V(0) = 1/2$, $F_2^{S,V}(0) = (\kappa_p \pm \kappa_n)/2$

- Sachs form factors: $G_E = F_1 + \frac{t}{4m^2} F_2$, $G_M = F_1 + F_2$

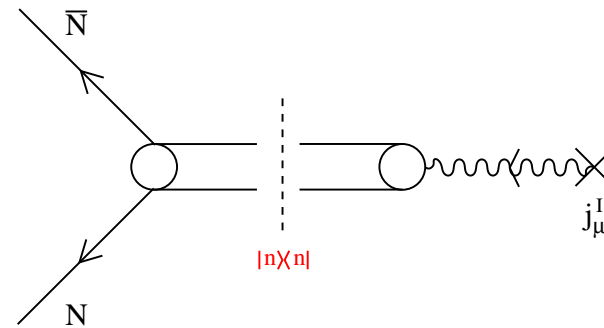
- Radii: $F(t) = F(0) [1 + t \langle r^2 \rangle / 6 + \dots]$

Starting Point: Spectral Decomposition

- **Crossing:** $\langle N(p') | j_\mu^I | N(p) \rangle \longleftrightarrow \langle \bar{N}(\bar{p}) N(p) | j_\mu^I | 0 \rangle$
- **Spectral decomposition:** (using microcausality, analyticity,...)
(Federbush, Goldberger, Treiman, Chew,...)

$$\text{Im} \langle \bar{N}(\bar{p}) N(p) | j_\mu^I | 0 \rangle \sim \sum_n \langle \bar{N}(\bar{p}) N(p) | n \rangle \langle n | j_\mu^I | 0 \rangle \implies \text{Im } F$$

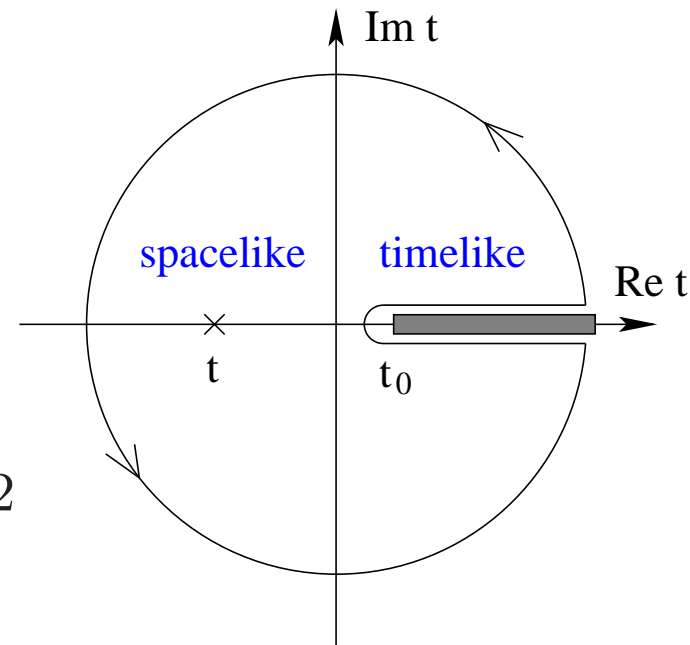
- **On-shell intermediate states**
 - **imaginary part**
 - relate physical matrix elements



- **Intermediate States:**
 - j_μ^S (isoscalar): $3\pi, 5\pi, \dots, K\bar{K}, K\bar{K}\pi, \dots$
 - j_μ^V (isovector): $2\pi, 4\pi, \dots$

Dispersion Relations

- Form factors have multiple cuts in interval $[t_n, \infty[$ ($n = 0, 1, 2, \dots$)
- Dispersion relation for F :
 - Apply Cauchy's formula in complex t -plane
 - Subtractions?



$$F_i(t) = \frac{1}{\pi} \int_{t_0}^{\infty} \frac{\text{Im} F_i(t')}{t' - t} dt', \quad i = 1, 2$$

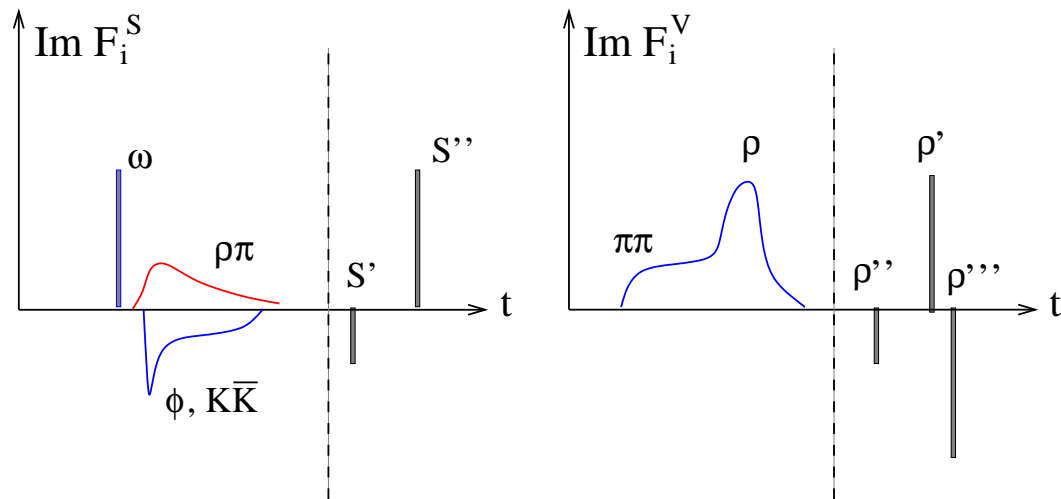
- Higher mass intermediate states are suppressed
- Spectral functions are central objects \longrightarrow from where to take?

EM Spectral Functions

- Include 2π and $K\bar{K}$, $\rho\pi (\rightarrow 3\pi)$ continua as independent input
- Higher mass states: dominated/parametrized by vector meson poles (widths can be included)

$$\text{Im}F_i(t) = \sum_v \pi a_i^v \delta(t - M_v^2), \quad a_i^v = \frac{M_v^2}{f_v} g_{vNN} \Rightarrow F_i(t) = \sum_v \frac{a_i^v}{M_v^2 - t}$$

- EM Spectral Function:



Spectral Function: 2π -, $K\bar{K}$ -, $\rho\pi$ -Continua

- Continuum contributions:
 - 2π : analytic continuation of πN scattering data, $F_\pi(t)$ (Höhler, Pietarinen, '75, Belushkin, HWH, Meißner, '05)
 - $K\bar{K}$: KN scattering data, $F_K(t)$ (HWH, Ramsey-Musolf, '99)
 - $\rho\pi$ ($\rightarrow 3\pi$): Jülich NN model (Meißner et al., '97)

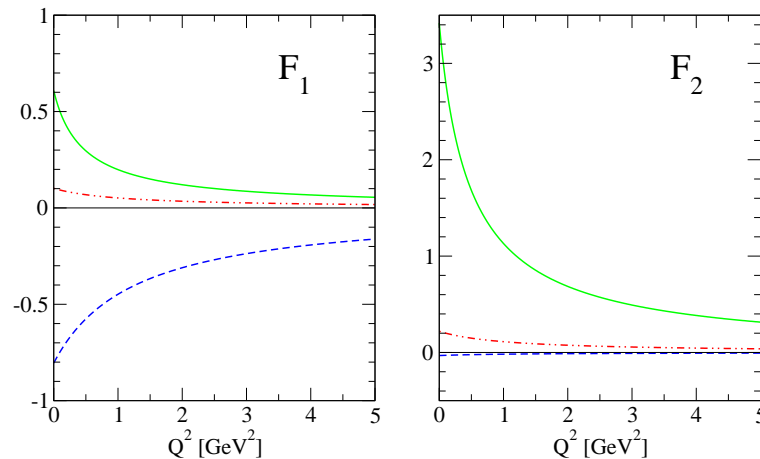
- Evaluate with DR

→ Form factor contributions

⋯⋯⋯ $K\bar{K}$

- - - $\rho\pi$

— 2π



Asymptotic Behavior

- pQCD prediction (modulo logs): $F_1 \sim 1/t^2$, $F_2 \sim 1/t^3$ as $t \rightarrow \infty$
- Various ways to implement asymptotic behavior from pQCD:
 - Superconvergence relations only
→ add broad resonance to generate imaginary part for $t \geq 4m^2$

$$F_i^{(I,broad)}(t) = \frac{a_i^I (M_I^2 - t)}{(M_I^2 - t)^2 + \Gamma_I^2}, \quad i = 1, 2, \quad I = S, V,$$

- Explicit pQCD term in addition to SC relations

$$F_i^{(I,pQCD)} = \frac{a_i^I}{1 - c_i^2 t + b_i^2 (-t)^{i+1}}, \quad i = 1, 2, \quad I = S, V,$$

-

Error Bands

- General Problem: What are the errors in dispersion analyses?
 - Generate theory error bands for best fits
 - Difficult since problem is highly non-linear
- Perform Monte Carlo sampling of fits with χ^2 in interval

$$[\chi_{min}^2, \chi_{min}^2 + \delta\chi^2], \quad \delta\chi^2 \simeq 1.04$$

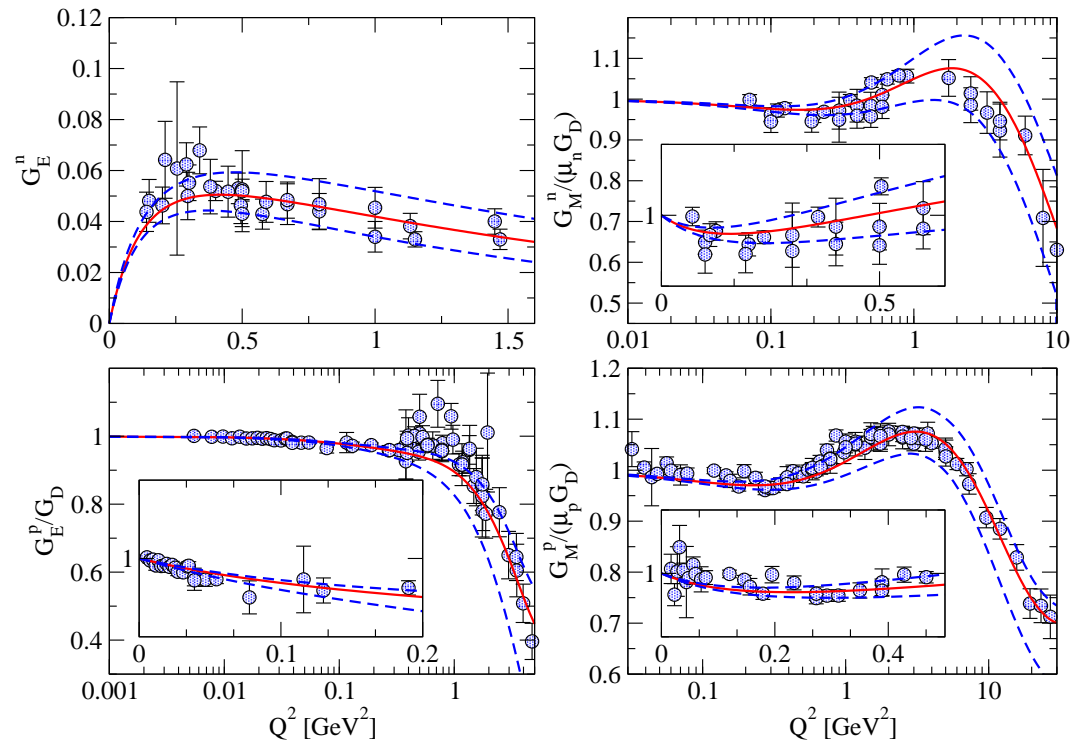
→ keep form factor values with maximal deviation from best fit

- Results: Belushkin, HWH, Meißner, Phys. Rev. C **75** (2007) 035202

Superconvergence Approach: Space-Like

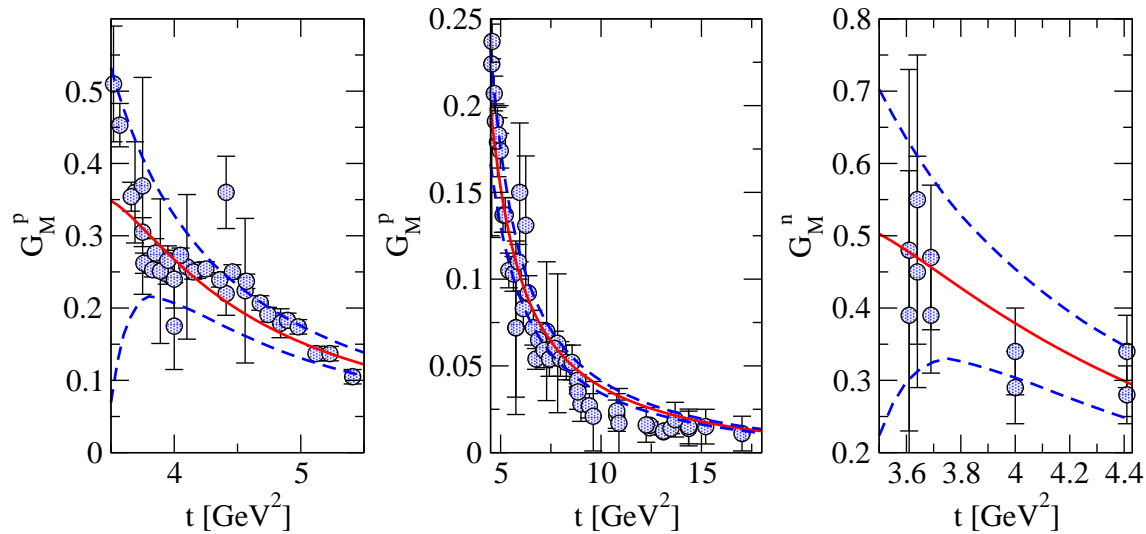
- 17 Parameters
 - ω, ϕ
 - 2 effective IS poles
 - 5 effective IV poles
 - $\chi^2/\text{dof} = 1.8$
- good description of data

— best fit
 - - - 1σ band



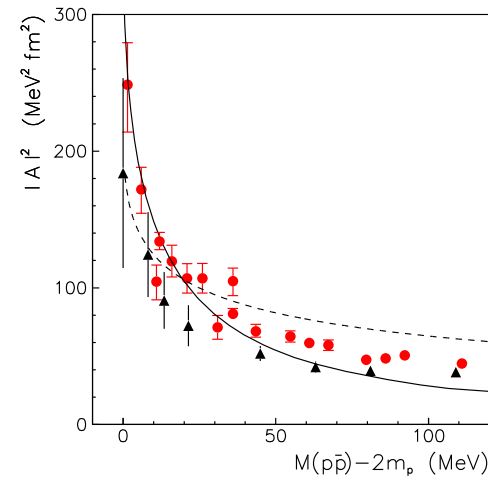
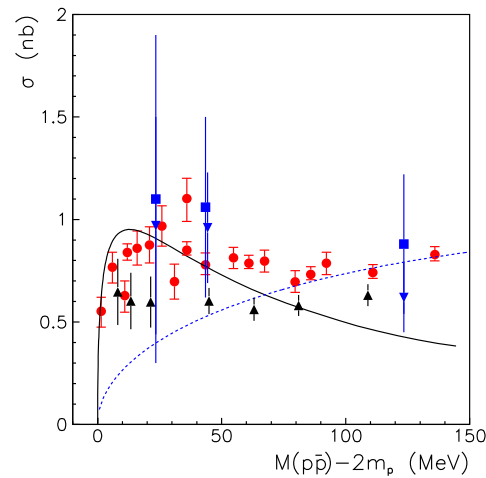
Superconvergence Approach: Time-Like

- Experimental extraction ambiguous (E/M separation)
 - Subthreshold resonance? (Antonelli et al., PRD **71** ('05) 054010)
- ⇒ $N\bar{N}$ final-state interaction
(Haidenbauer, HWH, Meißner, Sibirtsev, Phys. Lett. B **643** (2006) 29)



$N\bar{N}$ Final-State Interaction

- Consider cross section/transition amplitude for $e^+e^- \rightarrow N\bar{N}$
(\rightarrow source of timelike form factor data)



- Steep rise described by Jülich $N\bar{N}$ model/Watson-Migdal treatment
(Haidenbauer, HWH, Meißner, Sibirtsev, PLB **643** ('06) 29)
- Consistent with $J/\Psi \rightarrow \gamma p\bar{p}$, $\omega p\bar{p}$, $B \rightarrow p\bar{p}K$
(Haidenbauer, Meißner, Sibirtsev, '06, '08)

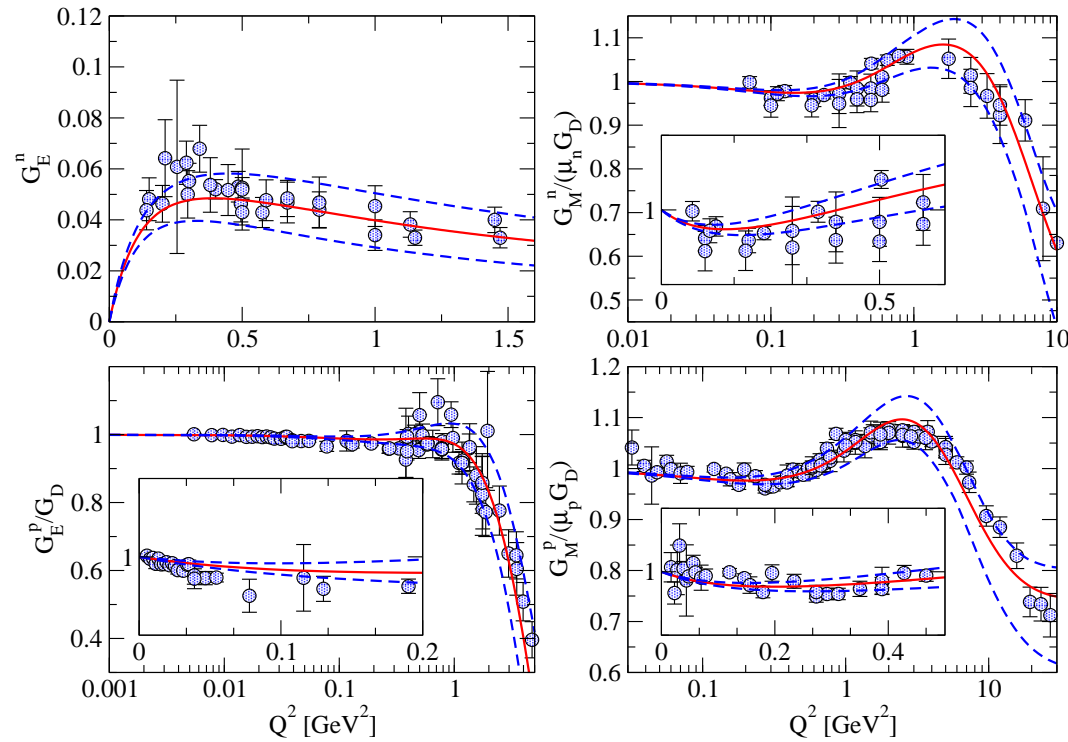
Explicit pQCD Approach: Space-Like

- 14 Parameters

- ω, ϕ
- 1 effective IS poles
- 3 effective IV poles
- explicit pQCD term
- $\chi^2/\text{dof} = 2.0$

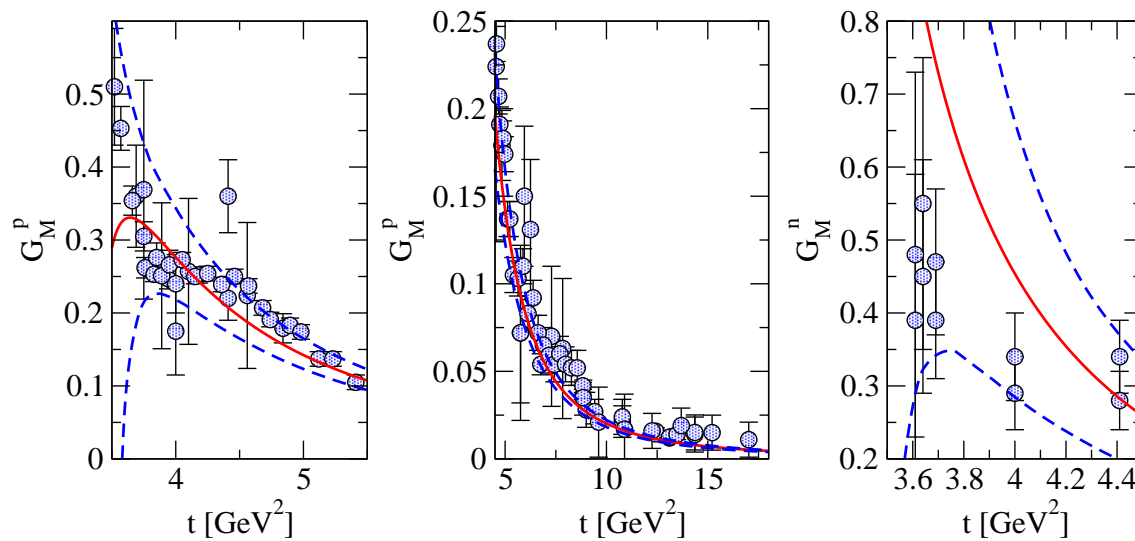
- good description of data

— best fit
 - - - 1σ band



Explicit pQCD Approach: Time-Like

- Timelike neutron data is not included in the fit (also in SC)
- No predictive power at threshold



General Comments/Radii

- Successive improvement by reduction of number of poles (stability constraint)
- Theoretical/systematic uncertainties? (2γ physics, consistency of data,..)
- Extraction of radii

	SC	ex. pQCD	recent determ.
r_E^p [fm]	0.84...0.85	0.82...0.84	0.886(15) [1,2,3]
r_M^p [fm]	0.85...0.86	0.84...0.85	0.855(35) [2,4]
$(r_E^n)^2$ [fm ²]	-0.11...-0.13	-0.11...-0.13	-0.115(4) [5]
r_M^n [fm]	0.85...0.87	0.86...0.87	0.873(11) [6]

[1] Rosenfelder, Phys. Lett. B **479** ('00) 381

[2] Sick, Phys. Lett. B **576** ('03) 62

[3] Melnikov, van Ritbergen, Phys. Rev. Lett. **84** ('00) 1673

[4] Sick, private communication

[5] Kopecky et al., Phys. Rev. C **56** ('97) 2229

[6] Kubon et al., Phys. Lett. B **524** ('02) 26

CLAS data: Space-Like

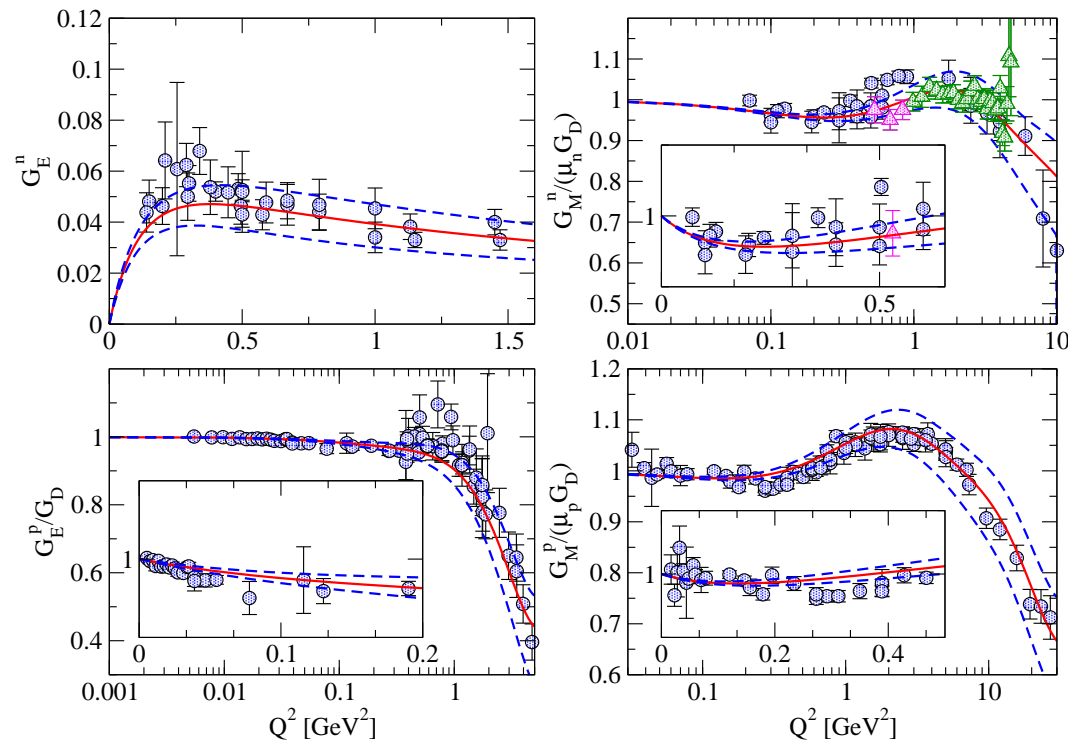
cf. CLAS collaboration, arXiv:0811.1716v1; W. Brooks, private communication

- 14 Parameters

- ω
- 2 effective IS poles
- 3 effective IV poles
- explicit pQCD term
- $\chi^2/\text{dof} = 2.2$

- good description of data

— best fit
 - - - 1σ band



Two-Photon Corrections

- Discrepancy between Rosenbluth and polarization transfer (PT) data
⇒ Two-Photon Exchange Effects
- Direct model calculations
(Blunden, Melnichouk, Tjon, 2003, 2005; Afanasev et al., 2004, 2005, ...)
⇒ right direction, effect too small
- Model-independent extraction from data?
- Assumption: no significant two-photon effects in PT data ⇒
Estimate hard 2γ corrections from comparison of our previous analysis
(mainly PT data) and direct analysis of Rosenbluth cross sections for
proton (including Coulomb corrections → soft 2γ corrections)

Two-Photon Corrections

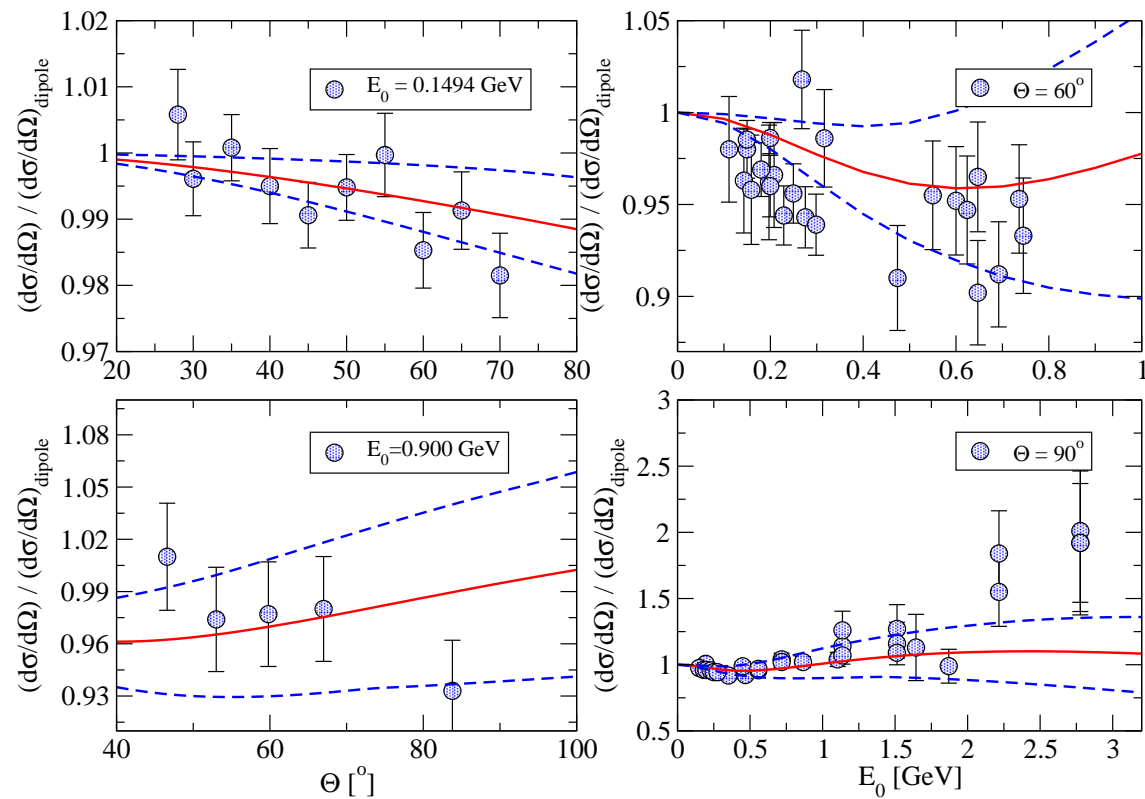
- **Hybrid analysis:** FF data for neutron, cross sections for proton
(Belushkin, HWH, Meißner, Phys. Lett. B **658** (2008) 138)
- **Easiest to compare at cross section level**
⇒ reconstruct “PT cross section” from FF data

$$\left(\frac{d\sigma}{d\Omega}\right) (1 + \delta^{2\gamma}) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Ros}}$$

- **Comparison with direct calculation** (Blunden et al.)
→ add in Coulomb correction $\Delta^{2\gamma} = \delta^{2\gamma} + \delta^C$
(cf. Arrington, Sick, Phys. Rev. C **70** (2004) 028203)

Cross Section Analysis

- Example: cross section analysis in SC approach



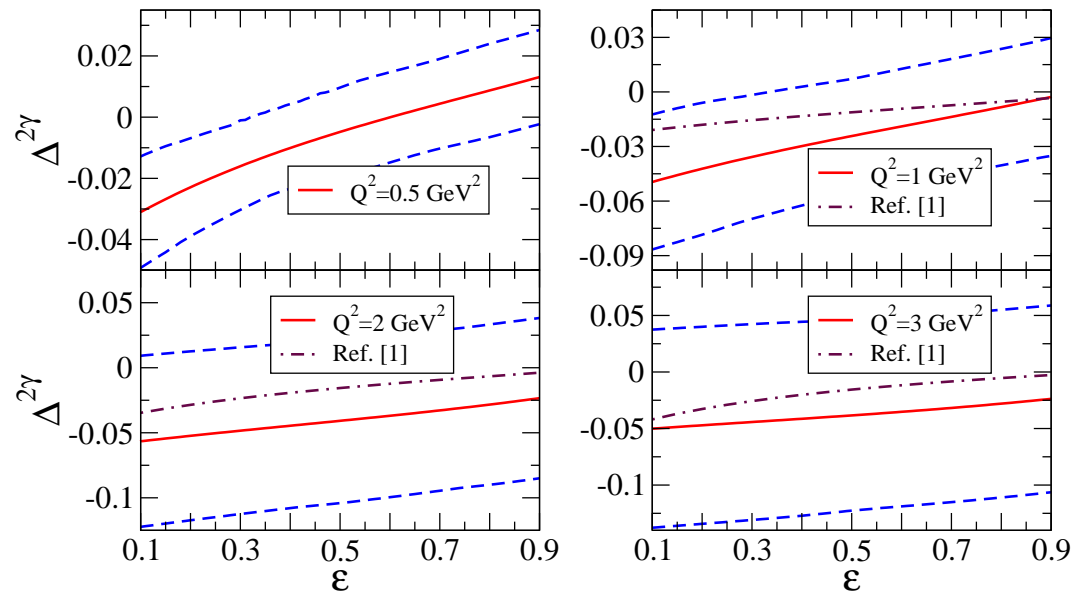
Two-Photon Corrections

- Extracted two-photon correction term

- $\Delta^{2\gamma} = \delta^{2\gamma} + \delta^C$

- — SC

- - · - · Blunden et al.

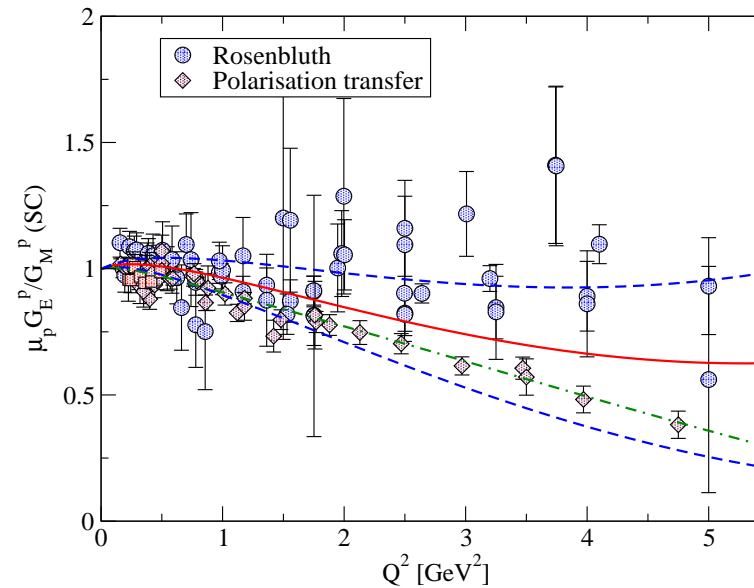


cf.

Blunden, Melnitchouk, Tjon, Phys. Rev. C **72** (2005) 034612

Comparison with Form Factor Ratios

- Form factor ratio from cross section analysis (SC)



- Consistent within error bars
- Form factor data not included in analysis

Summary

- Dispersion analysis of nucleon EM form factors
- Improved spectral functions: 2π , $K\bar{K}$, $\rho\pi$ continua
- Consistent description of EM FF data
- Radii: agreement with other analyses except r_E^p
→ not likely explained by 2γ physics (Blunden, Sick, '05)
- Model independent extraction of hard two-photon effects
→ no discrepancy between Ros and PT data within errors
- Currently investigating subtracted dispersion relations

Additional Transparencies

CLAS data: Space-Like II

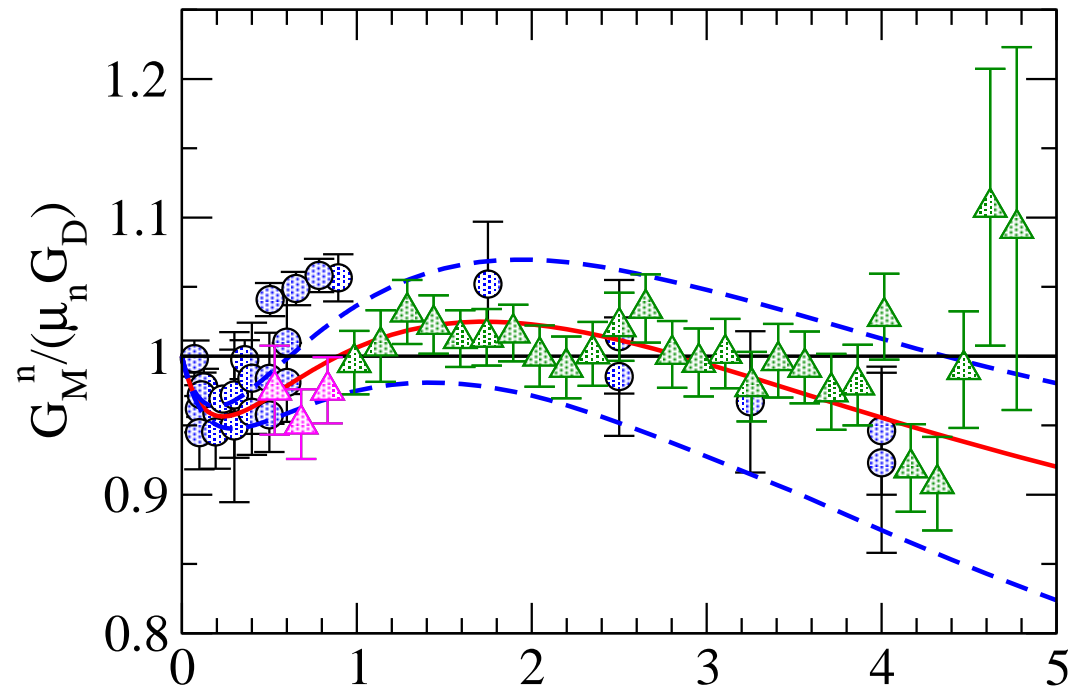
cf. CLAS collaboration, arXiv:0811.1716v1; W. Brooks, private communication

- 14 Parameters

- ω
- 2 effective IS poles
- 3 effective IV poles
- explicit pQCD term
- $\chi^2/\text{dof} = 2.2$

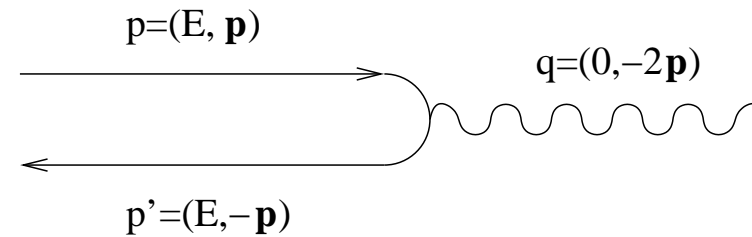
- good description of data

- best fit
- - - 1σ band



Interpretation of Radii

- Nucleon FF's in space-like region:
can always find reference frame where no energy is transferred (**Breit frame**)



$$F(q^2) = F(-\mathbf{q}^2) = \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} \rho(r)$$

$$\longrightarrow \langle r^2 \rangle = \frac{4\pi}{F(0)} \int dr r^4 \rho(r), \quad F(0) = 4\pi \int dr r^2 \rho(r)$$

- Interpretation:
 - G_E (G_M): **FT of charge (magnetization) distribution** ($q^2 \ll m^2$)
 - F_1, F_2 : only formal definition

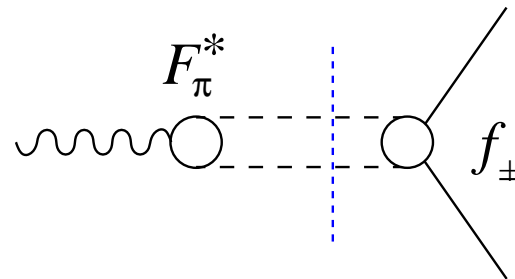
Isvector Spectral Function: 2π -Continuum

- 2π -contribution to spectral functions: (Frazer, Fulco, '59 \rightarrow predicted ρ)

$$\text{Im}G_E^V(t) = \frac{q_t^3}{m\sqrt{t}} F_\pi(t)^* f_+^1(t)$$

$$\text{Im}G_M^V(t) = \frac{q_t^3}{\sqrt{2t}} F_\pi(t)^* f_-^1(t)$$

$$\text{where } q_t = \sqrt{t/4 - M_\pi^2}$$



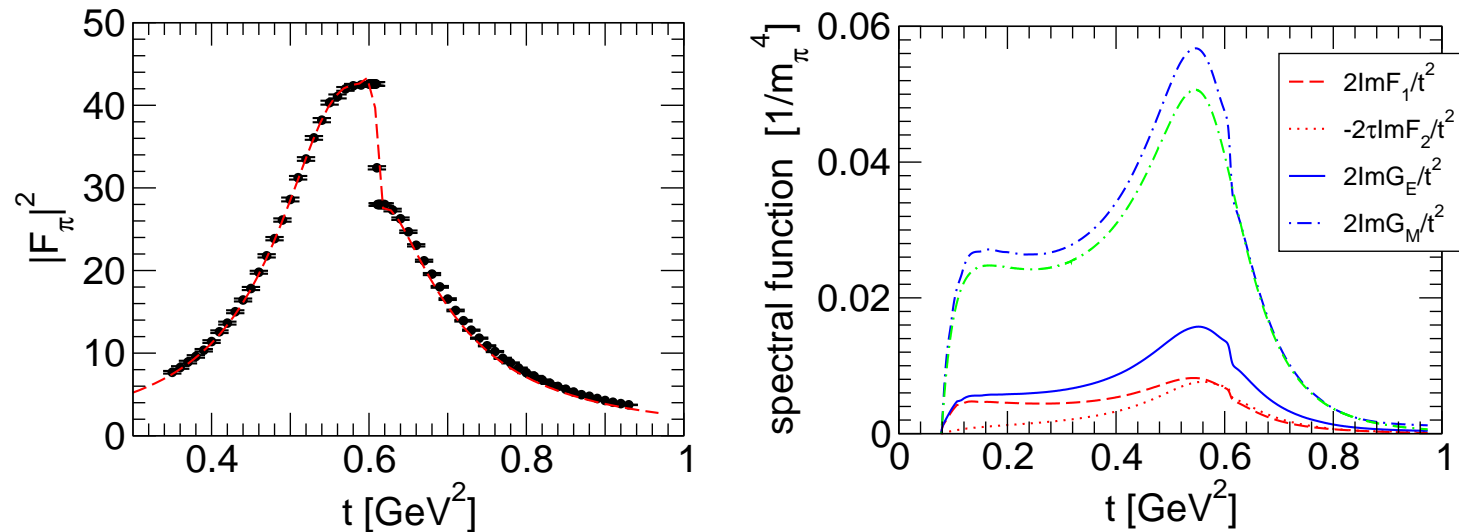
- $\pi\pi \rightarrow \bar{N}N$ P-waves: $f_\pm^1(t)$

\longrightarrow from analytic continuation of πN data (Höhler, Pietarinen, '75)

- Singularity close to threshold: $t_c \approx 3.98M_\pi^2 \rightarrow$ isovector radii
- Pion EM form factor $F_\pi(t)$: from $e^+e^- \rightarrow \pi^+\pi^-$

Isvector Spectral Function: 2π -Continuum

- New data for pion form factor (CMD, KLOE, SMD)
- New determination of 2π -continuum (Belushkin, HWH, Meißner, '06)

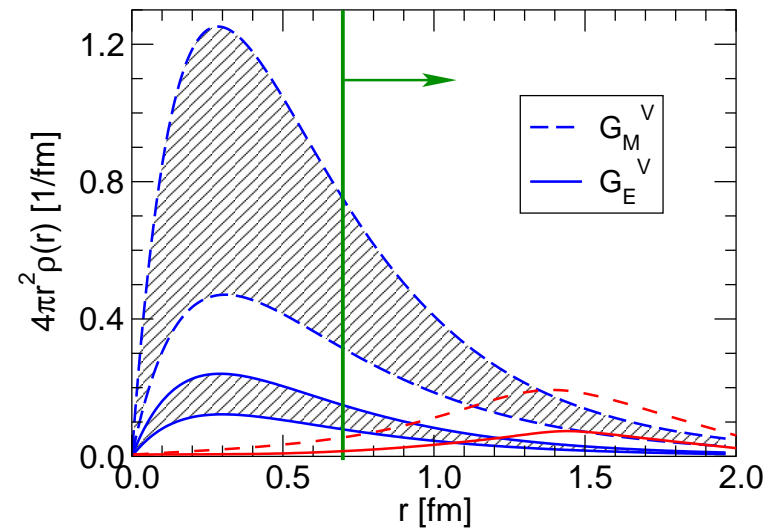


- Pronounced ρ peak with strong $\rho - \omega$ mixing
- Contains information on long-range pion cloud (cf. Drechsel et al., '04)

On the Pion Cloud of the Nucleon

- FW find very long-ranged pion cloud contribution: $r \simeq 2$ fm
Friedrich, Walcher, EPJA **17** ('03) 607
- Long-range pion contribution given by 2π -continuum

HWH, Drechsel, Meißner, PLB **586** ('04) 291



$$\rho_i^V(r) = \frac{1}{4\pi^2} \int_{4M_\pi^2}^{40M_\pi^2} dt \operatorname{Im} G_i^V(t) \frac{e^{-r\sqrt{t}}}{r}, \quad i = E, M$$

- Maxima around $r_{\max} \approx 0.3$ fm \longleftrightarrow FW: $r_{\max} \approx 1.5$ fm
- Smaller pion cloud contribution beyond $r \sim 1$ fm compared to FW
- Independent of contribution from $t > 40M_\pi^2$

Bump-Dip Structure in G_E^n

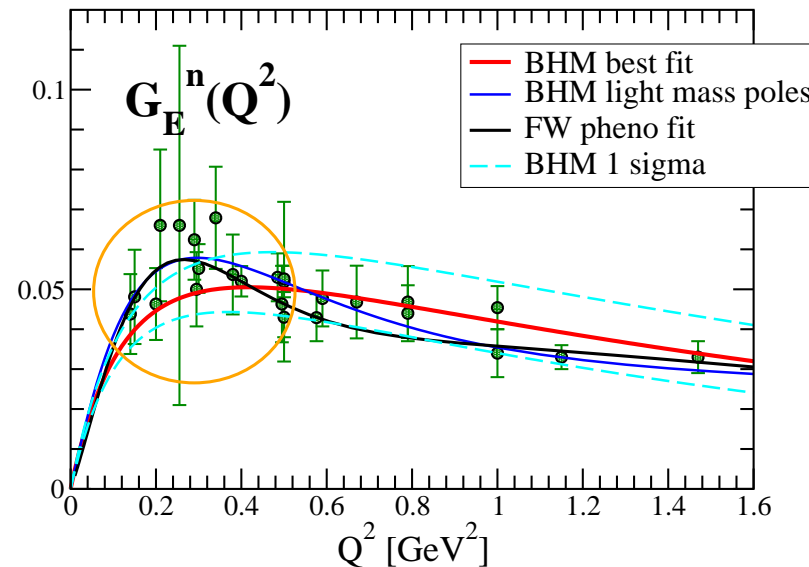
- Can structure be generated in dispersive approach?

→ low mass strength required!

→ e.g. low-mass poles:

$$M_S^2 = 0.7 \text{ GeV}^2$$

$$M_V^2 = 0.1 \text{ GeV}^2$$



- No known vector mesons in this region
- Vector meson dominance applicable for $t \leq 1 \text{ GeV}^2$
- Higher mass continua? ($|3\pi\rangle$: $t_{th} \approx 0.17 \text{ GeV}^2$, $|4\pi\rangle$: $t_{th} \approx 0.31 \text{ GeV}^2$)
 $\Leftrightarrow |3\pi\rangle$ small in ChPT (Bernard, Kaiser, Meißner, Nucl. Phys. A **611** ('96) 429)

Bump-Dip Structure: Space-Like Only

- 22 Parameters

- ω
- 3 effective IS poles
- 4 effective IV poles
- explicit pQCD term
- $\chi^2/\text{dof} = 0.9$
(only spacelike)

- good description of data

— best fit
 - - - 1σ band

