Tensor Polarization Observables in Deuteron Electrodisintegration

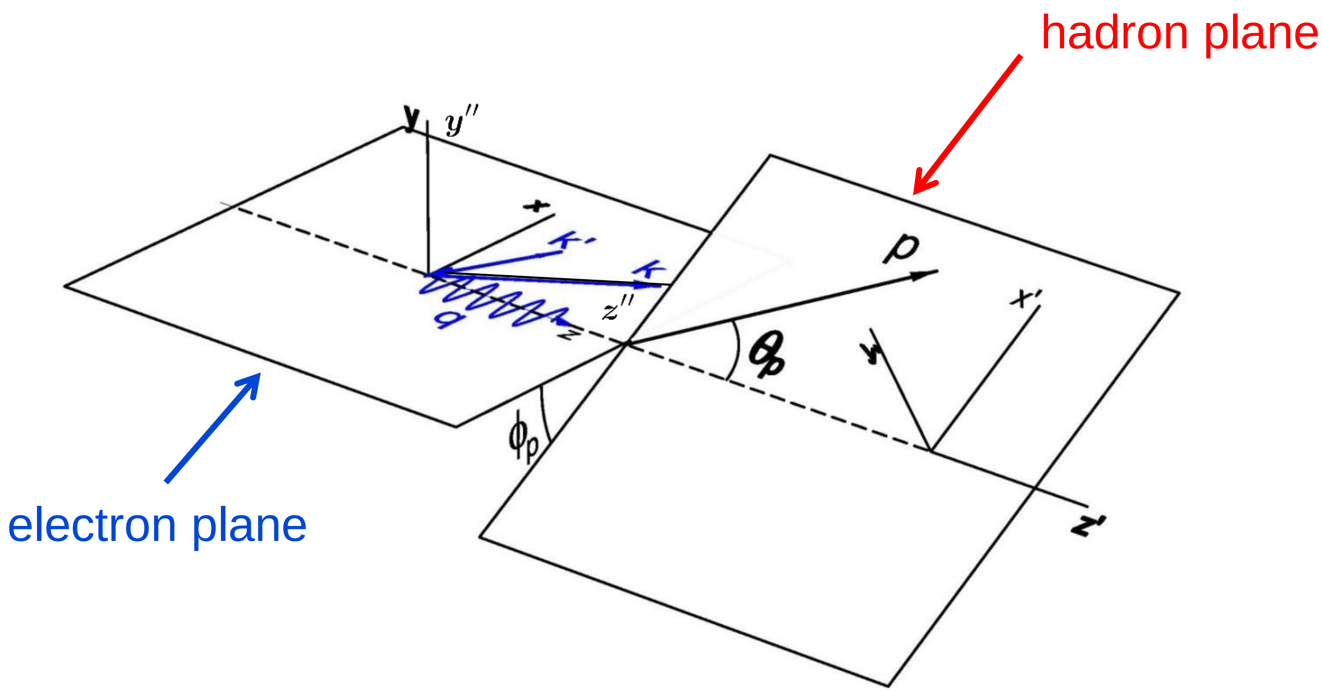
J. W. Van Orden
ODU/Jlab

W. P. Ford
University of Southern Mississippi

S. Jeschennek
OSU-Lima

Cross Section

$$\frac{d\sigma^5}{d\epsilon' d\Omega_e d\Omega_p} = \frac{m_p\, m_n\, p_p}{8\pi^3\, M_d}\, \sigma_{Mott}\, f_{rec}^{-1} \Big[v_L R_L + v_T R_T + v_{TT} R_{TT} + v_{LT} R_{LT} + h\, v_{LT'} R_{LT'} + h\, v_{T'} R_{T'} \Big]$$



S. Jeschennek and J. W. Van Orden, Phys. Rev. C **80**, 054001 (2009).

It is usually assumed that the *hadronic matrix elements* are quantized in the hadron plane designated by the set of coordinates *x', y'* and *z'*

The hadronic tensor in this frame in spherical components is given by

designates hadron plane

$$\overline{w}_{\lambda'_\gamma,\lambda_\gamma}(\overline{D}) = \sum_{s_1,s_2,\lambda_d,\lambda'_d} \overline{\langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (-)| J_{\lambda_\gamma} |\boldsymbol{P}\lambda_d\rangle}\, \overline{\rho}_{\lambda_d \lambda'_d} \times \overline{\langle \boldsymbol{P}\lambda'_d| J^\dagger_{\lambda'_\gamma} |\boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (-)\rangle}$$

The deuteron density matrix can be written in terms of spherical tensor operators as:

$$\overline{\boldsymbol{\rho}} = \frac{1}{3}\left\{\mathbf{1} + \sum_{J=1}^{2}\left[\overline{T}_{J0}\boldsymbol{\tau}_{J0} + \sum_{M=1}^{J}\left(\Re(\overline{T}_{JM})\boldsymbol{\tau}^{\Re}_{JM} + \Im(\overline{T}_{JM})\boldsymbol{\tau}^{\Im}_{JM}\right)\right]\right\}$$

V. Dmitrasinovic and Franz Gross, Phys. Rev. C **40**, 2479 (1989)

The dependence of the response functions on the azimuthal angle ϕ_p can now be made explicit by defining

$$\overline{w}_{\lambda'_\gamma,\lambda_\gamma}(\boldsymbol{\tau}_i^{(I,II)}) = \sum_{s_1,s_2,\lambda_d,\lambda'_d} \overline{\langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (-)| J_{\lambda_\gamma} |\boldsymbol{P}\lambda_d\rangle} (\boldsymbol{\tau}_i^{(I,II)})_{\lambda_d \lambda'_d} \overline{\langle \boldsymbol{P}\lambda'_d| J^\dagger_{\lambda'_\gamma} |\boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (-)\rangle}$$

where

$$\boldsymbol{\tau}_i^{(I)} \in \left\{\mathbf{1}, \boldsymbol{\tau}_{11}^{\Im}, \boldsymbol{\tau}_{20}, \boldsymbol{\tau}_{21}^{\Re}, \boldsymbol{\tau}_{22}^{\Re}\right\}$$
$$\boldsymbol{\tau}_i^{(II)} \in \left\{\boldsymbol{\tau}_{10}, \boldsymbol{\tau}_{11}^{\Re}, \boldsymbol{\tau}_{21}^{\Im}, \boldsymbol{\tau}_{22}^{\Im}\right\}$$

The response functions can then be written as

$$\begin{aligned}
R_L(\overline{D}) &= \overline{R}_L^{(I)}(\overline{D}) \\
R_T(\overline{D}) &= \overline{R}_T^{(I)}(\overline{D}) \\
R_{TT}(\overline{D}) &= \overline{R}_{TT}^{(I)}(\overline{D}) \cos 2\phi_p + \overline{R}_{TT}^{(II)}(\overline{D}) \sin 2\phi_p \\
R_{LT}(\overline{D}) &= \overline{R}_{LT}^{(I)}(\overline{D}) \cos \phi_p + \overline{R}_{LT}^{(II)}(\overline{D}) \sin \phi_p \\
R_{LT'}(\overline{D}) &= \overline{R}_{LT'}^{(I)}(\overline{D}) \sin \phi_p + \overline{R}_{LT'}^{(II)}(\overline{D}) \cos \phi_p \\
R_{T'}(\overline{D}) &= \overline{R}_{T'}^{(II)}(\overline{D})
\end{aligned}$$

The two classes of response functions can then be written as

$$\overline{R}_a^{(I,II)}(\overline{D}) = \sum_i \overline{R}_a^{(I,II)}(\boldsymbol{\tau}_i^{(I,II)})\overline{T}_i^{(I,II)}$$

where

$$\overline{T}_i^{(I)} \in \left\{U, \Im(\overline{T}_{11}), \overline{T}_{20}, \Re(\overline{T}_{21}), \Re(\overline{T}_{22})\right\}$$
$$\overline{T}_i^{(II)} \in \left\{\overline{T}_{10}, \Re(\overline{T}_{11}), \Im(\overline{T}_{21}), \Im(\overline{T}_{22})\right\}$$

Since ϕ_p may vary across detectors, it is more likely that the polarization of the target will be fixed relative to the electron plane rather than the hadron plane.

In this case, the polarization tensor must be:

1. Rotated through $-\phi_p$ about the z axis

2. Rotated through some angle θ about the y axis to the desired direction.

For polarization of the deuteron target along the direction of the beam, the polarization for the rank one contributions in the hadron plane are given by

Denotes polarization in the beam direction.

$$\begin{aligned}
\overline{T}_{10} &= \cos\theta_{kq}\widetilde{T}_{10} + \sqrt{2}\sin\theta_{kq}\Re\widetilde{T}_{11} \\
\Re(\overline{T}_{11}) &= -\frac{1}{\sqrt{2}}\sin\theta_{kq}\cos\phi_p\widetilde{T}_{10} + \cos\theta_{kq}\cos\phi_p\Re\widetilde{T}_{11} + \sin\phi_p\Im\widetilde{T}_{11} \\
\Im(\overline{T}_{11}) &= \frac{1}{\sqrt{2}}\sin\theta_{kq}\sin\phi_p\widetilde{T}_{10} - \cos\theta_{kq}\sin\phi_p\Re\widetilde{T}_{11} + \cos\phi_p\Im\widetilde{T}_{11}
\end{aligned}$$

where θ_{kq} is the angle between the beam direction and the momentum transfer.

If $\theta_{kq} \to 0$ then $\widetilde{T}_{1M} \to T_{1M}$

giving the polarization for the target polarized in the electron plane along the virtual photon direction.

Similarly, for the rank 2 contributions

$$
\begin{aligned}
\overline{T}_{20} &= \frac{1}{4}(1+3\cos 2\theta_{kq})\widetilde{T}_{20} + \sqrt{\frac{3}{2}}\sin 2\theta_{kq}\Re\widetilde{T}_{21} + \sqrt{\frac{3}{8}}\left(1-\cos 2\theta_{kq}\right)\Re\widetilde{T}_{22} \\
\Re(\overline{T}_{21}) &= -\sqrt{\frac{3}{8}}\sin 2\theta_{kq}\cos\phi_p\widetilde{T}_{20} + \cos 2\theta_{kq}\cos\phi_p\Re\widetilde{T}_{21} + \cos\theta_{kq}\sin\phi_p\Im\widetilde{T}_{21} \\
&\quad +\frac{1}{2}\sin 2\theta_{kq}\cos\phi_p\Re\widetilde{T}_{22} + \sin\theta_{kq}\sin\phi_p\Im\widetilde{T}_{22} \\
\Im(\overline{T}_{21}) &= \sqrt{\frac{3}{8}}\sin 2\theta_{kq}\sin\phi_p\widetilde{T}_{20} - \cos 2\theta_{kq}\sin\phi_p\Re\widetilde{T}_{21} + \cos\theta_{kq}\cos\phi_p\Im\widetilde{T}_{21} \\
&\quad -\frac{1}{2}\sin 2\theta_{kq}\sin\phi_p\Re\widetilde{T}_{22} + \sin\theta_{kq}\cos\phi_p\Im\widetilde{T}_{22} \\
\Re(\overline{T}_{22}) &= \sqrt{\frac{3}{32}}(1-\cos 2\theta_{kq})\cos 2\phi_p\widetilde{T}_{20} - \frac{1}{2}\sin 2\theta_{kq}\cos 2\phi_p\Re\widetilde{T}_{21} - \sin\theta_{kq}\sin 2\phi_p\Im\widetilde{T}_{21} \\
&\quad +\frac{1}{4}\left(3+\cos 2\theta_{kq}\right)\cos 2\phi_p\Re\widetilde{T}_{22} + \cos\theta_{kq}\sin 2\phi_p\Im\widetilde{T}_{22} \\
\Im(\overline{T}_{22}) &= -\sqrt{\frac{3}{32}}(1-\cos 2\theta_{kq})\sin 2\phi_p\widetilde{T}_{20} + \frac{1}{2}\sin 2\theta_{kq}\sin 2\phi_p\Re\widetilde{T}_{21} - \sin\theta_{kq}\cos 2\phi_p\Im\widetilde{T}_{21} \\
&\quad -\frac{1}{4}\left(3+\cos 2\theta_{kq}\right)\sin 2\phi_p\Re\widetilde{T}_{22} + \cos\theta_{kq}\cos 2\phi_p\Im\widetilde{T}_{22}
\end{aligned}
$$

For polarization along the beam axis with only

$$\widetilde{T}_{10} \neq 0 \quad \text{or} \quad \widetilde{T}_{20} \neq 0$$

four asymmetries can be defined as

$$A_d^V = \frac{v_L R_L(\widetilde{T}_{10}) + v_T R_T(\widetilde{T}_{10}) + v_{TT} R_{TT}(\widetilde{T}_{10}) + v_{LT} R_{LT}(\widetilde{T}_{10})}{\widetilde{T}_{10}\Sigma}$$

$$A_d^T = \frac{v_L R_L(\widetilde{T}_{20}) + v_T R_T(\widetilde{T}_{20}) + v_{TT} R_{TT}(\widetilde{T}_{20}) + v_{LT} R_{LT}(\widetilde{T}_{20})}{\widetilde{T}_{20}\Sigma}$$

=0 in PWIA

$$A_{ed}^V = \frac{v_{LT'} R_{LT'}(\widetilde{T}_{10}) + v_{T'} R_{T'}(\widetilde{T}_{10})}{\widetilde{T}_{10}\Sigma}$$

$$A_{ed}^T = \frac{v_{LT'} R_{LT'}(\widetilde{T}_{20}) + v_{T'} R_{T'}(\widetilde{T}_{20})}{\widetilde{T}_{20}\Sigma}$$

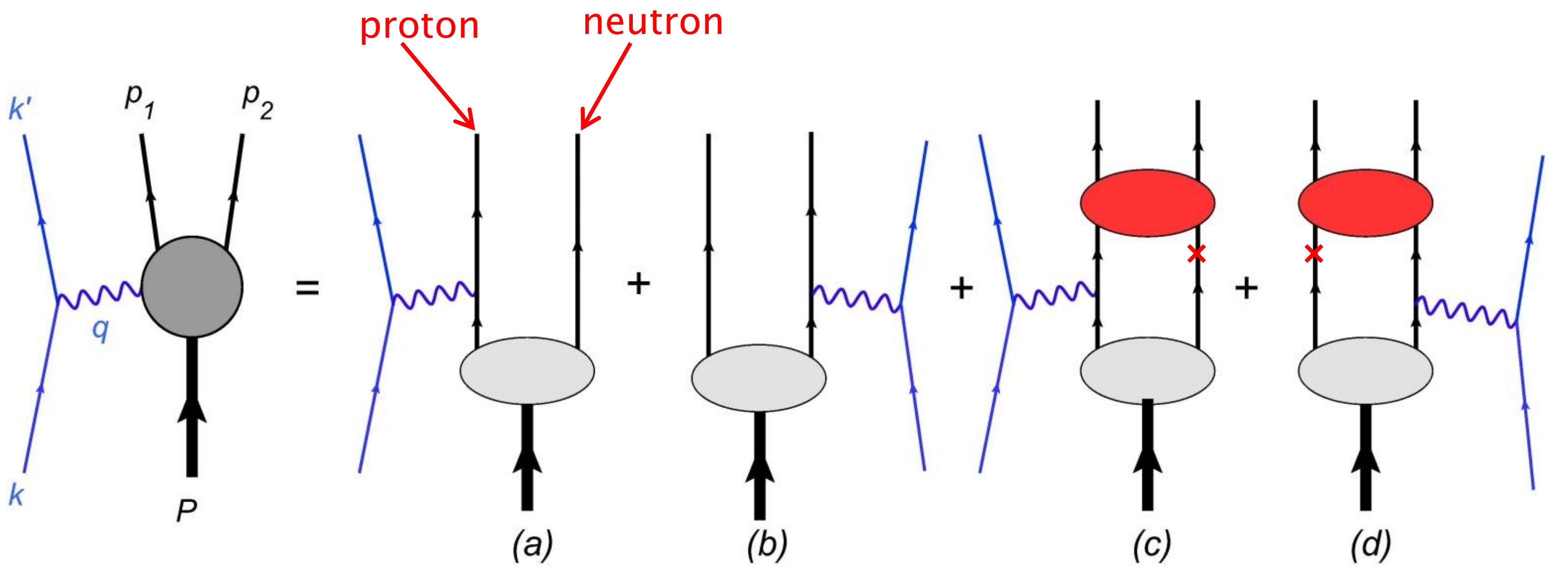
where

$$\Sigma = v_L R_L(U) + v_T R_T(U) + v_{TT} R_{TT}(U) + v_{LT} R_{LT}(U)$$

unpolarized response function

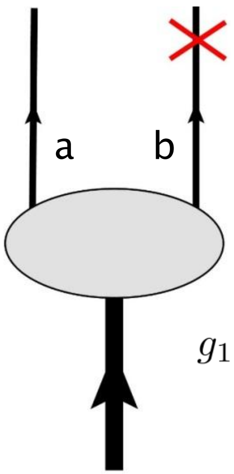
The Impulse Approximation

The impulse approximation to deuteron electrodisintegration is defined by the Feynman diagrams:



Note: Impulse approximation calculations does not conserve current.

Deuteron Vertex Function



$$
\begin{aligned}
[\Gamma_{\lambda_d}(p_2,P)]_{ab} &= \Bigg[g_1(p_2^2, p_2\cdot P)\gamma\cdot\xi_{\lambda_d}(P) + g_2(p_2^2,p_2\cdot P)\frac{p\cdot\xi_{\lambda_d}(P)}{m} \\
&\quad - \left(g_3(p_2^2,p_2\cdot P)\gamma\cdot\xi_{\lambda_d}(P) + g_4(p_2^2,p_2\cdot P)\frac{p\cdot\xi_{\lambda_d}(P)}{m}\right)\frac{\gamma\cdot p_1 + m}{m} C \Bigg]_{ba}
\end{aligned}
$$

The invariant functions g_i are given by

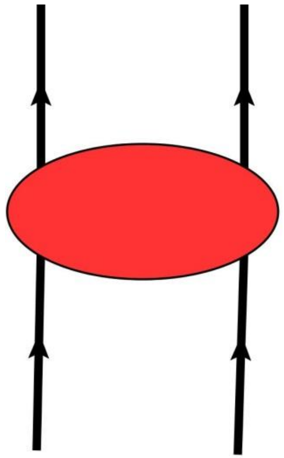
$$
\begin{aligned}
g_1(p_2^2,p_2\cdot P) &= \frac{2E_k - M_d}{\sqrt{8\pi}}\left[u(k) - \frac{1}{\sqrt{2}} w(k) + \sqrt{\frac{3}{2}}\frac{m}{k} v_t(k)\right] \\
g_2(p_2^2,p_2\cdot P) &= \frac{2E_k - M_d}{\sqrt{8\pi}}\left[\frac{m}{E_k+m} u(k) + \frac{m(2E_k+m)}{\sqrt{2}k^2} w(k) + \sqrt{\frac{3}{2}}\frac{m}{k} v_t(k)\right] \\
g_3(p_2^2,p_2\cdot P) &= \sqrt{\frac{3}{16\pi}}\frac{mE_k}{k} v_t(k) \\
g_4(p_2^2,p_2\cdot P) &= -\frac{m^2}{\sqrt{8\pi}M_d}\left[(2E_k - M_d)\left(\frac{1}{E_k+m} u(k) - \frac{E_k+2m}{\sqrt{2}k^2} w(k)\right) + \frac{\sqrt{3}M_d}{k} v_s(k)\right]
\end{aligned}
$$

where $k = \sqrt{\frac{(P\cdot p_2)^2}{P^2} - p_2^2}$ is the magnitude of the three-momentum in the deuteron rest frame.

Final State Interactions

There are no reliable meson-exchange models of *NN* scattering for the invariant masses where pions production channels are open.

The scattering amplitudes must be obtain from data.



The scattering amplitudes are obtained from a parameterization in terms of five Fermi invariants.

$$\hat{M} = \mathcal{F}_S(s,t)1^{(1)}1^{(2)} + \mathcal{F}_V(s,t)\gamma^{\mu(1)}\gamma_\mu^{(2)} + \mathcal{F}_T(s,t)\sigma^{\mu\nu(1)}\sigma_{\mu\nu}^{(2)}$$
$$- \mathcal{F}_P(s,t)(i\gamma_5)^{(1)}(i\gamma_5)^{(2)} + \mathcal{F}_A(s,t)(\gamma_5\gamma^\mu)^{(1)}(\gamma_5\gamma_\mu)^{(2)}$$

- A complete description of on-shell NN scattering.
- Lorentz invariant.
- Has complete spin dependence.

We use two approaches:

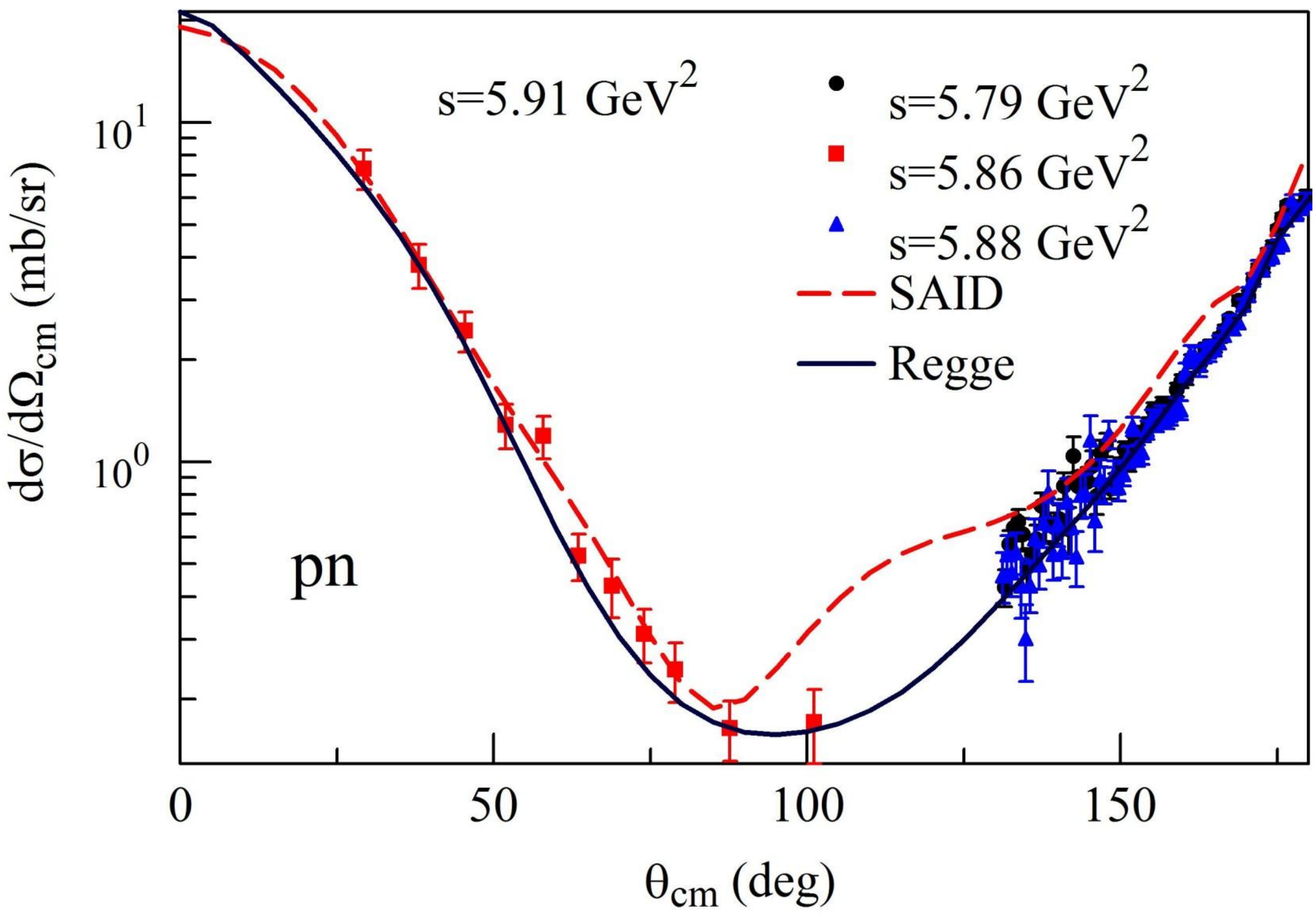
1. The invariant functions $\mathcal{F}_i(s,t)$ are constucted from the SAID helicity amplitudes. *np* amplitudes are available for $s<5.98$ GeV^2

2. We have recently performed a fit of the $\mathcal{F}_i(s,t)$ available *NN* data from $s=5.4$ GeV^2 to $s=4000$ GeV^2 based on a Regge model.*

In the calculations shown here, only on-shell contributions from the *np* amplitudes are used.

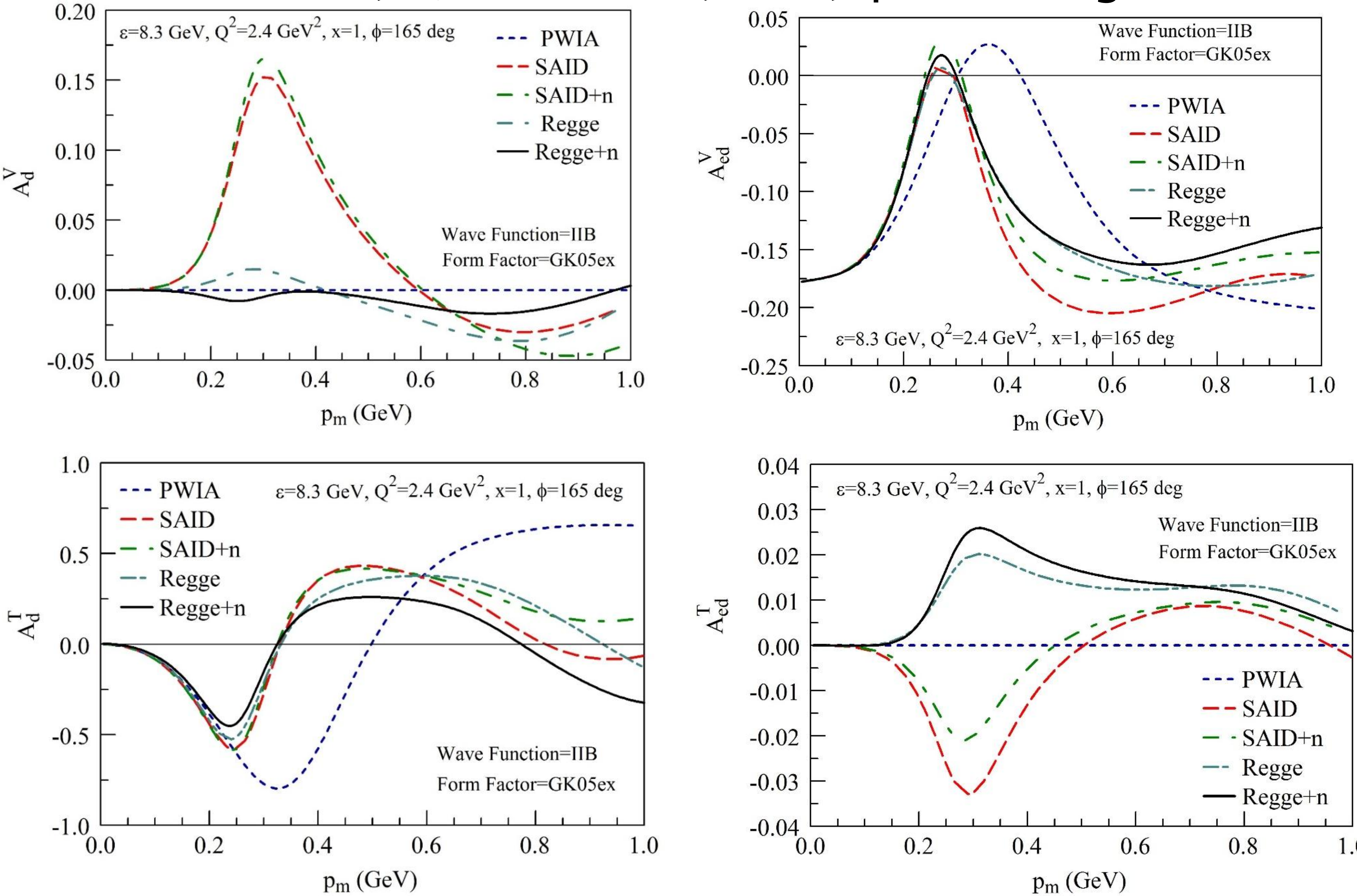
* W. P. Ford and J. W. Van Orden, Phys. Rev. C 87, 014004 (2013).
W. P. Ford, Ph.D. Dissertation, http://arxiv.org/abs/1310.0871

pn scattering

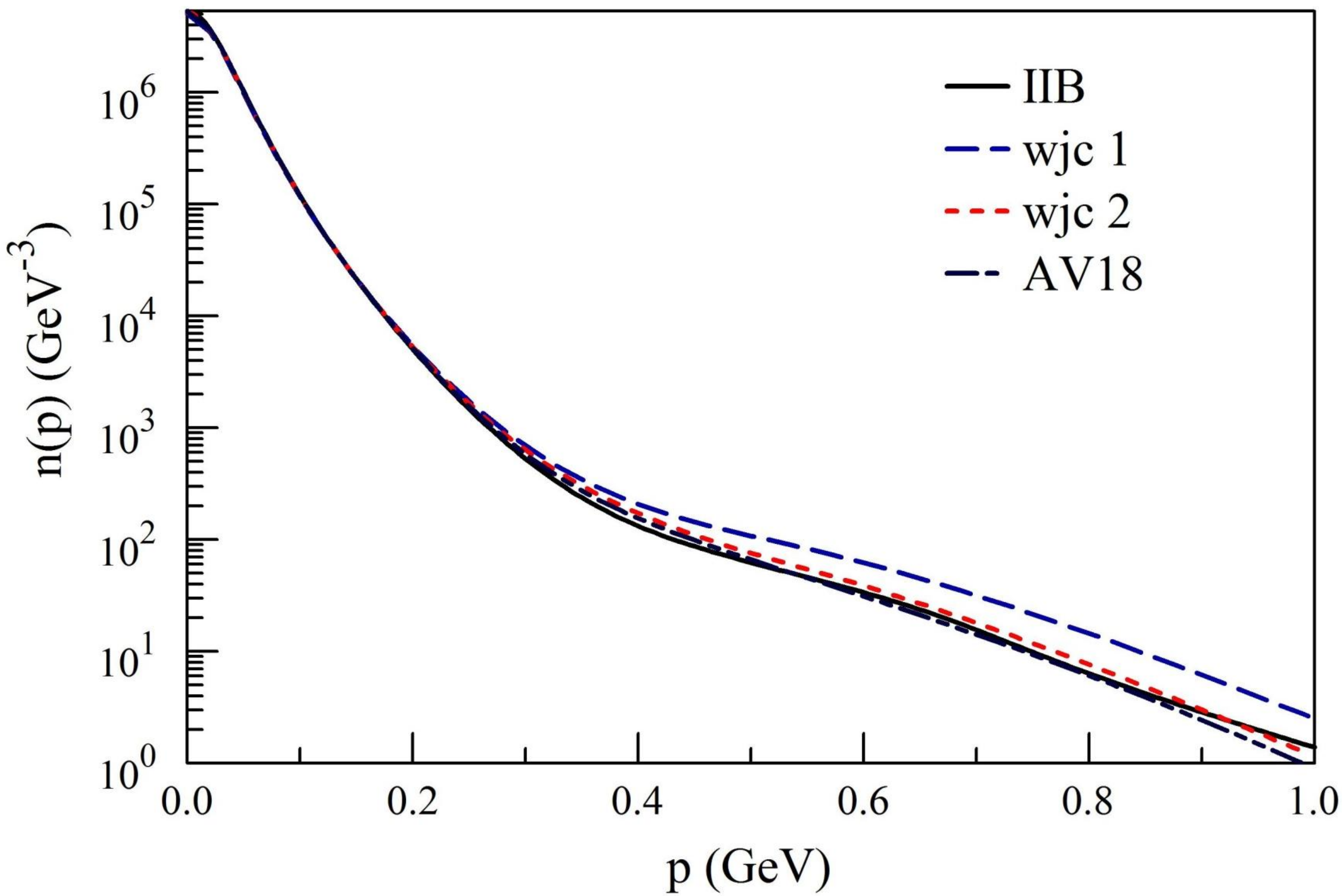


FSI Effects, *x*=1

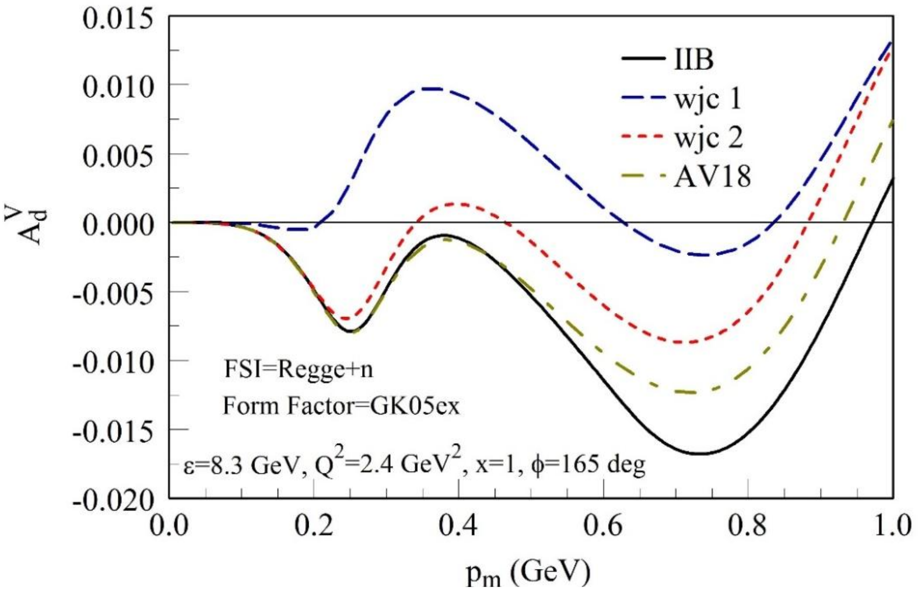
ϵ=8.3 GeV, Q^2=2.4 GeV^2, x=1, ϕ=165 deg.

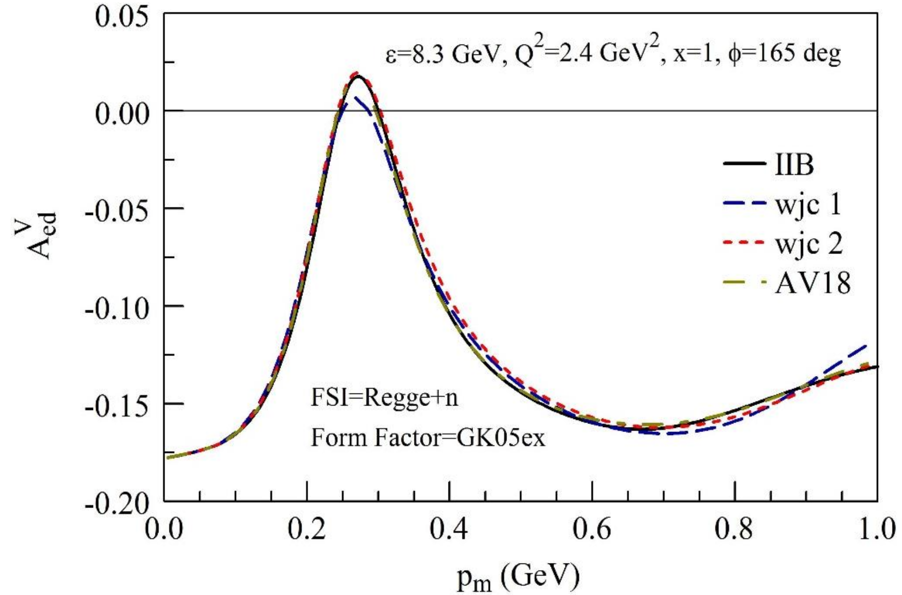


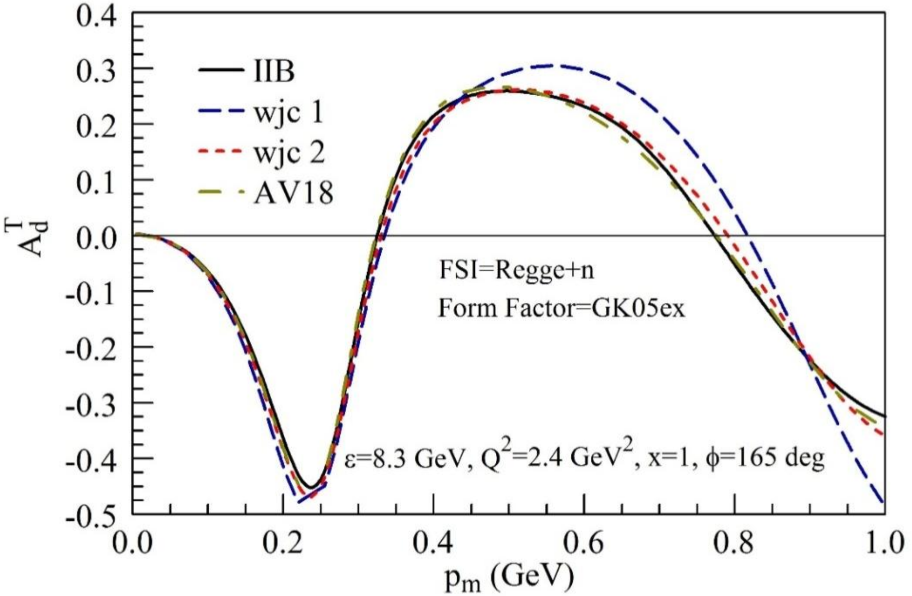
Momentum Distributions

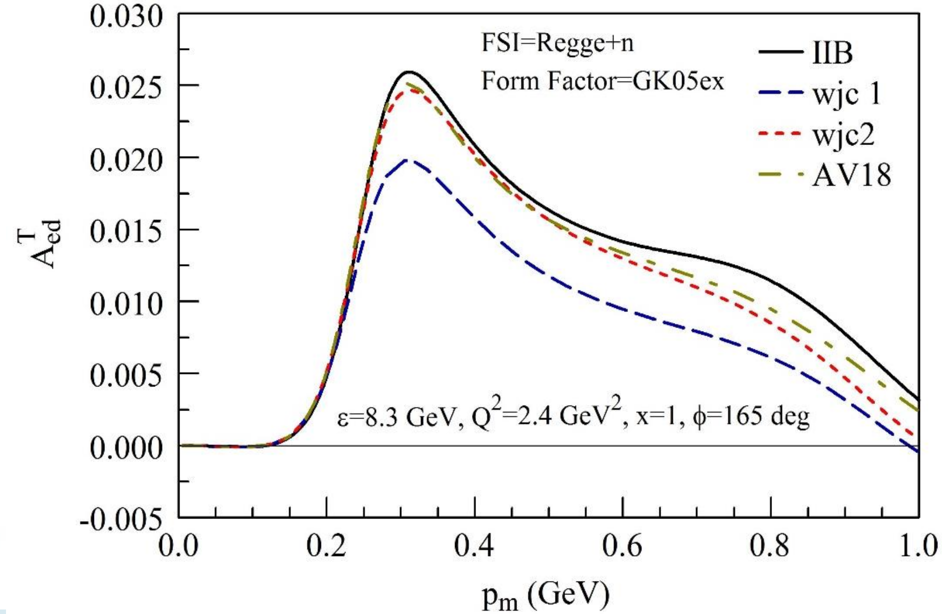


“Wave Function” Effects, x=1

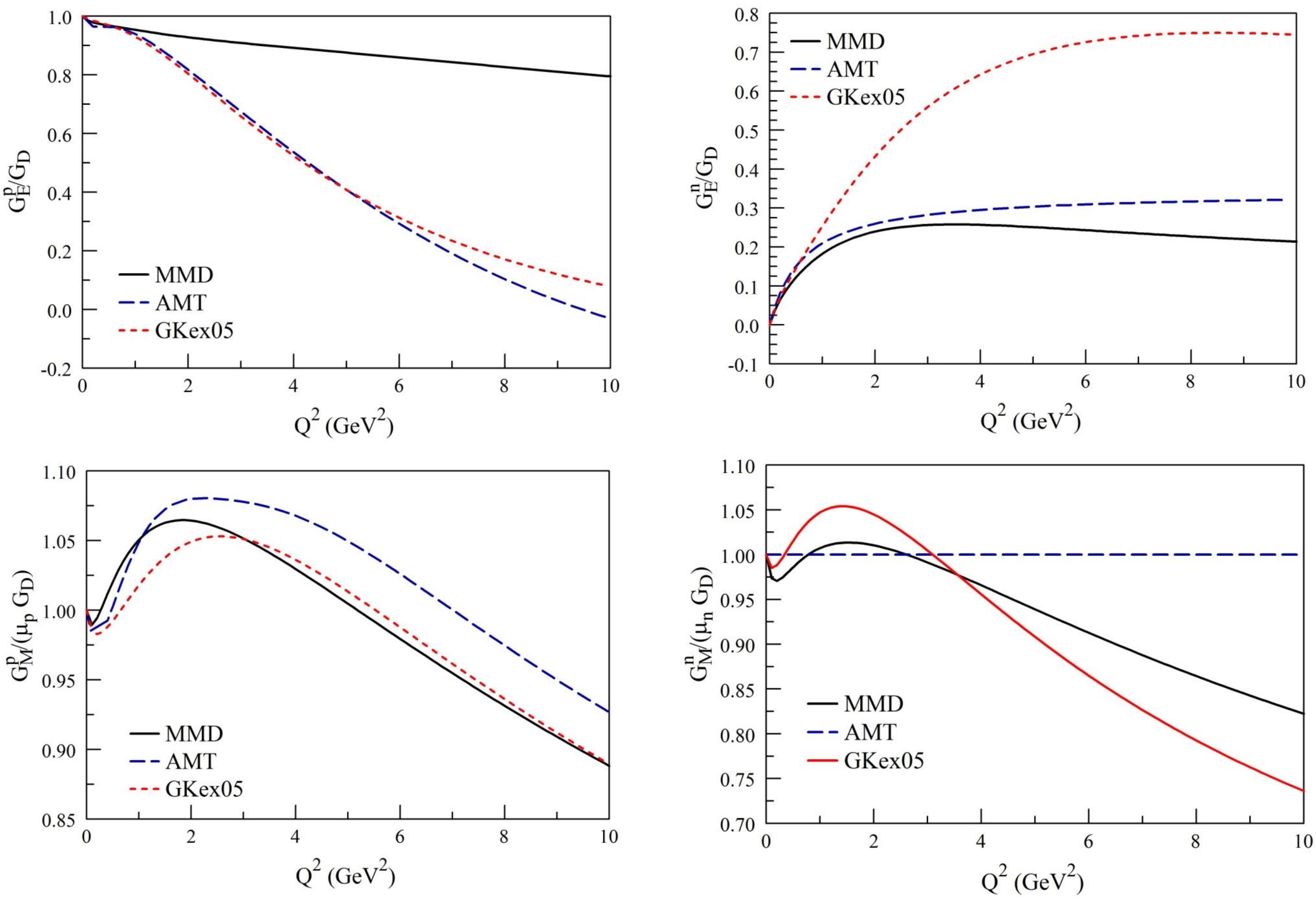




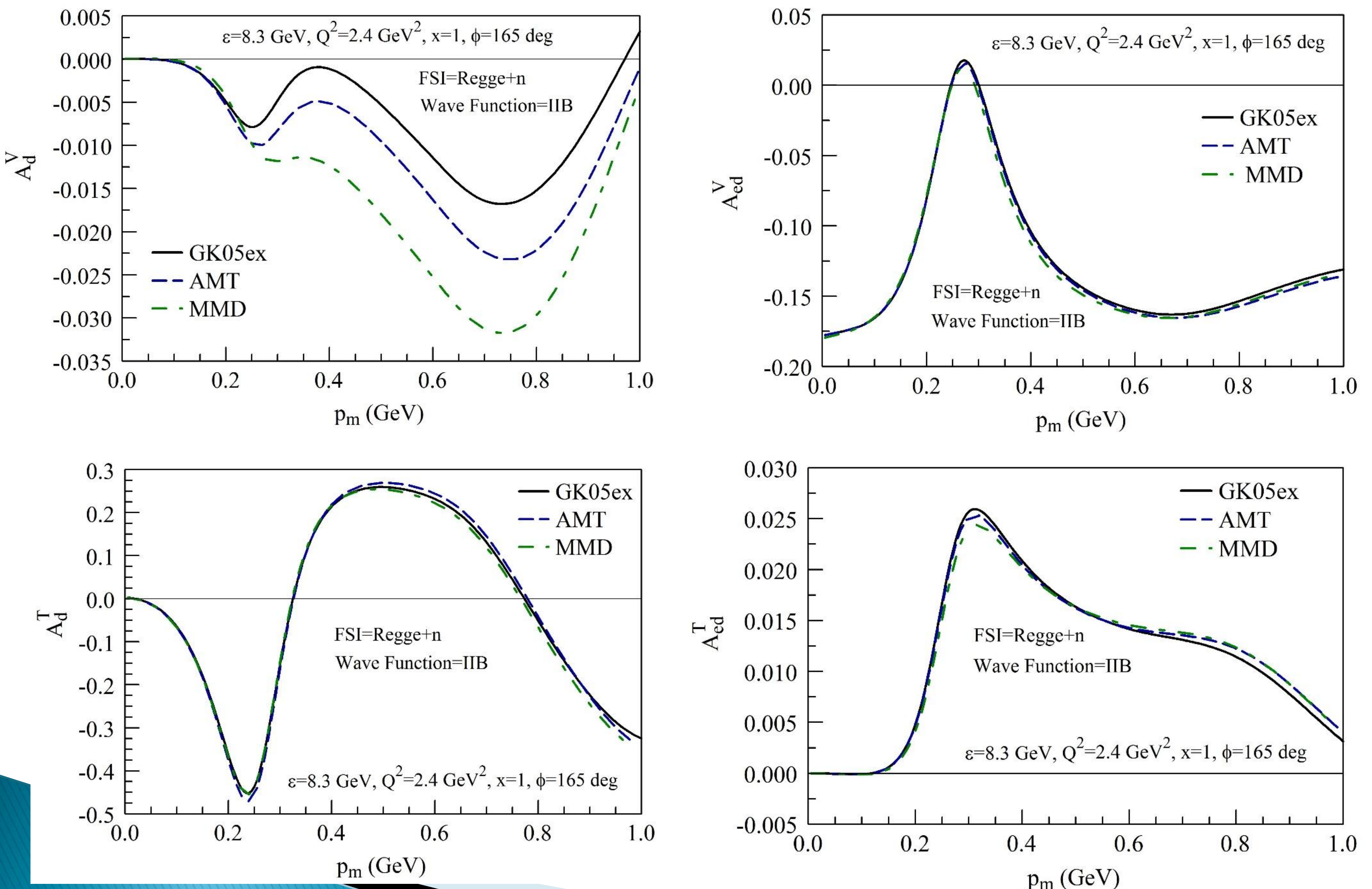




Nucleon Form Factors

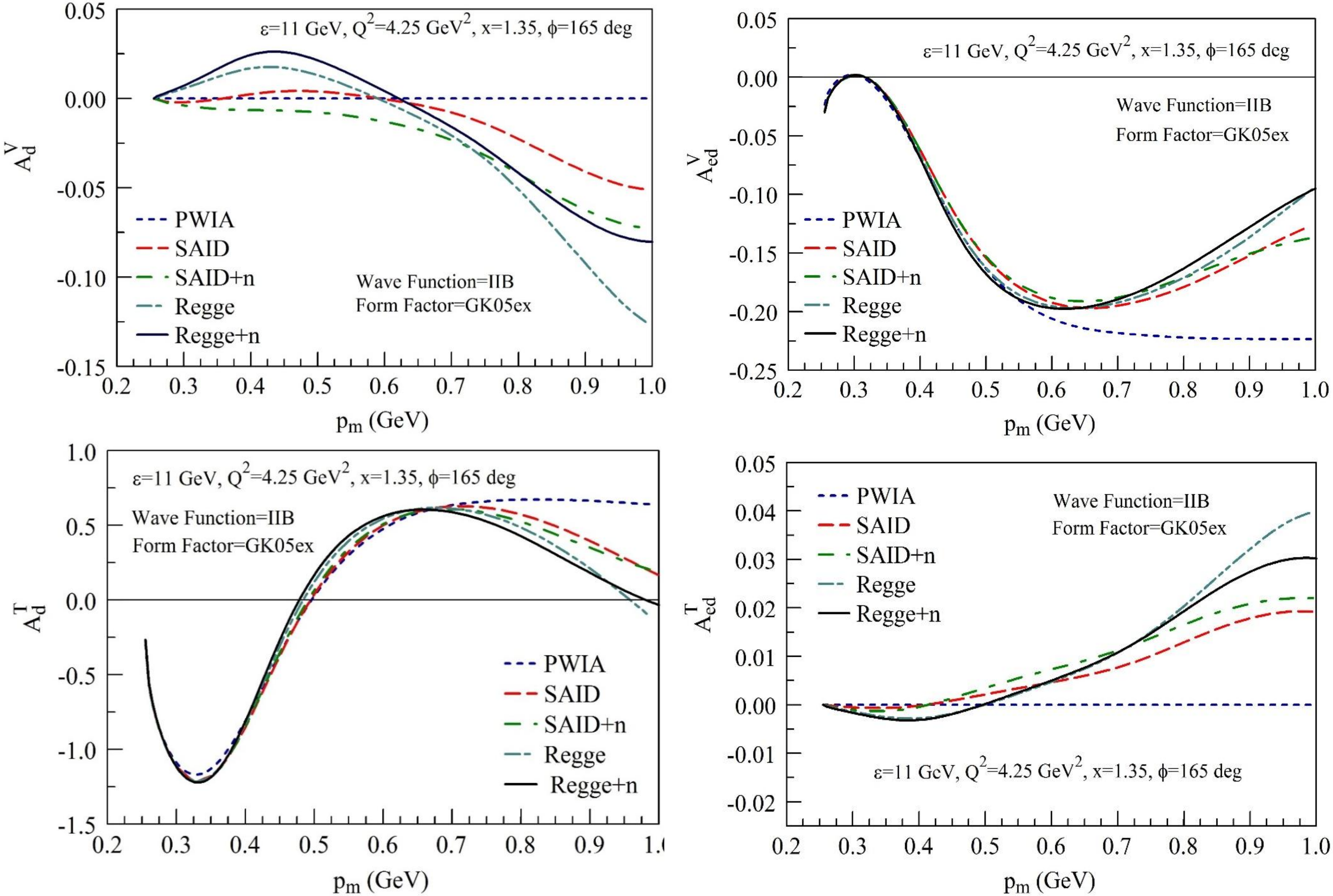


Form Factor Effects, $x=1$



FSI Effects, x=1.35

ϵ=11 GeV, Q^2=4.25 GeV^2, x=1.35, ϕ=165 deg.



Summary

The effects of FSI are large for x=1 and at large missing momenta.

Contributions from scattering on the neutron are generally important.

The variation with the choice of wave functions is largely consistent with variations in their momentum distributions.

The observables are largely insensitive the the choice electromagnetic form factors except for those observables that are very small.

$$T_{10} = \sqrt{\frac{3}{2}} P_z$$

$$\Re(T_{11}) = -\sqrt{\frac{3}{2}} P_x$$

$$\Im(T_{11}) = -\sqrt{\frac{3}{2}} P_y$$

$$T_{20} = \frac{1}{\sqrt{2}} P_{zz}$$

$$\Re(T_{21}) = -\frac{1}{\sqrt{3}} P_{xz}$$

$$\Im(T_{21}) = -\frac{1}{\sqrt{3}} P_{yz}$$

$$\Re(T_{22}) = \frac{1}{\sqrt{12}} (P_{xx} - P_{yy})$$

$$\Im(T_{22}) = \frac{1}{\sqrt{3}} P_{xy}$$