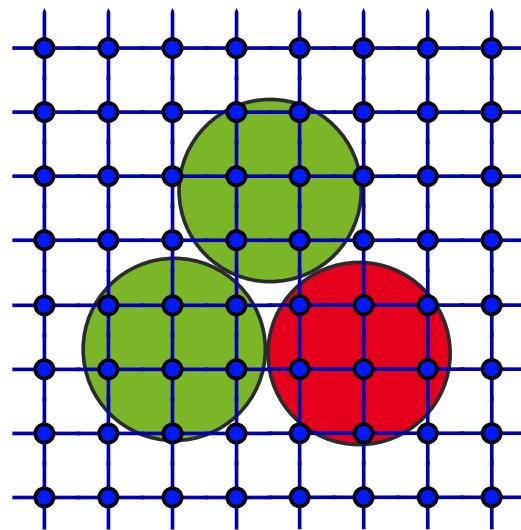


Hadron, hadron-hadron and hadron-hadron-hadron properties from lattice QCD

Silas Beane

University of New Hampshire







- Tom Luu (Livermore)
- Kostas Orginos (William and Mary/JLab)
- Assumpta Parreño (Barcelona)
- Martin Savage (Washington)
- Aaron Torok (New Hampshire)
- André Walker-Loud (Maryland)
- Silas Beane (New Hampshire)

Outline

- Funding Sources
- Current Resources
- Motivation
- Physics
 - $\pi\pi$
 - $\pi\pi\pi$
 - NN
 - ΛN
- Conclusion

Funding Sources

- Department of Energy (USQCD/INCITE): Fermilab, JLab
- National Science Foundation (LRAC: 2,000,000 SU): NCSA
- Centro Nacional de Supercomputación de España: Mare Nostrum
- Lawrence Livermore: commodity clusters

Current Resources

Hybrid of staggered sea quarks ([MILC](#)) and domain-wall valence quarks ([LHPC+NPLQCD](#))

(2+1) dynamical flavors

Lattices “chopped” from 64 to 32 with [sources](#) displaced in time and space

Coarse Config Set	Dimensions	bm_l	bm_s	bm_{dwf}	m_π	# configs	# sources
2064f21b676m007m050	$20^3 \times 64$	0.007	0.05	0.0081	294 MeV	468	16
2064f21b676m010m050	$20^3 \times 64$	0.010	0.05	0.0138	348 MeV	658	20
2064f21b679m020m050	$20^3 \times 64$	0.020	0.05	0.0313	484 MeV	486	24
2064f21b681m030m050	$20^3 \times 64$	0.030	0.05	0.0474	565 MeV	564	8

$$b_{MILC}^{coarse} = 0.1243 \pm 0.0015 \text{ fm} \quad (\text{Sommer})$$

$$L \sim 2.5 \text{ fm}$$

Motivation

Why all the unphysical parameters??

Motivation

Why all the unphysical parameters??

$$\text{COST} \sim (L)^4 (b)^{-6.5} (M_q)^{-2.5}$$

Motivation

Why all the unphysical parameters??

$$\text{COST} \sim (L)^4 (b)^{-6.5} (M_q)^{-2.5}$$

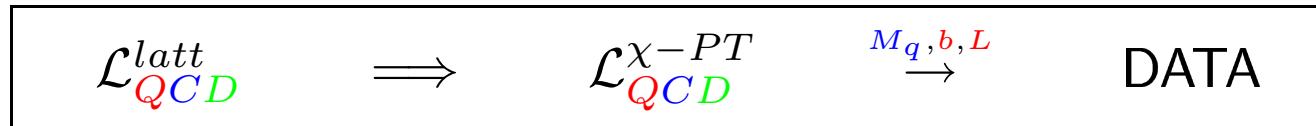
How then do we approach nature??

Motivation

Why all the unphysical parameters??

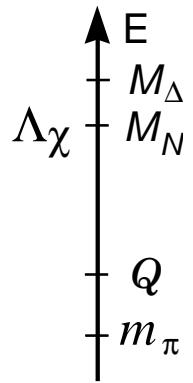
$$\text{COST} \sim (\textcolor{red}{L})^4 (\textcolor{red}{b})^{-6.5} (\textcolor{blue}{M}_q)^{-2.5}$$

How then do we approach nature??



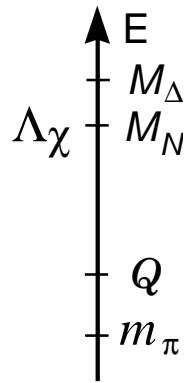
χ -**PT**

QCD:



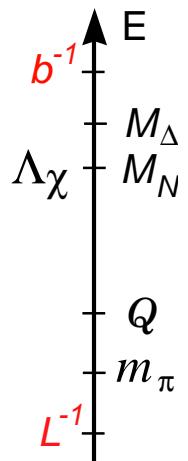
$$\frac{Q}{\Lambda_\chi}, \quad \frac{m_\pi}{\Lambda_\chi}, \quad \frac{M_\Delta - M_N}{\Lambda_\chi}, \quad \dots$$

QCD:

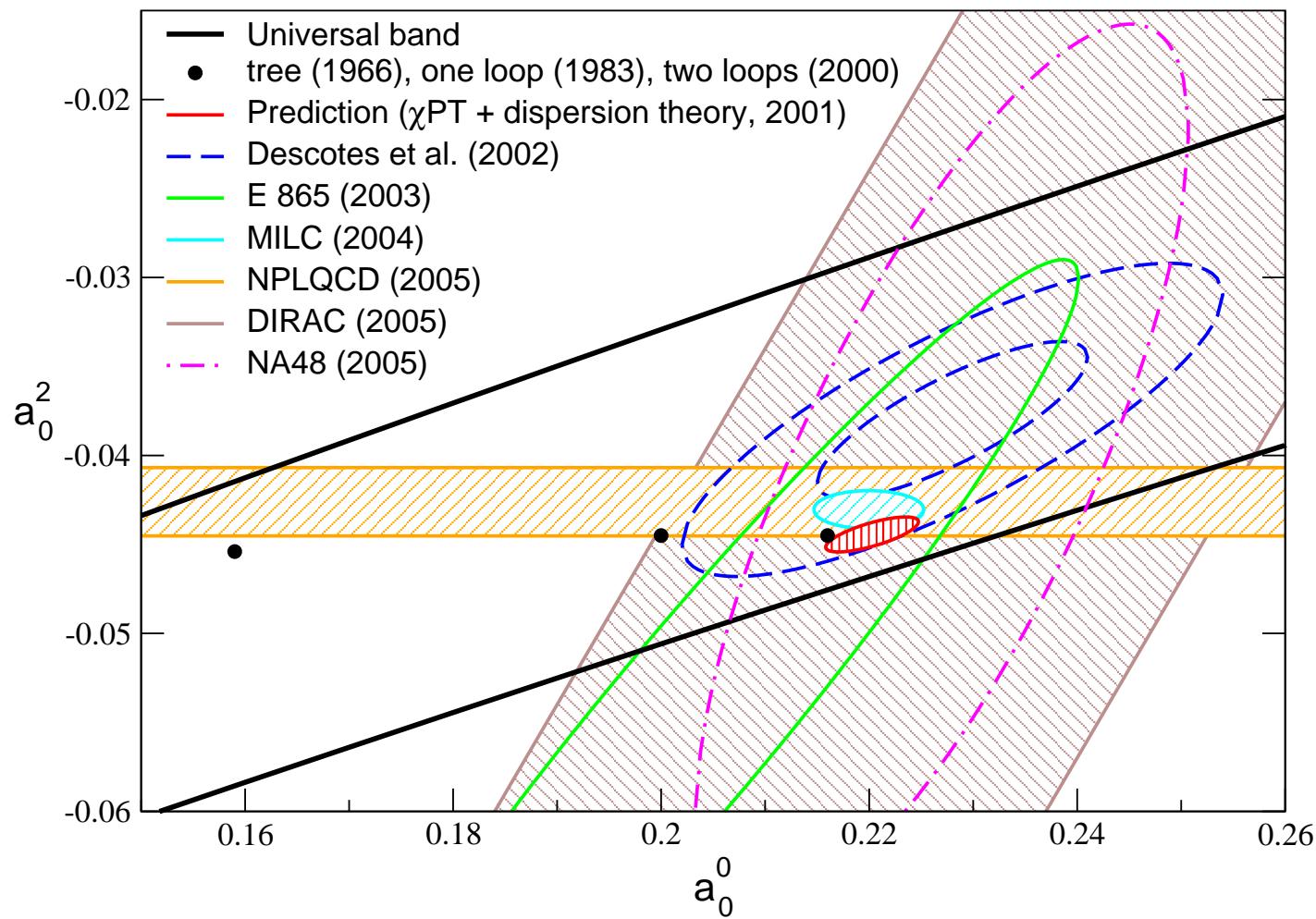


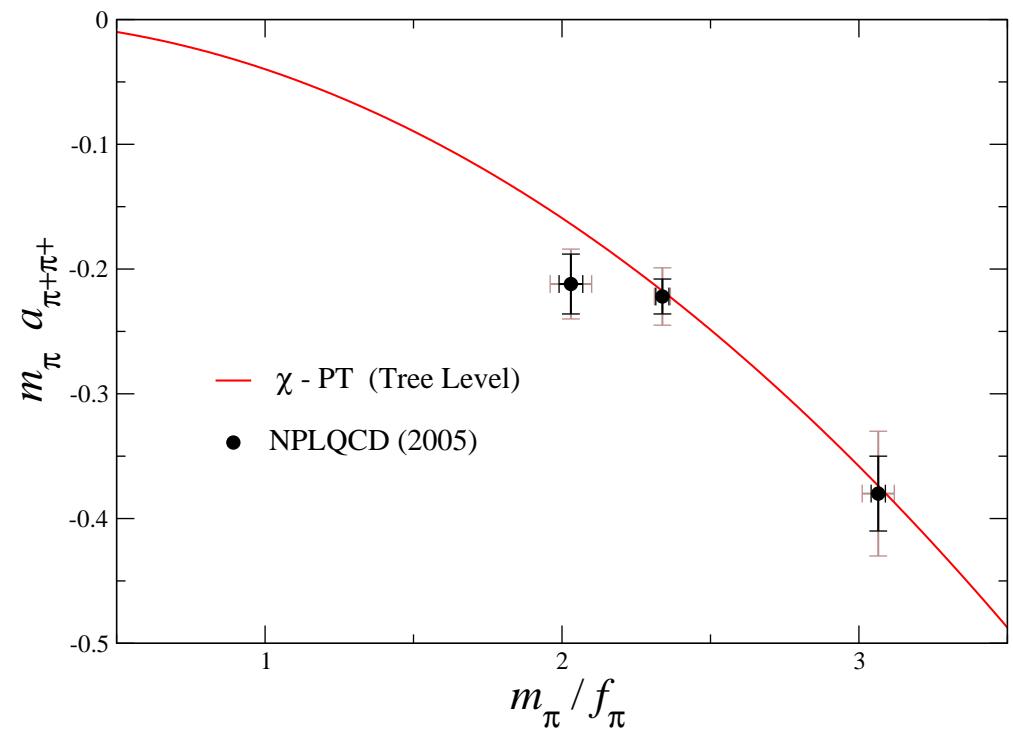
$$\frac{Q}{\Lambda_\chi}, \quad \frac{m_\pi}{\Lambda_\chi}, \quad \frac{M_\Delta - M_N}{\Lambda_\chi}, \quad \dots$$

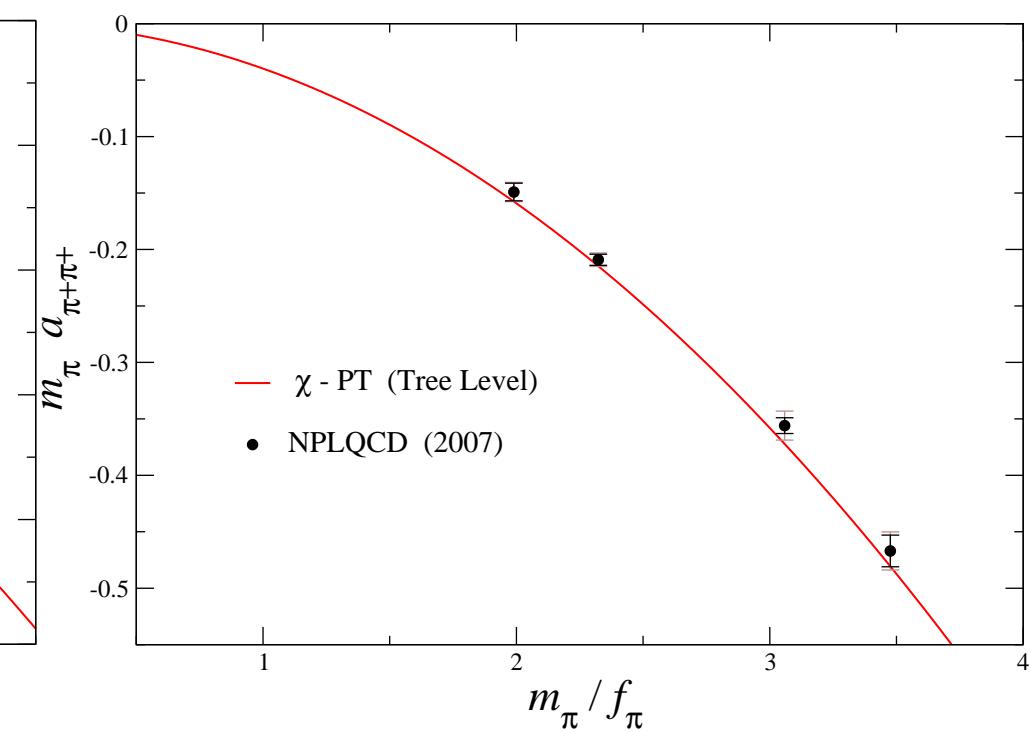
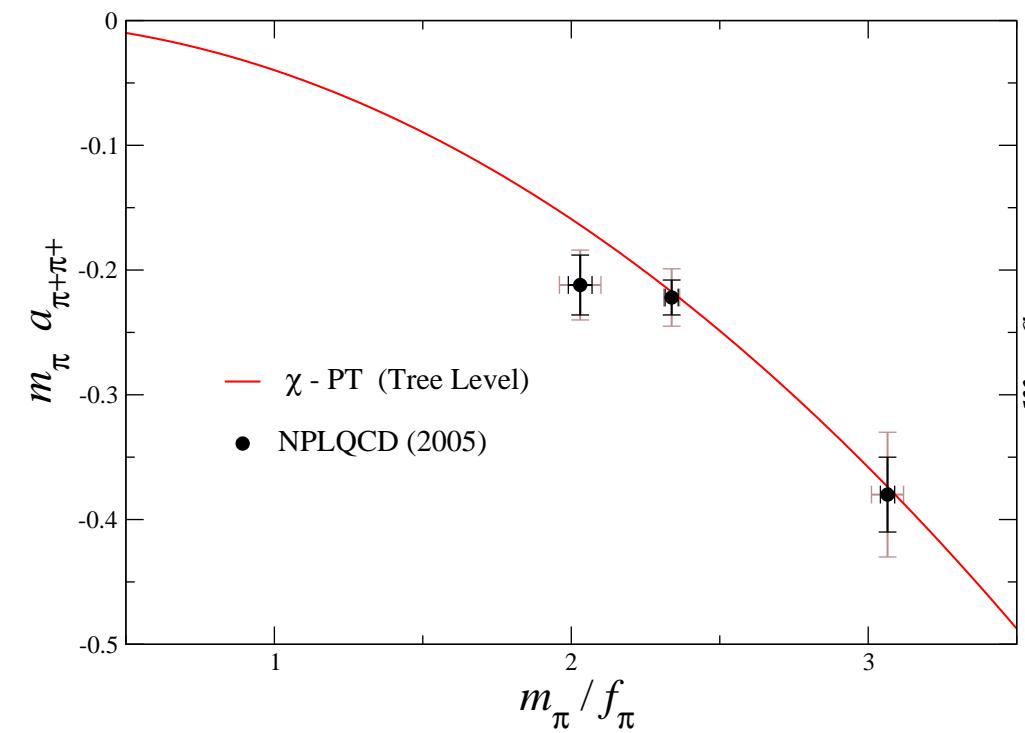
Lattice QCD :

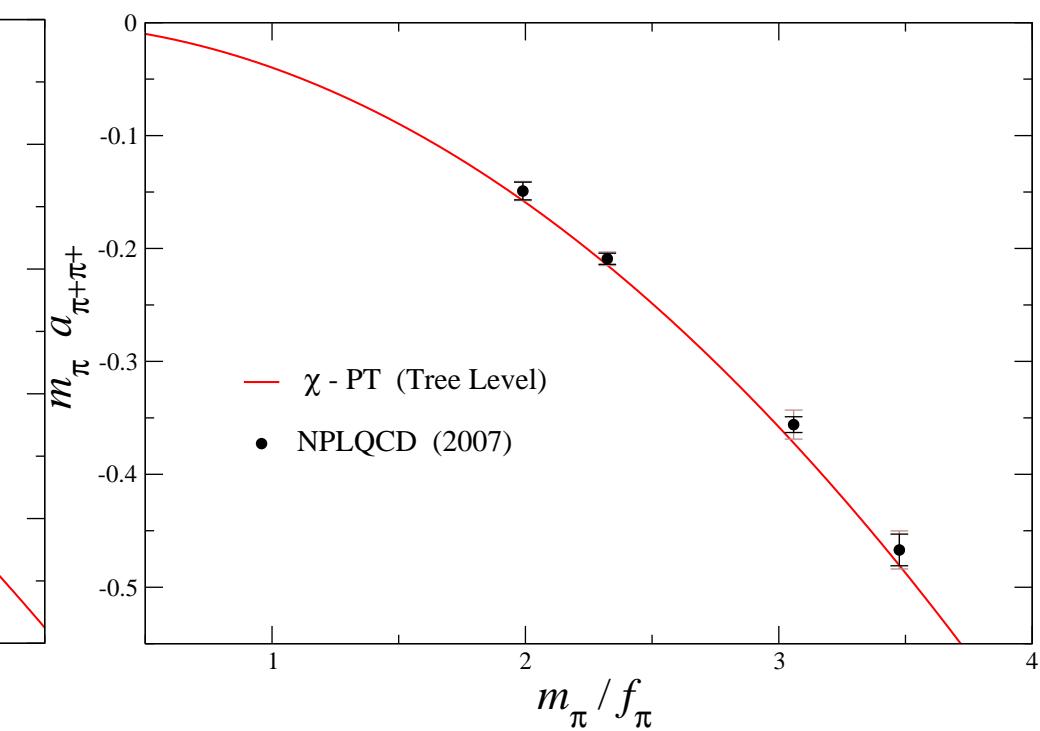
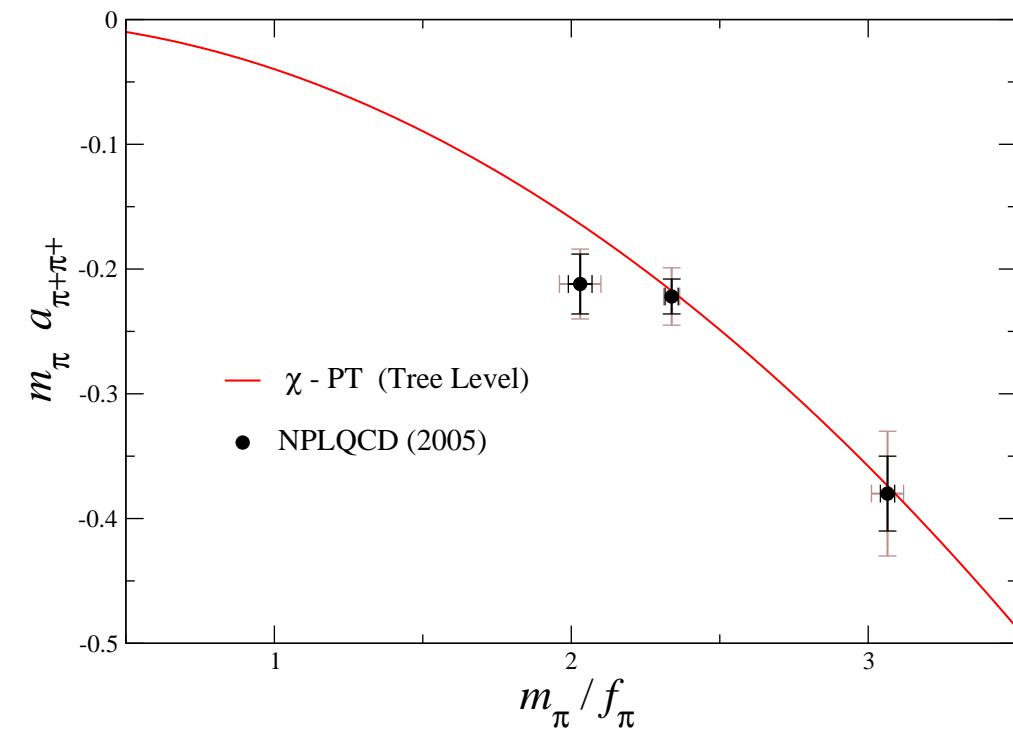


$$b m_\pi, \quad e^{-m_\pi L}, \quad m_\pi L, \quad \frac{1}{L \Lambda_\chi}, \quad \dots$$









Motivation for an update:

- Twenty-fold increase in statistics
- Mixed-action χ PT now exists

Presently can only compute $a_{\pi\pi}^{I=2} = a_{\pi^+\pi^+}$ *using lattice QCD*

$$C_{\pi^+\pi^+}(p, t) = \langle 0 | \sum_{|\mathbf{p}|=p} \sum_{\mathbf{x}, \mathbf{y}} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \mathcal{O}_{\pi^-}(t, \mathbf{x}) \mathcal{O}_{\pi^-}(t, \mathbf{y}) \mathcal{O}_{\pi^+}(0, \mathbf{0}) \mathcal{O}_{\pi^+}(0, \mathbf{0}) | 0 \rangle$$

$$\frac{C_{\pi^+\pi^+}(p, t)}{C_{\pi^+}(t)C_{\pi^+}(t)} \rightarrow \sum_{n=0}^{\infty} \mathcal{A}_n e^{-\Delta E_n t}$$

Presently can only compute $a_{\pi\pi}^{I=2} = a_{\pi^+\pi^+}$ *using lattice QCD*

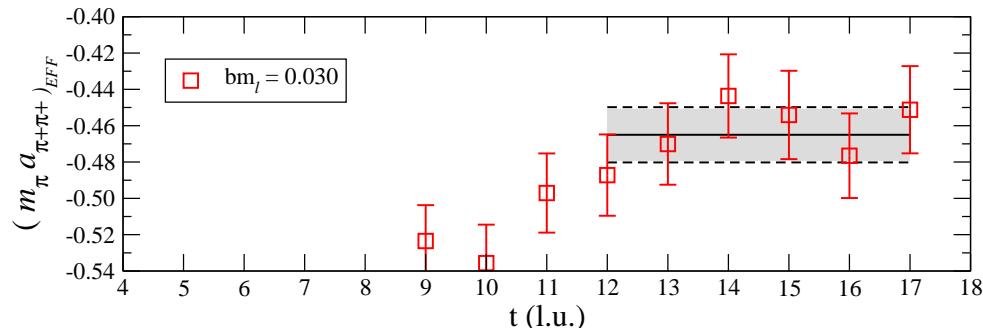
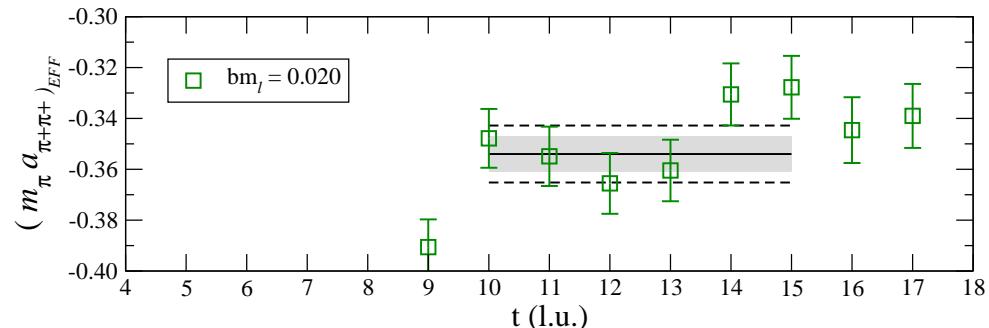
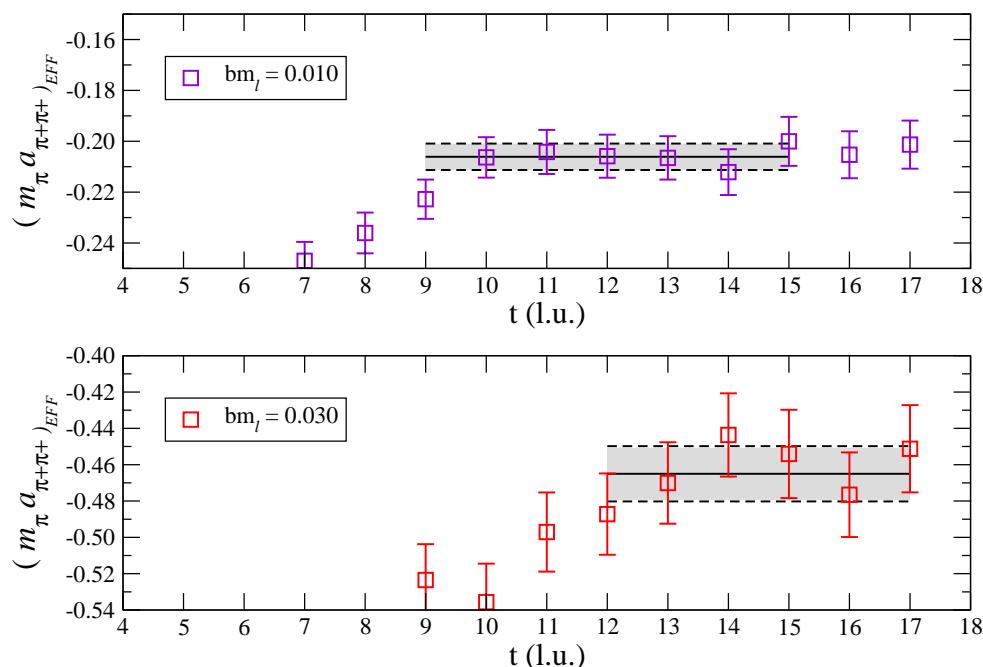
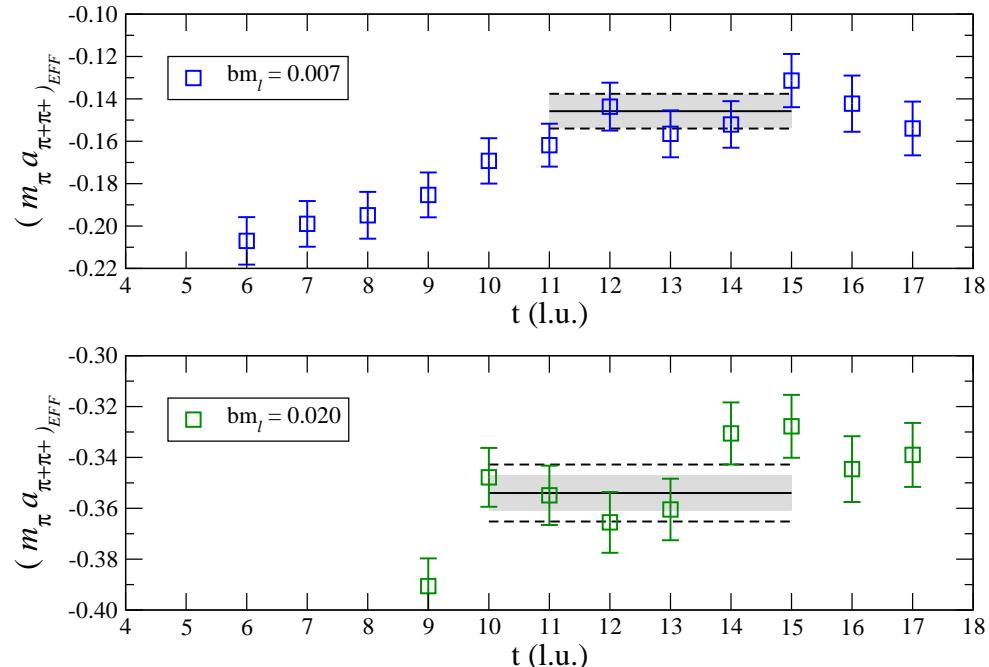
$$C_{\pi^+\pi^+}(p, t) = \langle 0 | \sum_{|\mathbf{p}|=p} \sum_{\mathbf{x}, \mathbf{y}} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \mathcal{O}_{\pi^-}(t, \mathbf{x}) \mathcal{O}_{\pi^-}(t, \mathbf{y}) \mathcal{O}_{\pi^+}(0, \mathbf{0}) \mathcal{O}_{\pi^+}(0, \mathbf{0}) | 0 \rangle$$

$$\frac{C_{\pi^+\pi^+}(p, t)}{C_{\pi^+}(t)C_{\pi^+}(t)} \rightarrow \sum_{n=0}^{\infty} \mathcal{A}_n e^{-\Delta E_n t}$$

$$\Delta E_n \equiv E_n - 2m_\pi = 2 \sqrt{\vec{p}_n^2 + m_\pi^2} - 2m_\pi$$

$$\Delta E_0 = -\frac{4\pi a_{\pi\pi}}{m_\pi L^3} \left[1 + c_1 \frac{a_{\pi\pi}}{L} + c_2 \left(\frac{a_{\pi\pi}}{L} \right)^2 \right] + \mathcal{O}\left(\frac{1}{L^6}\right)$$

$$c_1 = \frac{1}{\pi} \sum_{\mathbf{j} \neq \mathbf{0}}^{|j| < \Lambda} \frac{1}{|\mathbf{j}|^2} - 4\Lambda = -2.837297 , \quad c_2 = c_1^2 - \frac{1}{\pi^2} \sum_{\mathbf{j} \neq \mathbf{0}} \frac{1}{|\mathbf{j}|^4} = 6.375183$$



Quantity	$m_l = 0.007$	$m_l = 0.010$	$m_l = 0.020$	$m_l = 0.030$
Fit Range	8 – 12	8 – 13	7 – 13	9 – 12
m_π (l.u.)	0.18454(58)(51)	0.222294(31)(09)	0.31132(28)(21)	0.37407(49)(12)
m_π/f_π	1.990(11)(14)	2.3230(57)(30)	3.0585(49)(95)	3.4758(98)(60)
Fit Range	11 – 15	9 – 15	10 – 15	12 – 17
$\Delta E_{\pi\pi}$ (l.u.)	0.00779(47)(14)	0.00745(20)(07)	0.00678(18)(20)	0.00627(23)(10)
$m_\pi a_{\pi\pi}^{I=2} (b \neq 0)$	-0.1458(78)(25)(14)	-0.2061(49)(17)(20)	-0.3540(68)(89)(35)	-0.465(14)(06)(05)
$ \mathbf{p} /m_\pi$	0.2032(60)(18)	0.1836(25)(09)	0.1480(17)(23)	0.1298(24)(10)

CHIRAL AND CONTINUUM EXTRAPOLATION: Mixed-action χ -PT at one loop

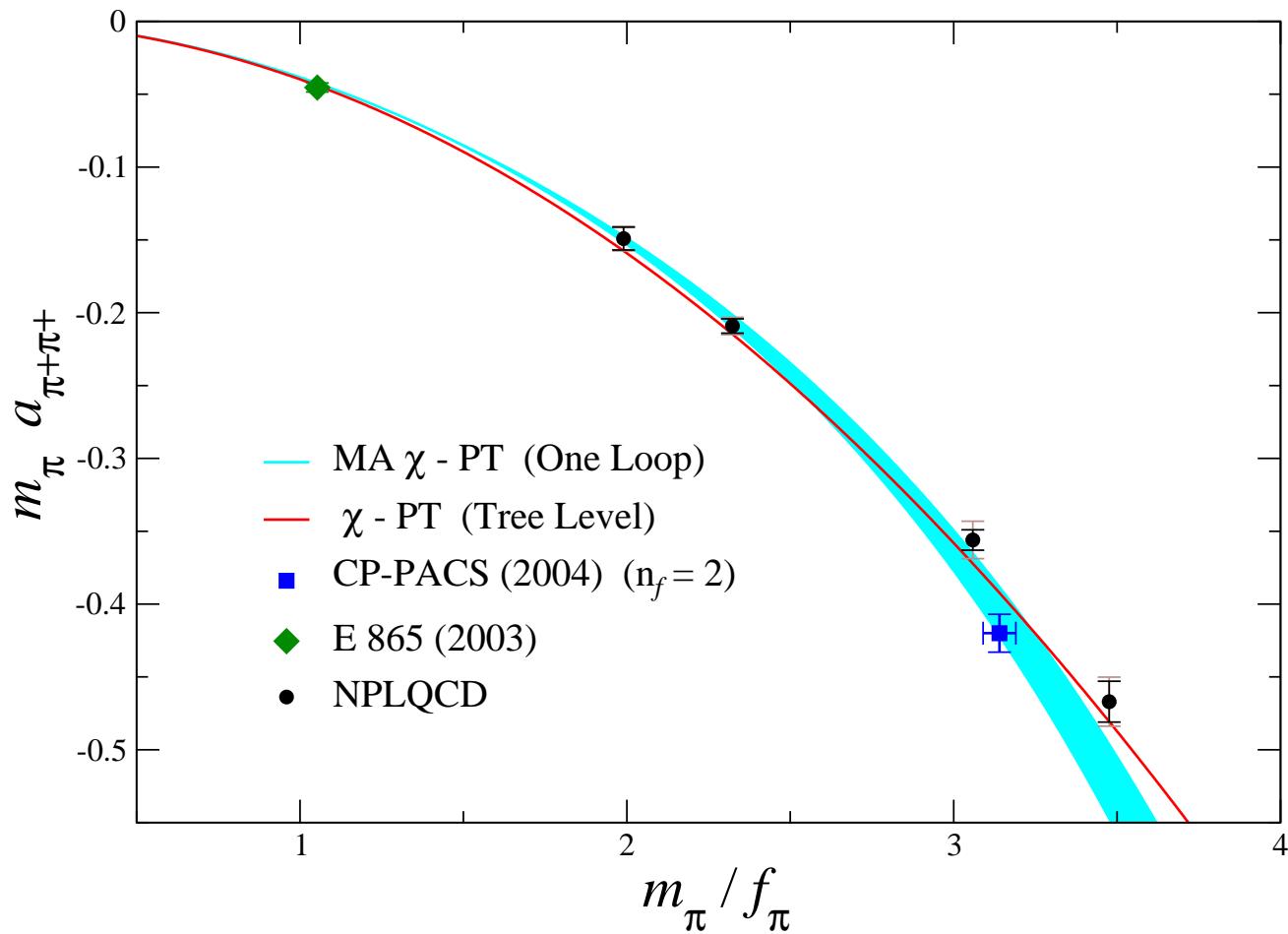
$$m_\pi a_{\pi\pi}^{I=2}(b \neq 0) = -\frac{m_\pi^2}{8\pi f_\pi^2} \left[1 + \frac{m_\pi^2}{16\pi^2 f_\pi^2} \left(3 \log \frac{m_\pi^2}{16\pi^2 f_\pi^2} - 1 - l_{\pi\pi}^{I=2} \right) \right]$$

CHIRAL AND CONTINUUM EXTRAPOLATION: Mixed-action χ -PT at one loop

$$m_\pi a_{\pi\pi}^{I=2}(b \neq 0) = -\frac{m_\pi^2}{8\pi f_\pi^2} \left[1 + \frac{m_\pi^2}{16\pi^2 f_\pi^2} \left(3 \log \frac{m_\pi^2}{16\pi^2 f_\pi^2} - 1 - l_{\pi\pi}^{I=2} \right) \right] \\ + \frac{m_\pi^2}{8\pi f_\pi^2} \left[\frac{1}{(4\pi f_\pi)^2} \left[\frac{\tilde{\Delta}_{ju}^4}{6m_\pi^2} \right] \right]$$

$$\tilde{\Delta}_{ju}^2 \equiv \tilde{m}_{jj}^2 - m_{uu}^2 = 2B_0(m_j - m_u) + b^2 \Delta_I + \dots = 0.0769(22) \quad \text{MILC(2004)}$$

- Contains all $\mathcal{O}(m_\pi^2 b^2)$ and $\mathcal{O}(b^4)$ lattice artifacts.
- m_π and f_π are the lattice-physical parameters.



FIT	$l_{\pi\pi}^{I=2}(\mu = f_\pi)$	$m_\pi \ a_{\pi\pi}^{I=2}$ (extrapolated)	χ^2/dof
A	$6.43 \pm 0.23 \pm 0.26$	$-0.043068 \pm 0.000076 \pm 0.000085$	1.17
B	$5.97 \pm 0.29 \pm 0.42$	$-0.043218 \pm 0.00009 \pm 0.00014$	0.965
C	$4.89 \pm 0.64 \pm 0.68$	$-0.04357 \pm 0.00021 \pm 0.00022$	0.054

Systematic Errors

- Higher-order effects in MA χ PT:

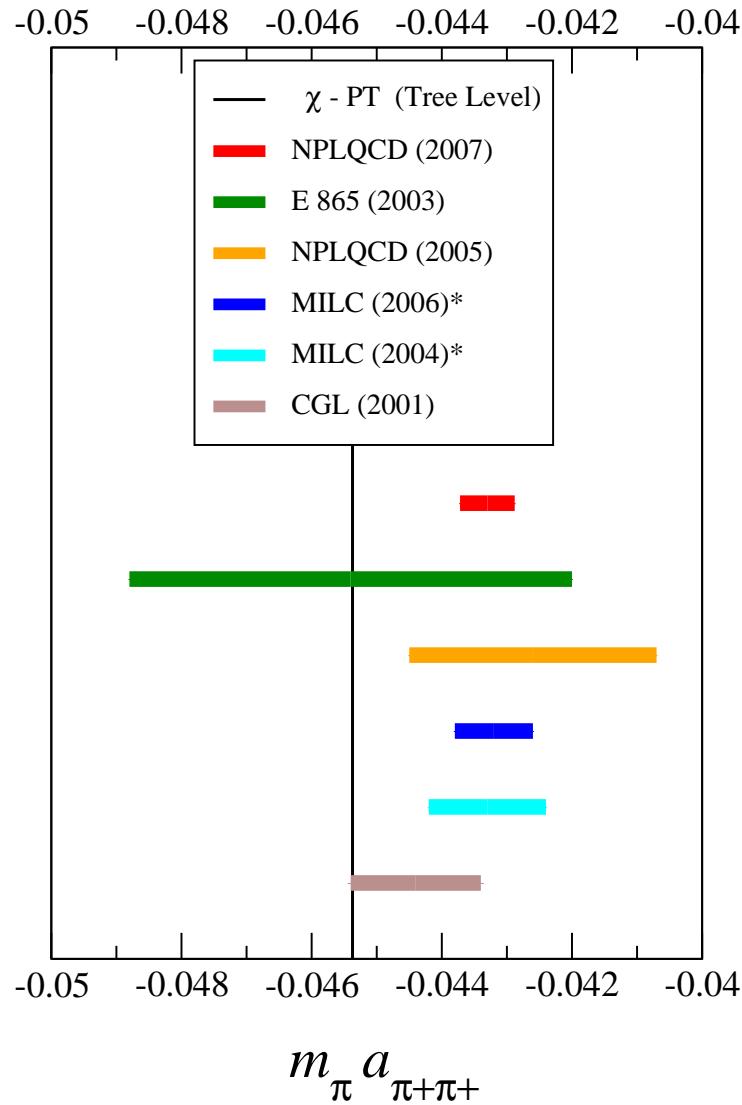
$$\mathcal{O}(m_\pi^4 b^2) \sim \frac{2\pi m_\pi^4}{(4\pi f_\pi)^4} \frac{b^2 \Delta_I}{(4\pi f_\pi)^2} < 1\%$$

- Finite-volume effects: $\sim 4\%$ at lightest mass.
- Residual chiral symmetry breaking:

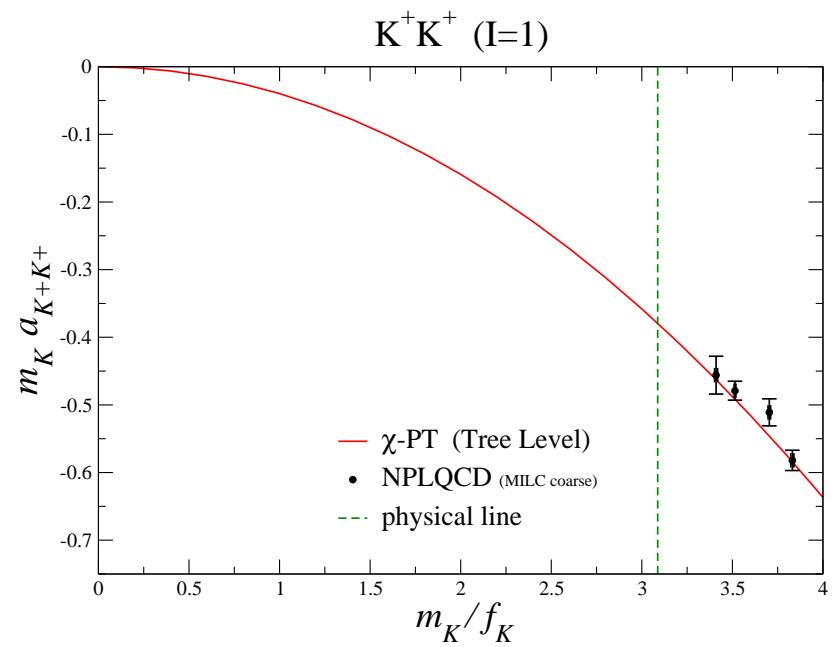
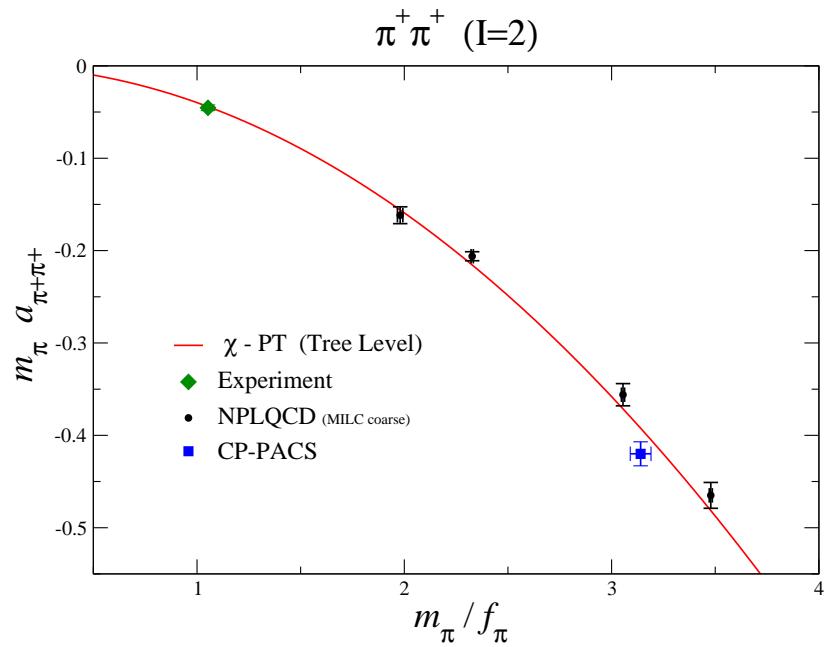
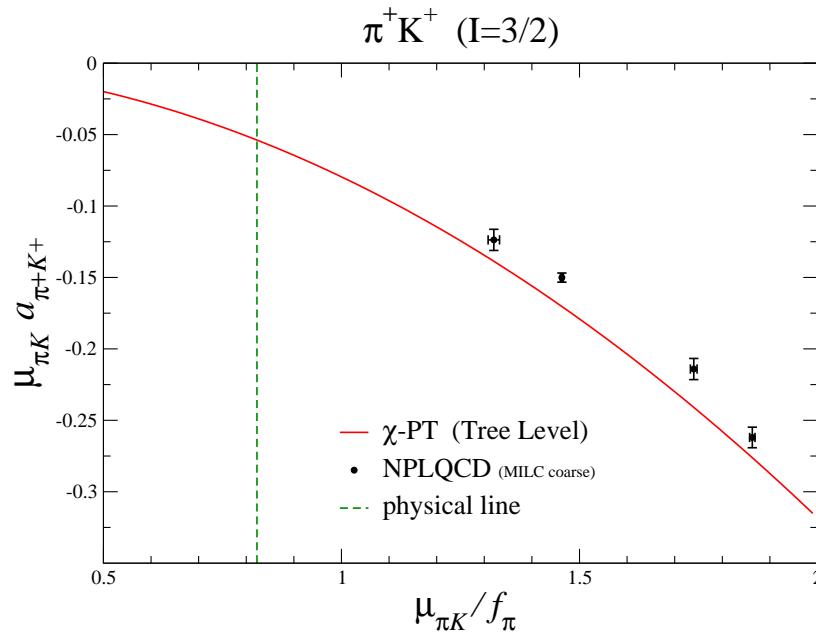
$$\frac{8\pi m_\pi^4}{(4\pi f_\pi)^4} \frac{m_{res}}{m_l} \sim 3\%$$

- Range corrections:
$$\frac{(m_\pi a_{\pi\pi}^{I=2})^2 p^2}{2m_\pi^2} \sim 1\%$$
- Isospin violation: Only issue if compare to experiment!

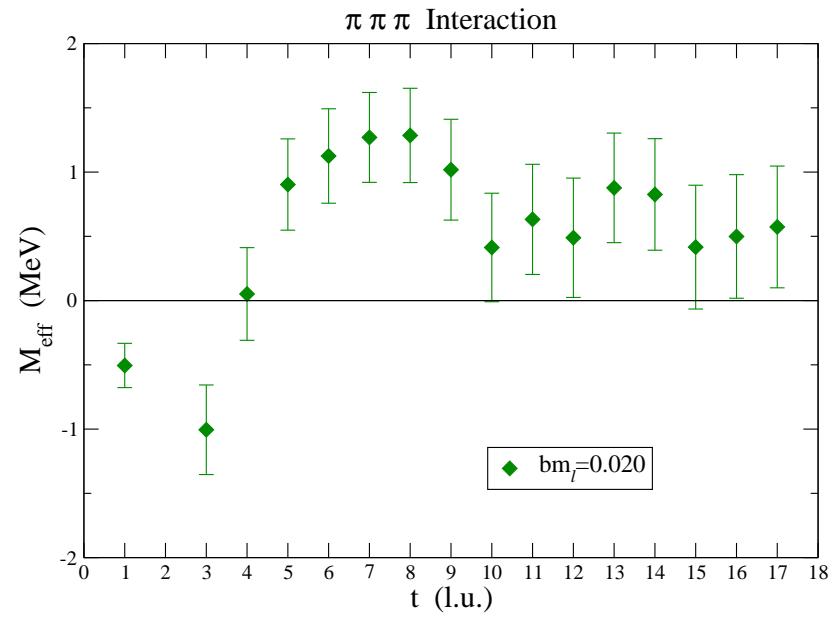
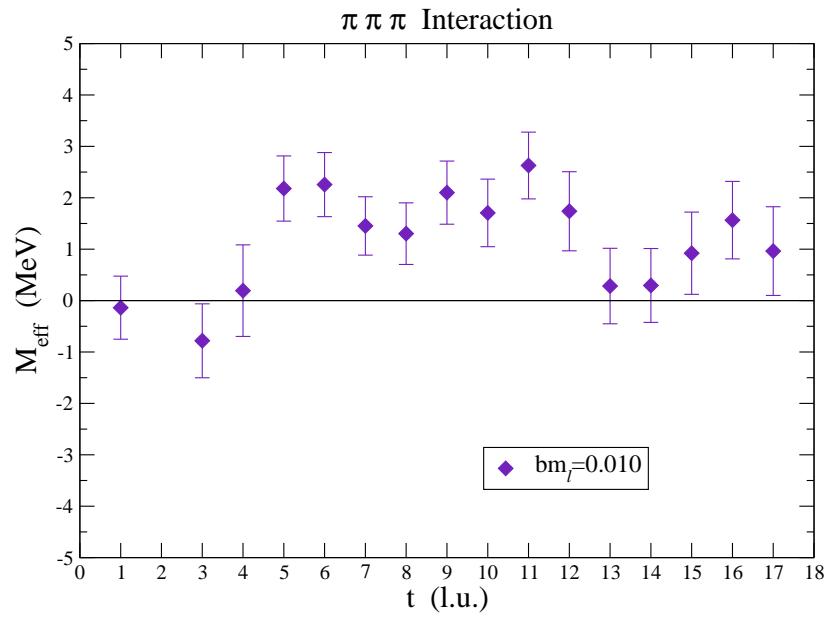
Status of $\pi\pi$



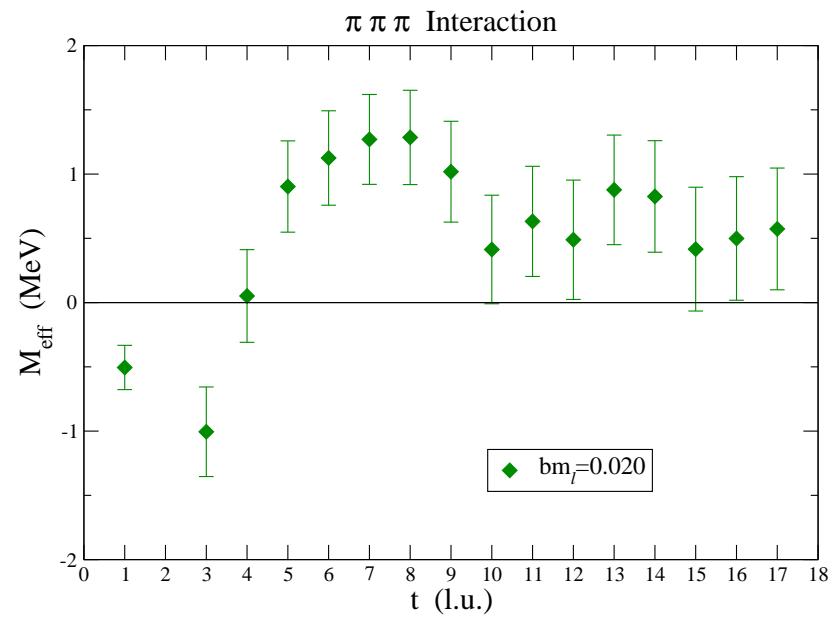
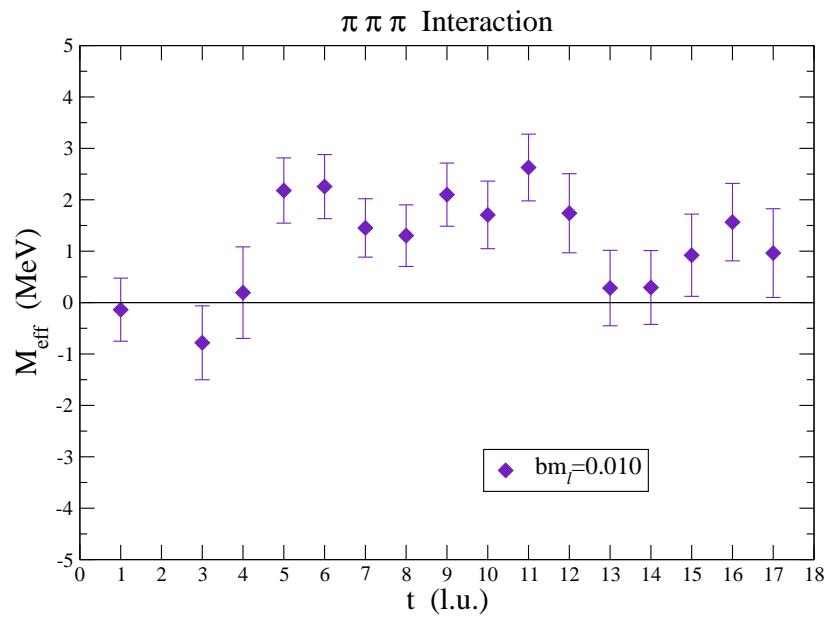
$m_\pi a_{\pi\pi}^{I=2}$	
χ PT (Tree Level)	-0.04438
NPLQCD (2007)	-0.04330 ± 0.00042
E 865 (2003)	$-0.0454 \pm 0.0031 \pm 0.0010 \pm 0.0008$
NPLQCD (2005)	$-0.0426 \pm 0.0006 \pm 0.0003 \pm 0.0018$
MILC (2006)*	-0.0432 ± 0.0006
MILC (2004)*	-0.0433 ± 0.0009
CGL (2001)	-0.0444 ± 0.0010



$\pi\pi\pi$



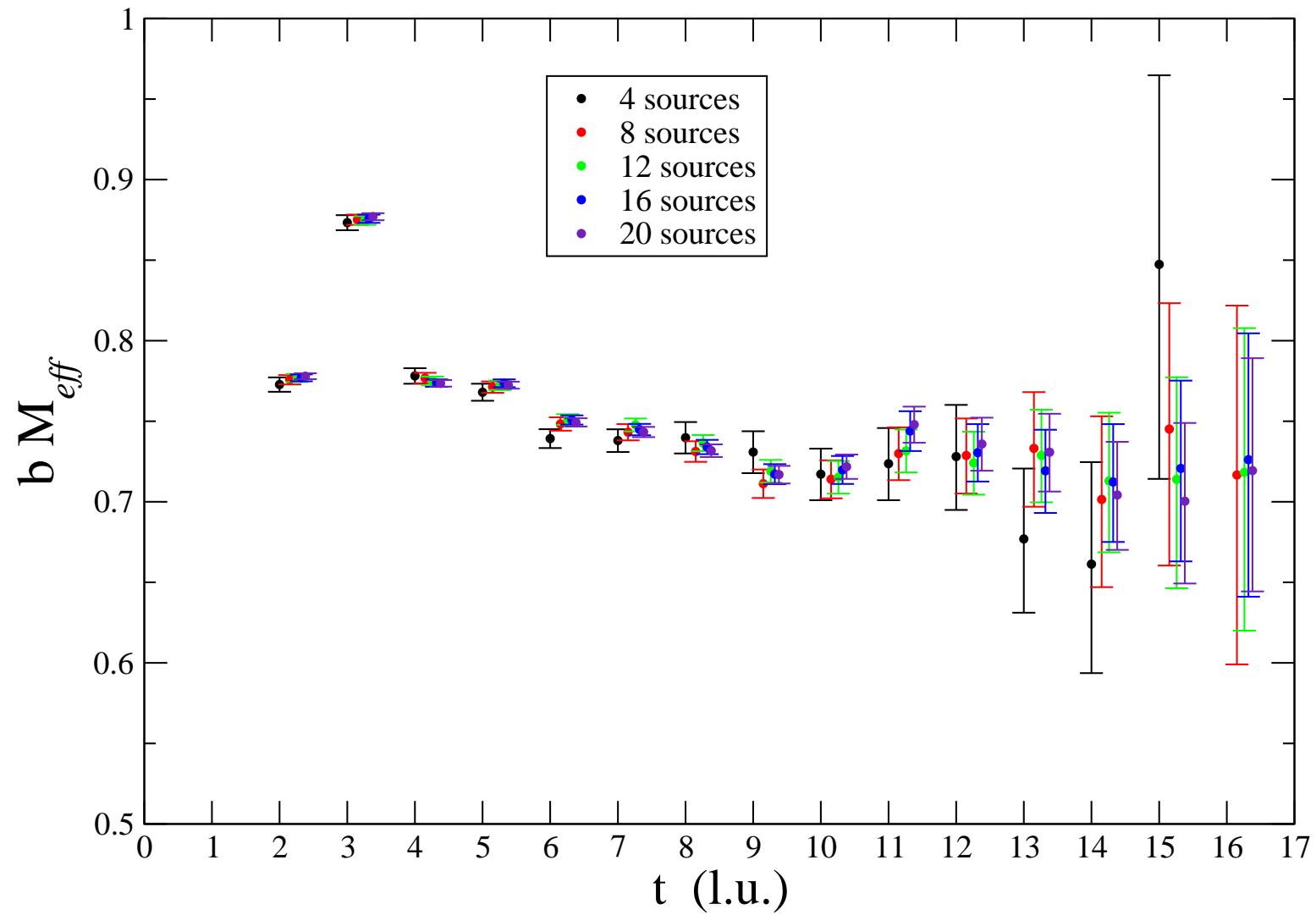
$\pi\pi\pi$



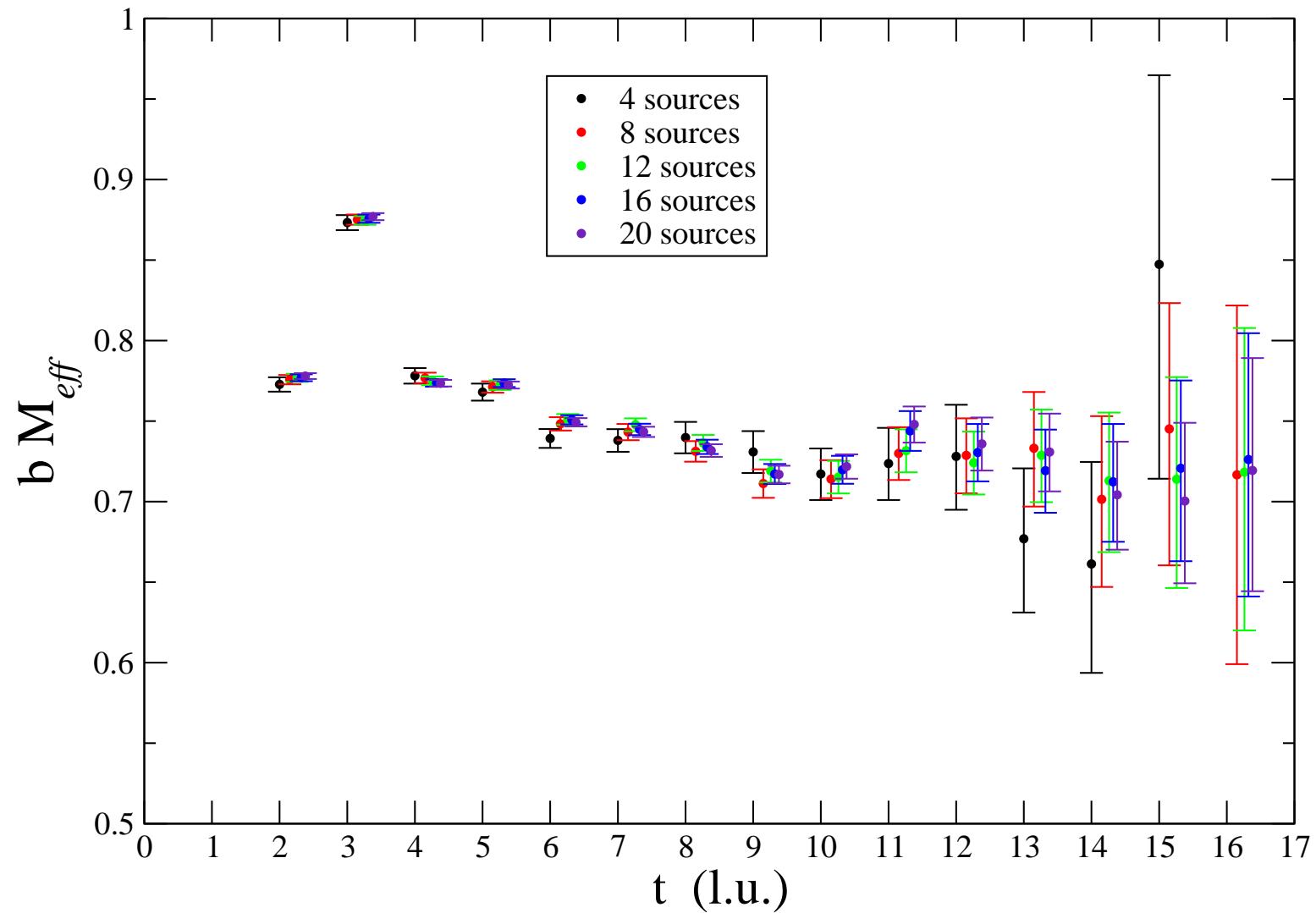
Three particles in a finite volume:

$$\Delta E_0^{3\text{-body}} = -3 \frac{4\pi a}{m_\pi L^3} \left[1 + c_1 \frac{a}{L} + (2c_1^2 - c_2) \left(\frac{a}{L} \right)^2 + \frac{2\pi a^2 r}{L^3} \right] + \frac{\eta_3}{L^6} + \mathcal{O}\left(\frac{1}{L^6}\right)$$

PROTON [$bm_l = 0.010$]

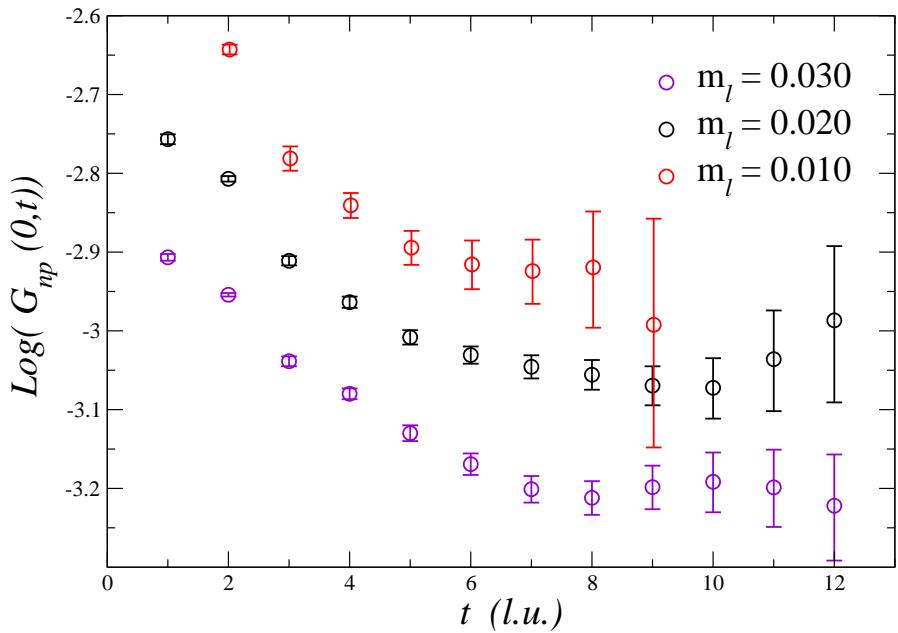
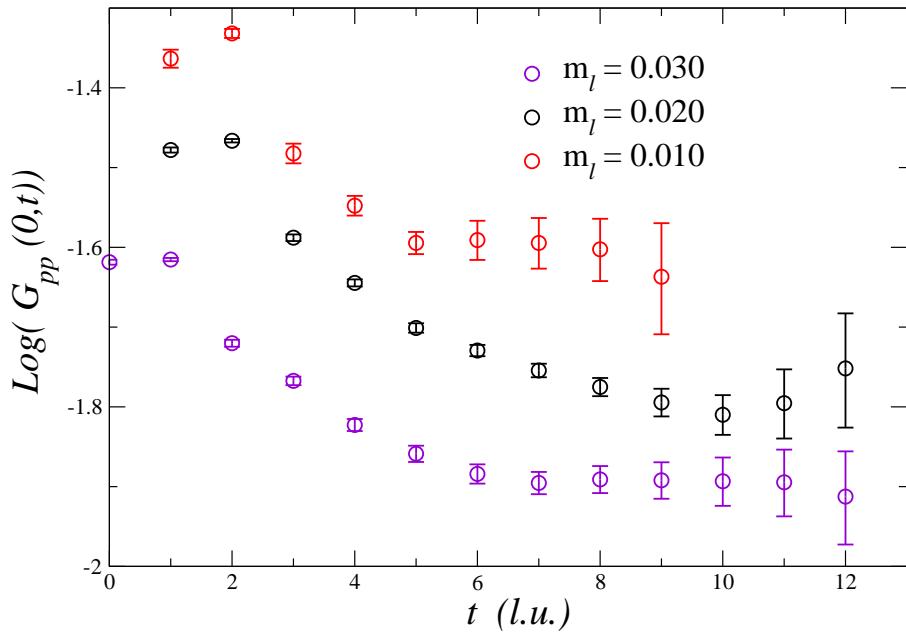


PROTON [$bm_l = 0.010$]



signal/noise $\sim \sqrt{N} e^{-(M_N - 3m_\pi/2)t}$!

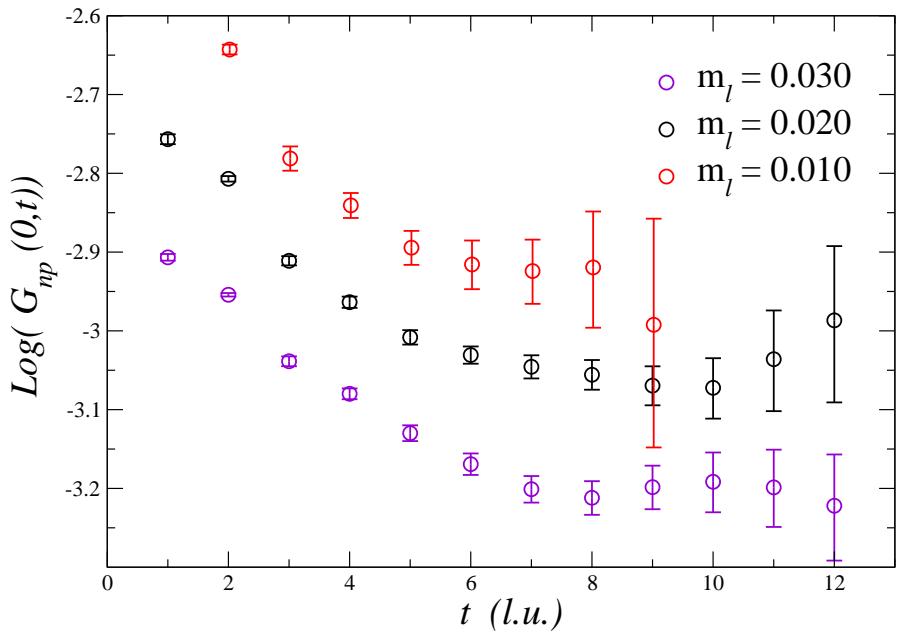
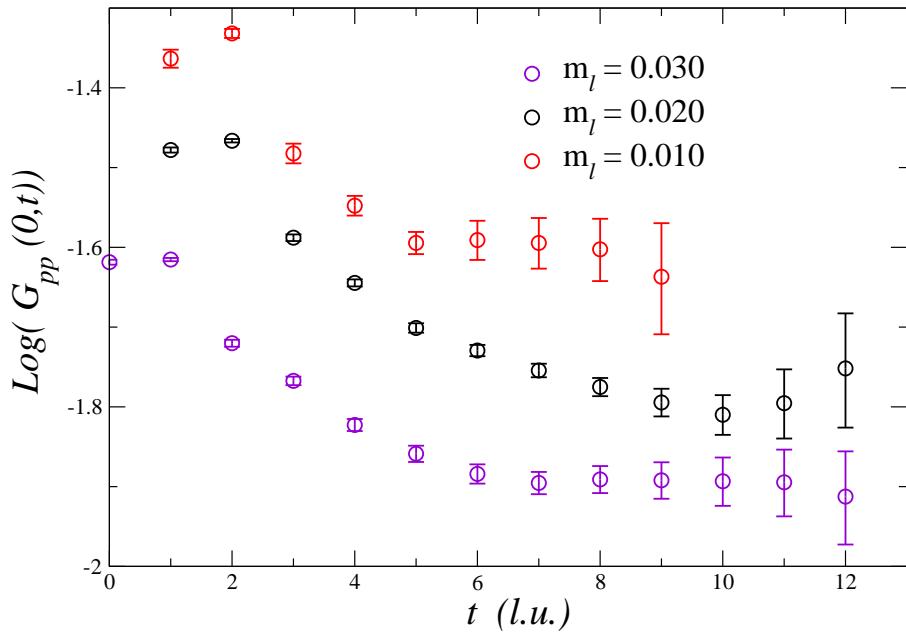
NN



$$G(t) = C_{NN}^{IS}(t) / (C_N(t))^2 \rightarrow \mathcal{A} e^{-E_0 t} + \dots$$

$$C_{NN}^{IS}(t) = X_{\alpha\beta\sigma\rho}^{ijkl} \sum_{\mathbf{x},\mathbf{y}} \langle N_i^\alpha(t,\mathbf{x}) N_j^\beta(t,\mathbf{y}) N_k^{\sigma\dagger}(0,\mathbf{0}) N_l^{\rho\dagger}(0,\mathbf{0}) \rangle$$

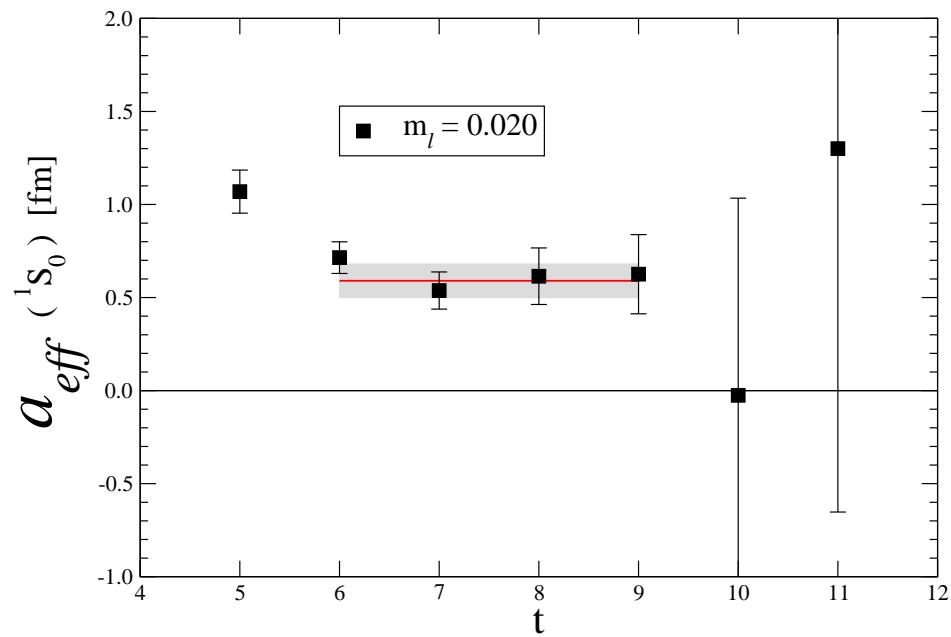
NN

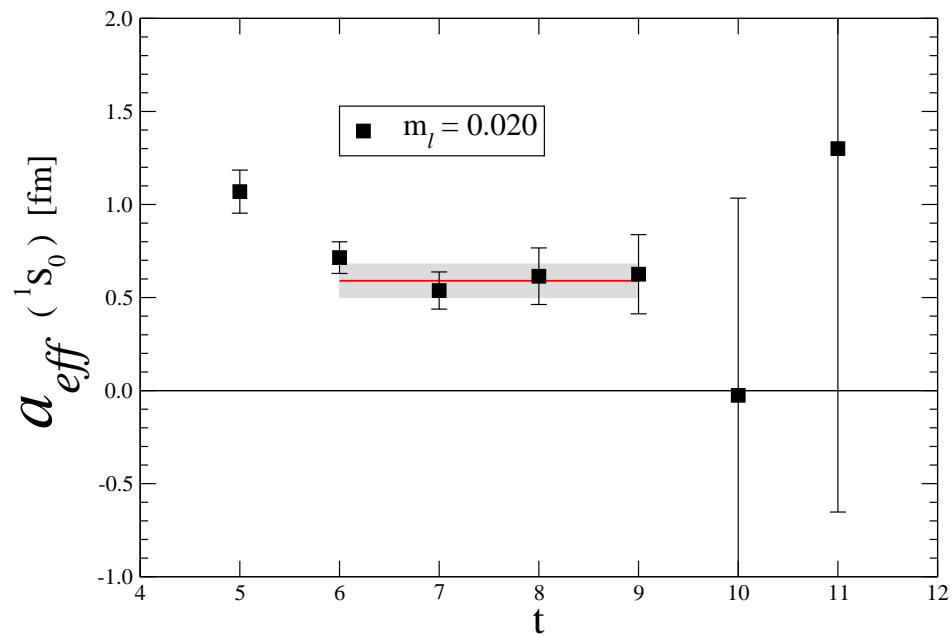


$$G(t) = C_{NN}^{IS}(t) / (C_N(t))^2 \rightarrow \mathcal{A} e^{-E_0 t} + \dots$$

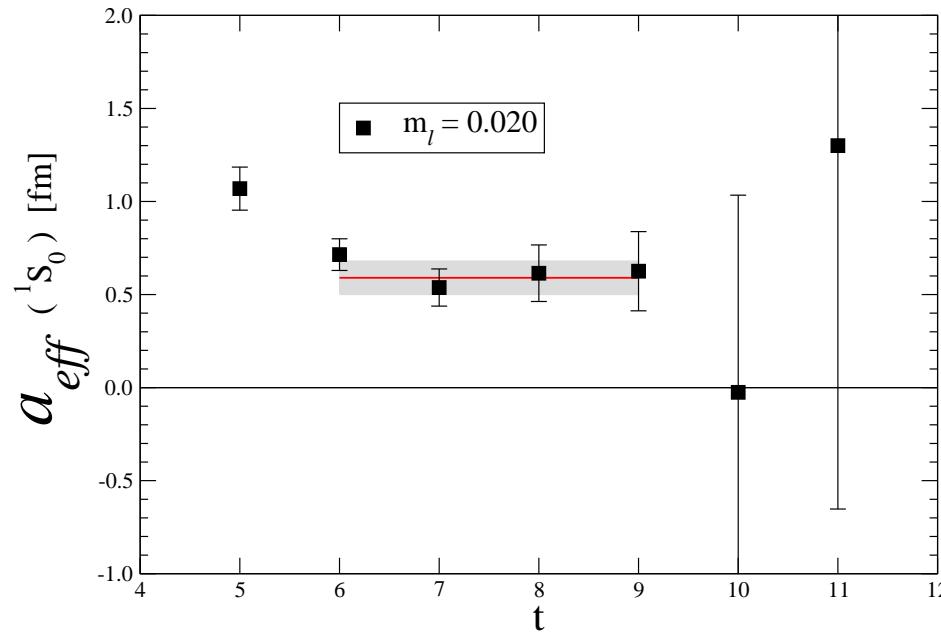
$$C_{NN}^{IS}(t) = X_{\alpha\beta\sigma\rho}^{ijkl} \sum_{\mathbf{x},\mathbf{y}} \langle N_i^\alpha(t,\mathbf{x}) N_j^\beta(t,\mathbf{y}) N_k^{\sigma\dagger}(0,\mathbf{0}) N_l^{\rho\dagger}(0,\mathbf{0}) \rangle$$

signal/noise $\sim \sqrt{N} e^{-2(M_N - 3m_\pi/2)t}$!





All scattering lengths are “natural” size for $m_\pi \geq 350$ MeV!



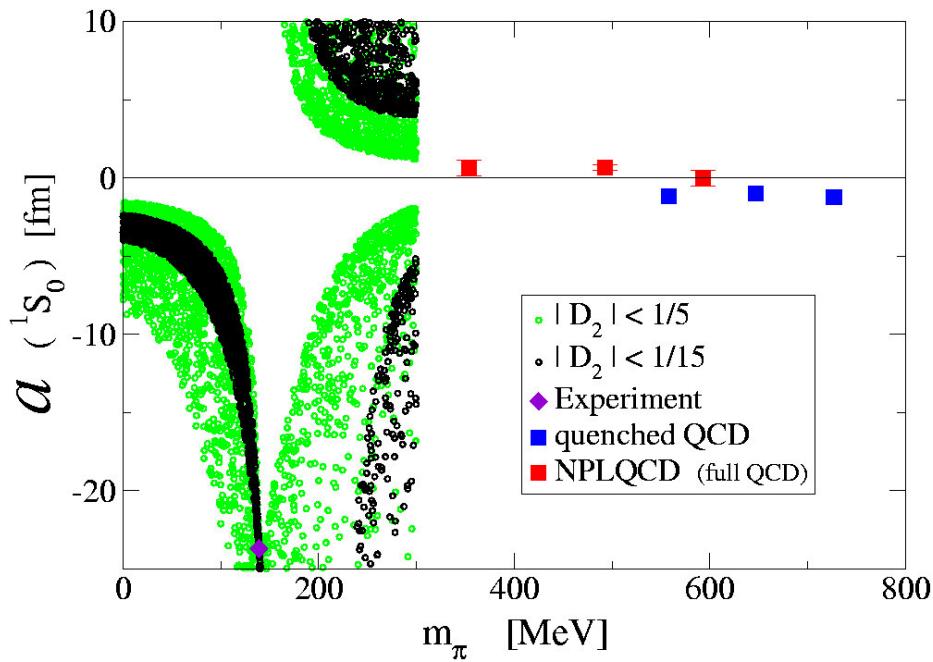
All scattering lengths are “natural” size for $m_\pi \geq 350$ MeV!

Experiment:

$$\begin{aligned}
 a_s^{1S_0} &= -23.714 \text{ fm} & r_s^{1S_0} &= 2.73 \text{ fm} \\
 a_s^{3S_1} &= 5.425 \text{ fm} & r_s^{3S_1} &= 1.749 \text{ fm}
 \end{aligned}$$

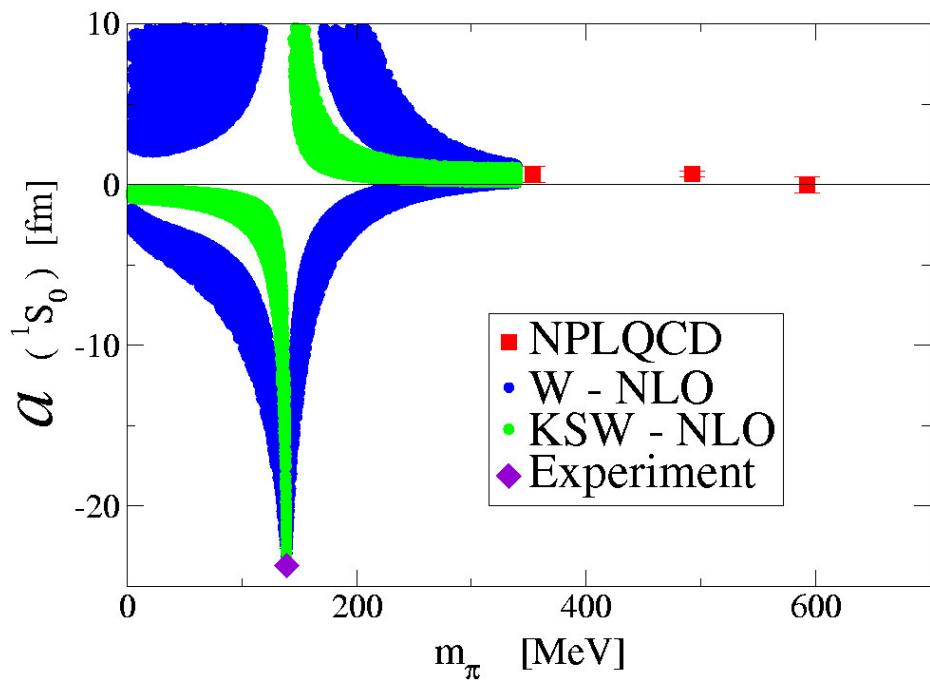
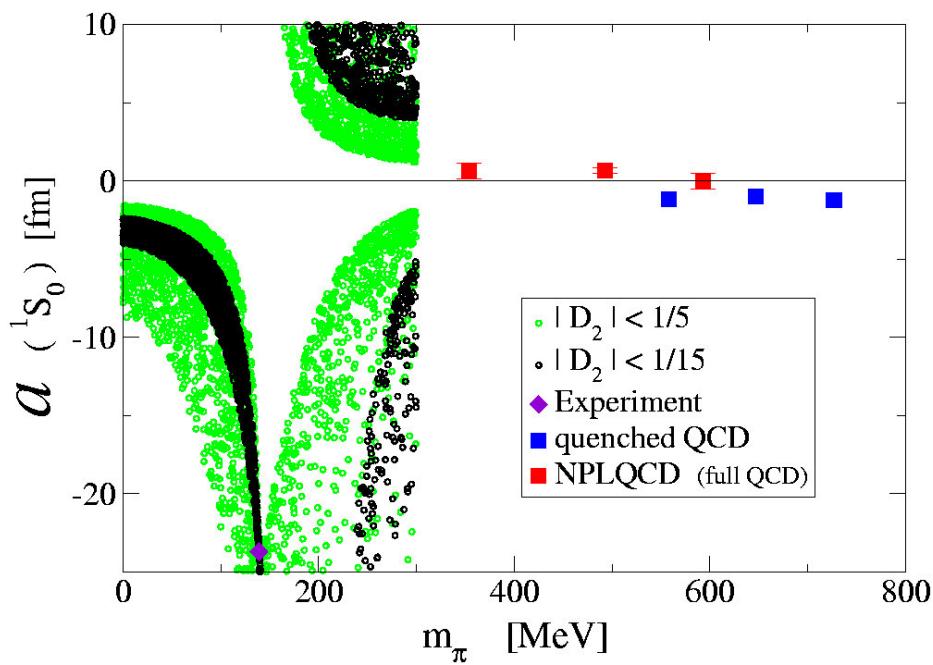
$$a_s \gg \Lambda_{QCD}^{-1} !!$$

1S_0 of NN



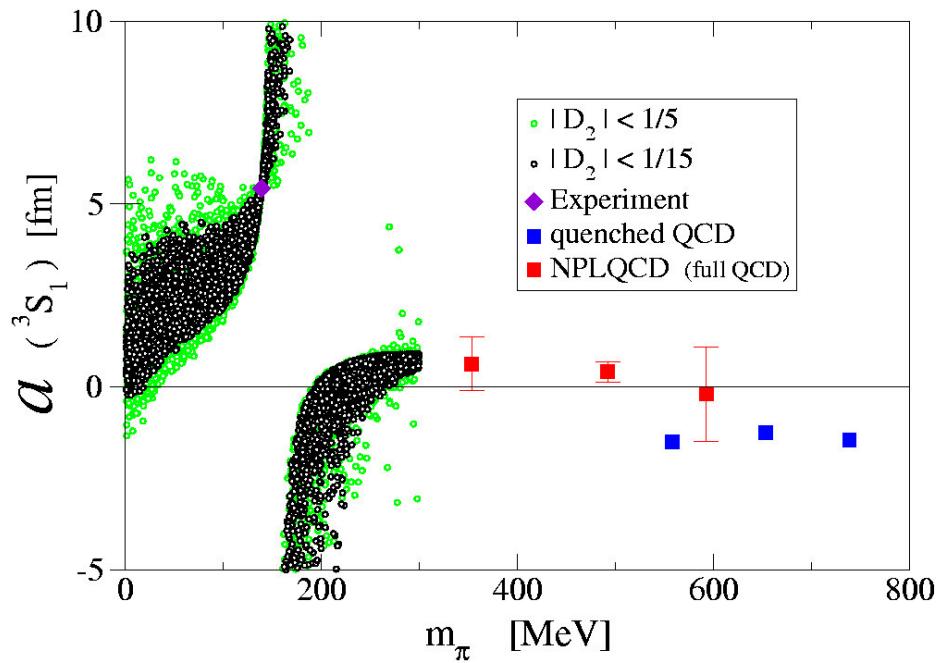
m_π (MeV)	$a(^1S_0)$ (fm)
353.7 ± 2.1	$0.63 \pm 0.50 \pm 0.2$
492.5 ± 1.1	$0.65 \pm 0.18 \pm 0.2$
593.0 ± 1.6	$0.0 \pm 0.5 \pm 0.2$

1S_0 of NN



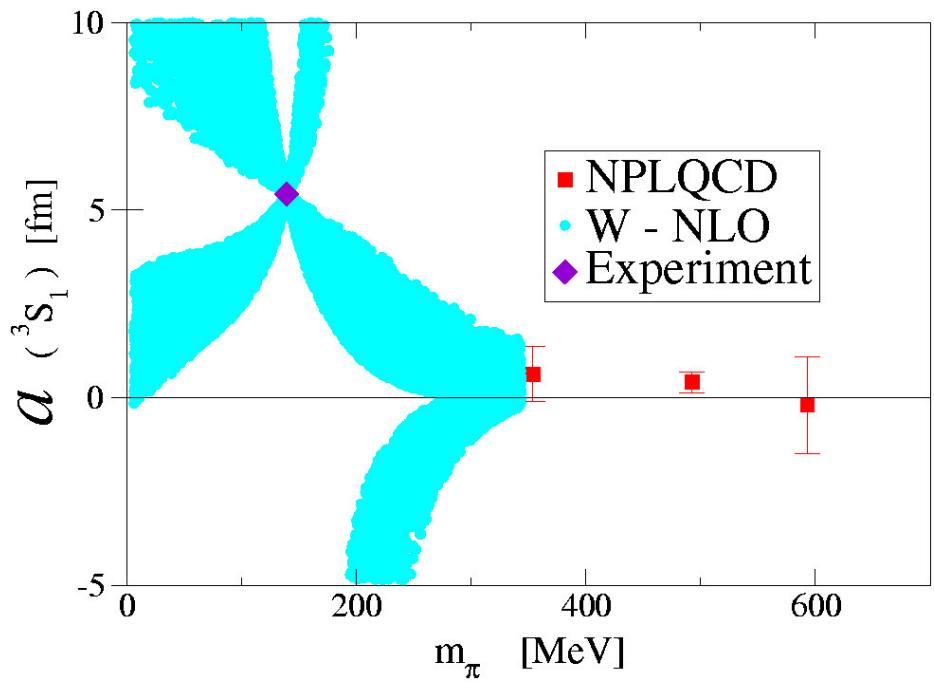
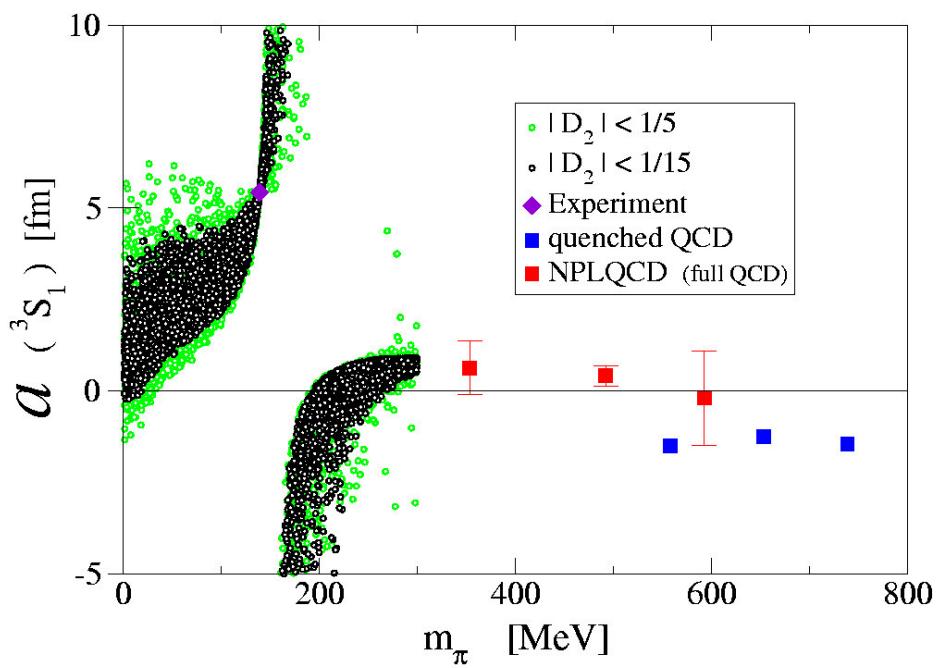
m_π (MeV)	$a(^1S_0)$ (fm)
353.7 ± 2.1	$0.63 \pm 0.50 \pm 0.2$
492.5 ± 1.1	$0.65 \pm 0.18 \pm 0.2$
593.0 ± 1.6	$0.0 \pm 0.5 \pm 0.2$

3S_1 of NN



m_π (MeV)	$a(^3S_1)$ (fm)
353.7 ± 2.1	$0.63 \pm 0.74 \pm 0.2$
492.5 ± 1.1	$0.41 \pm 0.28 \pm 0.2$
593.0 ± 1.6	$-0.2 \pm 1.3 \pm 0.2$

3S_1 of NN



m_π (MeV)	$a(^3S_1)$ (fm)
353.7 ± 2.1	$0.63 \pm 0.74 \pm 0.2$
492.5 ± 1.1	$0.41 \pm 0.28 \pm 0.2$
593.0 ± 1.6	$-0.2 \pm 1.3 \pm 0.2$

ΛN Scattering

$\Lambda p \rightarrow \Lambda p$ from experiment?

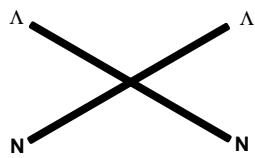
$$0.0 > a^{(1S_0)} > -15 \text{ fm}$$

$$-0.6 > a^{(3S_1)} > -3.2 \text{ fm}$$

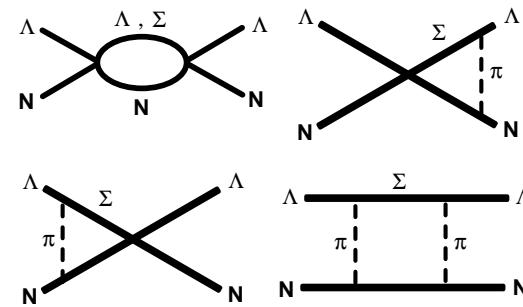
$$0.0 < r^{(1S_0)} < 15 \text{ fm}$$

$$2.5 < r^{(3S_1)} < 15 \text{ fm}$$

LO :

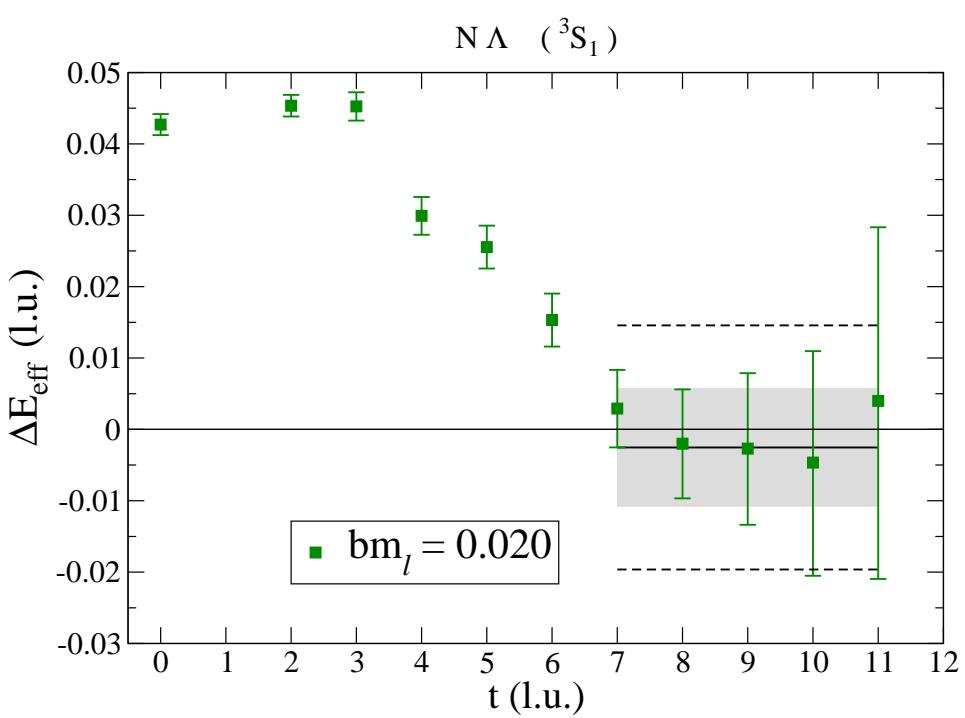
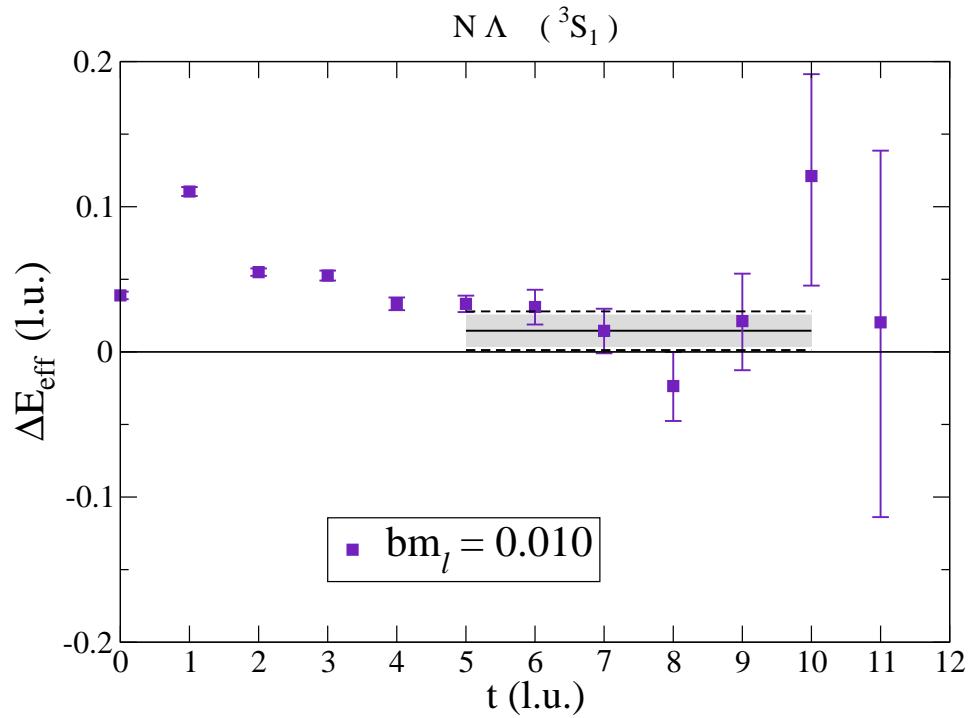
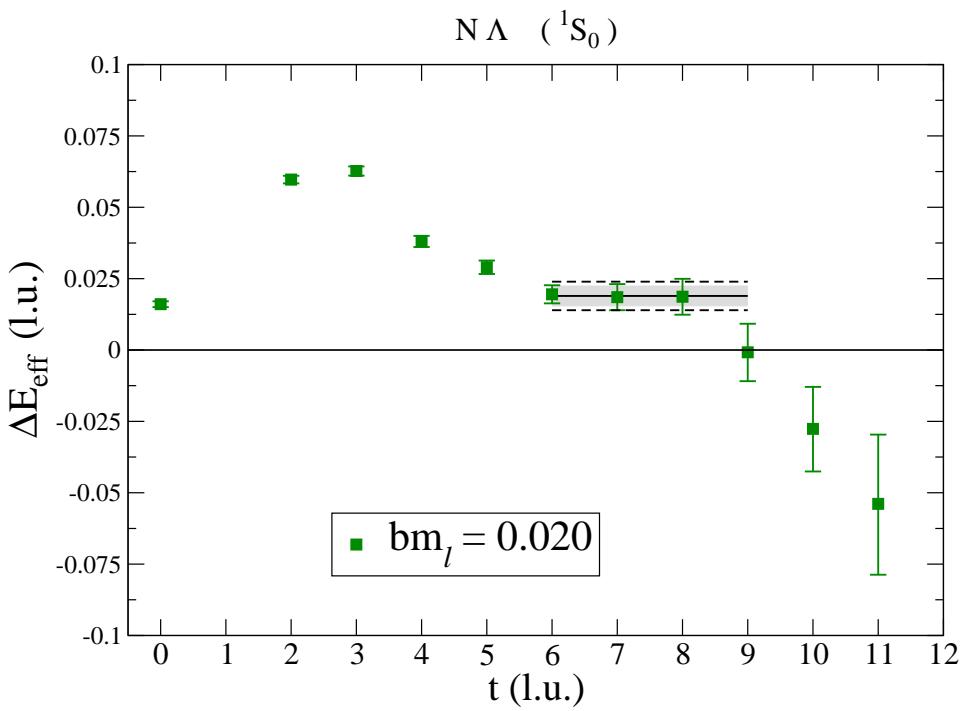
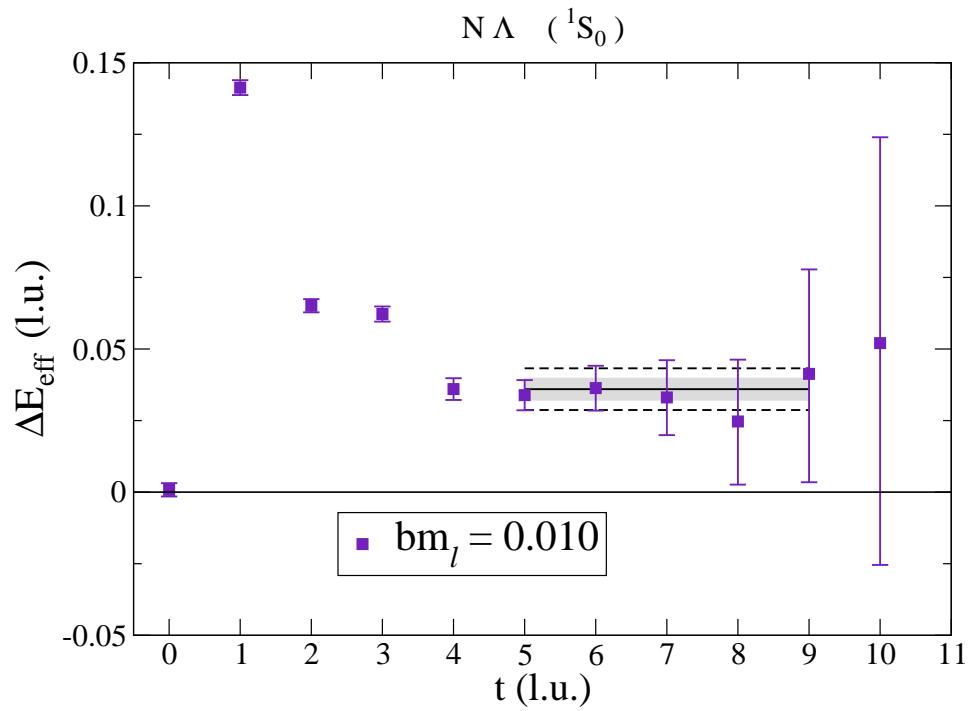


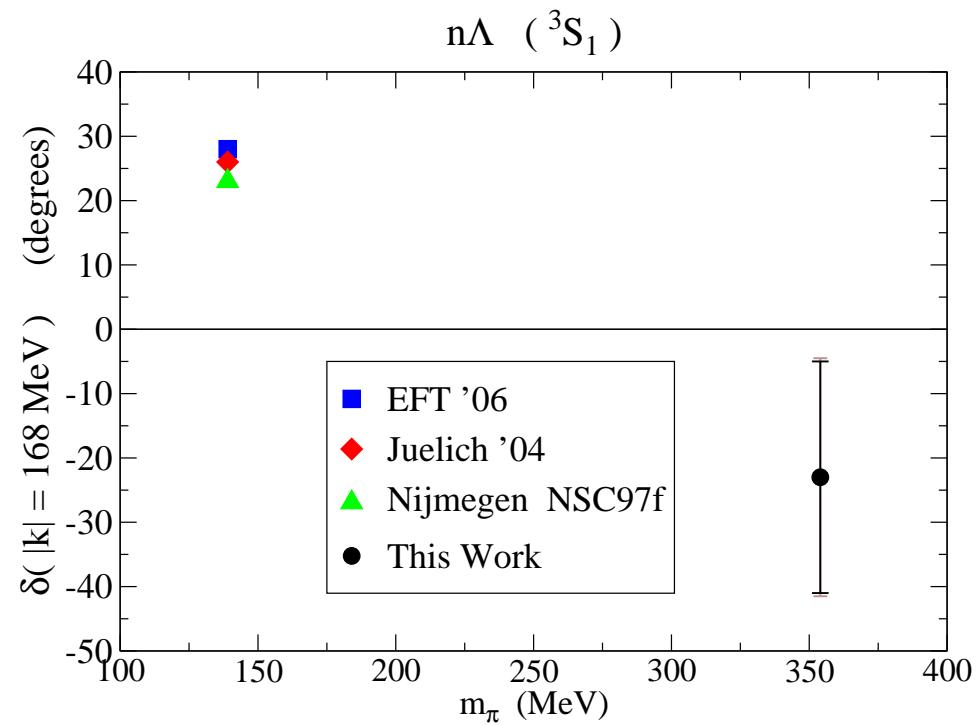
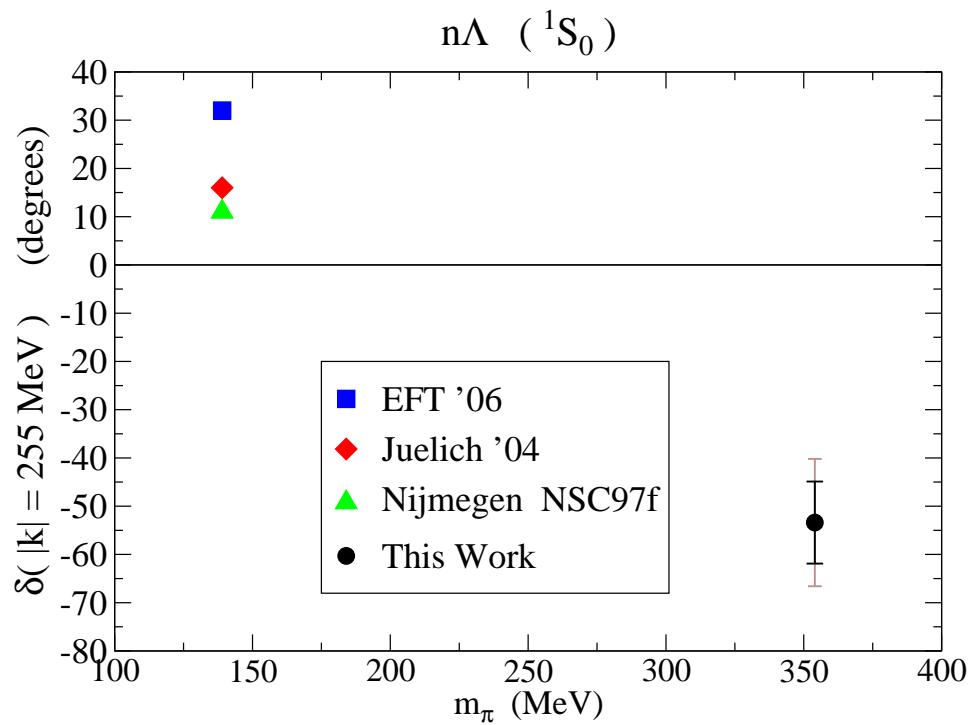
NLO :



$$\Sigma\Lambda C_0, \Lambda\Lambda C_0 \longrightarrow a^{()} , r^{()}$$

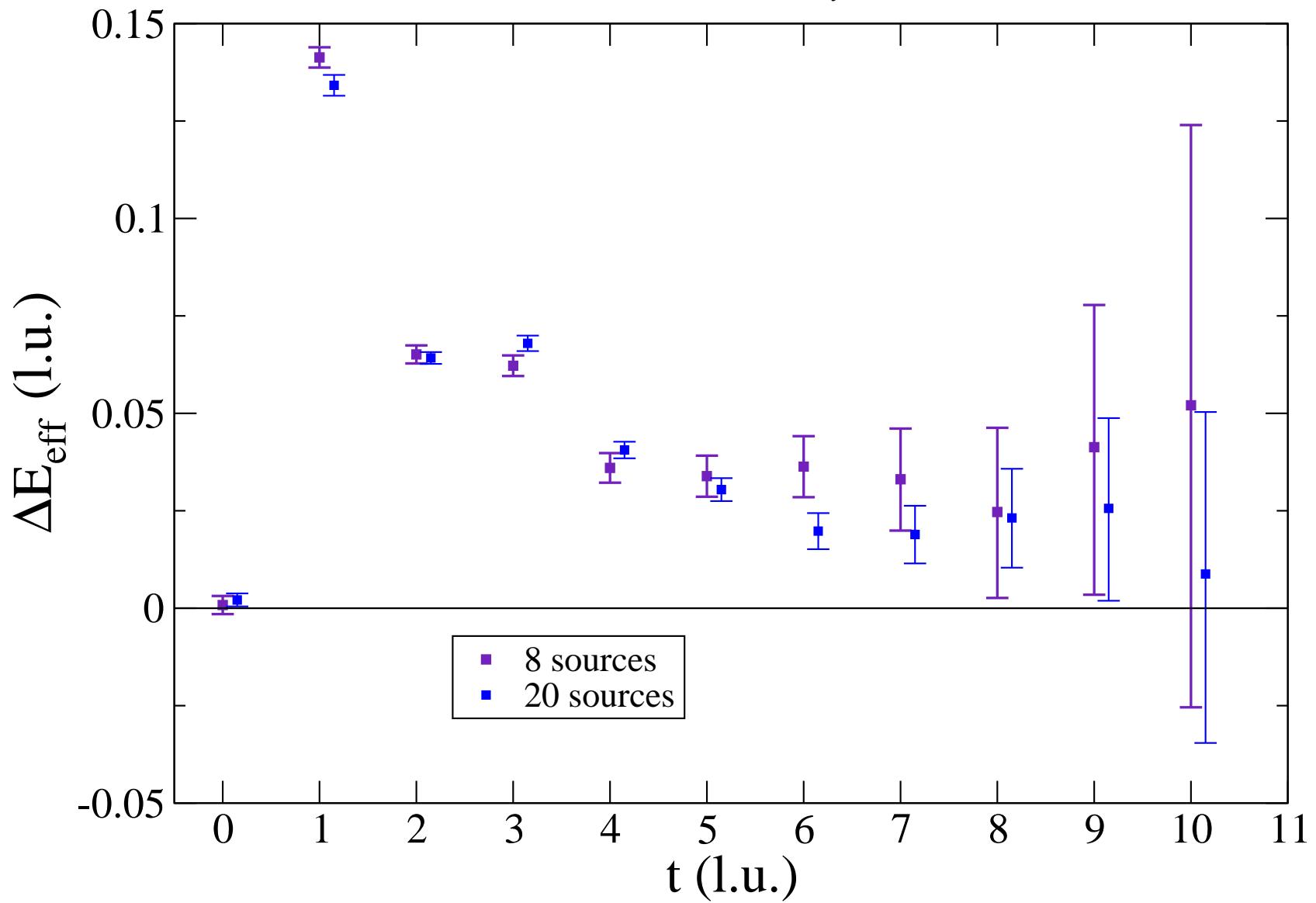
Lattice QCD may soon compete with experiment





- If observed state is ground state in the lattice volume, interactions are all repulsive.
- There may exist states of lower negative energies \Rightarrow bound states in the continuum.
- Poor statistics \Rightarrow nothing definitive can be said about the existence of such states.

$N\Lambda$ (1S_0) [$bm_l = 0.010$]



Conclusion

- Meson-meson scattering from lattice QCD is a precision science.
- Meson-meson-meson interactions are under investigation.
Three-body finite-volume analysis is in place.
- First contact between QCD and nuclear physics has been achieved.
- Resources are being gathered to run lattice simulations at another volume and at another lattice spacing. Critical for identification of bound states; e.g. deuterium.
- Need to be smarter about increasing signal/noise

