QCD results from NuTeV

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Outline:

- NuTeV experiment
- Methods of extraction of the Structure Functions
- NLO Theory Models for the QCD fits
- QCD results
- Remarks on the Target Mass Corrections
- Conclusions
**NuTeV Experiment:**

- A precision $\nu - Fe$ DIS experiment:
  - Iron Calorimeter + Muon Spectrometer
- Data taking: 1996-97 FNAL fixed target
  - $< E_\nu > \sim 120$ GeV, $< Q^2 > \sim 25$ GeV$^2$
- Sign Selected Beams:
  - 99.9% pure $\nu_\mu$, and 99.7% pure $\bar{\nu}_\mu$
- Calibration beam throughout run:
  - $\mu$, $e^-$, hadrons (4.5 - 190 GeV)
    - Muons: $\frac{\delta E_\mu}{E_\mu} = 0.70\%$
    - Hadrons: $\frac{\delta E_{HAD}}{E_{HAD}} = 0.43\%$
- CC events: $8.6 \times 10^5 \nu$ and $2.3 \times 10^5 \bar{\nu}$
CC Deep Inelastic Neutrino Scattering:

- Lorentz-invariant quantities in terms of measured $E_\mu$, $\theta_\mu$, $E_{had}$:

\[
\begin{align*}
Q^2 &= 4(E_\mu + E_{had})E_\mu \sin^2 \frac{\theta_\mu}{2} \\
x &= \frac{Q^2}{2ME_{had}} \\
y &= \frac{E_{had}}{E_\mu + E_{had}} \\
\nu &= E_{had}
\end{align*}
\]

Neutrino Differential Cross-Section:

\[
\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx dy} = \frac{G_F^2 ME_\nu}{\pi(1 + \frac{Q^2}{M_W^2})^2} \left[ \left(1 - y - \frac{Mxy}{2E_\nu}\right) F_2^{\nu(\bar{\nu})} + \frac{y^2}{2} 2xF_1^{\nu(\bar{\nu})} \pm y(1 - \frac{y}{2})xF_3^{\nu(\bar{\nu})} \right]
\]

Neutrino Structure Functions in terms of quark compositions of target:

- $2xF_1^{\nu(\bar{\nu})}(x, Q^2) = \Sigma \left[ xq^{\nu(\bar{\nu})} + \bar{x}q^{\nu(\bar{\nu})} \right]$  \\
- $F_2^{\nu(\bar{\nu})}(x, Q^2) = \Sigma \left[ xq^{\nu(\bar{\nu})} + x\bar{q}^{\nu(\bar{\nu})} + 2xk^{\nu(\bar{\nu})} \right]$  \\
- $xF_3^{\nu(\bar{\nu})}(x, Q^2) = \Sigma \left[ xq^{\nu(\bar{\nu})} - x\bar{q}^{\nu(\bar{\nu})} \right]$  \\
- powerful tool for testing pQCD and measuring $\Lambda_{QCD}$
Extracting Differential Cross Section:

Differential Cross Section in terms of flux and number of events:
\[
\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx dy} = \frac{1}{\Phi(E_\nu)} \frac{d^2N^{\nu(\bar{\nu})}}{dx dy}
\]

Data:
- Flux Sample: data set of \( E_{had} < 20 \) GeV
- Cross Section Sample:
  - \( E_\mu > 15 \) GeV, \( E_{had} > 10 \) GeV, \( E_\nu \in (30, 360) \) GeV, \( Q^2 > 1 \) GeV\(^2\)

Monte Carlo:
- for acceptance and smearing corrections only

[ref: M.Tzanov et al., hep-ex/0509010]
Extracting Differential Cross Section:

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[\text{M.Tzanov et al., hep-ex/0509010}]
Extracting Structure Functions:

GOAL: determine $\Lambda_{QCD}$ from the NLO QCD fits to SF's

For the $xF_3$ only NLO QCD fit: 1p fits to the Diff. Cross sections:

$$\left[ \frac{d^2\sigma^\nu}{dxdy} - \frac{d^2\sigma^{\nu'}}{dxdy} \right] \frac{\pi}{2MG^2 E_\nu} \approx \left( y - \frac{y^2}{2} \right) xF_3^{avg}$$

$$\Delta F_2 = F_2^\nu - F_2^{\nu'} \approx 0; \hspace{1cm} \text{no model input needed}$$

For the $xF_3$ and $F_2$ NLO QCD fit: 2p fits to $y$-dependence of cross sections:

- use cross section error matrix:
- obtain correlations between $F_2$ and $xF_3$

$$\frac{d^2\sigma^\nu}{dxdy} = \frac{2MG^2 E_\nu}{\pi} \left[ \left( 1 - y - \frac{M xy}{2E} + \frac{1 + \frac{4M^2 x^2}{Q^2} y^2}{1 + RL} \right) \left( F_2^{avg} + \frac{\Delta F_2}{2} \right) + y \left( 1 - \frac{y}{2} \right) \left( xF_3^{avg} + \frac{\Delta xF_3}{2} \right) \right]$$

$$\frac{d^2\sigma^{\nu'}}{dxdy} = \frac{2MG^2 E_\nu}{\pi} \left[ \left( 1 - y - \frac{M xy}{2E} + \frac{1 + \frac{4M^2 x^2}{Q^2} y^2}{1 + RL} \right) \left( F_2^{avg} - \frac{\Delta F_2}{2} \right) + y \left( 1 - \frac{y}{2} \right) \left( xF_3^{avg} - \frac{\Delta xF_3}{2} \right) \right]$$

$$F_2^{avg}(x, Q^2) = \frac{1}{2} \left( F_2^\nu(x, Q^2) + F_2^{\nu'}(x, Q^2) \right) \hspace{1cm} xF_3^{avg}(x, Q^2) = \frac{1}{2} \left( xF_3^\nu(x, Q^2) + xF_3^{\nu'}(x, Q^2) \right)$$

$$\Delta xF_3 = xF_3^\nu - xF_3^{\nu'}; \hspace{1cm} \Delta F_2 = F_2^\nu - F_2^{\nu'}; \hspace{1cm} \text{Input model for } RL[\text{fit to world data}], \Delta xF_3[\text{NLO TR-VFNS}]$$
NLO Theory models:

- massless $\overline{MS}$: J.F. Owens’ program

- massive ACOT scheme: F. Olness, S. Kretzer base program
  [ref: F. Olness, private communication; Phys. Rev. D50 (1994)]

  - belongs to the VFNS factorization schemes
  - uses CWZ renormalization method: a hybrid of $\overline{MS}$ & zero momentum renorm.

- smooth transition from the $M_H >> \mu_F$ to $M_H << \mu_F$ regions

- massless evolution: [ref: J. F. Owens’ program]
  - no gain of information from a mass dependent evolution [ref: F.Olness, Phys. Rev. D V57 (1998)]
  - starts at $Q_0^2 = 5 \text{ GeV}^2$

- $m_b = 4.3 \pm 0.2 \text{ GeV}, m_c = 1.4 \pm 0.2 \text{ GeV}
Preliminary NLO QCD fits:

\( x F_3 \) from non-singlet fit results:

- Perform NLO QCD fits to:
  1. \( x F_3(x, Q^2) \) only
  2. \( F_2(x, Q^2) \) and \( x F_3(x, Q^2) \)

- PDF's evolved using DGLAP equations
- \( \Lambda_{QCD}^{n_f=4} \) free parameter in the fit

Requirements:
- \( Q^2 > 5 \text{ GeV}^2, x < 0.8, W^2 > 10 \text{ GeV}^2 \)
- apply Target Mass Correction [Next]
Perform NLO QCD fits to:
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Remarks on the Target Mass corrections:

\[ F_{1TM}^{TM}(x, Q^2) = \frac{x F_{1}^{(0)}(\xi, Q^2)}{k} + \frac{M_p^2 x^2}{Q^2 k^2} \int_{\xi}^{1} \frac{F_{2}^{(0)}(u, Q^2)}{u^2} du + \frac{2M_p^4 x^3}{Q^4 k^3} \int_{\xi}^{1} du \int_{u}^{1} \frac{F_{2}^{(0)}(v, Q^2)}{v^2} dv \]

\[ F_{2TM}^{TM}(x, Q^2) = \frac{x^2 F_{2}^{(0)}(\xi, Q^2)}{k^3} + \frac{6M_p^2 x^3}{Q^2 k^4} \int_{\xi}^{1} \frac{F_{2}^{(0)}(u, Q^2)}{u^2} du + \frac{12M_p^4 x^4}{Q^4 k^5} \int_{\xi}^{1} du \int_{u}^{1} \frac{F_{2}^{(0)}(v, Q^2)}{v^2} dv \]

\[ F_{3TM}^{TM}(x, Q^2) = \frac{x F_{3}^{(0)}(\xi, Q^2)}{k^2} + \frac{2M_p^2 x^2}{Q^2 k^3} \int_{\xi}^{1} \frac{F_{3}^{(0)}(u, Q^2)}{u} du \]

\[ k = \sqrt{1 + \frac{4x^2 M_p^2}{Q^2}}, \quad \xi = \frac{2x}{1 + k} \]

- General relations, holding to all orders in \( \alpha_S \):
  - no use of the Callan - Gross relation
  - \( F_{iTM}^{TM} \) are functions of \( (x, Q^2) \)
- Quark masses effects included in \( F_{i}^{(0)} \).
- \( F_{i}^{(0)} \) are experimental SF's in the limit \( M_P \to 0 \)
Remarks on the Target Mass corrections:

\[ F_{1}^{TM}(x, Q^2) = \frac{x}{\xi} \frac{F_{1}^{(0)}(\xi, Q^2)}{k} + \frac{M_p^2 x^2}{Q^2 k^2} \int_{\xi}^{1} \frac{F_{2}^{(0)}(u, Q^2)}{u^2} du + \frac{2 M_p^4 x^3}{Q^4 k^3} \int_{\xi}^{1} du \int_{u}^{1} \frac{F_{2}^{(0)}(v, Q^2)}{v^2} dv \]

\[ F_{2}^{TM}(x, Q^2) = \frac{x^2}{\xi^2} \frac{F_{2}^{(0)}(\xi, Q^2)}{k^3} + \frac{6 M_p^2 x^3}{Q^2 k^4} \int_{\xi}^{1} \frac{F_{2}^{(0)}(u, Q^2)}{u^2} du + \frac{12 M_p^4 x^4}{Q^4 k^5} \int_{\xi}^{1} du \int_{u}^{1} \frac{F_{2}^{(0)}(v, Q^2)}{v^2} dv \]

\[ F_{3}^{TM}(x, Q^2) = \frac{x}{\xi} \frac{F_{3}^{(0)}(\xi, Q^2)}{k^2} + \frac{2 M_p^2 x^2}{Q^2 k^3} \int_{\xi}^{1} \frac{F_{3}^{(0)}(u, Q^2)}{u} du \]

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**NLO QCD results:**

<table>
<thead>
<tr>
<th>SFs</th>
<th>Parameter</th>
<th>$xF_3$ only</th>
<th>$F_2 + xF_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\overline{MS}$</td>
<td>ACOT</td>
</tr>
<tr>
<td>NS</td>
<td>$\Lambda^{(n_f=4)}$(MeV)</td>
<td>476 ± 60</td>
<td>488 ± 59</td>
</tr>
<tr>
<td></td>
<td>$A_{1u_v}$</td>
<td>0.73 ± 0.01</td>
<td>0.73 ± 0.01</td>
</tr>
<tr>
<td></td>
<td>$A_{2u_v}$</td>
<td>3.47 ± 0.06</td>
<td>3.47 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>$A_{0u_v} + A_{0d_v}$</td>
<td>4.74+2.37</td>
<td>4.73+2.36</td>
</tr>
<tr>
<td>S</td>
<td>$A_{0ud}$</td>
<td></td>
<td>0.68 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>$A_{2ud}$</td>
<td></td>
<td>6.74 ± 0.20</td>
</tr>
<tr>
<td>G</td>
<td>$A_{0g}$</td>
<td></td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td>$A_{2g}$</td>
<td></td>
<td>4.83 ± 1.38</td>
</tr>
<tr>
<td></td>
<td>$\chi^2/dof$</td>
<td>78/59</td>
<td>77/59</td>
</tr>
<tr>
<td></td>
<td>$\alpha_S(M_{Z^0})$</td>
<td>0.1257 ± 0.0029</td>
<td>0.1260 ± 0.0028</td>
</tr>
</tbody>
</table>

- Errors include statistical and all experimental system. uncertainties
- Full Covariance Error Matrix taking into account all the correlations has been used
**Contributing Systematics**

**Experimental systematics:**
- energy scales: $E_\mu, E_{had}$
- energy smearing models
- flux uncertainties: $\frac{B}{A}, m_c$
- input models: $\Delta x F_3, R_W$

<table>
<thead>
<tr>
<th>$xF_3$ (MeV)</th>
<th>scale</th>
<th>smear model</th>
<th>flux uncert.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>stat.</td>
<td>$E_\mu$</td>
<td>$E_{had}$</td>
</tr>
<tr>
<td>$\Lambda_{ACOT}^{nf=4} = 454$</td>
<td>$\pm 57$</td>
<td>$+76$</td>
<td>$-10$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$F_2 &amp; xF_3$ (MeV)</th>
<th>scale</th>
<th>smear model</th>
<th>flux uncert.</th>
<th>input model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>stat.</td>
<td>$E_\mu$</td>
<td>$E_{had}$</td>
<td>$E_\mu$</td>
</tr>
<tr>
<td>$\Lambda_{ACOT}^{nf=4} = 434$</td>
<td>$\pm 22$</td>
<td>$+21$</td>
<td>$-10$</td>
<td>$-22$</td>
</tr>
</tbody>
</table>

**Theoretical Uncertainty:**
- scale dependence: $\mu_F^2 = C_i Q^2$, $C_i = 1/2, 1, 2...$

<table>
<thead>
<tr>
<th>$C_i$</th>
<th>$xF_3$ only</th>
<th>$xF_3 + F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$+74$ MeV</td>
<td>$+61$ MeV</td>
</tr>
<tr>
<td>0.5</td>
<td>$-113$ MeV</td>
<td>$-87$ MeV</td>
</tr>
</tbody>
</table>
NuTeV (ACOT scheme) Preliminary Results:

$xF_3$ Fit Result:

$\alpha_S(M_Z) = 0.1260 \pm 0.0028 (exp) + 0.0034 (th)$

$F_2 + xF_3$ Fit Result:

$\alpha_S(M_Z) = 0.1247 \pm 0.0020 (exp) + 0.0030 (th)$

WORLD AVERAGE

$\alpha_S(M_Z) = 0.1176 \pm 0.0020$ [PDG 2005]

(excluding Lattice QCD:

$\alpha_S(M_Z) = 0.1185 \pm 0.0020$)

DIS points from the scaling violations are summarized here by the MRST NLO(and NNLO) global fits analysis [PDG 2005]

MRST04 NLO: $\alpha_S(M_Z) = 0.1205 \pm 0.004$
**Conclusions:**

- First $\Lambda_{QCD}^{n_f=4}$ from $\nu - DIS$ including full NLO treatment of charm production
- NLO QCD fits to $xF_3(x, Q^2)$ and $F_2(x, Q^2)$
- Use of the full covariance differential cross section error matrix [NuTeVpack - M.Tzanov et al., hep-ex/0509010];
- Results above other DIS measurements;
- Error dominated by the theoretical uncertainty
  - largest experimental uncertainty due to the $E_\mu$ scale [$\frac{\delta E_\mu}{E_\mu} = 0.7\%$]
- Target Mass Corrections:
  - incorporated into the NLO $\sin^2\theta_W$ results
  - plan a brief report in collaboration with H. Reno, S. Kretzer, F. Olness, I. Schienbein, C. Keppel, W. Melnitchouk, ...
- Many thanks to J.F. Owens and F. Olness
Parametrization of the PDF's:

\[ xq^{NS} = xu_v + xd_v = (A_{0u_v} + A_{0d_v})x^{A_{1u_v}}(1 - x)^{A_{2u_v}} \]

\[ xq^S = xu_v + xd_v + 2A_{0ud}(1 - x)^{A_{2ud}} \]

\[ xG = A_{0g}(1 - x)^{A_{2g}} \]

\( A_{0u_v}, A_{0d_v}, \) and \( A_{0g} \) are constrained by the QCD sum rules.
Backup 2: NuTeV vs. CCFR

CCFR quotes: \( \alpha_S(M_Z) = 0.119 \pm 0.002 (\text{stat} + \text{syst}) \pm 0.001 (HT) \pm 0.004 (th) \)

prelim. NuTeV: \( \alpha_S(M_Z) = 0.125 \pm 0.002 (\text{stat} + \text{syst}) \pm 0.003 (th) \)

Notes on the comparisons:

CCFR analysis used a different method of extracting the SF’s and had used a LO correction for charm production

no QCD constraints used in CCFR analysis

consistent way to compare: use CCFR \( F_2 \) (PMI) \([\text{Phys.Rev.Lett.86(2001)}]\) vs. NuTeV \( F_2 \):

CCFR: \( \Lambda^{n_f=4} = 330 \pm 64 \text{ MeV} \); NuTeV: \( \Lambda^{n_f=4} = 425 \pm 71 \text{ MeV} \)

(errors added in quadrature)

theoretical uncertainty for CCFR is quoted from MRST paper, NuTeV evaluates it by varying the factorization scale in the program