Introduction to Quantum Computing

Stephen Casey – NASA Langley Research Center

Slide template creator – Krysta Svore

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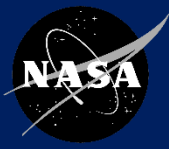

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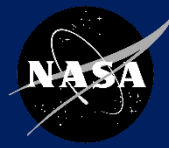

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$|0\rangle$ $|1\rangle$

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$$|\psi\rangle = \sqrt{a}\,|0\rangle + e^{i\phi}\sqrt{1-a}\,|1\rangle$$

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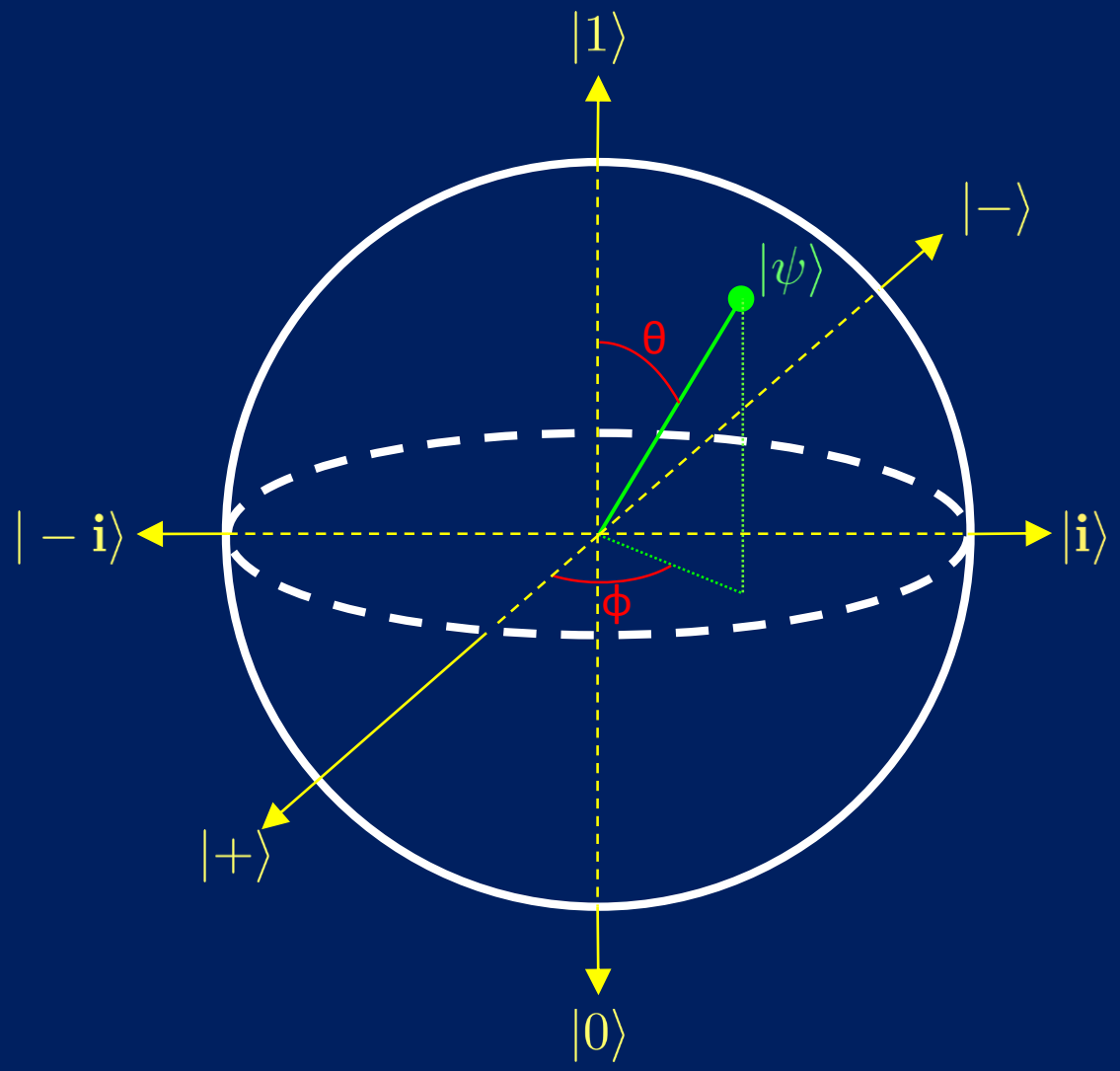
Bloch Sphere



Hadamard basis

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$|\mathbf{i}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + \mathbf{i}|1\rangle)$$

$$|-\mathbf{i}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - \mathbf{i}|1\rangle)$$

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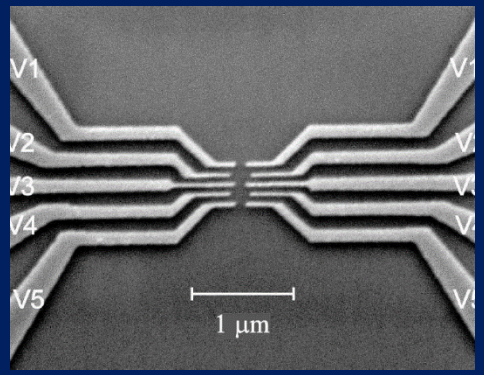
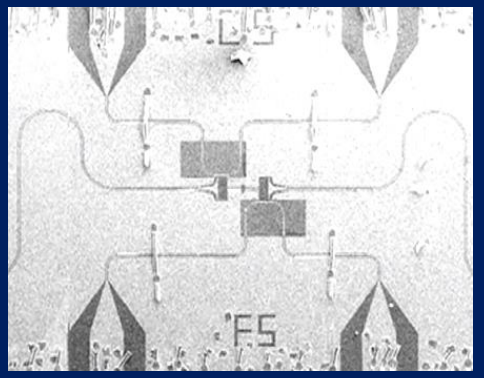
Quantum Hardware Technologies

Quantum dots

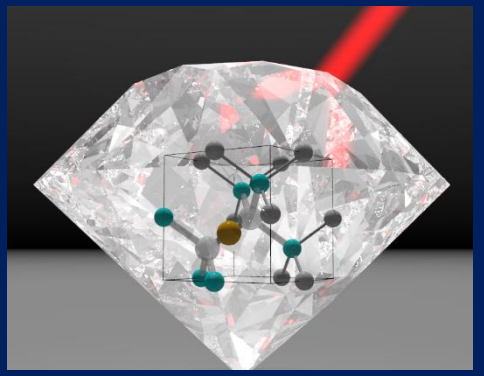


Superconductors

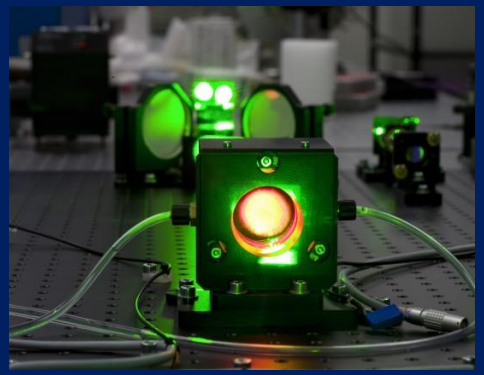


Ion traps

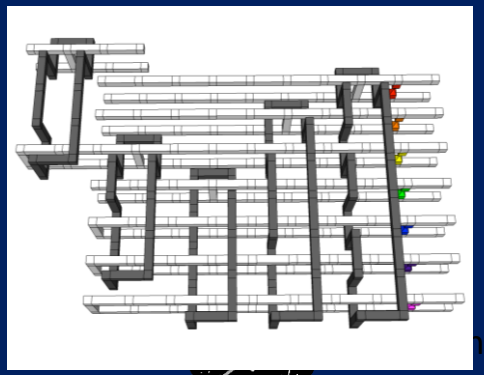
Nitrogen vacancy centers



Optical photons



Topological



Center

Unitary gates

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Pauli-X

$$\begin{matrix}\alpha|0\rangle \\ \beta|1\rangle\end{matrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{matrix}\beta|0\rangle \\ \alpha|1\rangle\end{matrix}$$

Pauli-Y

$$\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

Pauli-Z

$$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Hadamard

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

Phase

$$\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}$$

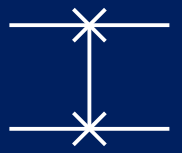
Rotation

$$\begin{pmatrix}1 & 0\\ 0 & e^{i\phi}\end{pmatrix}$$

Controlled-NOT

$$\begin{matrix}|00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle\end{matrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix} = \begin{matrix}|00\rangle \\ |01\rangle \\ |11\rangle \\ |10\rangle\end{matrix}$$

Swap



$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

Controlled-swap

$$\begin{pmatrix}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0\\ 0 & 0 & 0 & 0 & 0 & \mathbf{1} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1}\end{pmatrix}$$

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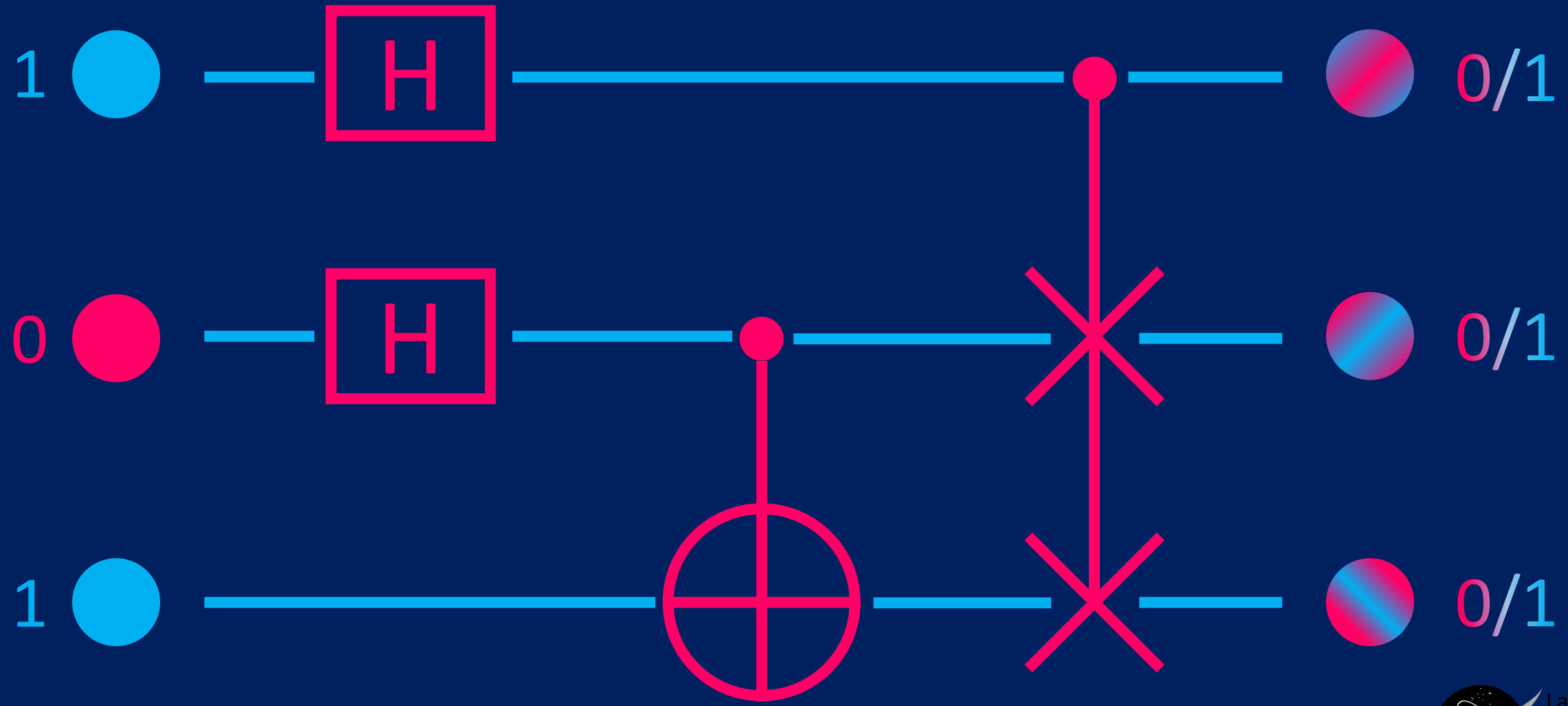
Quantum circuit model



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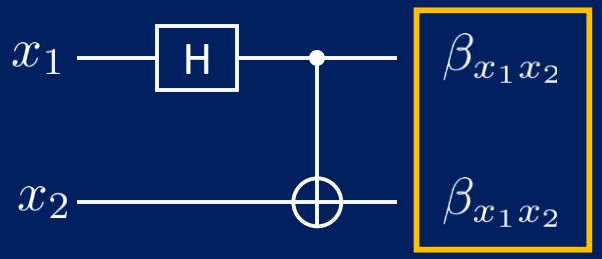
Entanglement

$$|x_1\rangle \otimes |x_2\rangle \equiv |x_1\rangle|x_2\rangle \equiv |x_1 x_2\rangle$$

Entangled



$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\beta_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$|\beta_{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

← Bell states

$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$$

$$|\psi_1\rangle \otimes |\psi_2\rangle = (\alpha_1|0\rangle + \beta_1|1\rangle) \otimes (\alpha_2|0\rangle + \beta_2|1\rangle)$$

$$= \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

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Quantum Fourier Transform

$$|j\rangle \rightarrow \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{\frac{2\pi i}{N} jk} |k\rangle$$

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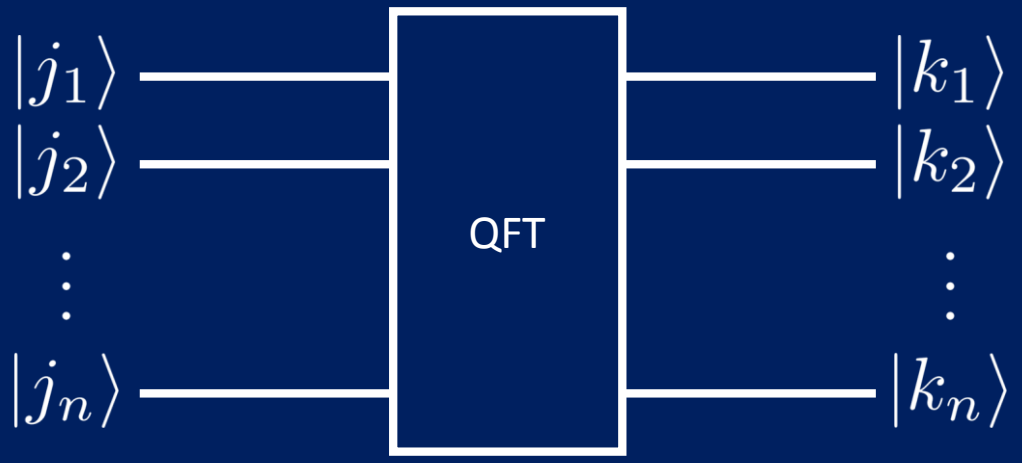
$$QFT = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega^1 & \omega^2 & \omega^2 & \cdots & \omega^{(N-1)} \\ 1 & \omega^2 & \omega^4 & \omega^6 & \cdots & \omega^{2(N-1)} \\ 1 & \omega^3 & \omega^6 & \omega^9 & \cdots & \omega^{3(N-1)} \\ & & & & & \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ & & & & & \\ 1 & \omega^{(N-1)} & \omega^{2(N-1)} & \omega^{3(N-1)} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix}$$

$$\omega = e^{\frac{2\pi i}{N}}$$



Exponential Speedup

$$\text{Classical}: \ \Theta(n2^n)$$

$$\text{Quantum}: \ \Theta(n^2)$$

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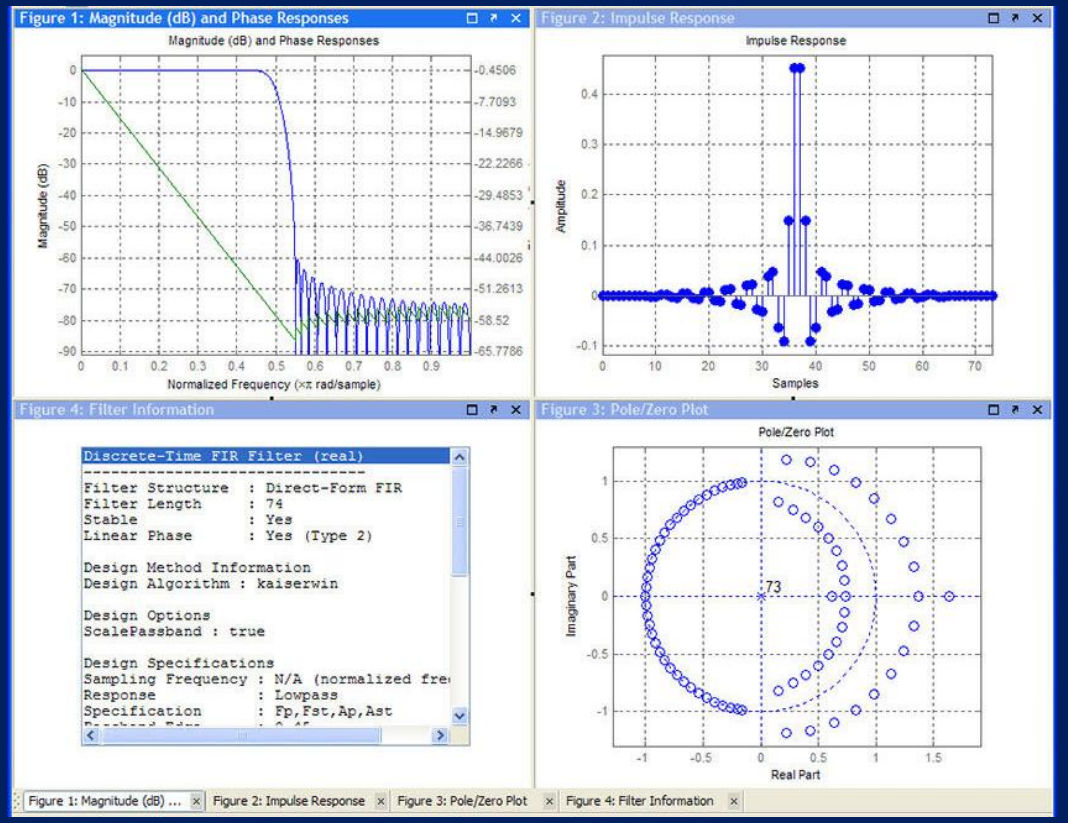
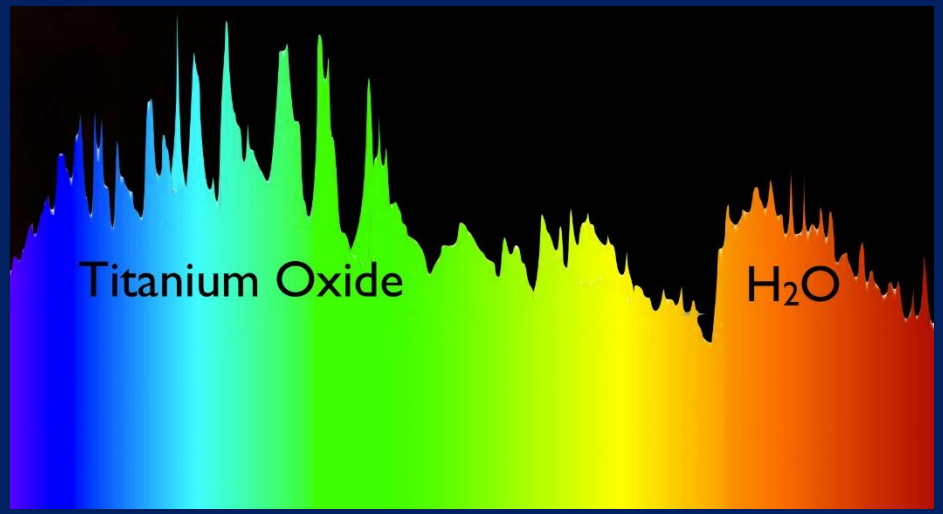
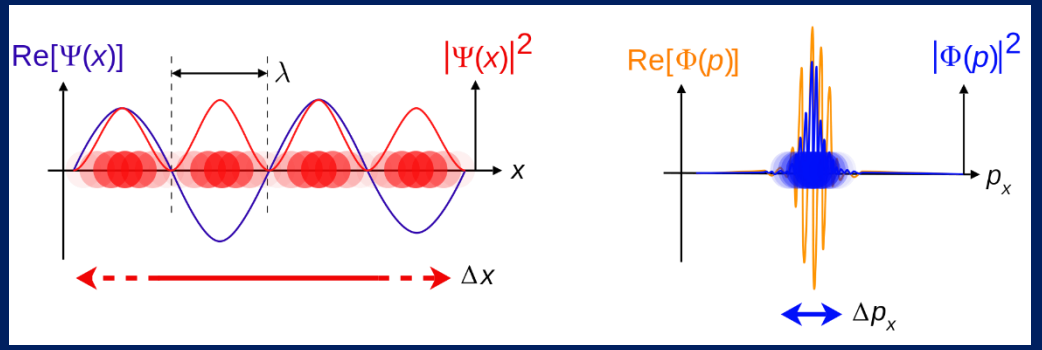
Classical FFT:
1GB → 10 billion operations







Quantum FFT:
1GB → 27 operations

Spectroscopy | Acoustics | Video compression | Quantum mechanics | Signal processing

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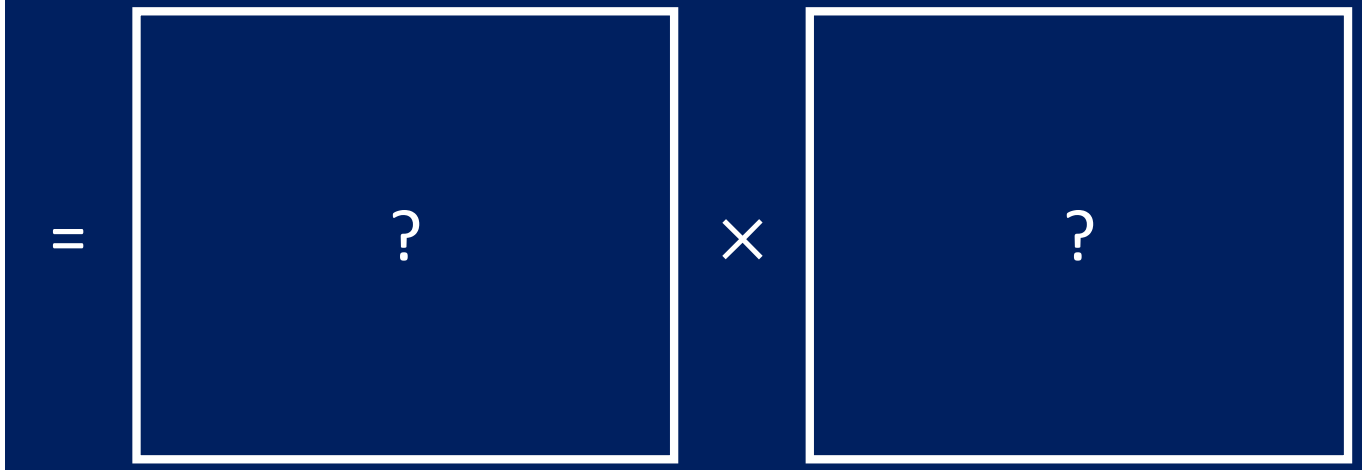

Slide credit: John Preskill

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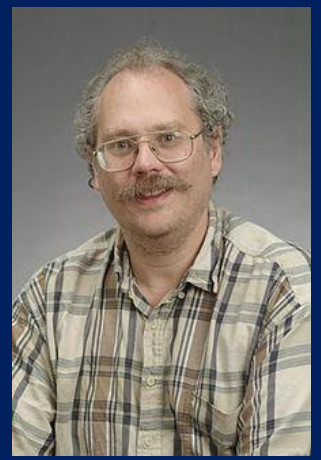

Peter Shor

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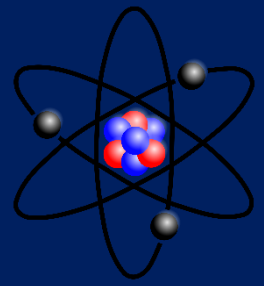
Classical Computer	Quantum Computer
	
193 digits: 30 CPU-years (2.2 GHz)	193 digits: 0.1 seconds
500 digits: 10^{12} CPU-years	500 digits: 2 seconds
RSA SECURITY® UNBREAKABLE	RSA SECURITY® BREAKABLE

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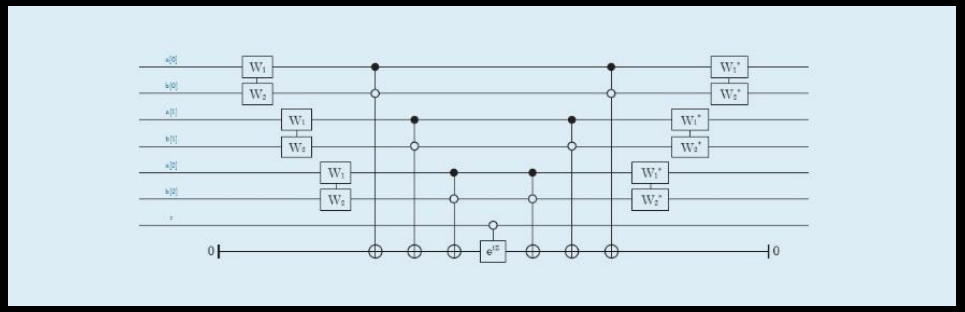
Programming languages

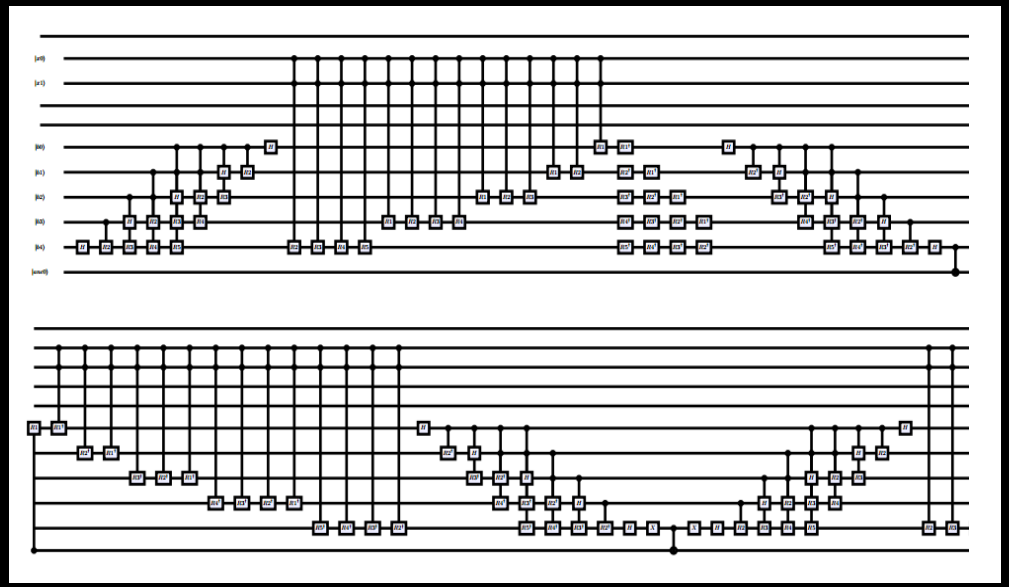
Quipper

```
import Quipper

w :: (Qubit,Qubit) -> Circ (Qubit,Qubit)
w = named_gate "W"

toffoli :: Qubit -> (Qubit,Qubit) -> Circ Qubit
toffoli d (x,y) =
            qnot d 'controlled' x .==. 1 .&&. y .==. 0

eiz_at :: Qubit -> Qubit -> Circ ()
eiz_at d r =
            named_gate_at "eiZ" d 'controlled' r .==. 0
circ :: [ (Qubit,Qubit) ] -> Qubit -> Circ ()
circ ws r = do
            label (unzip ws,r) (("a","b","r")
            with_ancilla $ \d -> do
            mapM_ w ws
            mapM_ (toffoli d) ws
            eiz_at d r
            mapM_ (toffoli d) (reverse ws)
            mapM_ (reverse_generic w) (reverse ws)
            return ()

main = print_generic EPS circ (replicate 3 (qubit,qubit)) qubit
```



Liqui|>

```
CCAdd a cbs             // Perform the initial Add
AddA' N bs              // Invert the add
QFT' bs                 // Convert out of Fourier space
CNOT [bMx ; anc]        // Remember the overflow bit
QFT bs                  // Return to Fourier space
CAddA N (anc :: bs)     // Do the add based on overflow
ccAdd' a cbs            // Undo the add
QFT' bs                 // Get out of Fourier space
X [bMx]                 // Use the top bit as a flag
CNOT [bMx ; anc]        // Clean up the Ancilla
X [bMx]                 // Reverse use of the top bit
QFT bs                  // Return to Fourier space
CCAdd a cbs             // Do the final version of the add
```



QCL, Q, qGCL, QFC, QPL, QML, and others!

D-Wave Two

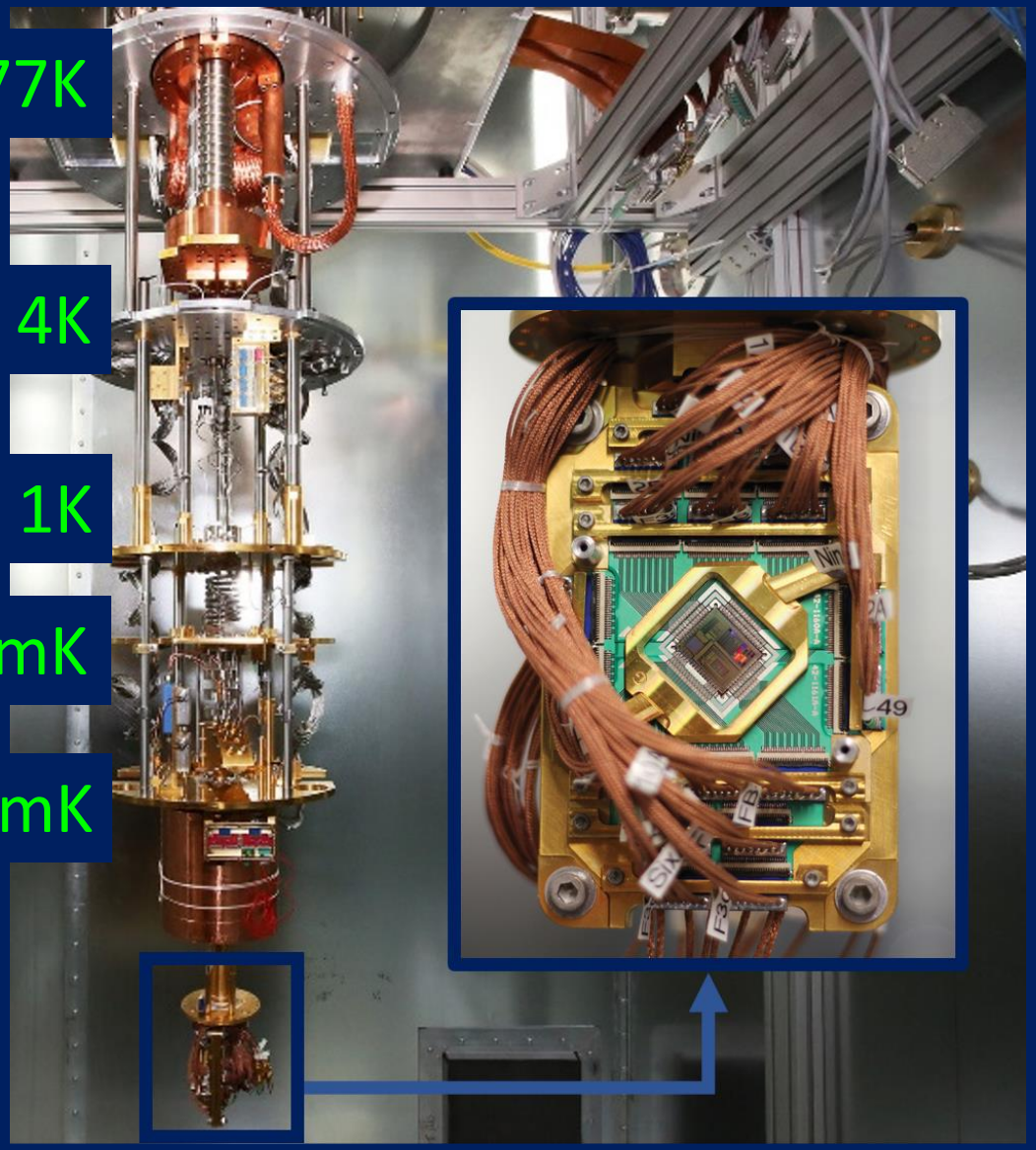
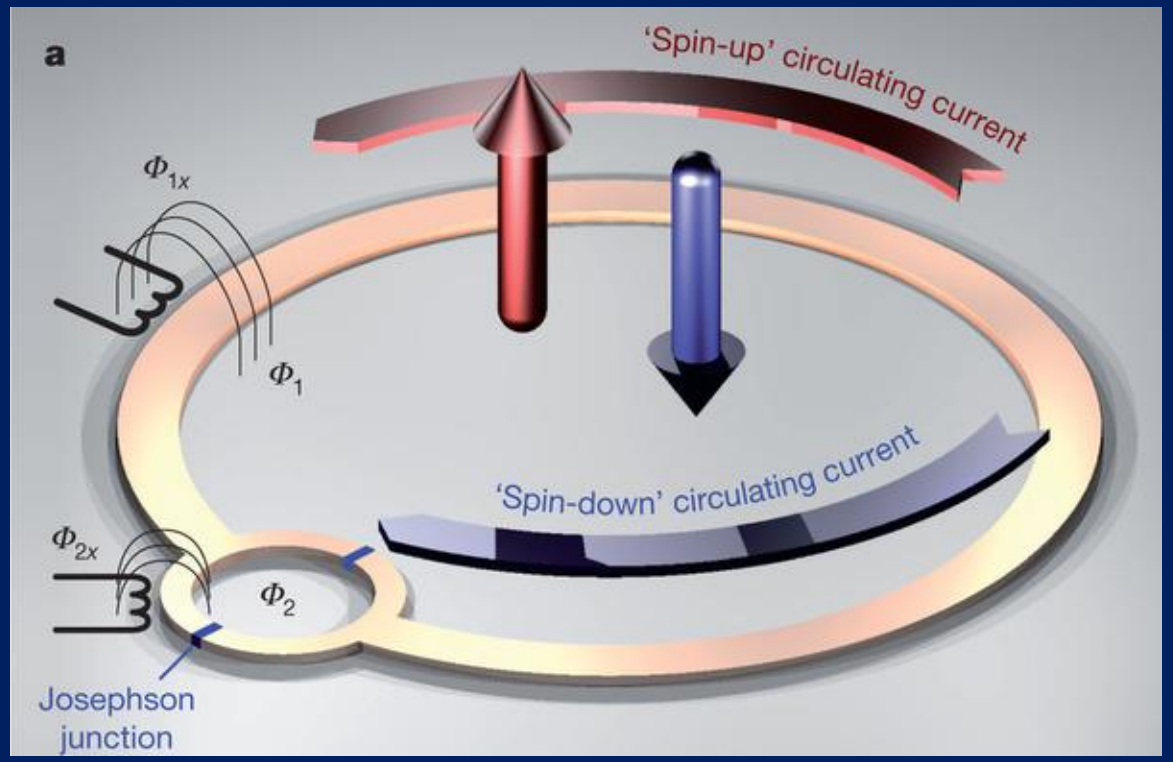
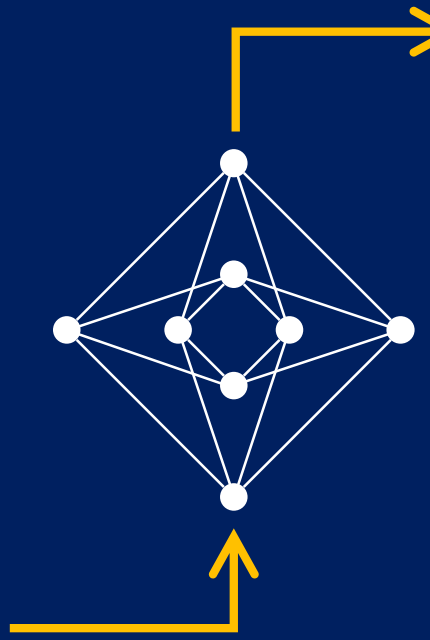


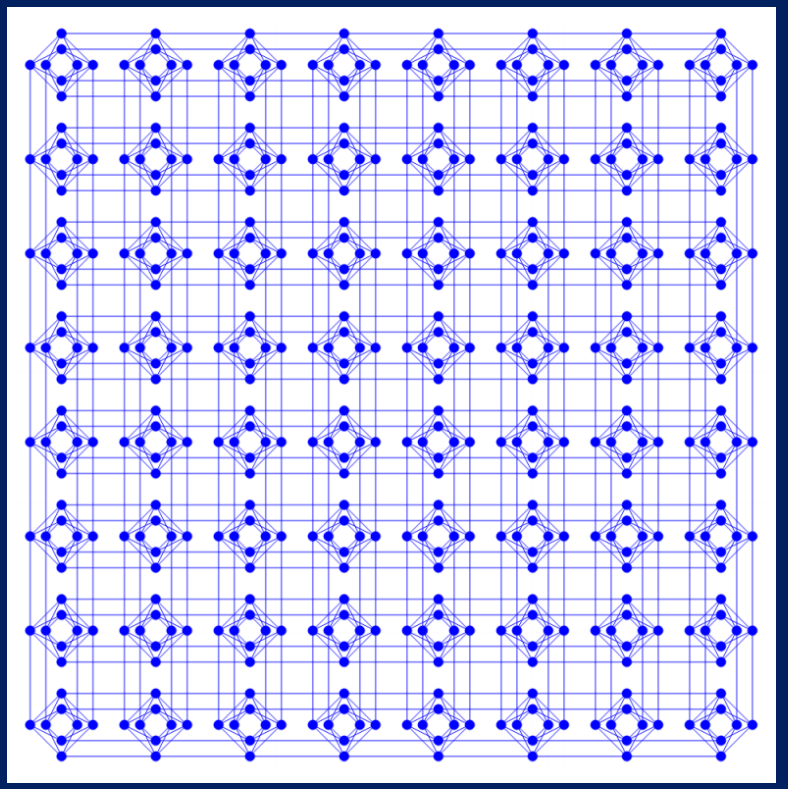
Image credit: D-Wave Systems, Inc.

Processor Architecture

Chimera structure







Superconducting flux qubits

Ising model

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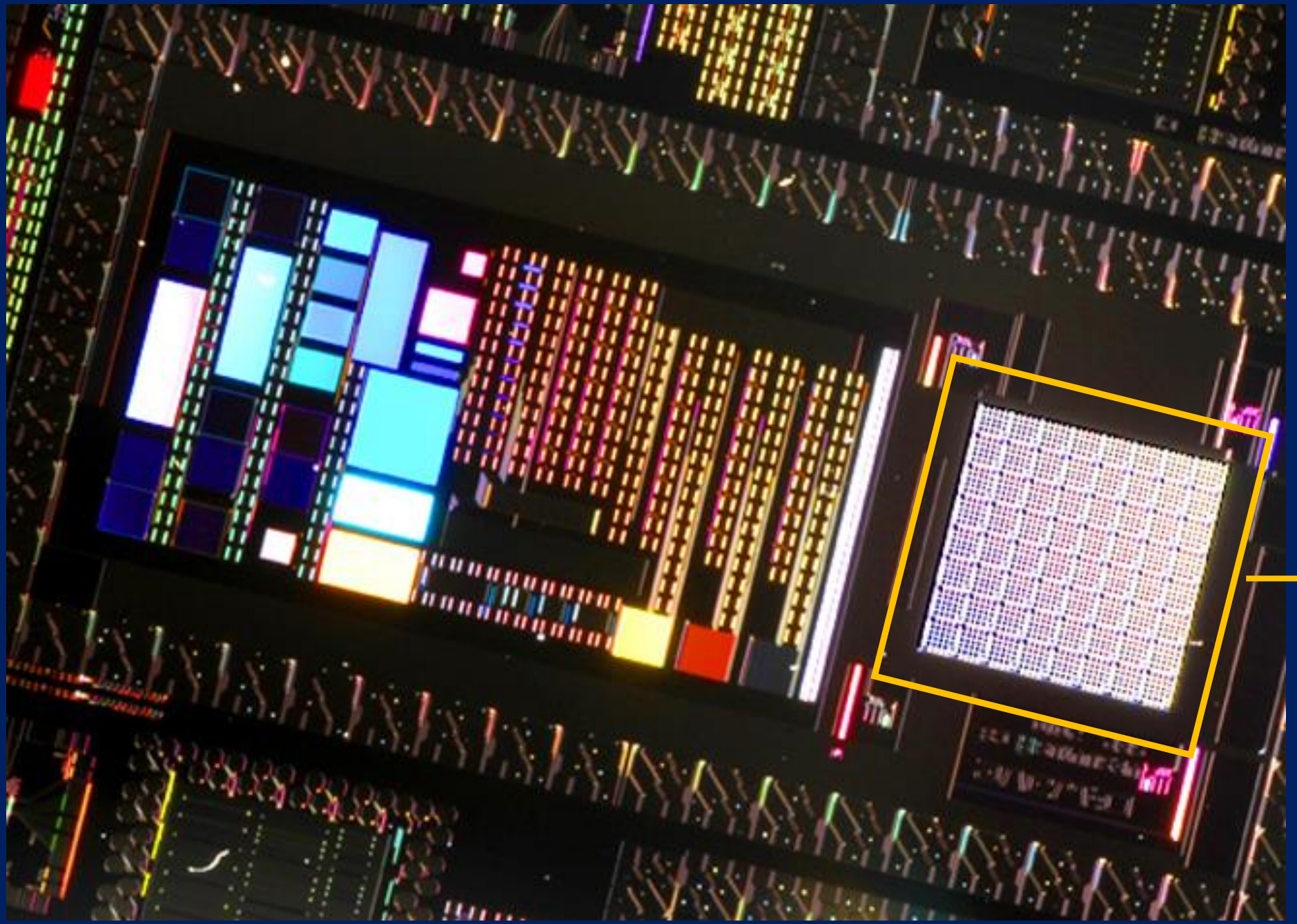

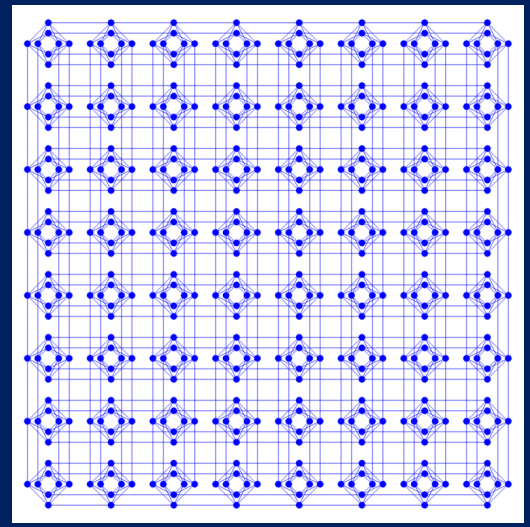

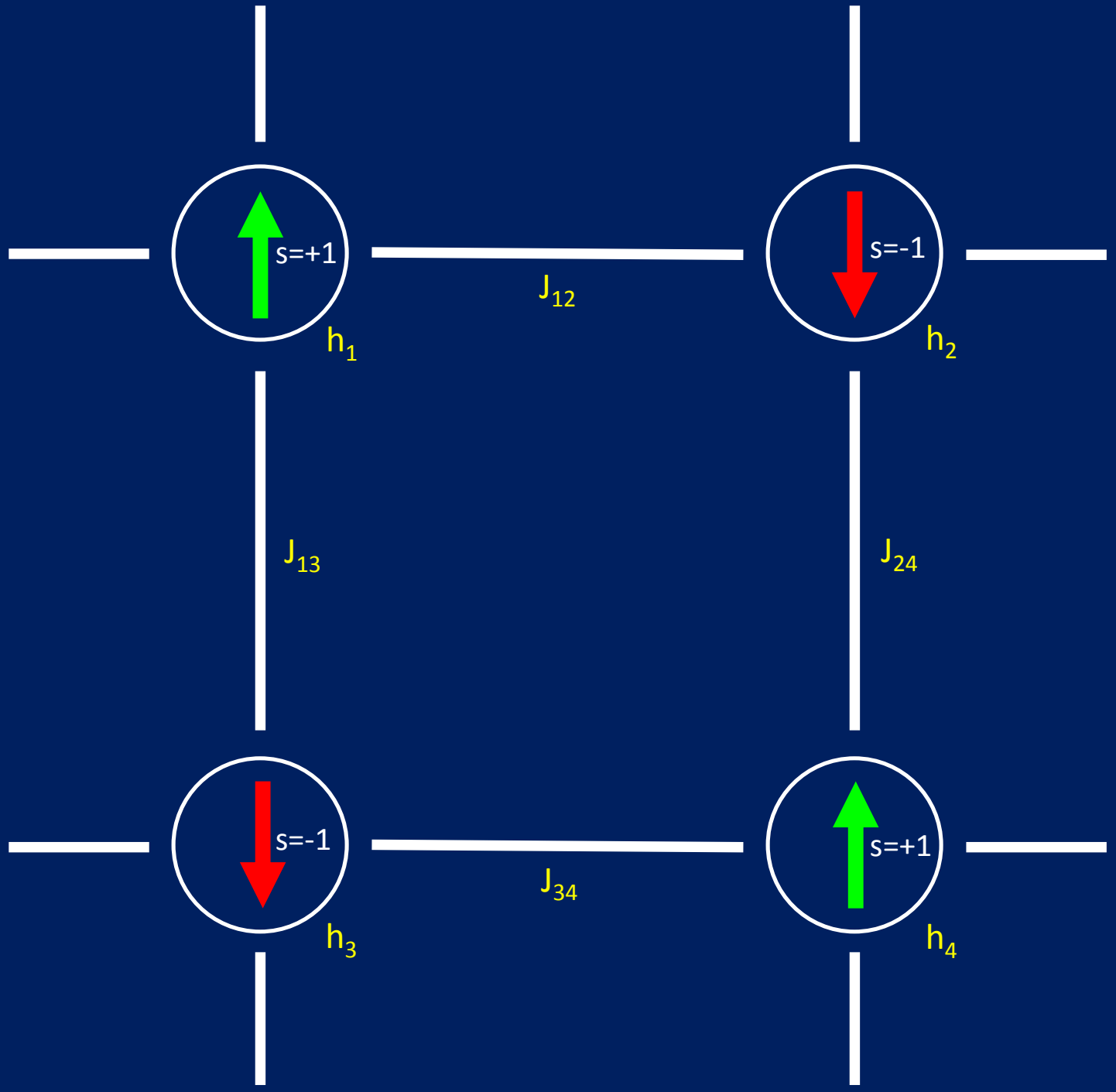

$$H(s) = \sum_i h_i s_i + \sum_{i<j} J_{ij} s_i s_j$$

Find optimum *s* to minimize *H(s)*

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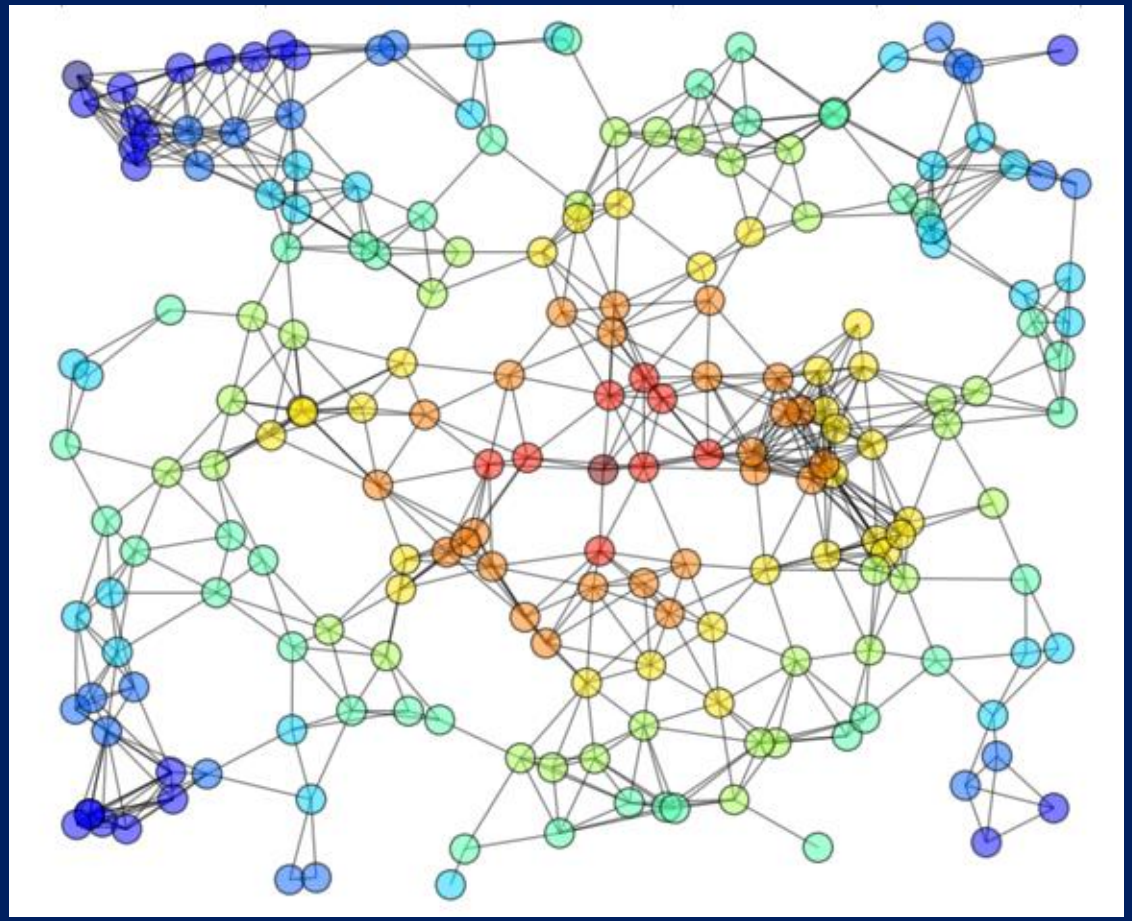
Graph embedding



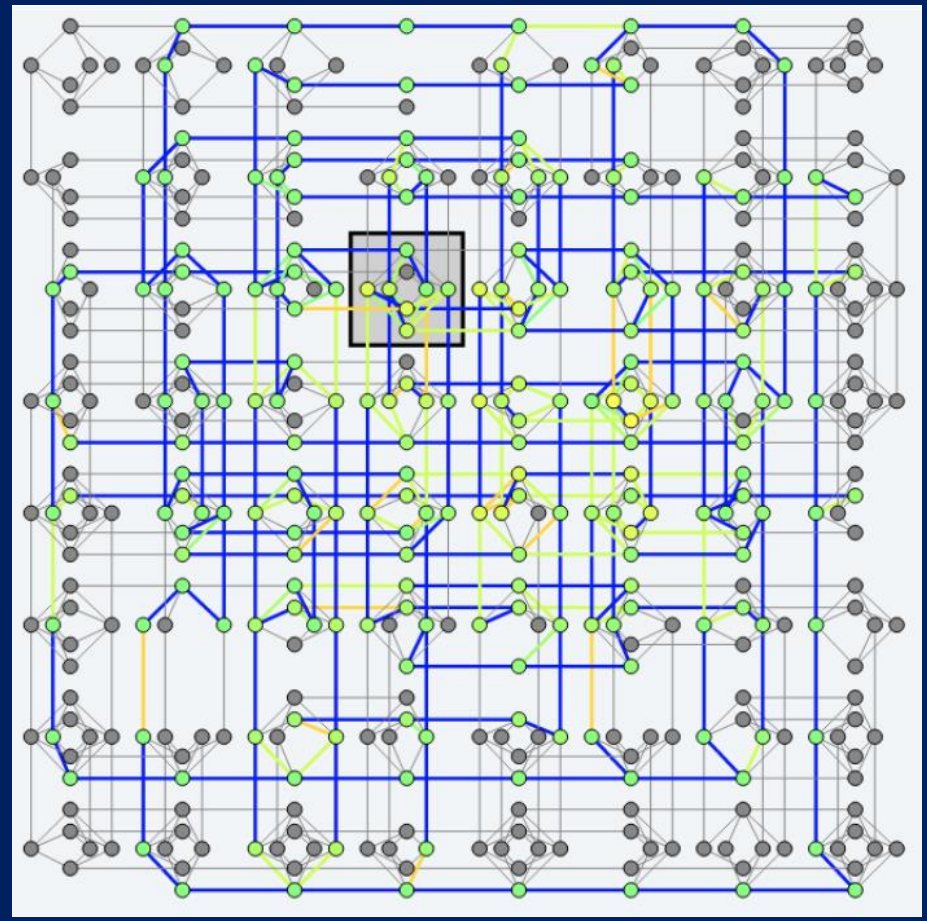
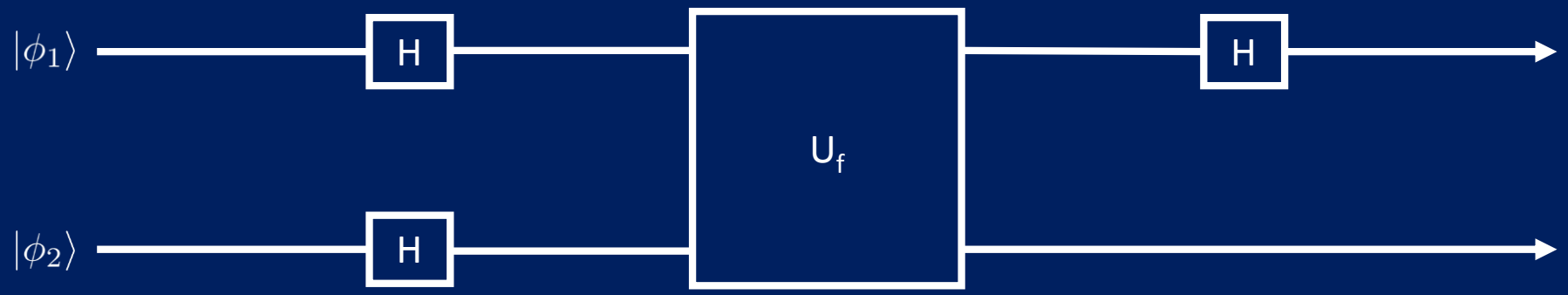
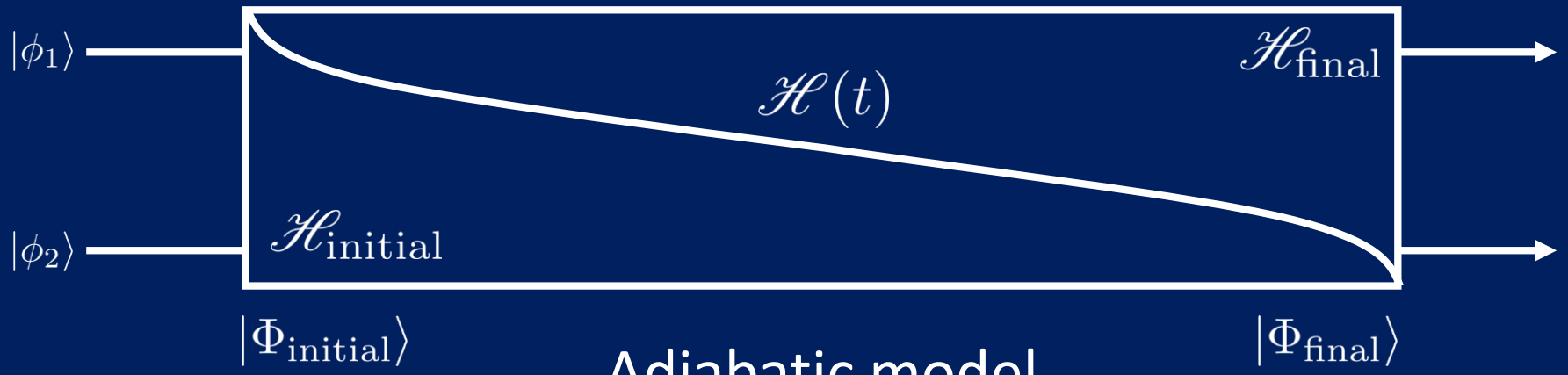


Image credit: Dridi and Alghassi, 2015

$$\mathscr{H}(t) = s(t)\mathscr{H}_{\text{initial}} + (1 - s(t))\mathscr{H}_{\text{final}} \qquad s: 1 \to 0$$



Gate model



Adiabatic model

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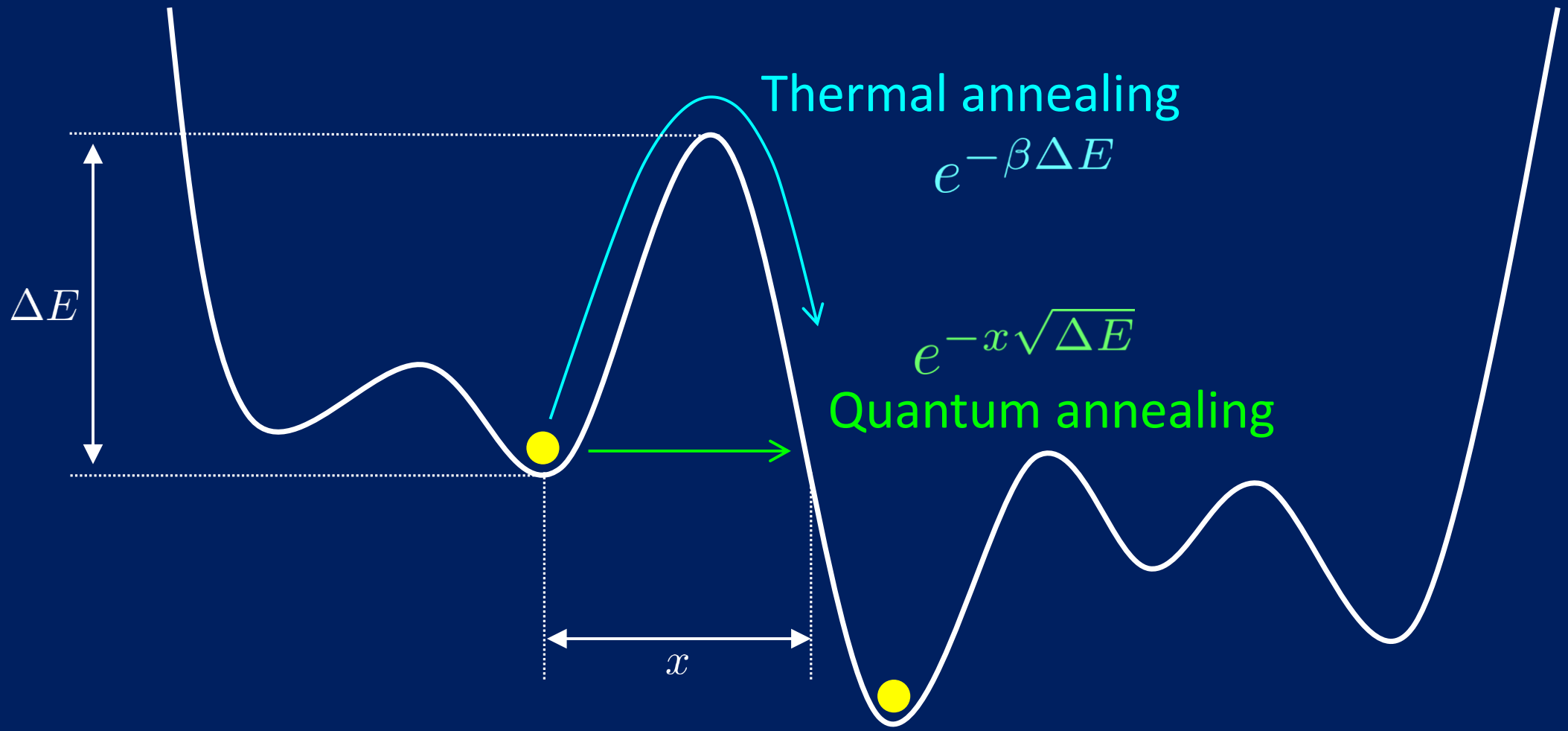

Optimized solution = global minimum energy

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Optimization equals …

Machine Learning

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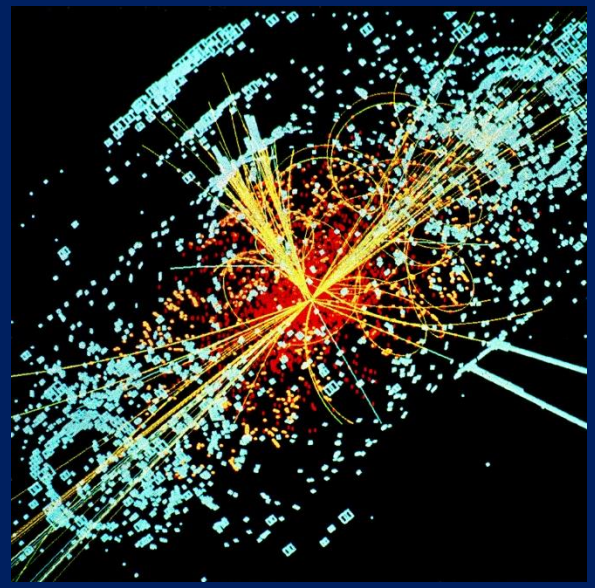
Large Hadron Collider



Searching for Exotic Particles in High-Energy Physics with Deep Learning

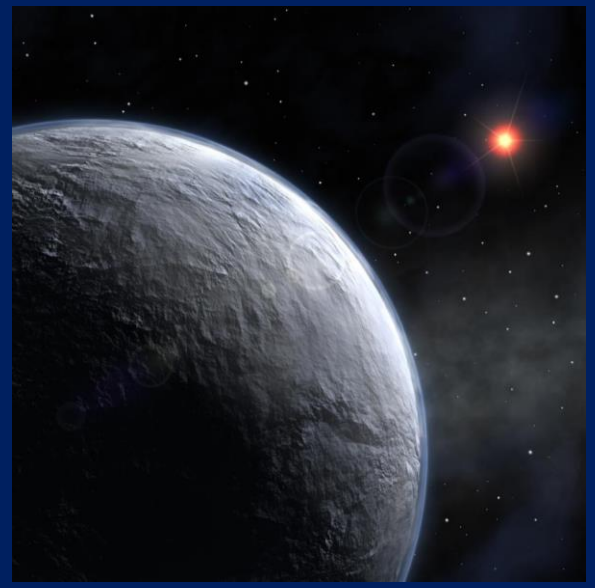
Baldi et al., 2014

Boosted Decision Trees

Shallow Neural Networks

Deep Neural Networks

NASA Quantum Artificial Intelligence Lab (QuAIL)



Bayesian Network Structure Learning Using Quantum Annealing

O'Gorman et al., 2014

NASA Kepler mission's search for habitable, Earth-sized planets

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Other Applications

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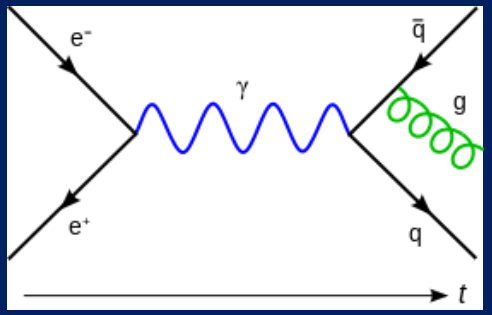
Quantum field theory

- Relativistic scattering amplitudes in four-dimensional spacetime
- Jordan et at. (2012)
- Exponential speedups

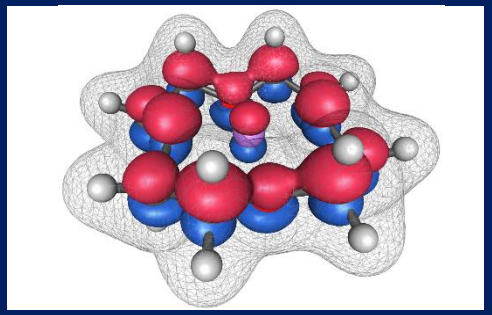


Searching large databases

- Grover's algorithm
- Grover (1996)
- Quadratic speedups

Quantum simulation

- Quantum chemistry, materials science, large physical systems
- Feynman (1982); Lanyon et al. (2009)
- Exponential speedups



Encryption

- Breaks RSA, DSA, ElGamal, and elliptic curve signature protocols
- Unbreakable encryption – BB84, E91, Lo-Chau, KMB09 protocols
- Shor (1994), Bennett and Brassard (1984), Ekert (1991)
- Quantum key networks exist in Boston, LANL, Vienna, Geneva, and Tokyo

What happens next?

Thanks to Markus Diefenthaler

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