Introduction to spectroscopy on the lattice

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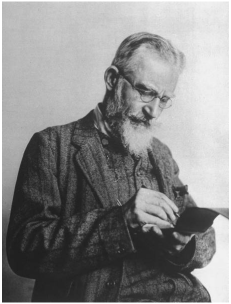
Divided by a common language?

"England and America are two countries divided by a common language"



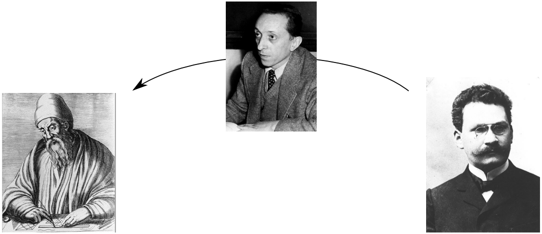
Can the same be said of QCD experimentalists and lattice theorists?

Overview

- Discretising QCD
- Monte Carlo methods
- Spectroscopy
- Scattering and decay physics
- Spectroscopy 2.0
- Conclusions

Minkowski, Wick and Euclid

- Some properties of theories in Minkowski space can be related by Wick rotation to corresponding theories in Euclidean space.
- Analytic continuation: $t \rightarrow i\tau$, $\frac{-i}{\hbar}S \rightarrow \frac{1}{\hbar}S$.
- Using Euclidean metric is needed for **numerical** path integration



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The symmetries of QCD

- QCD is the relativistic $SU(3)$ gauge theory of quarks

quarks	gluons
ψ_i^α	A_μ^a
$i = 1 \dots N_c$ colour	$a = 1 \dots 8$ adjoint colour
$\alpha = 1 \dots 4$ spin	$\mu = 1 \dots 4$ 4-vector

- The symmetries that define Euclidean QCD are
  - Gauge symmetry
  - Poincaré group (rotations, boosts and translations)
  - CPT (charge conjugation, parity and time-reversal)
  - Flavour $SU(N_f)$ (for N_f mass degenerate quark flavours)
  - Chiral $SU(N_f)_L \times SU(N_f)_R$ (for N_f massless quark flavours)
  - Conformal invariance for theory with only massless quarks
- The QCD vacuum spontaneously breaks some of these symmetries
- The lattice will explicitly break some of these symmetries. . .

Continuum gauge transformations

- Quark fields form a (fundamental) representation of the gauge group, $SU(3)$, that means they transform under a (space-time dependent) rotation as

$$\psi(x) \longrightarrow \psi^{(g)}(x) = \Lambda(x)\psi(x)$$

$$\bar{\psi}(x) \longrightarrow \bar{\psi}^{(g)}(x) = \bar{\psi}(x)\Lambda^\dagger(x)$$

  where $\Lambda(x)$ is the gauge transformation at x, and $\Lambda^\dagger(x)\Lambda(x) = 1$, $\det \Lambda(x) = 1$.
- To make a theory of fermion with this symmetry, another field is needed that transmits information about relative gauge transformations at nearby points.
- The derivative ∂_μ acting on the quark field must be replaced with a gauge covariant derivative D_μ with

$$D_\mu = \partial_\mu - igA_\mu$$

Continuum gauge transformations (2)

- A_μ is another field, that transforms according to

$$A_\mu \longrightarrow A_\mu^{(g)} = \frac{1}{ig}(\partial_\mu \Lambda)\Lambda^{-1} + \Lambda A_\mu \Lambda^{-1}$$

- Now under a gauge transformation, $D\psi$ transforms in the same way as ψ so the bilinear $\bar{\psi} D\psi$ is gauge invariant.
- A_μ forms an adjoint representation of the gauge transformation group.
- So A can be written in terms of an element of the Lie algebra of $SU(3)$: $A_\mu(x) = T^a A_\mu^a(x)$
- A field strength tensor can be written, which is analogous to the electromagnetic tensor (which contains electric and magnetic fields)

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

- The QCD field strength tensor has a commutator that is not present for QED, which leads to gluon self-interaction.

Gauge invariant actions

- The field strength tensor has simple transformation properties

$$F_{\mu\nu} \longrightarrow F^{(g)}_{\mu\nu} = \Lambda F_{\mu\nu} \Lambda^{-1}$$

- A gauge-invariant action on the gauge fields can be defined

$$S_g = \frac{1}{4} \int d^4x \ \mathrm{Tr} \ F_{\mu\nu} F_{\mu\nu}$$

- Similarly, for a quark field, a suitable action is

$$S_q = \int d^4x \ \bar{\psi}(\gamma_\mu D_\mu + m)\psi$$

- Here, we have wick-rotated the gamma-matrices so the quark fields form a spin-1/2 representation of $SO(4)$.

$$\{\gamma_\mu, \gamma_\nu\} = \delta_{\mu\nu}$$

Lattice fields - the quarks

- Quark fields are discretised in the simplest way; the fields are restricted to take values only on sites of the four-dimensional space-time lattice, $\psi(\underline{x}, t) \rightarrow \psi_{n_1,n_2,n_3,n_4}$.
- Each lattice site has $4 \times N_c = 12$ degrees of freedom per quark flavour.
- Gauge transforms will be defined for sites too: $\psi_{n_1,n_2,n_3,n_4} \longrightarrow \psi^{(g)}_{n_1,n_2,n_3,n_4} = \Lambda_{n_1,n_2,n_3,n_4}\psi_{n_1,n_2,n_3,n_4}$.
- In a path integral, fermions must be represented by elements of a grassmann algebra:

$$\int d\eta = 0, \int d\eta\ \eta = 1$$

- This will make life complicated for us when it comes to simulations.
- And more problems with quarks will arise when we try to define an action...

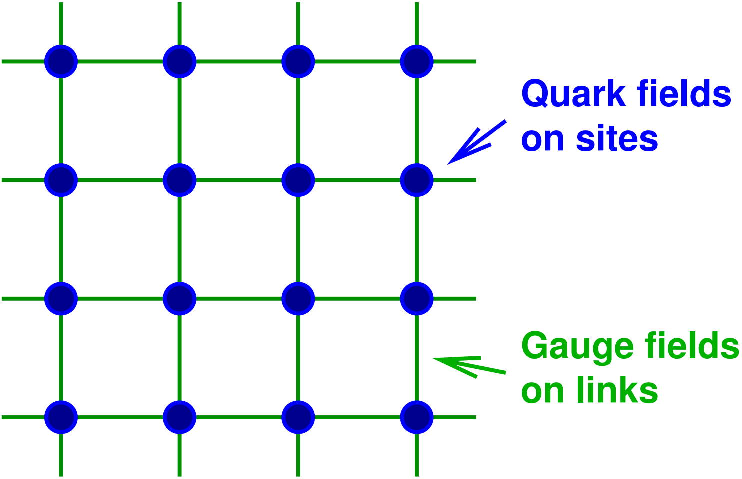
Lattice fields - the gluons

- Wilson recognised the way to build actions with a gauge symmetry on the lattice was to put the gluon field onto the lattice in a very different way: **gluons live on links.**
- Abandon the vector potential as the fundamental degree of freedom, use instead a small path-ordered exponential connecting adjacent sites on the lattice:

$$U_\mu(x) = \mathcal{P} exp \left\{ ig \int_x^{x+\hat{\mu}} ds \;\; A_\mu(s) \right\}$$

- Path-ordering is needed to give an unambiguous meaning to this expression since the gauge group is non-abelian ($A_\mu(x)$ does not commute with $A_\mu(y)$ when $x \neq y$).
- $U\mu \in SU(3)$ while $A_\mu \in \mathcal{L}(SU(3))$.
- To define a path-integral, we need to integrate over the $SU(3)$ group manifold; use an invariant Haar measure, $\mathcal{D}U$

Maintaining gauge invariance means . . .



Lattice gauge invariants

- Define the rules of gauge transformations so gauge invariants can be constructed out of lattice fields:

Gauge transformations of lattice fields

$$\begin{aligned}\psi(x) \longrightarrow \psi^{(g)}(x) &= \Lambda(x)\psi(x)\\ \bar{\psi}(x) \longrightarrow \bar{\psi}^{(g)}(x) &= \bar{\psi}(x)\Lambda^\dagger(x)\\ U_\mu(x) \longrightarrow U_\mu^{(g)}(x) &= \Lambda(x)U_\mu(x)\Lambda^\dagger(x+\hat{\mu})\end{aligned}$$

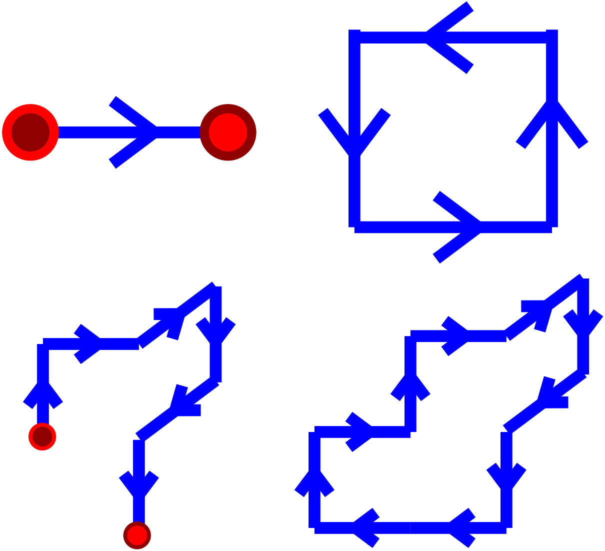
- Since $\Lambda^\dagger\Lambda = 1$, the following expressions are invariant under these transformations

Simple lattice gauge invariant functions

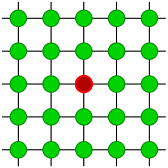
$$\bar{\psi}(x)U_\mu(x)\psi(x+\hat{\mu})$$

$$\text{Tr } U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)$$

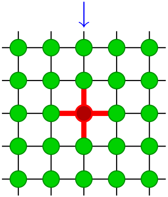
Lattice gauge invariants



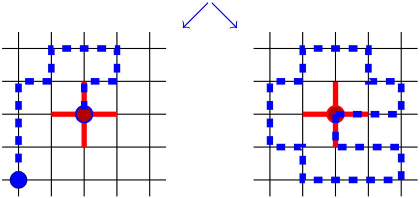
Gauge invariance



To rotate a quark field at site x, $\psi(x) \rightarrow \psi^g(x) = g(x)\psi(x)$ ...



... we must also rotate the gauge fields that start or end at the site $U_\mu(x) \rightarrow U^g_\mu(x) = g(x)U_\mu(x)g^\dagger(x+\hat{\mu})$



The gauge invariance of the special functions is seen

Lattice action - the gluons

- To define a path integral, we also need an action
- The simplest gauge invariant function of the gauge link variables alone is the *plaquette* (the trace of a path-ordered product of links around a 1×1 square).

$$S_G[U] = \frac{\beta}{N_c} \sum_{x,\mu<\nu} \mathrm{ReTr} \left(1 - U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)\right)$$

  This is the *Wilson gauge action*
- A path integral for the Yang-Mills theory of gluons would be

$$Z_{YM} = \int \prod_{\mu,x} \mathcal{D} U_\mu(x) e^{-S_G[U]}$$

- The coupling constant, g appears in $\beta = \frac{2N_c}{g^2}$
- No need to fix gauge; the gauge orbits can be trivially integrated over and the group manifold is compact.

Lattice action - the gluons

- A Taylor expansion in a shows that

$$S_G[U] = \frac{\beta}{N_c} \sum_{x,\mu<\nu} \mathrm{ReTr}\left(1 - U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)\right)$$
$$= \int d^4x \; -\frac{1}{4}\mathrm{Tr}\; F_{\mu\nu} F_{\mu\nu} + \mathcal{O}(a^2)$$

- All terms proportional to odd powers in the lattice spacing vanish because the lattice action preserves a discrete parity symmetry.
- The action is also invariant under a charge-conjugation symmetry, which takes $U_\mu(x) \to U_\mu^*(x)$.
- We have kept almost all of the symmetries of the Yang-Mills sector, but broken the $SO(4)$ rotation group down to the discrete group of rotations of a hypercube.

Lattice actions - the quarks

- The continuum action is a bilinear with a first-order derivative operator inside;

$$S_Q = \int d^4x \bar{\psi}(\gamma_\mu D_\mu + m)\psi$$

- When $m = 0$, the action has an extra, chiral symmetry:

$$\psi \longrightarrow \psi^{(\chi)} = e^{i\alpha\gamma_5}\psi, \bar{\psi} \longrightarrow \bar{\psi}^{(\chi)} = \bar{\psi} e^{i\alpha\gamma_5}$$

- The simplest lattice representation of a first-order derivative that preserves reflection symmetries is the central difference:

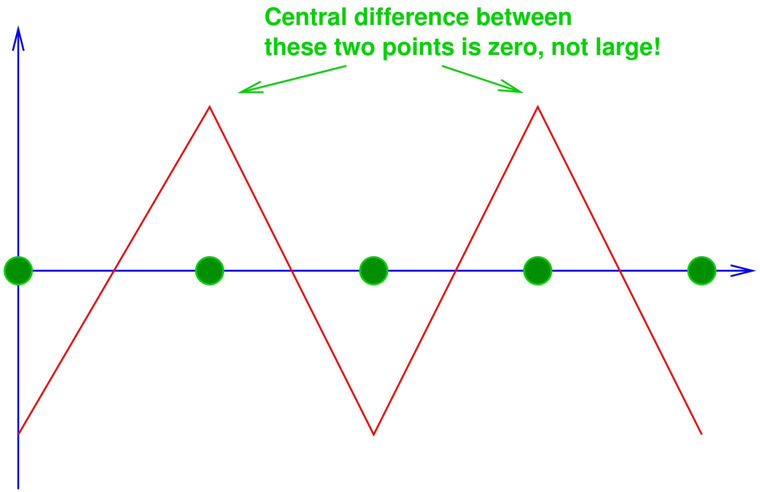
$$\partial_\mu \psi(x) = \frac{1}{2a}\left(\psi(x+\hat{\mu}) - \psi(x-\hat{\mu})\right)$$

- This can be made gauge covariant by including the gauge links:

$$D_\mu \psi(x) = \frac{1}{2a}\left(U_\mu(x)\psi(x+\hat{\mu}) - U_\mu(x-\hat{\mu})\psi(x-\hat{\mu})\right)$$

- **BUT** on closer inspection, there are more minima to this action than we want. Consider the case with no gauge fields, and when $\psi(x) = e^{ikx}$ with $k = \{\pi, 0, 0, 0\}$ or $\{\pi, \pi, 0, 0\}$ or $\{\pi, \pi, \pi, 0\}$ or $\ldots$.

Lattice doubling



Lattice actions - the quarks (3)

- This is the (in)famous **doubling problem**.

The Nielson-Ninomiya "no-go" theorem

There are **no** chirally symmetric, local, translationally invariant doubler-free fermion actions on a regular lattice.

- To put quarks on the lattice, more symmetry must be broken or else a theory with extra flavours of quarks must be simulated.
- A number of solutions are used, each with their advantages and disadvantages.
- The most commonly used are:
  - Wilson fermions
  - Kogut-Susskind (staggered) fermions
  - Ginsparg-Wilson fermions (overlap, domain wall, perfect...)
  - Twisted mass

Wilson's lattice quark action

- Wilson's original solution was to abandon chiral symmetry and add a lattice operator whose continuum limit is an irrelevant dimension-five operator. The term gives the doublers a mass $\propto 1/a$
- The extra term in the lattice action is the lattice representation of

$$a\sum_{\mu} D_\mu^2\psi \approx \sum_{\mu} U_\mu(x)\psi(x+\hat{\mu}) + U_\mu^\dagger(x-\mu)\psi(x-\hat{\mu})$$

- The breaking of chiral symmetry means the quark mass is not protected from additive renormalisations (short-distance gluons will now give quarks a large mass)
- Approaching the continuum limit requires fine-tuning to restore chiral symmetry and ensure quarks are light.
- Breaking chiral symmetry now introduces lattice artefacts at $\mathcal{O}(a)$.
- This action has a Symanzik-improved counterpart, the Sheikholeslami-Wohlert action, which removes all $\mathcal{O}(a)$ errors by a field redefinition and the addition of another dim-5 term, $\sigma_{\mu\nu}F_{\mu\nu}$

The Ginsparg-Wilson relation

- Actions that break chiral symmetry, but preserve a modified version can be constructed. The new chiral symmetry is

$$\{\gamma_5, \not{D}\} = 2a\,\not{D}\gamma_5\not{D} \text{ so } \{\gamma_5, \not{D}^{-1}\} = 2a\,\gamma_5$$

- In a propagator, chiral symmetry is broken by a contact term
- A number of realisations of this symmetry are in use. Neuberger's overlap uses an action

$$D = I - \frac{D_W}{\sqrt{D_W^\dagger D_W}}$$

  where D_W is the Wilson action with a large negative quark mass.
- Domain Wall quarks use a 5d lattice field (coupled to four-dimensional gluons). The boundaries in the 5^{th} dimension are set up so left- and right-handed quarks bind to different walls in 5d. Modes are separated so chiral symmetry is (almost) maintained.
- These quarks are expensive!

Staggered quarks

- Kogut and Susskind proposed an interesting partial solution to the doubling problem.
- A field redefinition is used to scatter the sixteen components of four flavours ("tastes") of quarks across the corners of a hypercube.
- On each lattice sites there are just N_c degrees of freedom
- A remnant of chiral symmetry remains which is sufficient to ensure there is no additive mass renormalisation.
- Simulations are fast; there is no fine-tuning so the fermion matrix is well-behaved and always positive which helps the simulation algorithms
- UV gluons can change the "taste" of a quark, so flavours mix
- Practitioners simulate theories with one or two flavours by taking fractional powers of the fermion path integral. It is still a matter of debate whether this is legitimate.

QCD on the computer - Monte Carlo integration

- On a finite lattice, with non-zero lattice spacing, the number of degrees of freedom is finite. The path integral becomes an "ordinary" high-dimensional integral.
- High-dimensional integrals can be estimated stochastically by Monte Carlo. Variance reduction is crucial, and can be achieved effectively provided the theory is simulated in the Euclidean space-time metric.
- No useful importance sampling weight can be written for the theory in Minkowski space.
- The Euclidean path-integral is a weighted average:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi \;\; \mathcal{O}[U, \bar{\psi}, \psi] \, e^{-S[U,\bar{\psi},\psi]}$$

- e^{-S} varies enormously; sample only the tiny region of configuration space that contributes significantly.

Dynamical quarks in QCD

- Monte Carlo integration with $N_f = 2$ (mass degenerate) quarks. Quark fields in the path integral obey a grassmann algebra which is difficult to manipulate in the computer.
- The quark action is a bilinear; the grassmann integrals can be done analytically and give

$$Z_Q[U] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \;\; e^{-\sum_f \bar{\psi}_f M[U]\psi} = \det M^{N_f}[U]$$

- The full partition function, including the gauge fields is

$$Z = \int \mathcal{D}U \;\; Z_Q[U] e^{-S_G[U]} = \int \mathcal{D}U \;\; \det M^{N_f}[U] e^{-S_G[U]}$$

- For (eg) $N_f = 2$ $\det M^2$ is positive and can be included in the importance sampling. It is a non-local function of the gauge fields, and expensive to compute. Using $M^\dagger = \gamma_5 M \gamma_5$, $\det M^2$ is re-written

$$Z_Q[U] = \int \mathcal{D}\phi \mathcal{D}\phi^* e^{-\phi^* [M^\dagger M]^{-1} \phi}$$

Dynamical quarks in QCD

- ϕ is an unphysical (non-local action) bosonic field with colour charge and spin structure (!) called the pseudofermion.
- Measuring the action requires applying the inverse of M a very large matrix
- M is sparse, and there are a set of linear algebra tricks (Krylov space solvers etc) that work effectively.
- Unfortunately, they require many applications of the matrix to a quark field, and so take a lot of computer time.
- This is where most computing power in lattice simulations goes; computing the effect of the quark fields acting on the gluons in the Monte Carlo updates.
- The alternative is the quenched approximation to QCD; ignore the fermion path integral completely - this is an unphysical approximation so its effects are hard to quantify.
- Inversion is needed again in the measurement stage too;

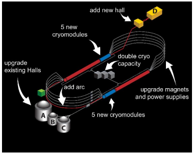
$$\langle \psi(x)\bar{\psi}(y)\rangle = M^{-1}[U](x,y)$$

Markov Chain Monte Carlo

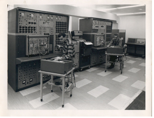
- How is the configuration space sampled?
- All techniques use a **Markov process**: this is a stochastic transition that takes the current state of the system and jumps randomly to a new state, such that the probability of the jump is independent of the past states of the system.
- Ergodic (positive recurrent, irreducible) Markov chains have unique stationary distributions; build the Markov process so it has our importance sampling distribution as its stationary state.
- If this can be done, then the sequence of configurations generated by the process is our importance sampling ensemble!
- Almost all algorithms exploit **detailed balance** to achieve this.

An analogy: experiment and lattice

Accelerator

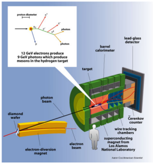


Configuration source



$$\{U^{(1)} \rightarrow U^{(2)} \rightarrow U^{(3)} \rightarrow \dots\}$$

Detector



Measurement on fields

$$C(t;U) = \text{Tr}\ G(U;t)\text{Tr}\ G(U;0)$$

Statistical analysis and fitting

Statistical analysis and fitting

Hadron spectroscopy (1)

- **Masses** of (colourless) QCD bound-states can be computed by measuring **two-point functions**. The Euclidean two-point function is

$$C(t) = \langle 0|\Phi(t)\Phi^\dagger(0)|0\rangle$$

- The time-dependence of the operator, Φ is given by $\Phi(t) = e^{Ht}\Phi e^{-Ht}$, so

$$C(t) = \langle \Phi|e^{-Ht}|\Phi^\dagger\rangle$$

  inserting a complete set of energy eigenstates gives

$$C(t) = \sum_{k=0}^{\infty}\langle \Phi|e^{-Ht}|k\rangle\langle k|\Phi^\dagger\rangle = \sum_{k=0}^{\infty}|\langle\Phi|k\rangle|^2 e^{-E_k t}$$

- Then $\lim_{t\to\infty} C(t) = Ze^{-E_0 t}$
- If the large-time exponential fall-off of the correlation function can be observed, the energy of the state can be measured.

Hadron spectroscopy (2)

- The energies of **excited states** can be computed reliably too.
- Tracking sub-leading exponential fall-off works sometimes but a more efficient method is to use a matrix of correlators. With a set of N operators $\{\Phi_1, \Phi_2, \dots\}$ (with the same quantum numbers), compute all elements of

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

- Now solve the generalised eigenvalue problem

$$C(t_1)v = \lambda C(t_0)v$$

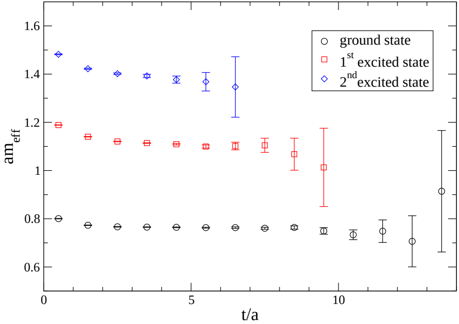
  for different t_0 and t_1.
- The method constructs an optimal linear combination to form a ground-state, and then constructs a set of operators that are orthogonal to it.
- The second eigenvector can not have overlap with the ground-state at large t, and will fall to the first excited energy.

Hadron spectroscopy (3)

- Lattice practitioners like to show this in an **"effective mass plot"**. The effective mass is

$$m_{\text{eff}}(t) = -\frac{1}{a}\log\frac{C(t+a)}{C(t)}$$

  and for times large enough such that C is dominated by the ground-state, the effective mass should become independent of time; a "plateau".



- Radial (?) excitations of a "static-light" meson.

Spin on the lattice

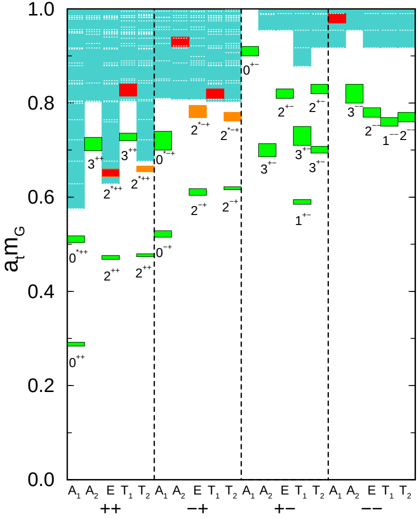
- Eigenstates of the hamiltonian simultaneously form irreducible representations of $SO(3)$, the rotation group. Spin is a good quantum number.
- The lattice hamiltonian does not have $SO(3)$ symmetry. It is symmetric under the discrete sub-group of **rotations of the cube**, O_h. This group has 48 elements (once parity is included) and ten irreducible representations.
- The eigenstates of the lattice hamiltonian therefore have a good "quantum letter"; $A_1^{u,g}, A_2^{u,g}, E^{u,g}, T_1^{u,g}, T_2^{u,g}$
- Can we deduce the continuum spin of a state? With some caveats, yes.
- A pattern of degeneracies must be found and matched against the representations of O_h subduced from $SO(3)$.

Spin on the lattice (2)

J	A_1	A_2	E	T_1	T_2
0	1	–	–	–	–
1	–	–	–	1	–
2	–	–	1	–	1
3	–	1	–	1	1
4	1	–	1	1	1
5	–	–	1	2	1
6	1	1	1	1	2
			⋮		

Example

The Yang-Mills glueball spectrum

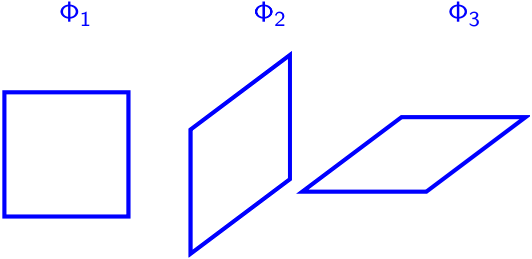


Creation operators: glueballs

- To measure the correlation functions, we need to measure appropriate creation operators on our ensemble.
- The operators should be functions of the fields on a time-slice and transform irreducibly according to an irrep of O_h (as well as isospin, charge conjugation etc.)
- First example: the glueball. An appropriate operator would be a gauge invariant function of the gluons alone: a closed loop trace.
- Link smearing greatly improved ground-state overlap.
- Apply smoothing filters to the links to extract just slowly varying modes that then have better overlap with the lowest states.

Creation operators: glueballs

- What do operators that transform irreducibly under O_h look like?

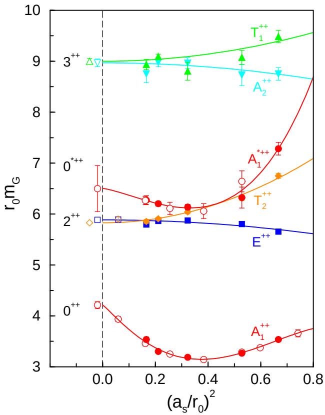


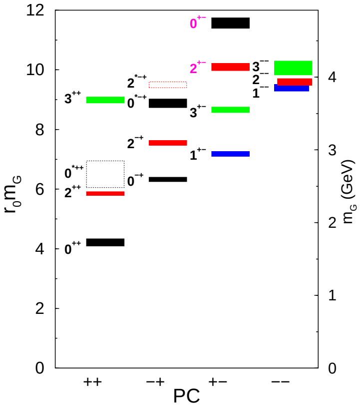
- Can make three operators by taking linear combinations of these loops.
- They form two irreducible representations (A_1^g and E_g).

$$\begin{array}{ccccccc} \Phi_{A_1^g} & = & \Phi_1 & + & \Phi_2 & + & \Phi_3 \\ \hline \Phi_{E^g}^{(1)} & = & \Phi_1 & - & \Phi_2 & & \\ \Phi_{E^g}^{(2)} & = & \frac{1}{\sqrt{3}}(\Phi_1 & + & \Phi_2 & - & 2\Phi_3) \end{array}$$

Creation operators: glueballs

- After running simulations at more than one lattice spacing, a **continuum extrapolation** ($a \to 0$) can be attempted.
- The expansion of the action can suggest the appropriate choice of extrapolating function.





Isovector meson correlation functions

- To create a meson, we need to build functions that couple to quarks.
- In the simplest model, a meson would be created by a quark bilinear, so the appropriate gauge invariant creation operator (for isospin $I = 1$) would be

$$\Phi_{\rm meson}(t) = \sum_{x} \bar{u}(\underline{x}, t)\Gamma U_{\mathcal{C}}(\underline{x}, \underline{y}; t) d(\underline{y}, t)$$

  where Γ is some appropriate Dirac structure, and $U_{\mathcal{C}}$ a product of (smeared) link variables.
- As before, appropriate operators that transform irreducibly under the lattice rotation group O_h are needed.
- The complication here is that we do not have direct access to the fermion integration variables in the computer.
- As with updating algorithms, the observation that the quark action is bilinear saves us:

$$\langle \psi_a^{\alpha}(\underline{x}, t)\bar{\psi}_b^{\beta}(\underline{y}, t')\rangle = [M^{-1}]_{ab}^{\alpha,\beta}(x, t; y, t')$$

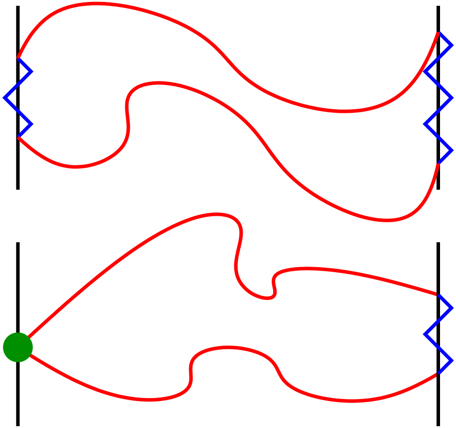
Isovector meson correlation functions (2)

- Now the elementary component in the correlation function is

$$\langle 0|\Phi(t)\Phi^\dagger(0)|0\rangle =$$
$$\langle \mathrm{Tr}\ M^{-1}(\underline{z},0;\underline{x},t)\Gamma U_{\mathcal{C}}(\underline{x},\underline{y},t)M^{-1}(\underline{y},t;\underline{w},0)\Gamma^\dagger U_{\mathcal{C}'}(\underline{w},\underline{z},0)\rangle$$

- In general, this is still expensive to compute, since it requires knowing many entries in the inverse of the fermion operator, M.
- If the choice of operator at the source is restricted and no momentum projection is made, only the bilinear at (eg) the origin on time-slice 0 is needed.
- Quark propagation from a single site to any other site is computed by solving $M\psi = e_0^{a,\alpha}$ where e_0 are the 12 vectors that only has non-zero components at the origin.
- Getting away from this restriction by estimating "all-to-all" propagators is an active research topic.

Isovector meson correlation functions (3)

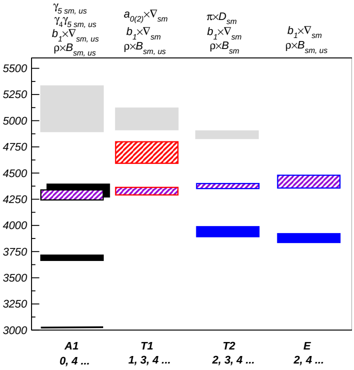


The most general operator.

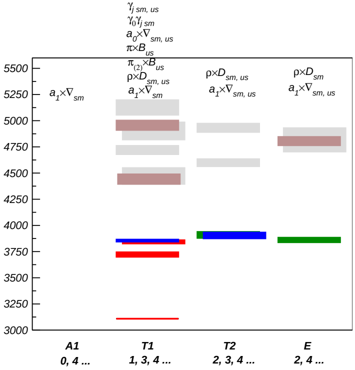
A restricted correlation function accessible to one point-to-all computation.

Charmonium spectroscopy

J. Dudek, R. Edwards, N. Mathur & D. Richards



$PC = -+$



$PC = --$

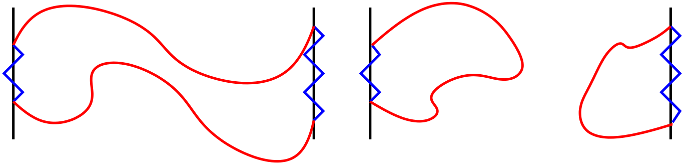
Isoscalar meson correlation functions (1)

- If we are interested in measuring isoscalar meson masses, extra diagrams must be evaluated, since four-quark diagrams become relevant. The Wick contraction yields extra terms, since

$$\langle \psi_i \bar{\psi}_j \psi_k \bar{\psi}_l \rangle = M^{-1}_{ij} M^{-1}_{kl} - M^{-1}_{il} M^{-1}_{jk}$$

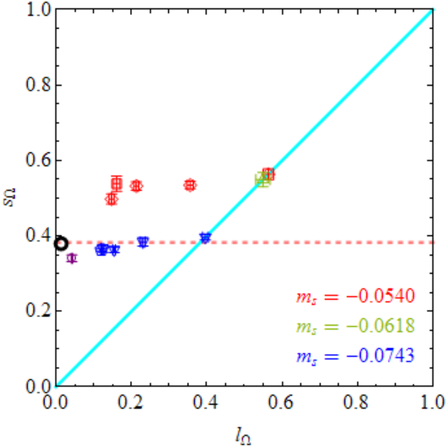
- Now

$$\langle 0 | \Phi_{I=0}(t) \Phi^\dagger_{I=0}(0) | 0 \rangle =$$
$$\langle 0 | \Phi_{I=1}(t) \Phi^\dagger_{I=1}(0) | 0 \rangle - \langle 0 | \mathrm{Tr}\ M^{-1} \Gamma U_{\mathcal{C}}(t) \mathrm{Tr}\ M^{-1} \Gamma U_{\mathcal{C}}(0) | 0 \rangle$$



Locating the physical quark masses

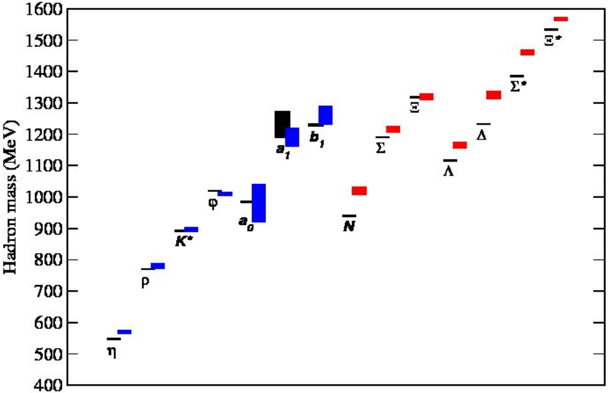
H-W Lin, S. Cohen, J. Dudek, R. Edwards, B. Joó, D. Richards, J. Bulava,
J. Foley, C. Morningstar, E. Engelson, S. Wallace, J. Juge, N. Mathur, MP & S. Ryan



$l_\Omega = 9m_\pi^2/4m_\Omega^2$ and $s_\Omega = 9(2m_K^2 - m_\pi^2)/4m_\Omega^2$

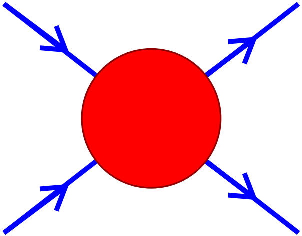
Light hadron spectrum

H-W Lin, S. Cohen, J. Dudek, R. Edwards, B. Joó, D. Richards, J. Bulava, J. Foley, C. Morningstar, E. Engelson, S. Wallace, J. Juge, N. Mathur, MP & S. Ryan



Discrepancy predominantly from extrapolation in light quark mass?

No-go: the Maiani-Testa theorem



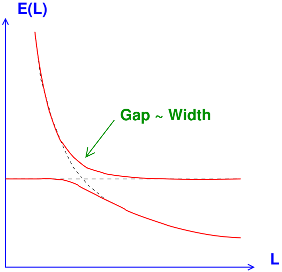
- Importance sampling Monte Carlo simulation only works efficiently for a path integral with a positive definite probability measure: Euclidean space.
- Maiani-Testa: Scattering matrix elements cannot be extracted from infinite-volume Euclidean-space correlation functions (except at threshold).
- Can the lattice tell us anything about low-energy scattering or states above thresholds?

Scattering lengths indirectly: Lüscher's method

- Scattering lengths can be inferred indirectly given the right measurements in Euclidean field theory.
- In a three-dimensional box with finite size L, the spectrum of low-lying states is discrete, even above thresholds (since the momenta of daughter mesons are quantised).
- Precise data on the dependence of the energy spectrum on L can be used to compute low-energy scattering (below inelastic threshold).
- This requires measuring energies of multi-hadron states.

Resonance energies and widths

- Above inelastic threshold, even less is known precisely.
- Resonant states will appear as "avoided level crossings" in the spectrum.
- Width can be inferred from the gap at the point where the energy levels get closest.



- Example: two states, $|\phi\rangle$ and $|\chi(p)\chi(-p)\rangle$ with $p = 2\pi/L$.

Resonance energies and widths

- Modelling these level crossings can be used to predict the energy and width of the resonance. Extracting these parameters from Monte Carlo data will require a precise scan of the energy of many states (ground-state, first excited, second . . .) in a given symmetry channel to be carried out at a number of lattice volumes.

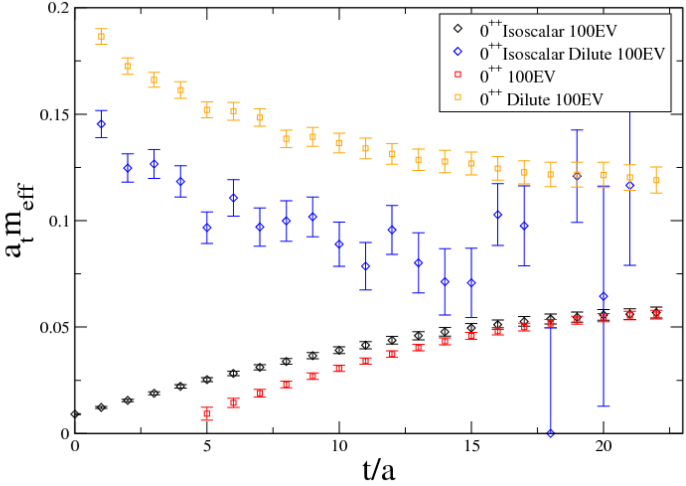
Requirements for measuring decay widths in QCD

- Light, dynamical quarks (to ensure unitarity)
- Accurate spectroscopy in appropriate channels
- Access to excited states in these channels
- Ability to create multi-hadron states

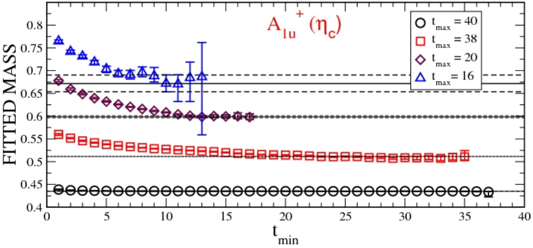
Quark propagation revisited

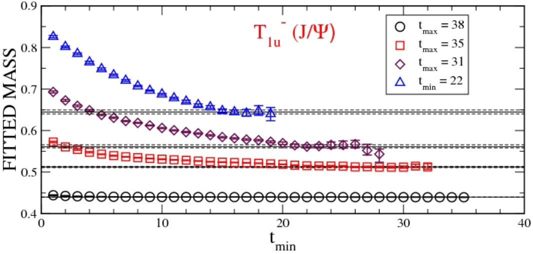
- For high-precision spectroscopy, we need to go beyond traditional "point-to-all" propagator methods.
- The restriction arises because M^{-1} is too large to compute and store in its entirity. We are able to apply it to a particular vector; $w = M^{-1}v$, so algorithms must start from this building block
- "All-to-all" techniques have been an active research topic for a long time, and are now entering mainstream spectroscopy calculations.
- The essential idea is to use Monte Carlo for the quark propagation phase too.
- Take a vector, η with all entries set randomly (and independently) to ± 1.
- Clearly $E[\eta_i \eta_j] = \delta_{ij}$ and so we have a stochastic representation of the identity operator in the vector space
- Now compute $\psi = M^{-1}\eta$ and so $E[\psi_i \eta_j] = M^{-1}_{ij}$

Stochastic “all-to-all” isoscalar data

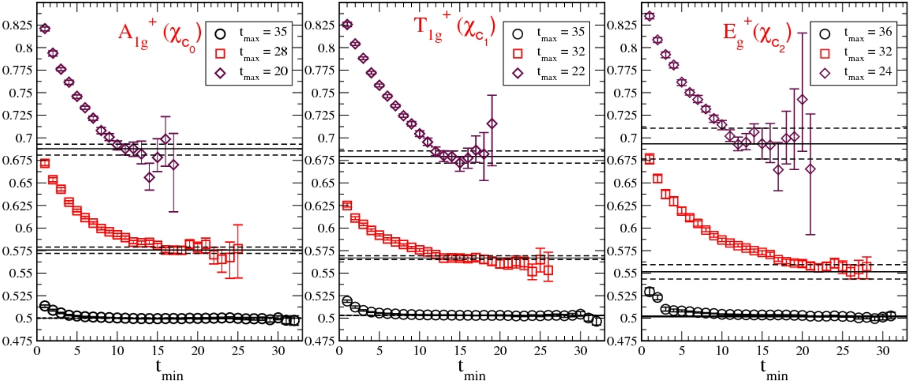


S waves: $\eta_c(0^{-+})$ and $J/\Psi(1^{--})$



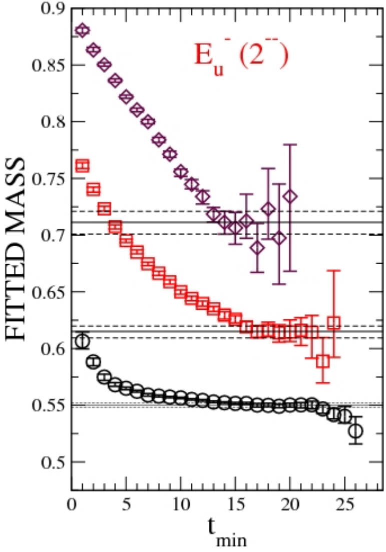


P waves

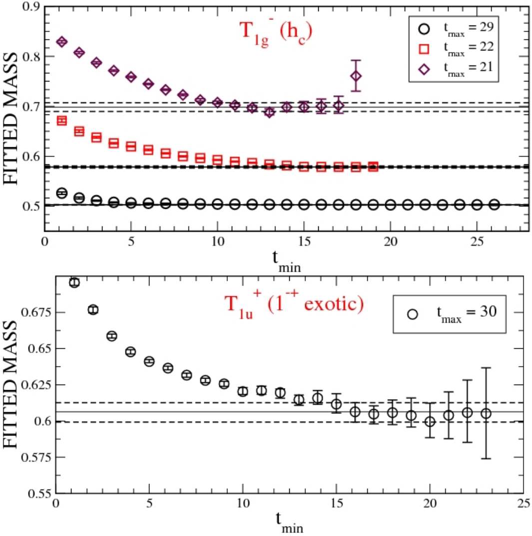


$A_{1g}^{+}(\chi_{c_0})$
$t_{max} = 35$
$t_{max} = 28$
$t_{max} = 20$
$T_{1g}^{+}(\chi_{c_1})$
$t_{max} = 35$
$t_{max} = 32$
$t_{max} = 22$
$E_{g}^{+}(\chi_{c_2})$
$t_{max} = 36$
$t_{max} = 32$
$t_{max} = 24$
t_{min}
0.825
0.8
0.775
0.75
0.725
0.7
0.675
0.65
0.625
0.6
0.575
0.55
0.525
0.5
0.475
0
5
10
15
20
25
30

D waves

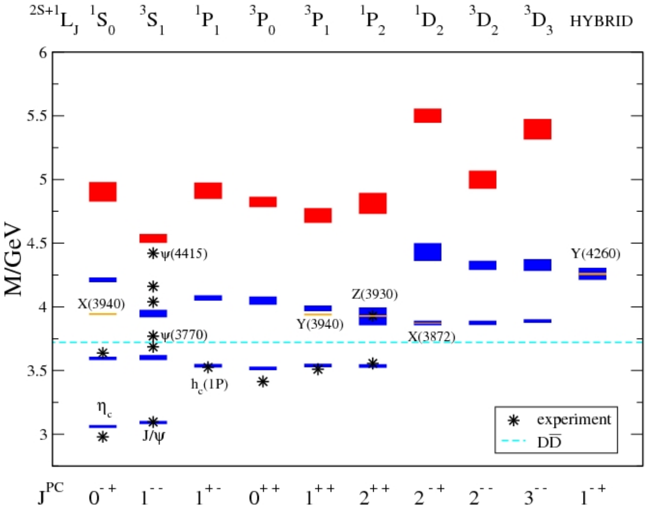

FITTED MASS
E_u^- (2^{--})
T_{1u}^- (3^{--})
t_{min}
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
5
10
15
20
25

h_c and the hybrid, 1^{-+}



Charmonium spectrum from all-to-all techniques

PRELIMINARY



Conclusions

- The lattice defines field theory without pertubative expansions, and regulates quantum fields
- Physics of lattice field theories can be computed numerically on (large) computers
- To do effective Monte Carlo, the Euclidean version of the field theory is needed; scattering and decay physics is difficult
- At present, simulations are starting to approach the physical quark masses
- Developing better methods to do spectroscopy is an active area of research; should be able to handle two-mesons states and isoscalar mesons with some precision soon.