Model independent charge densities from nucleon form factors

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What do form factors really measure?

Relation to orbital angular momentum of nucleon?

What is charge density at the center of the neutron?

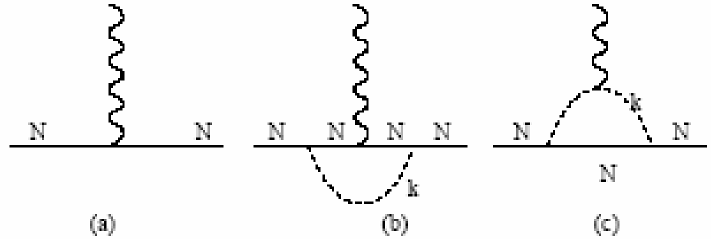
- Neutron has no charge, but charge density need not vanish
- Is central density positive or negative?

$n \to p\, \pi^-$

Neutron: Need π cloud effect at low Q^2

TTM

Cloudy Bag Model 1980



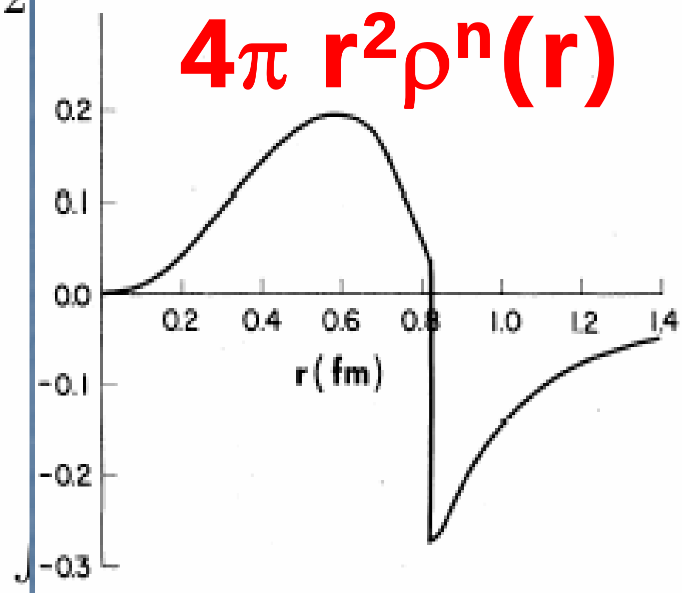


FIG. 11. Neutron charge density.

Relativistic treatment needed Feynman graphs,

Light front cloudy bag model LFCBM 2002

One gluon exchange also gives positive central charge density

Enough models- Today

model **ind**ependent information

Outline

- Electromagnetic form factors
- Light cone coordinates, kinematic subgroup
- GPDs + Bit of math
- Two dimensional Fourier transform of F_1 gives $\rho(b)$, Soper '77
- Data analysis, Interpretation (anyone's game)

Definitions

$$\langle p', \lambda' | J^\mu(0) | p, \lambda \rangle = \bar{u}(p', \lambda') \left(\gamma^\mu F_1(Q^2) + i \frac{\sigma^{\mu\alpha}}{2M} q_\alpha F_2(Q^2) \right) u(p, \lambda)$$

$$G_E(Q^2) \equiv F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \; G_M(Q^2) \equiv F_1(Q^2) + F_2(Q^2)$$

Old Interpretation- Breit frame $\vec{p}\,' = -\vec{p}$

G_E is helicity flip matrix element of J^0

Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density

Correct non-relativistic

Non-relativistic two particle :

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, t) = e^{i\mathbf{P}\cdot\mathbf{R} - i(\frac{P^2}{2M} - \epsilon)t}\phi(\mathbf{r})$$

ϕ invariant under Galilean transformation

Relativity: $(\mathbf{r}_1, t_1), (\mathbf{r}_2, t_2)$ $t_1 \neq t_2$

$\bar{e}^{i\,H(t_1 - t_2)}$ Interactions!

ϕ is frame dependent, interpretation of Sachs wrong

Why relativity if $Q^2 \ll M^2$

QCD- photon hits $\approx$ massless quarks

No matter how small Q^2 is, there is a boost correction that is $\propto Q^2$

$G_E^{\,n} \sim Q^2 \ (\int d^3r\ r^2|\psi|^2 + C/(m_q)^2)$

Light cone coordinates

"Time" $x^+ = (ct + z)/\sqrt{2} = (x^0 + x^3)/\sqrt{2}$

"Evolution" $\quad p^- = (p^0 - p^3)/\sqrt{2}$

"Space" $x^- = (ct - z)/\sqrt{2} = (x^0 - x^3)/\sqrt{2}$, If $x^+ = 0, \; x^- = -\sqrt{2}z$

"Momentum" $\quad p^+ = (p^0 + p^3)/\sqrt{2}$

Transverse : "Position" $\mathbf{b}$ "Momentum" $\mathbf{p}$

Relativistic formalism-kinematic subgroup of Poincare

- Lorentz transformation –transverse velocity v

$$k^+ \longrightarrow k^+, \;\; \mathbf{k} \longrightarrow \mathbf{k} - k^+\mathbf{v}$$

k^- such that k^2 not changed

Transverse boosts like non-relativistic

Generalized Parton Distribution

$$H_q(x,t) = \int \frac{dx^-}{4\pi} \langle p^+, \mathrm{p}', \lambda | \bar{q}(-\frac{x^-}{2}, 0) \gamma^+ q(\frac{x^-}{2}, 0) | p^+, \mathrm{p}, \lambda \rangle e^{ixp^+x^-}$$

$$H_q(x, \xi = 0, t) \equiv H_q(x, t)$$

$A^+=0$, $t=(p-p')^2 = -Q^2 = -(\mathbf{p}' - \mathbf{p})^2$

$$H_q(x,t) \equiv \int \frac{dx^-}{4\pi} \langle p^+, \mathrm{p}', \lambda | \bar{q}(-\frac{x^-}{2}, 0) \gamma^+ q(\frac{x^-}{2}, 0) | p^+, \mathrm{p}, \lambda \rangle e^{ixp^+x^-}$$

$H_q(x,0) = q(x)$

$$F_1(t) = \sum_q e_q \int dx H_q(x,t)$$

transverse center of mass R

$$\left| p^+, \mathbf{R} = \mathbf{0}, \lambda \right\rangle \equiv \mathcal{N} \int \frac{d^2\mathbf{p}}{(2\pi)^2} \left| p^+, \mathbf{p}, \lambda \right\rangle$$

$$\hat{O}_q(x, \mathrm{b}) \equiv \int \frac{dx^-}{4\pi} q_+^\dagger \left(-\frac{x^-}{2}, \mathrm{b} \right) q_+ \left(\frac{x^-}{2}, \mathrm{b} \right) e^{ixp^+x^-}$$

$$H_q(x,t) = \int \frac{dx^-}{4\pi} \langle p^+, \mathrm{p}', \lambda | \bar{q}(-\frac{x^-}{2}, 0) \gamma^+ q(\frac{x^-}{2}, 0) | p^+, \mathrm{p}, \lambda \rangle e^{ixp^+x^-}$$

$$\hat{O}_q(x,\mathbf{b}) \equiv \int \frac{dx^-}{4\pi} q_+^\dagger \left(-\frac{x^-}{2}, \mathbf{b}\right) q_+ \left(\frac{x^-}{2}, \mathbf{b}\right) e^{ixp^+x^-}$$

$$q(x,\mathbf{b}) \equiv \left\langle p^+, \mathbf{R} = \mathbf{0}, \lambda \right| \hat{O}_q(x,\mathbf{b}) \left| p^+, \mathbf{R} = \mathbf{0}, \lambda \right\rangle .$$

Burkardt

$$q(x,\mathbf{b}) = \int \frac{d^2 q}{(2\pi)^2} e^{i\,\mathbf{q}\cdot\mathbf{b}} H_q(x, t = -\mathbf{q}^2),$$

Integrate on x, Left: sets $x^- = 0$ $\longrightarrow$ $q_+^\dagger(0,b)\, q_+(0,b)$
DENSITY; right 2 Dim. Fourier T. of F_1

RESULT

$$\rho(b) \equiv \sum_q e_q \int dx\, q(x,\mathbf{b}) = \int \frac{d^2q}{(2\pi)^2} F_1(Q^2 = \mathbf{q}^2) e^{i\,\mathbf{q}\cdot\mathbf{b}}.$$

Density

Soper '77

$$\boxed{\rho(b) = \int_0^\infty \frac{dQ\,Q}{2\pi} J_0(Qb) \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau}}$$

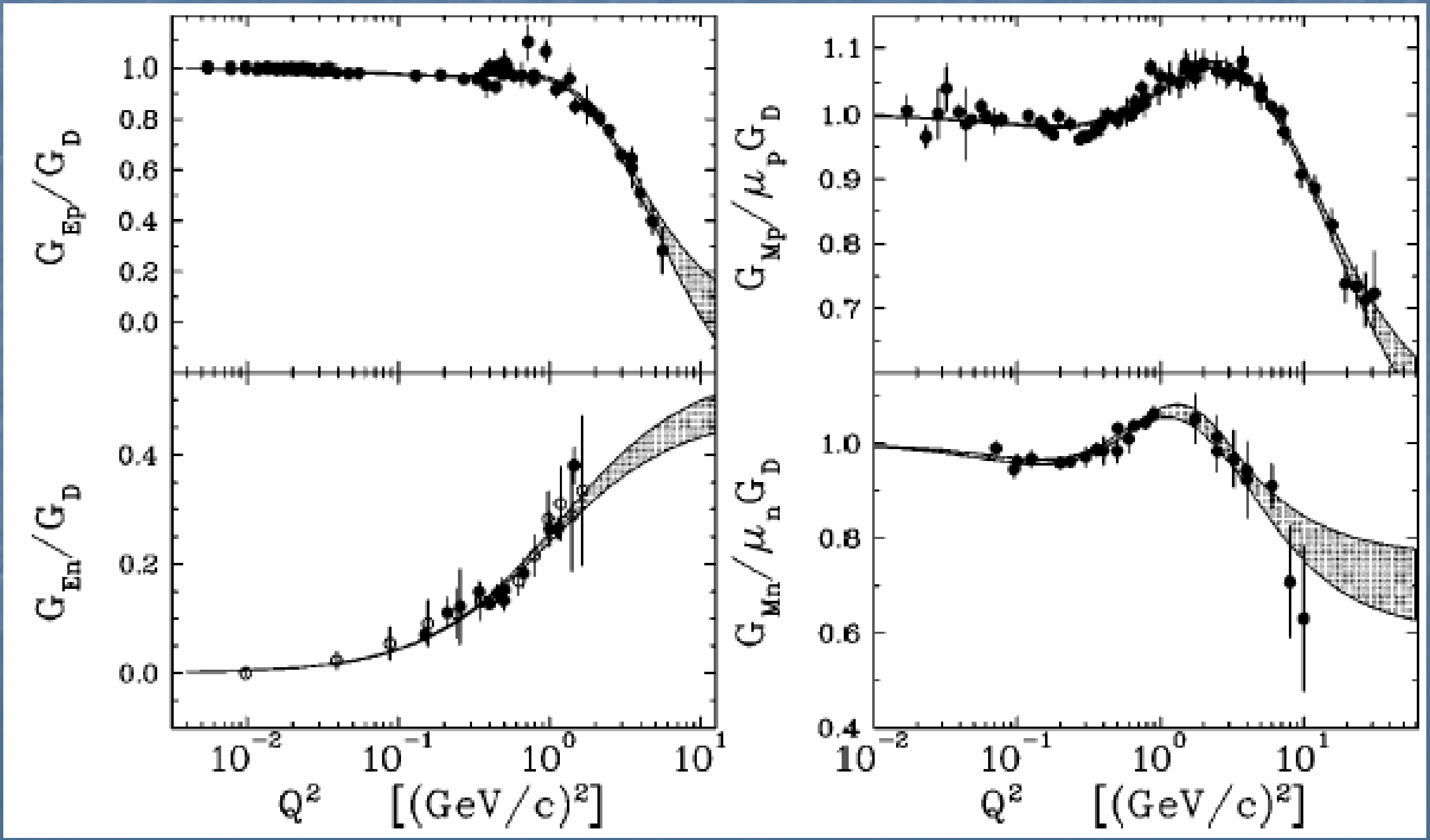
$\tau = Q^2/4M^2$

For neutron $\tau\, G_M \approx G_E$ at low Q^2

PHYSICAL REVIEW C 70, 068202 (2004)

Simple parametrization of nucleon form factors

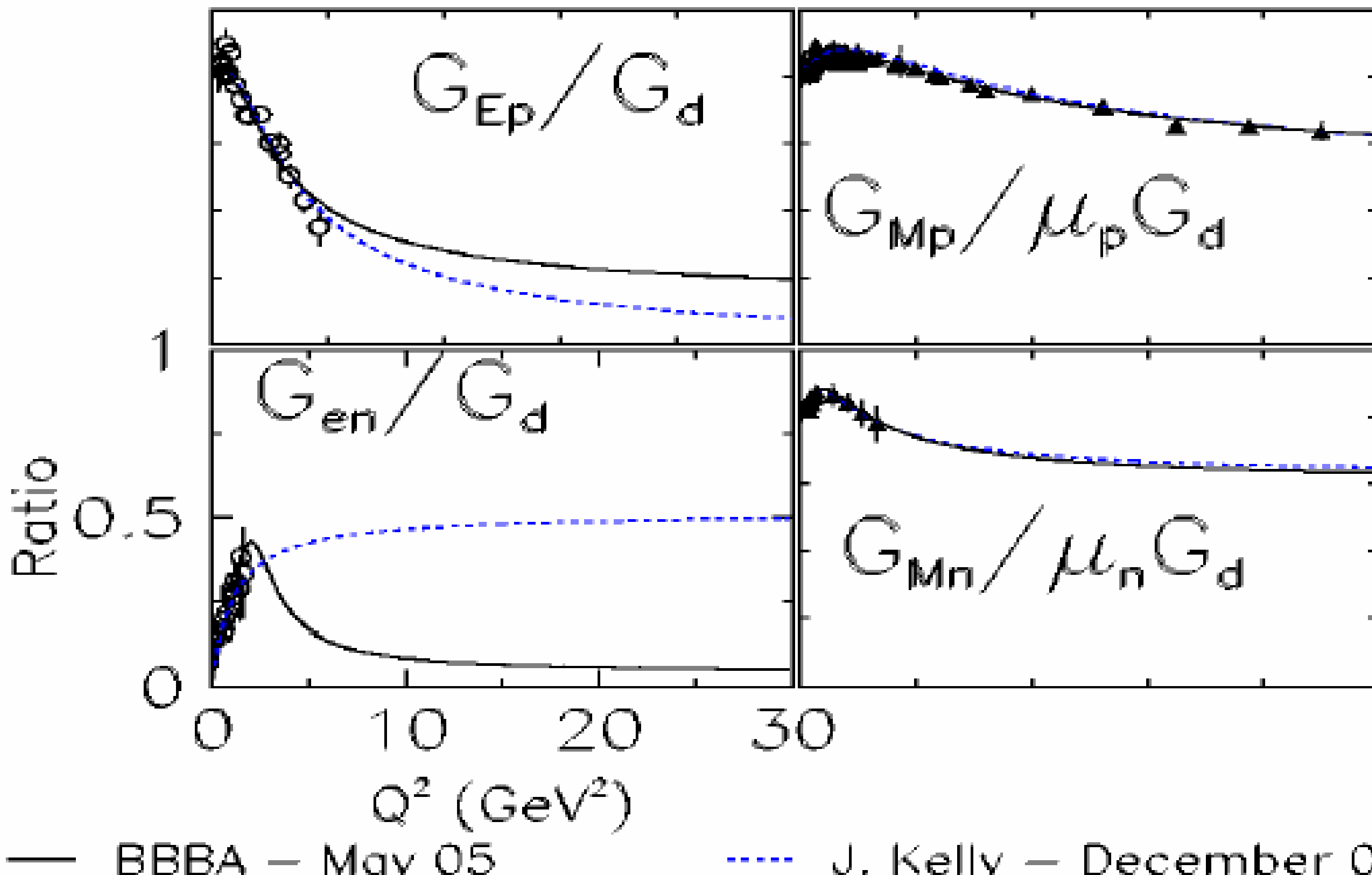
J. J. Kelly



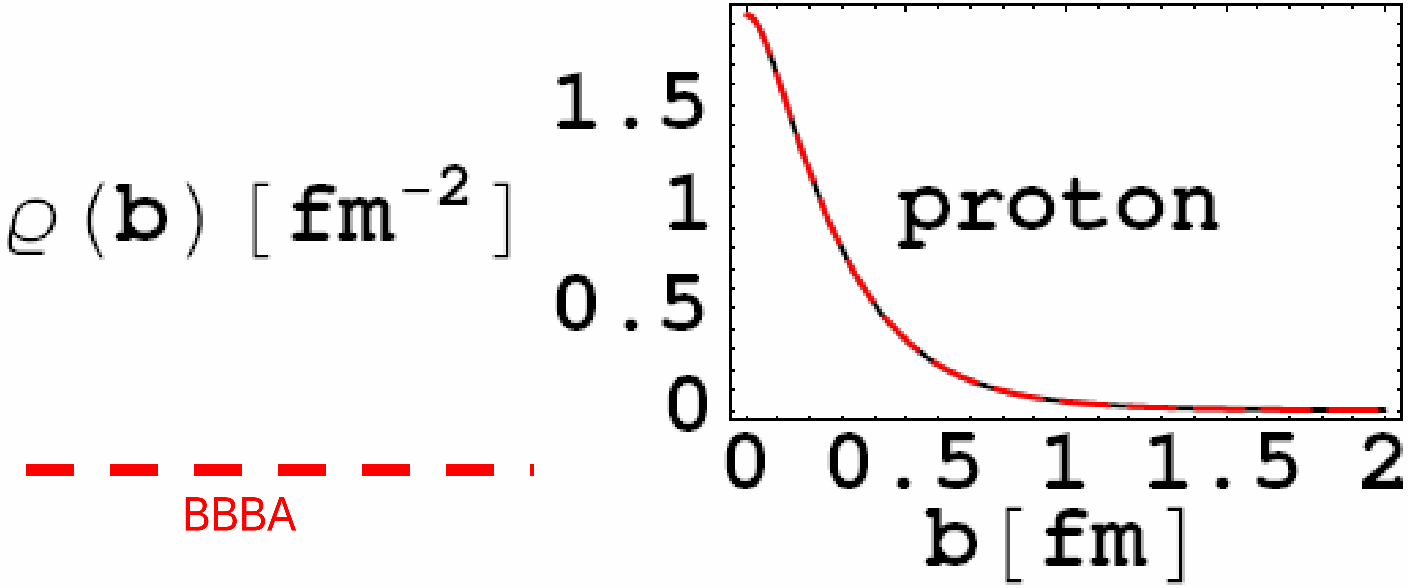
A New Parameterization of the Nucleon Elastic Form Factors

R. Bradford,[a] A. Bodek,[a] H. Budd,[a] and J. Arrington[b]

hep-ex/0602017

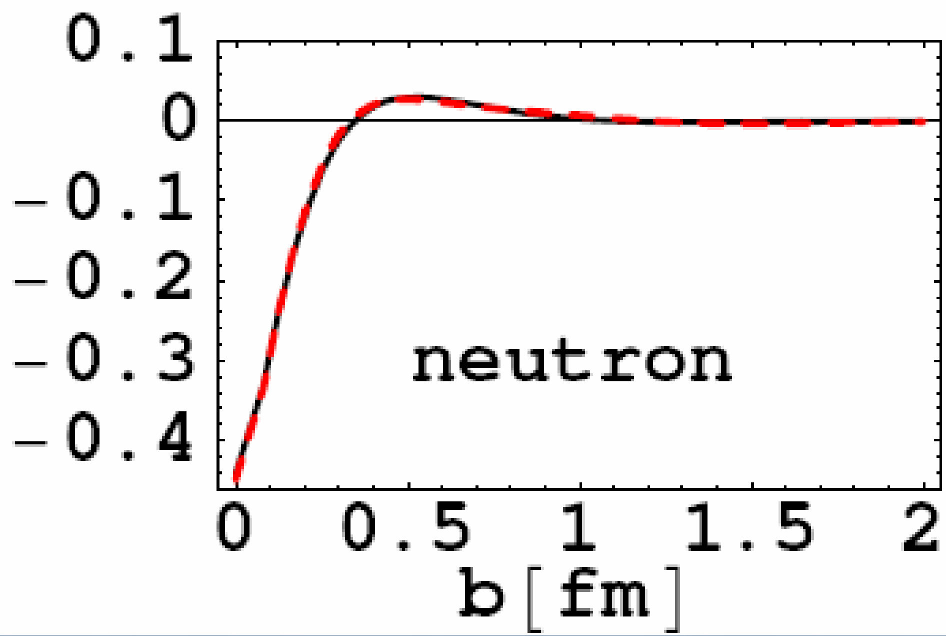


Results

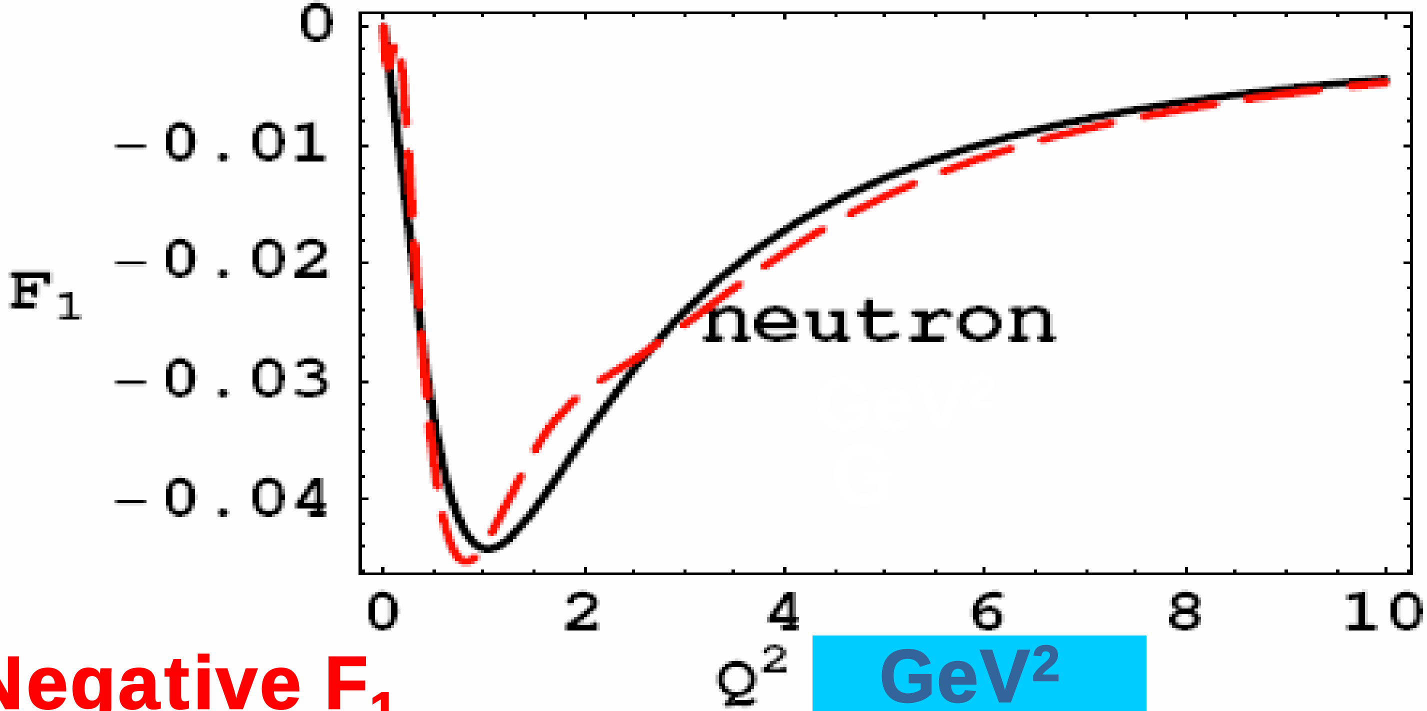


BBBA

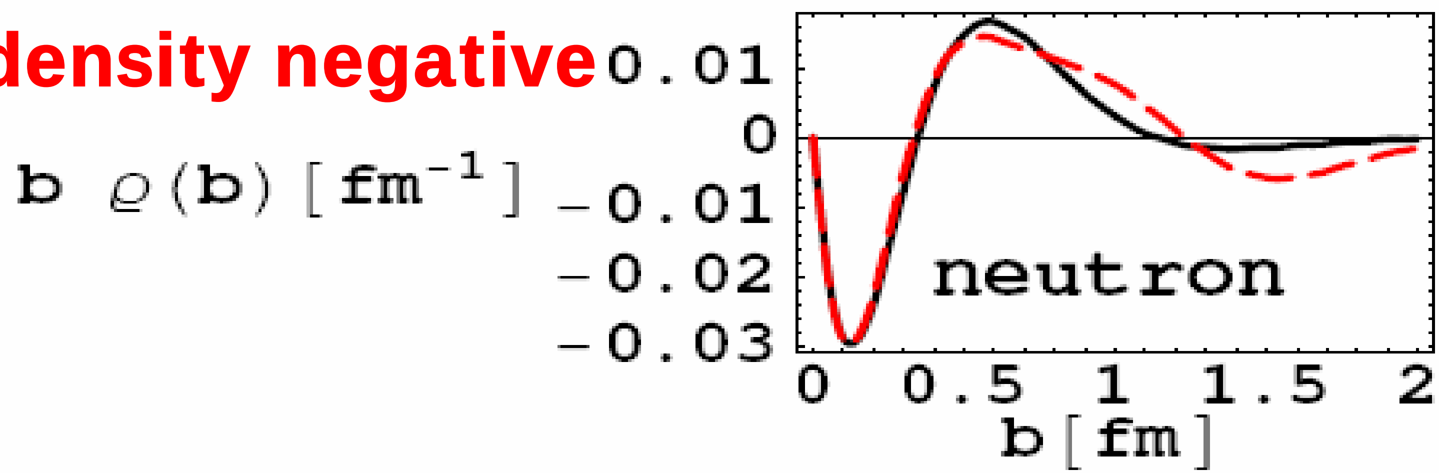
Kelly



Negative



Negative F_1 means central density negative



Resolution - G_E vs F_1 ∞ Momentum Frame IMF

$$J^\mu(0) = \bar{u}(p')\left(\gamma^\mu(F_1 + F_2) - \frac{p^\mu + p'^\mu}{2M}\right)u(p)$$

Breit frame (helicity flip) $J^0(0) = G_E$

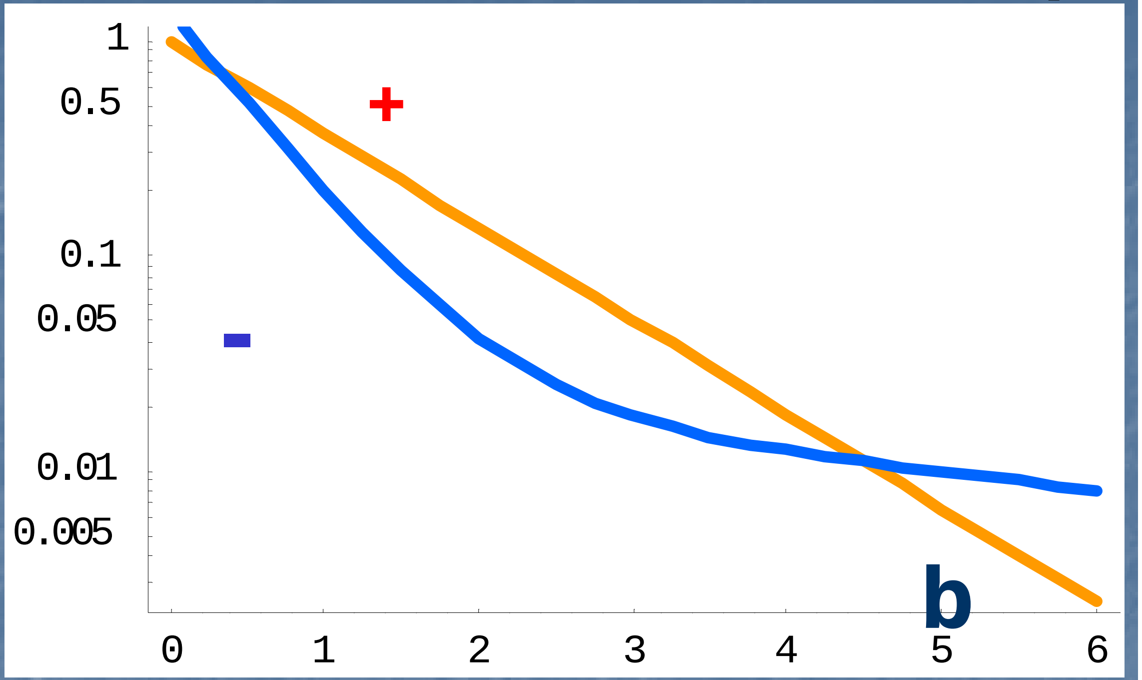
Boost to IMF $J^0 \sim (J^0 + J^3) = J^+$,

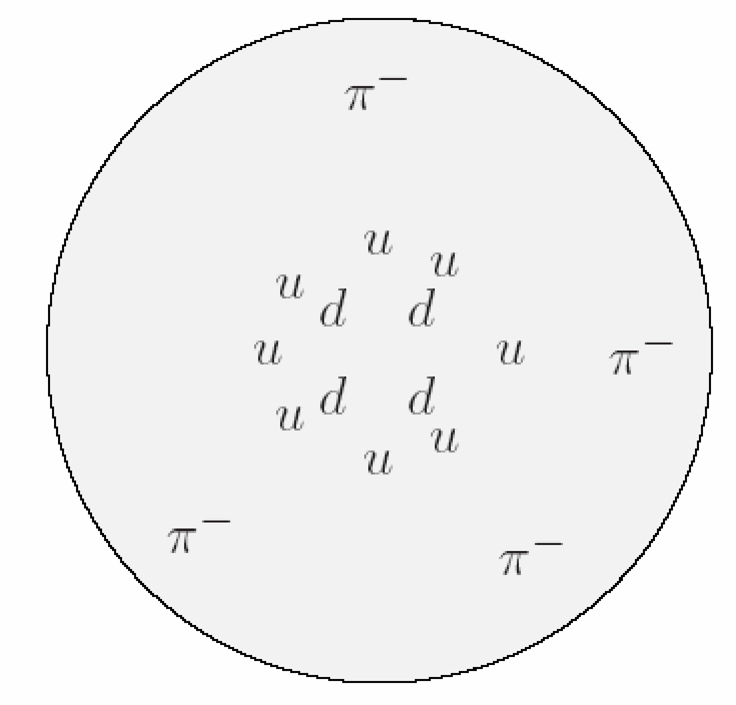
$$J^+(0) = \bar{u}(p')\left(\gamma^+(F_1 + F_2) - \frac{p^+}{M}\right)u(p)$$

Boost spinors to IMF, $\longrightarrow$ helicity non-flip,

$$J^+(0) = F_1$$

Neutron Interpretation

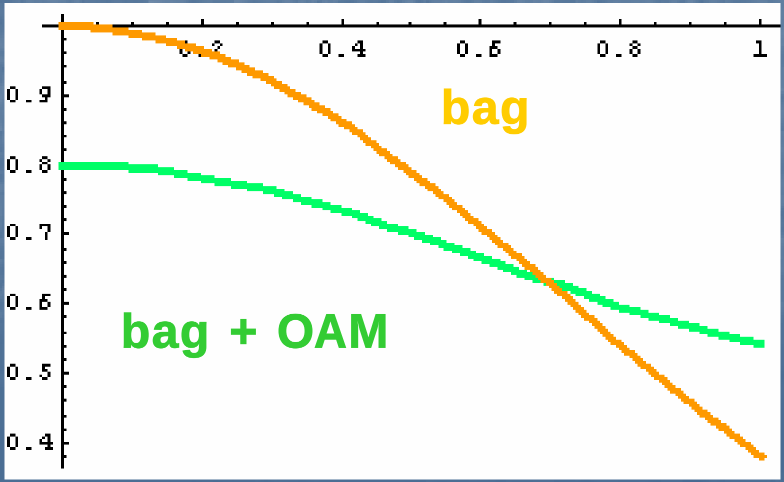




? π^- at short distance ?

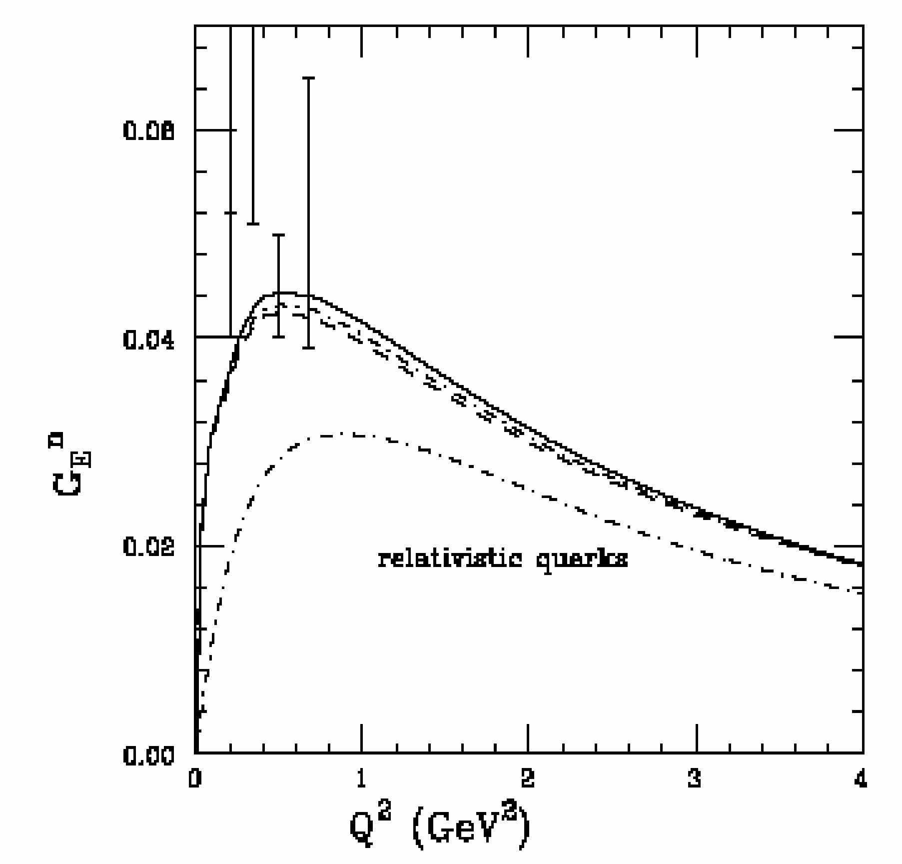
Central quark density reduced

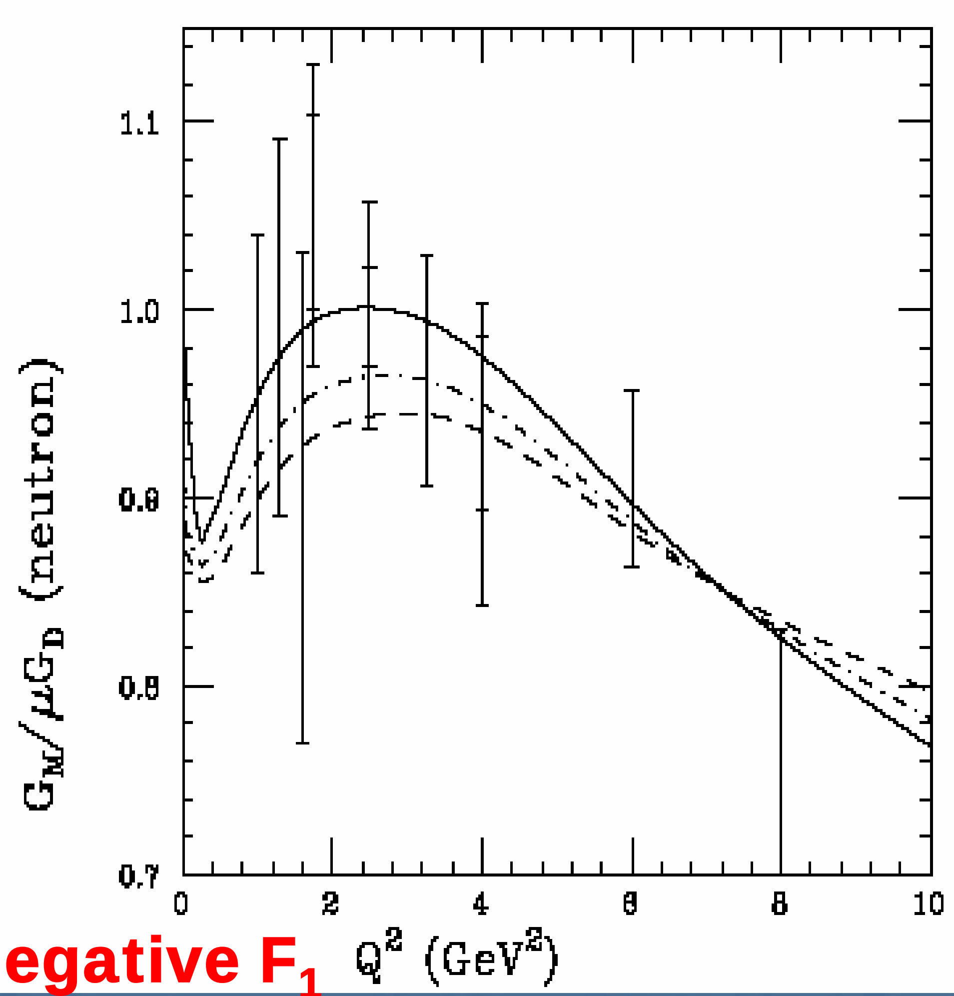
by orbital ang. momentum OAM?



Neutron Form Factors in LFCBM

Miller 2002

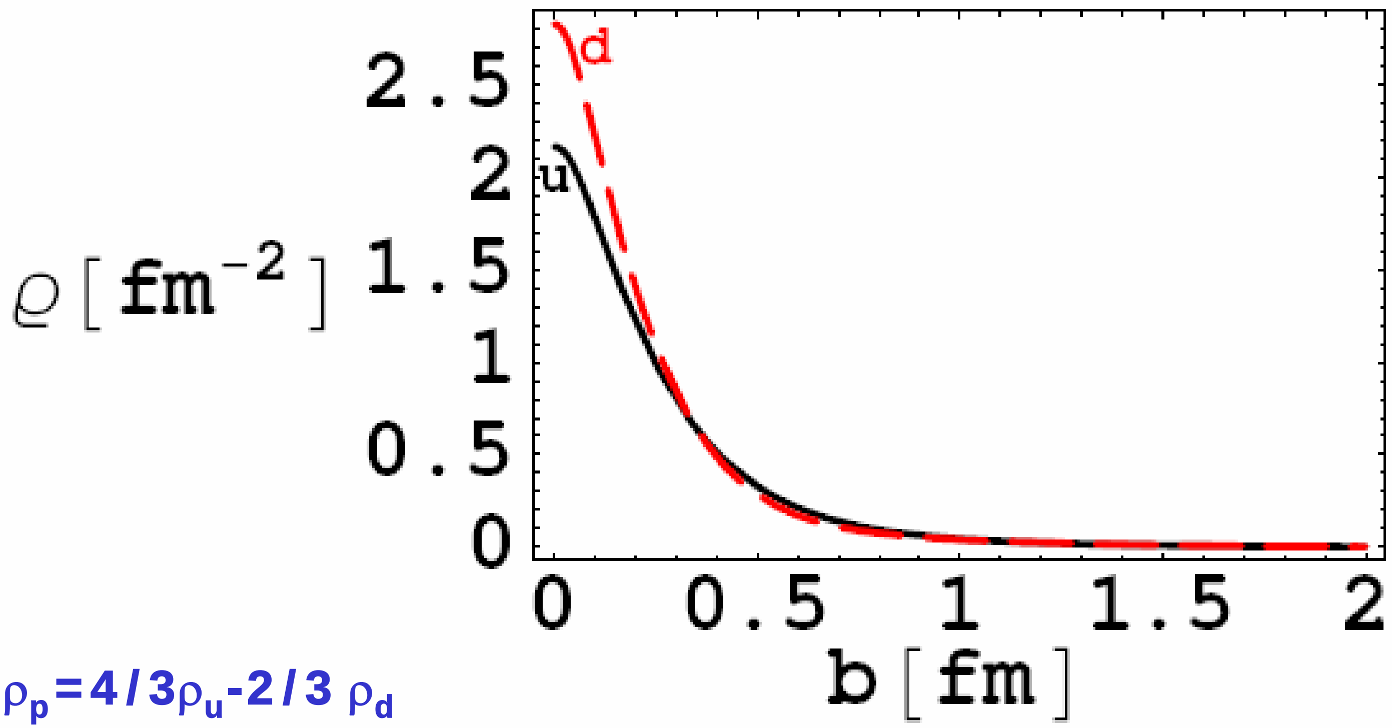




These give negative F_1

Charge symmetry: u in proton is d in neutron, d in proton is u in neutron

$\rho_u = \rho_p - \rho_n/2 \quad \rho_d = \rho_p - 2\rho_n$



$\rho_p = 4/3\rho_u - 2/3\ \rho_d$

Summary

- Model independent information on charge density

$$\rho(b) \equiv \sum_q e_q \int dx\, q(x,\mathbf{b}) = \int d^2q F_1(Q^2 = \mathbf{q}^2) e^{i\,\mathbf{q}\cdot\mathbf{b}}$$

• Central charge density of neutron is negative

• Pion cloud at large b