

# Electron and Photon Interactions with Deuterium in Chiral Perturbation Theory



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# OUTLINE

- Chiral perturbation theory for nuclear forces
- ChiPT predictions for deuteron electromagnetic form factors
- Precision predictions for  $G_C/G_Q$
- Compton scattering on deuterium and nucleon polarizability extractions
- Conclusion

# Chiral perturbation theory

- Chiral perturbation theory is the most general  $\mathcal{L}(\mathcal{N}, \pi)$  consistent with the  $SU(2)_L \times SU(2)_R$  of QCD and the pattern of its breaking.  $\mathcal{L}(\mathcal{N}, \pi)$  is an expansion in
$$P \equiv \frac{p, m_\pi}{m_\rho, 4\pi f_\pi, M}$$
- Unknown coefficients at a given order in  $P$  need to be determined from lattice or experimental data
- ChiPT is the low-energy effective theory of QCD. It is **model independent** and **systematically improvable**.
- Many successful applications in A=0 and A=1

# ChiPT for nuclear forces

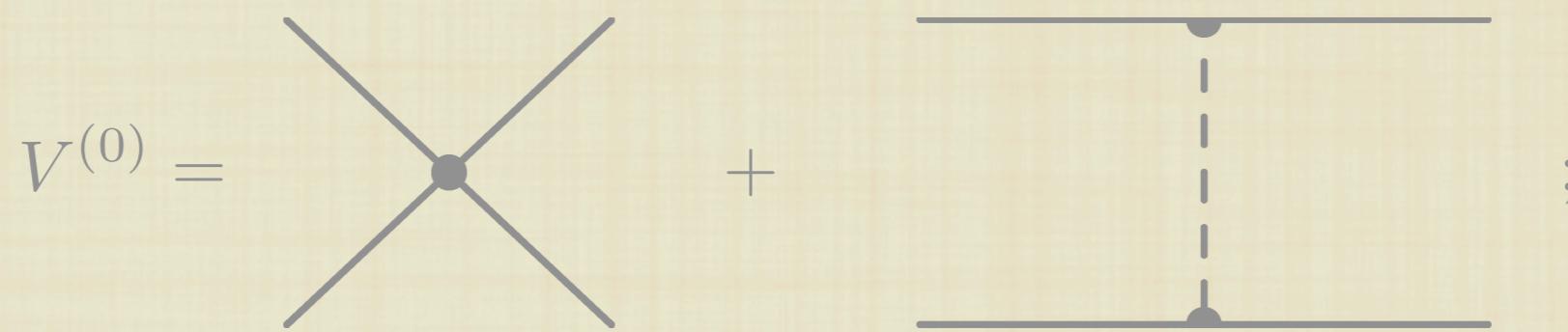
- ChiPT $\Rightarrow$ pion interactions are weak at low energy.

Weinberg (1990), apply ChiPT to V:

$$(E - H_0)|\psi\rangle = V|\psi\rangle$$

$$V = V^{(0)} + V^{(2)} + V^{(3)} + \dots$$

(Ordonez, Ray, van Kolck; Epelbaum, Meissner, Gloeckle; Entem, Machleidt)

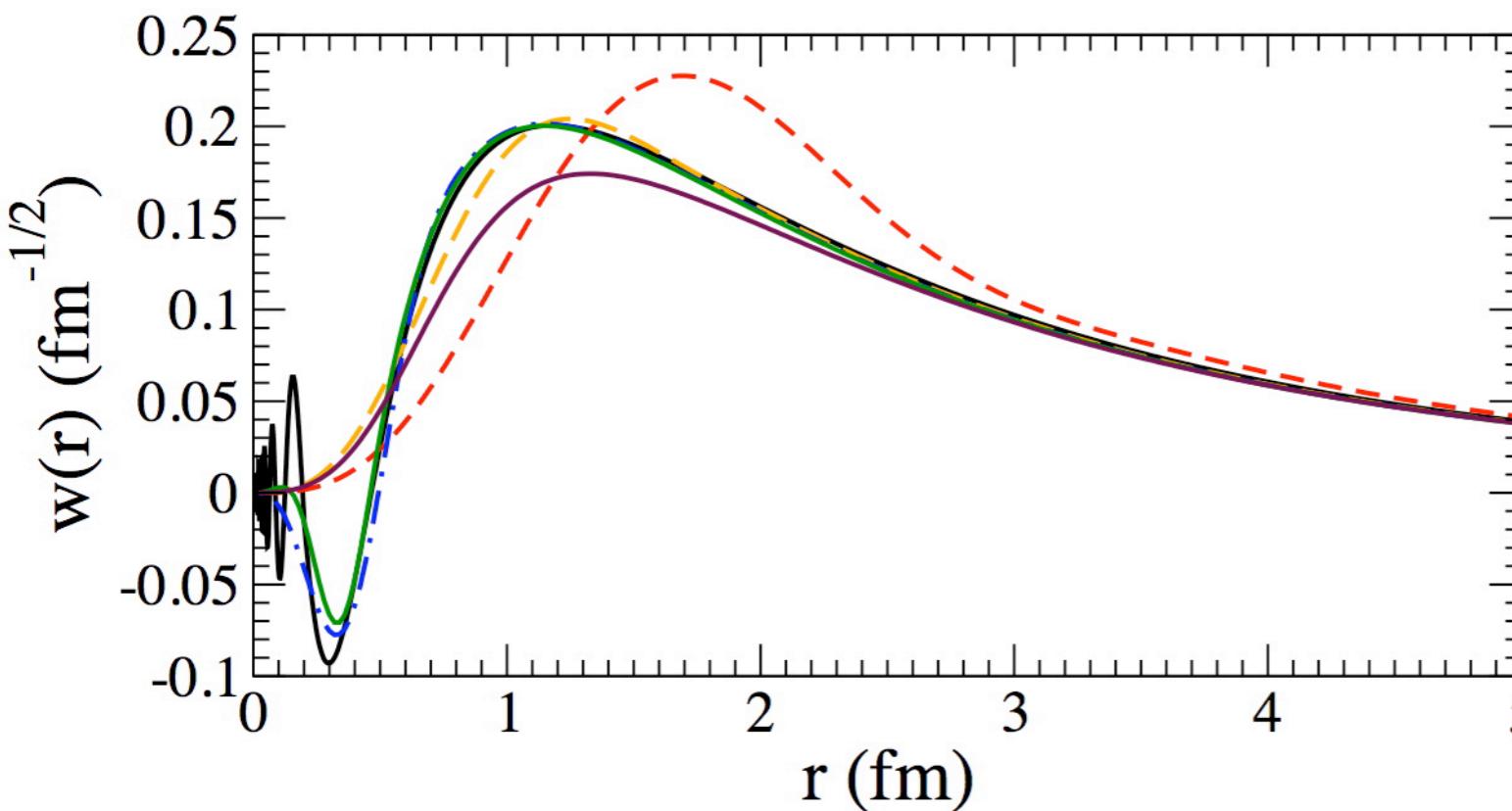
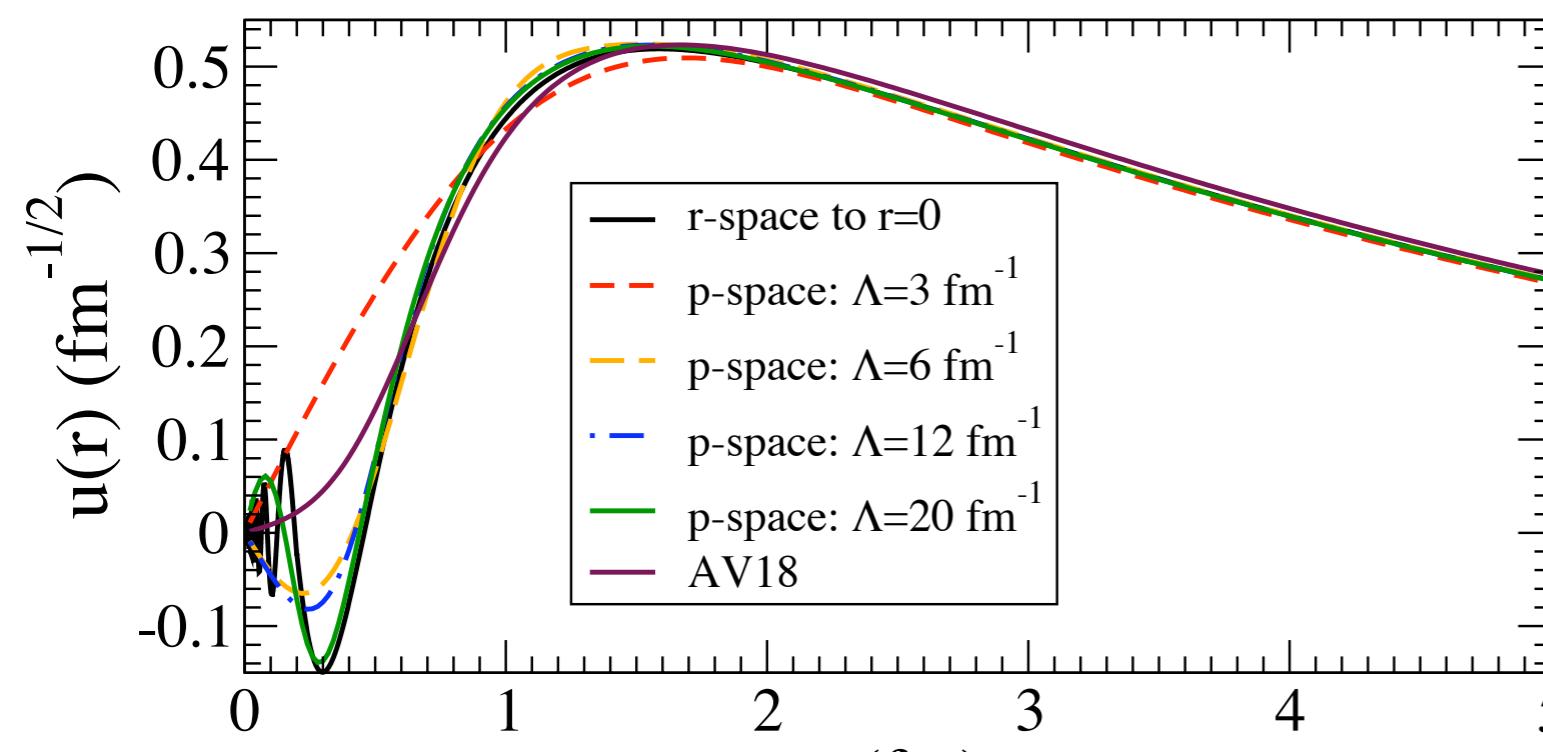


- Requires regularization and renormalization

- Cutoff  $\Lambda$ . Fit  $C(\Lambda)$  to, e.g. deuteron binding energy.

(Kaplan, Savage, Wise; Beane, Bedaque, Savage, van Kolck; Pavon Valderrama, Ruiz Arriola; Nogga, Timmermans, van Kolck; Birse)

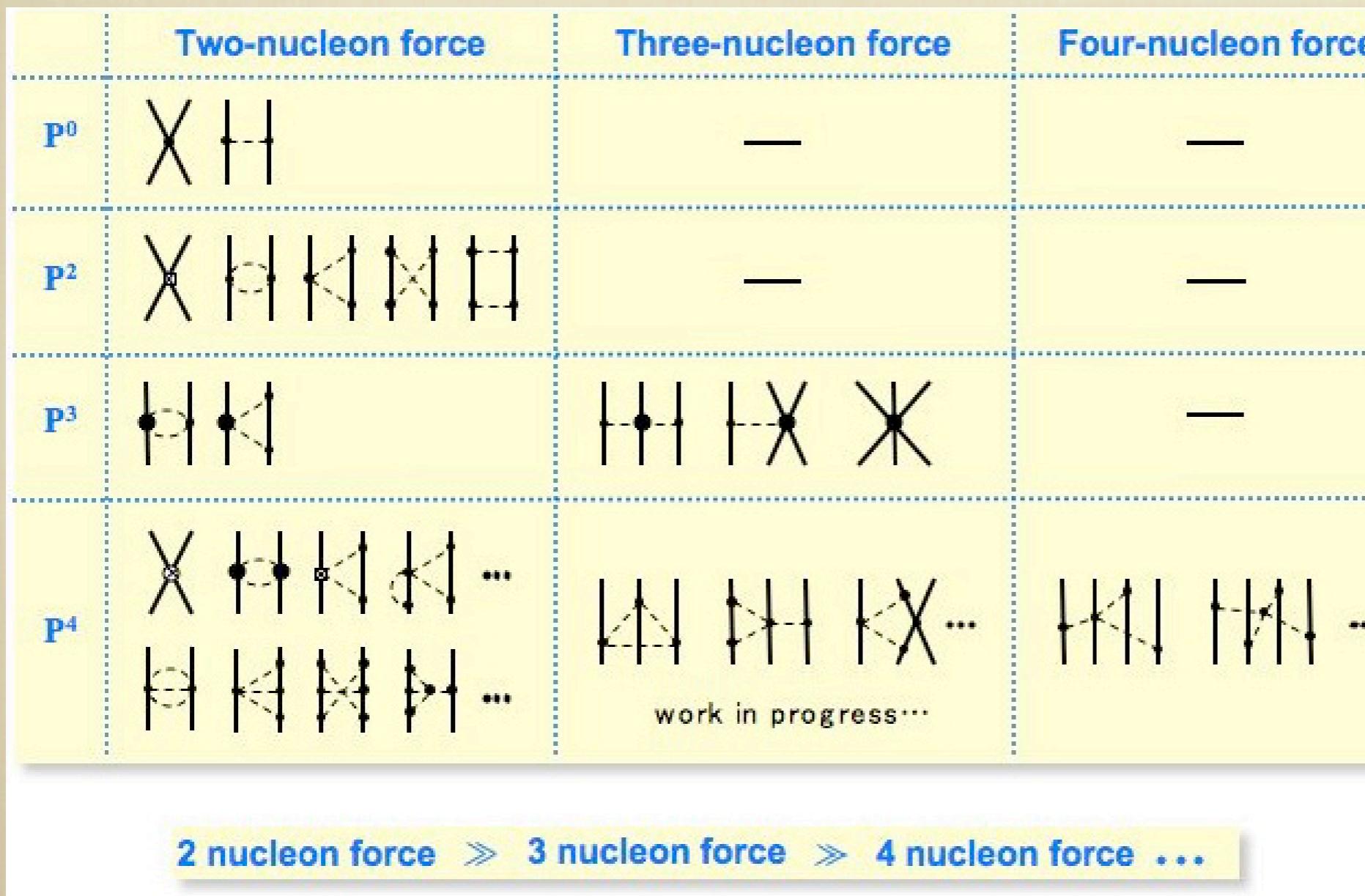
# ChiPT deuteron wave functions at leading order



- Converges to definite result
- Nodes associated with deeply-bound states
- $r <$  about 1 fm  
⇒ ChiPT invalid
- TPE corrections at NLO

# Higher orders in V

- Two-pion exchange, additional contact terms at NLO
- $N^3LO$  yields fit to NN data comparable to, e.g. AV18s

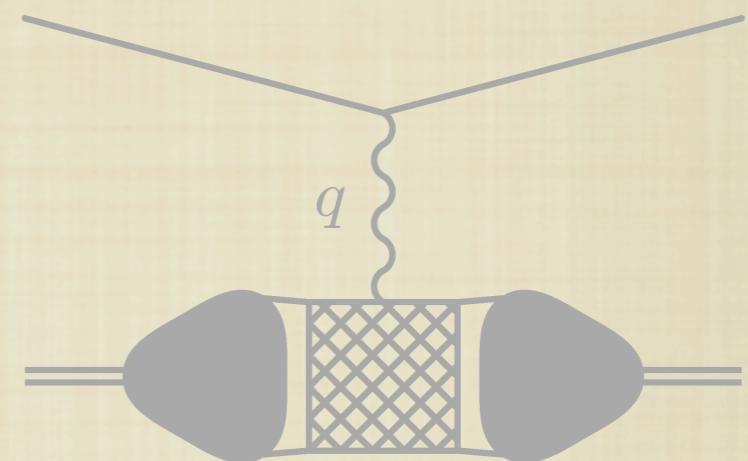


Consistent  
3NFs, 4NFs!

Courtesy  
E. Epelbaum

# Testing NN forces in elastic electron-deuteron scattering

- LO QED: electron couples to  $\mathbf{J}_\mu$
- LO ChiPT:  $J_0(\mathbf{r}) = |e| \delta^{(3)}(\mathbf{r} - \mathbf{r}_p)$
- Deuteron form factor:  $G_C(|\mathbf{q}|) = \int dr j_0 \left( \frac{|\mathbf{q}| r}{2} \right) [u^2(r) + w^2(r)]$
- Change  $Q^2 = -\mathbf{q}^2 \Rightarrow$  change spatial resolution
- Prediction of QED and NN force model



# Electron-deuteron observables

$$G_C = \frac{1}{3|e|} (\langle 1 | J^0 | 1 \rangle + \langle 0 | J^0 | 0 \rangle + \langle -1 | J^0 | -1 \rangle),$$

$$G_Q = \frac{1}{2|e|\eta M_d^2} (\langle 0 | J^0 | 0 \rangle - \langle 1 | J^0 | 1 \rangle)$$

$$G_M = -\frac{1}{\sqrt{2\eta}|e|} \langle 1 | J^+ | 0 \rangle; \quad \eta = \frac{Q^2}{4M_d^2}$$

**THEORY**

$$A = G_C^2 + \frac{2}{3}\eta G_M^2 + \frac{8}{9}\eta^2 M_d^4 G_Q^2,$$

$$B = \frac{4}{3}\eta(1+\eta)G_M^2,$$

$$\begin{aligned} T_{20} = & -\frac{1}{\sqrt{2}} \frac{1}{A(Q^2) + B(Q^2) \tan^2(\frac{\theta_e}{2})} \left[ \frac{8}{3}\eta G_C(Q^2) G_Q(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2) \right. \\ & \left. + \frac{1}{3}\eta \left\{ 1 + 2(1+\eta) \tan^2 \left( \frac{\theta_e}{2} \right) \right\} G_M^2(Q^2) \right]. \end{aligned}$$

**EXPERIMENT**

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[ A(Q^2) + B(Q^2) \tan^2 \left( \frac{\theta_e}{2} \right) \right]; \quad T_{20}(Q^2; \theta_e)$$

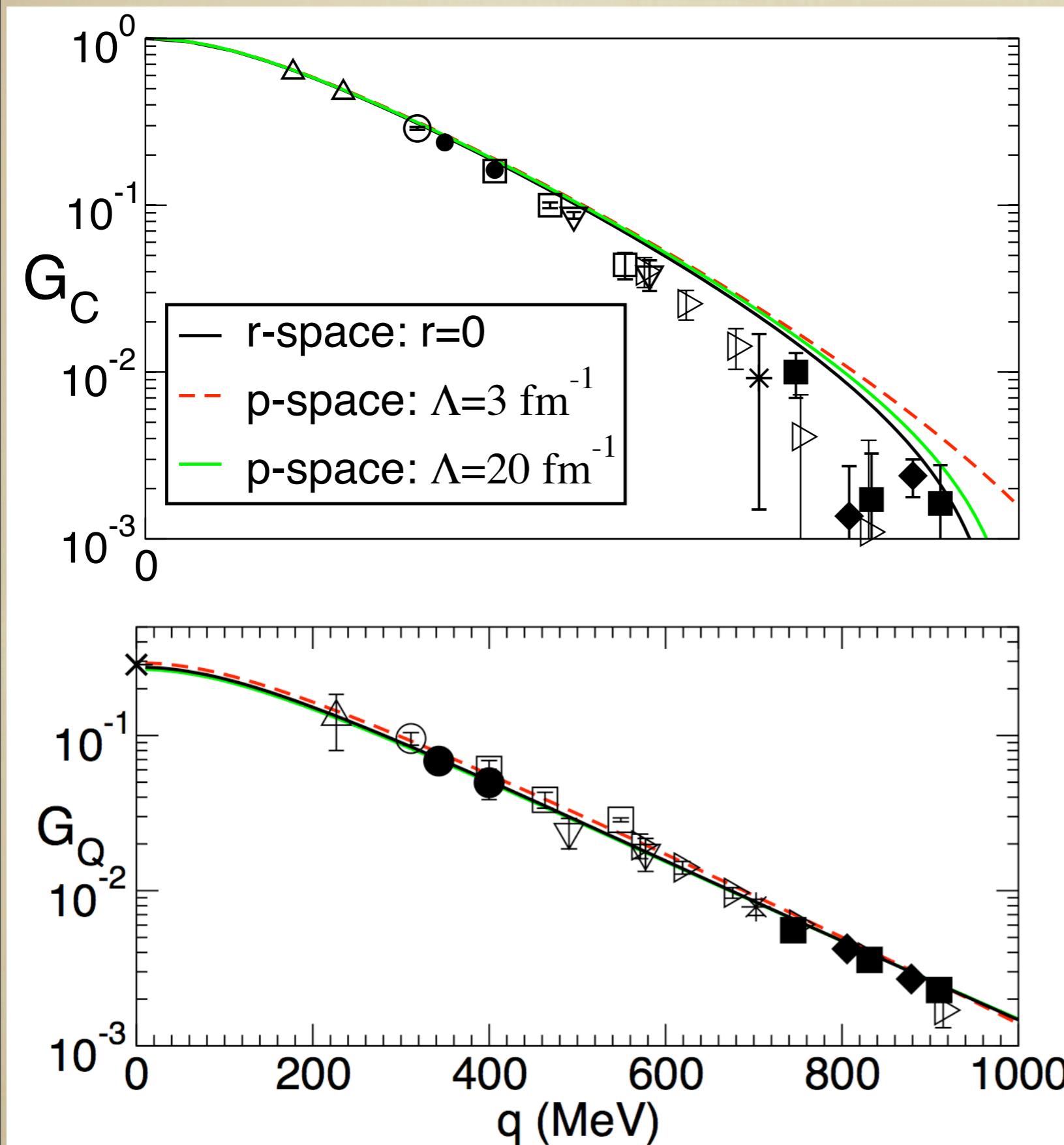
# Recent experiments; strategy

- JLab Hall A:  $A(Q^2)$   $Q^2=0.7\text{-}6 \text{ GeV}^2$ ,  $B(Q^2)=?$
- JLab Hall C:  $T_{20}(Q^2)$ ,  $A(Q^2)$   $Q^2=0.66\text{-}1.8 \text{ GeV}^2$
- Novosibirsk:  $T_{20}(Q^2)$   $Q^2=0.32\text{-}0.84 \text{ GeV}^2$
- BLAST:  $T_{20}(Q^2)$   $Q^2=0.137\text{-}0.667 \text{ GeV}^2$  **PRELIMINARY**
- JLab Hall A:  $A(Q^2)=0.04\text{-}0.64 \text{ GeV}^2$  **IN PROGRESS**
  - $B$  gives  $G_M$
  - $T_{20}$  gives  $G_C/G_Q$ ;  $A$  yields  $G_C^2 + G_Q^2$

## STRATEGY

Abbott et al., Eur. Phys. J. A47, 421 (2000) up to  $Q^2=1.4 \text{ GeV}^2$

# Results for $G_C$ and $G_Q$ at leading order



Pavon Valderrama, Ruiz Arriola,  
Nogga, DP (2007)

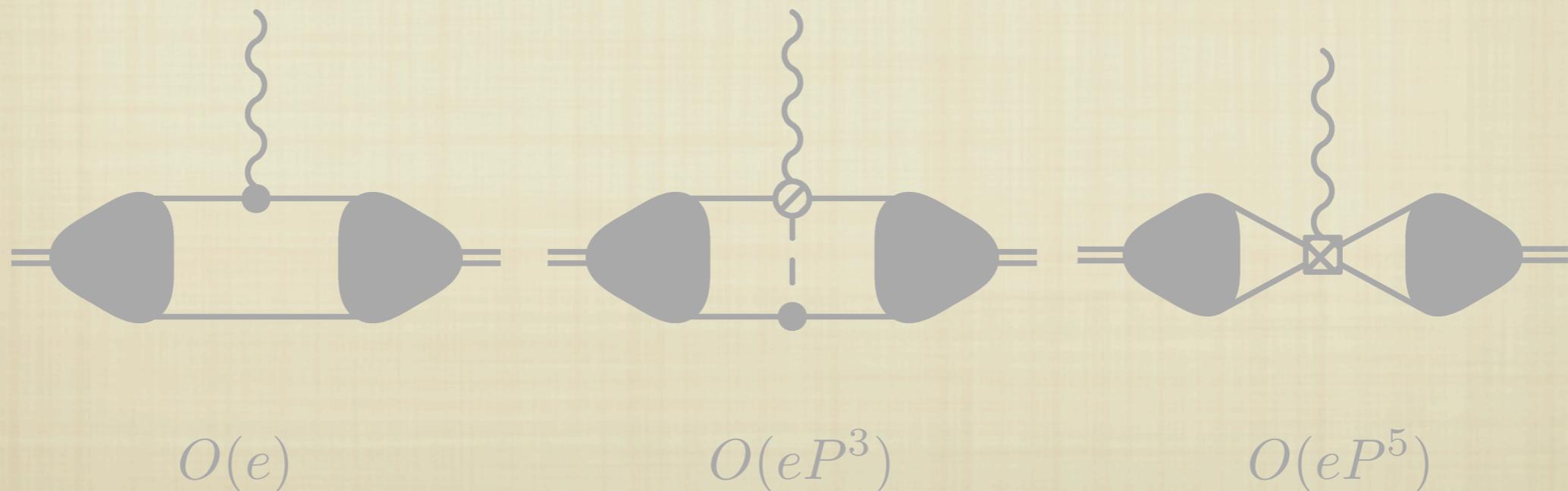
Nucleon  
form factors  
included via:

$$\frac{G_C}{G_E^{(s)}} = \langle \psi | e | \psi \rangle + O(P^2)$$

Two-pion  
exchange at  
 $O(P^2)$ ; large  
at  $O(P^3)$ .

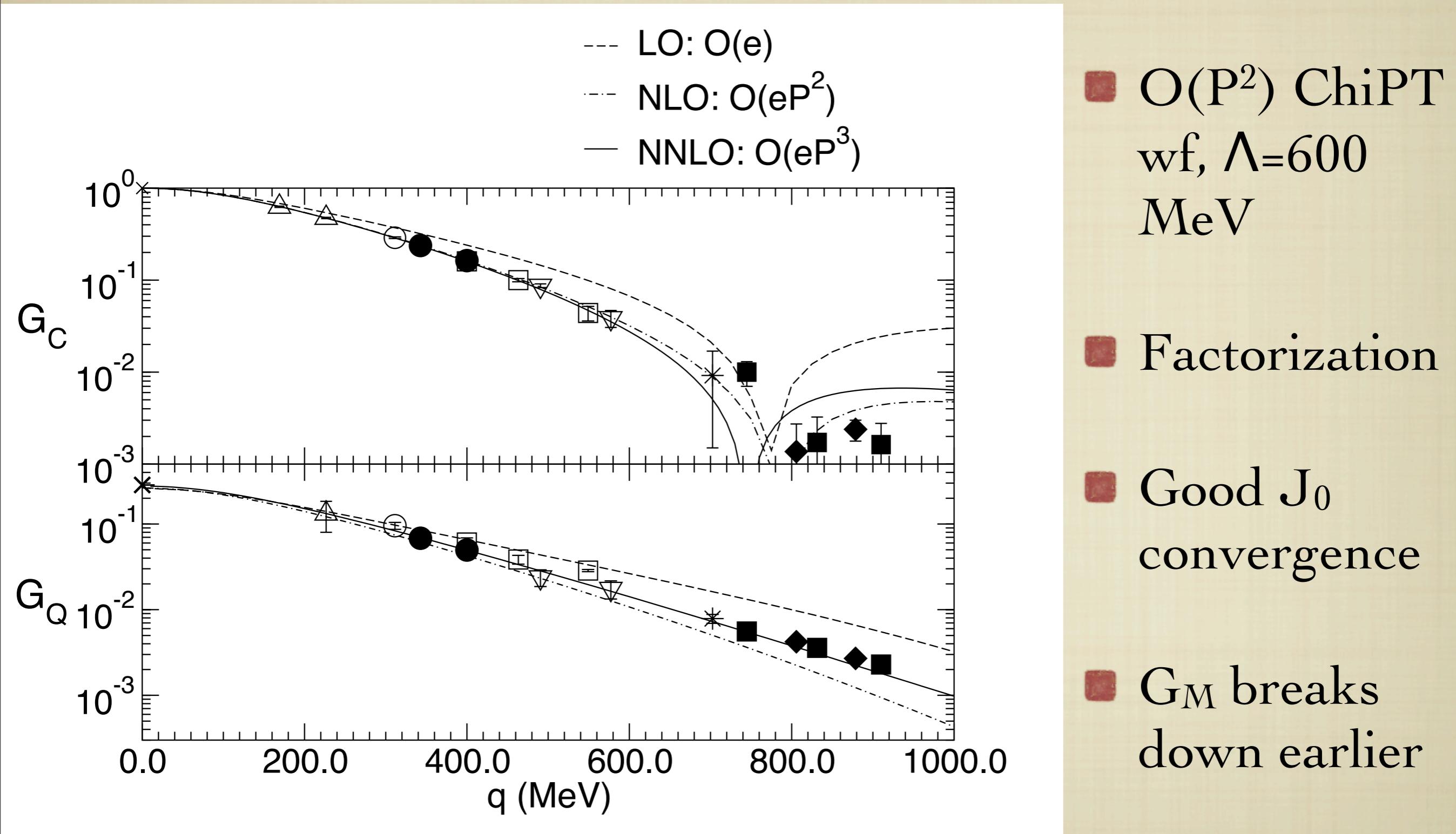
# Consistent charge operator

- To test  $O(P^2)$  &  $O(P^3)$  TPE need  $J_0$  to  $O(eP^3)$
- $O(eP^2)$ : Nucleon structure,  $1/M^2$  effects
- $O(eP^3)$ : 2B mechanism enters, but no free parameters
- $O(eP^4)$ : Two-pion exchange pieces of  $J_0$

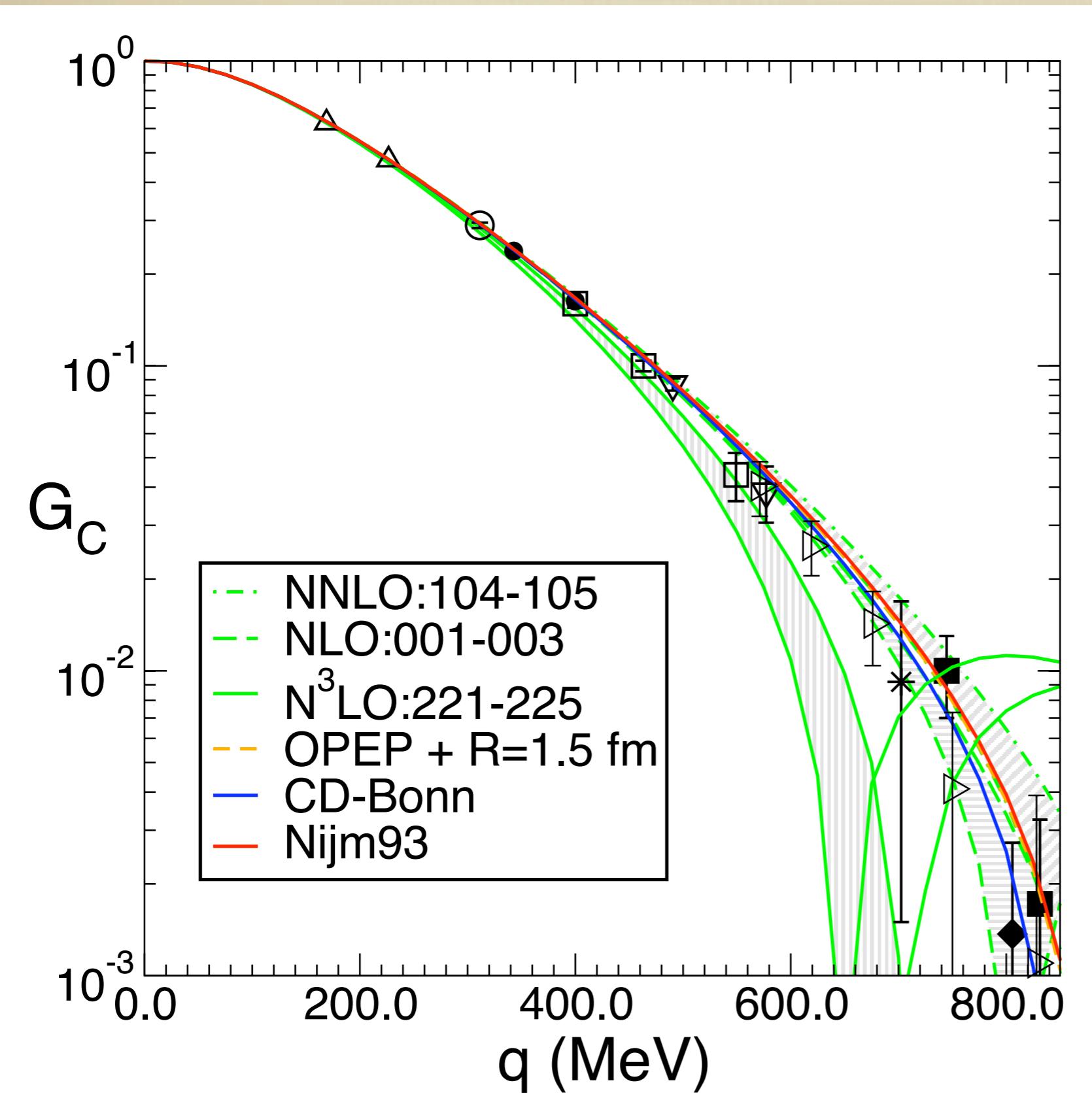


DP and Cohen (1999); Park et al. (1999); Meissner and Walzl (2001); DP (2003, 2006)

# Results for form factors

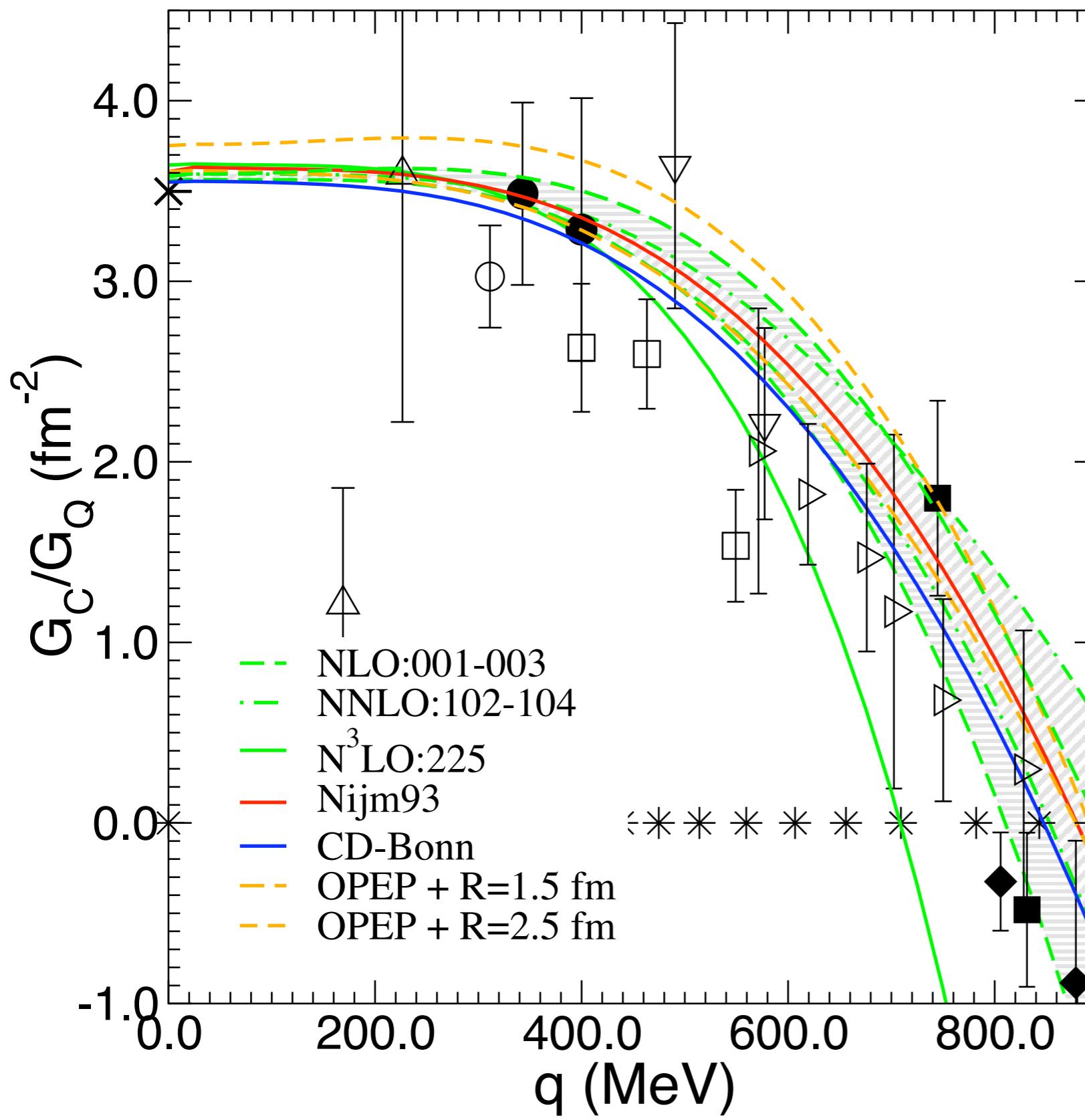


# ChiPT results for $G_C$



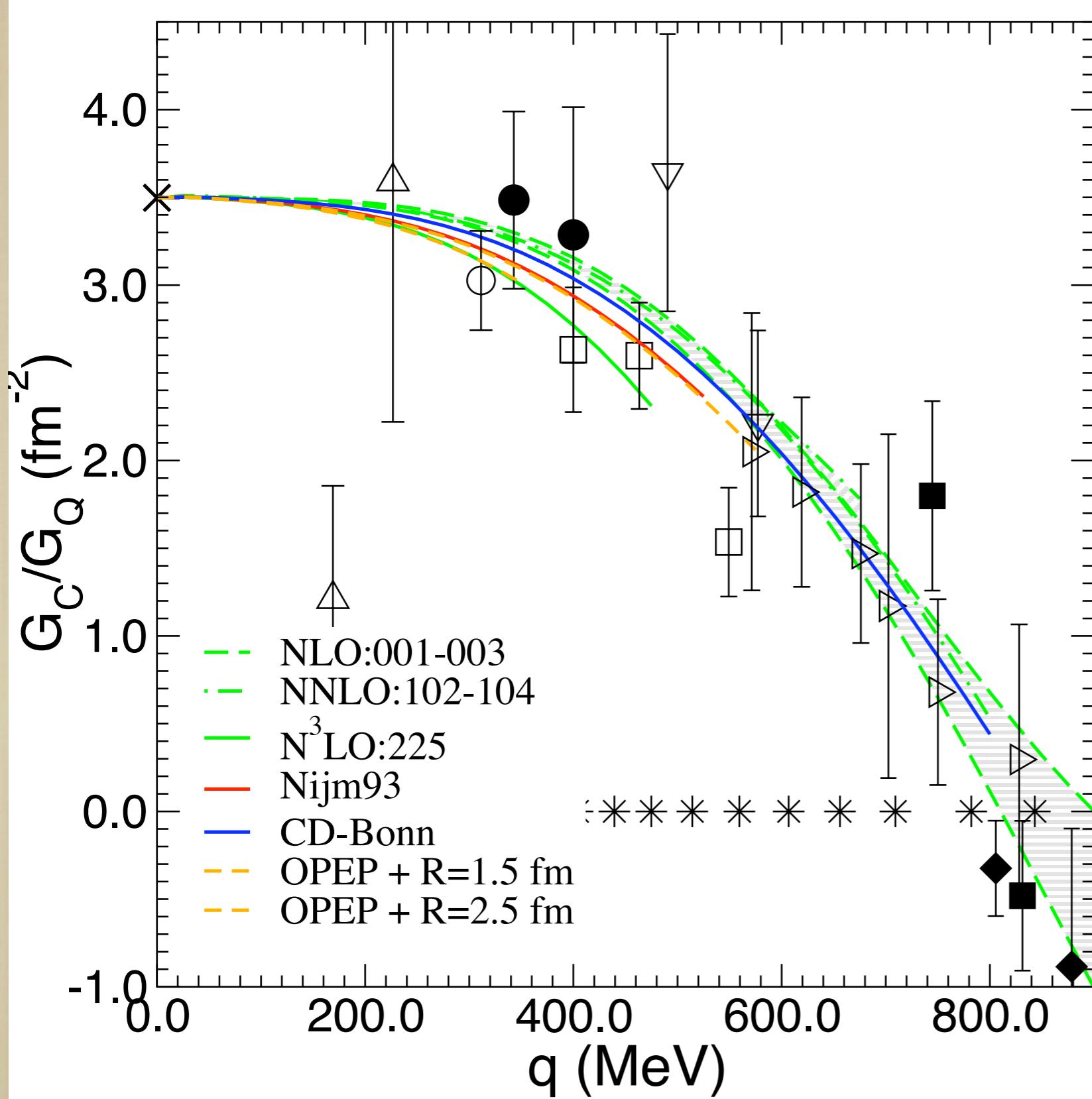
- Insensitive to  $r \sim 1/\Lambda$  physics
- Agreement with data: data questions for  $q = 0.2 - 0.4$  GeV
- Sensitivity to  $\pi N$  LECs
- ChiPT wf at  $O(P^4)$ ?

# Precision for $G_C/G_Q$ ?



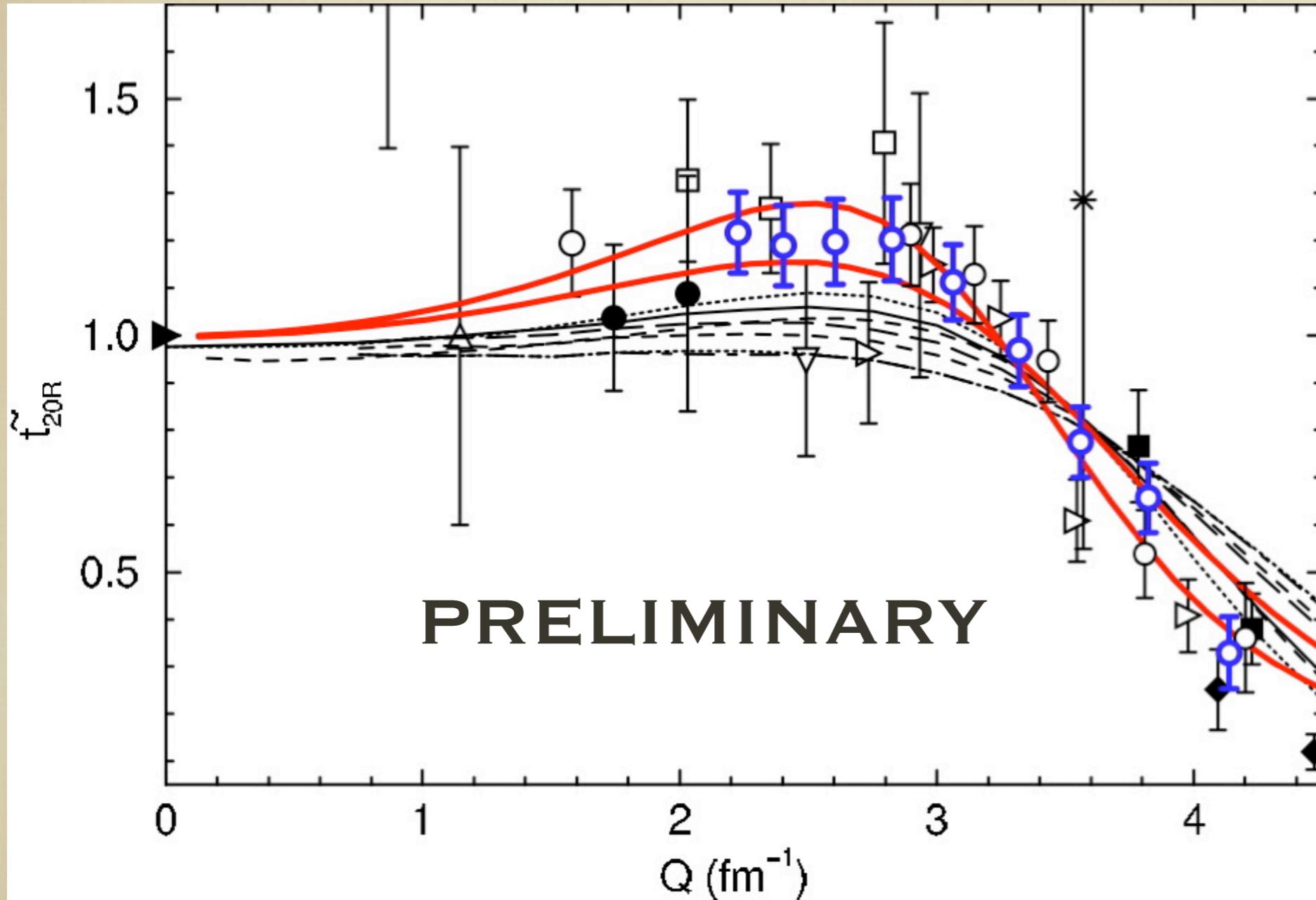
- Nucleon structure cancels out
- Variation in value of  $Q_d$  associated with physics at  $r \sim 1/\Lambda$ .

# Renormalized $G_C/G_Q$



- Adjust  $\mathcal{O}(eP^5)$  counterterm to reproduce  $Q_d$ : natural size
- Shape then largely model independent for  $q < 600 \text{ MeV}$
- $G_C/G_Q$  to 3% at  $q = 0.39 \text{ GeV}$

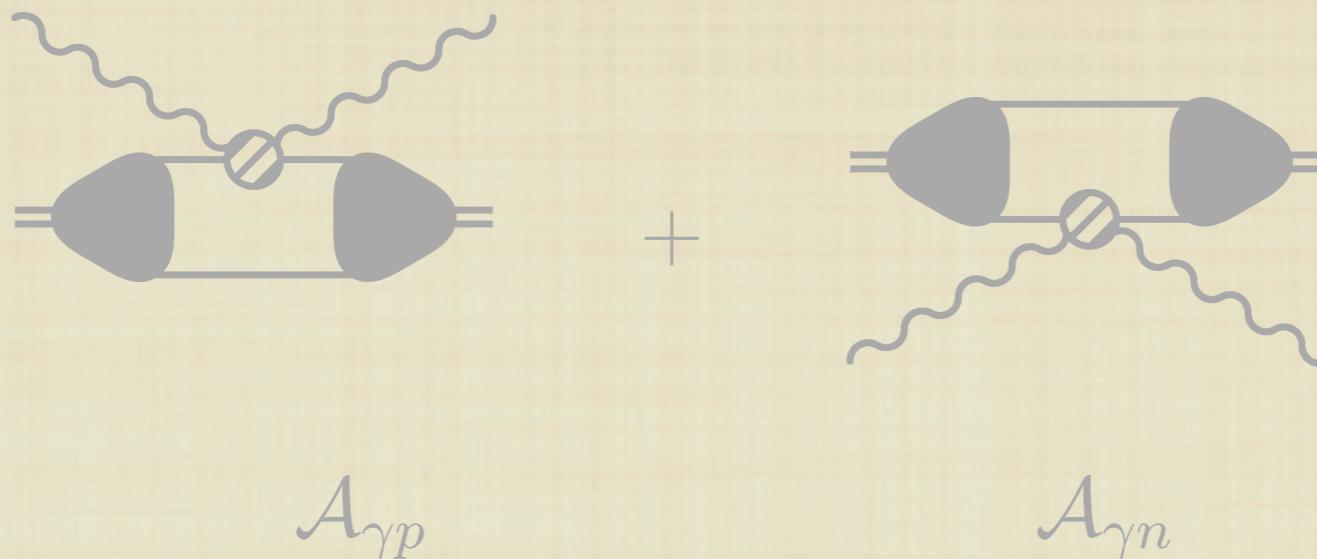
# BLAST data on $t_{20}$



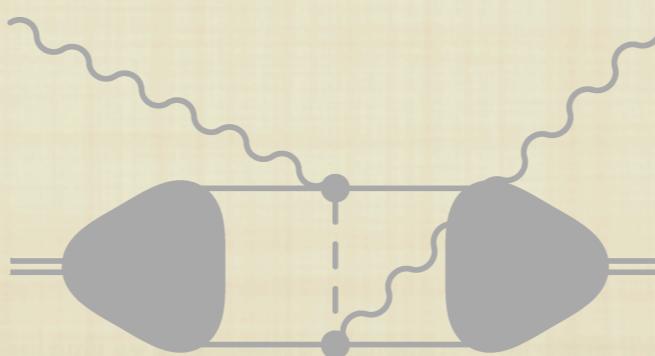
$$\tilde{t}_{20R} = -\frac{3\tilde{t}_{20}}{\sqrt{2}Q_d Q^2} \leftrightarrow G_C/G_Q$$

# Why Compton Scattering from Deuterium?

Goal is to investigate neutron Compton scattering

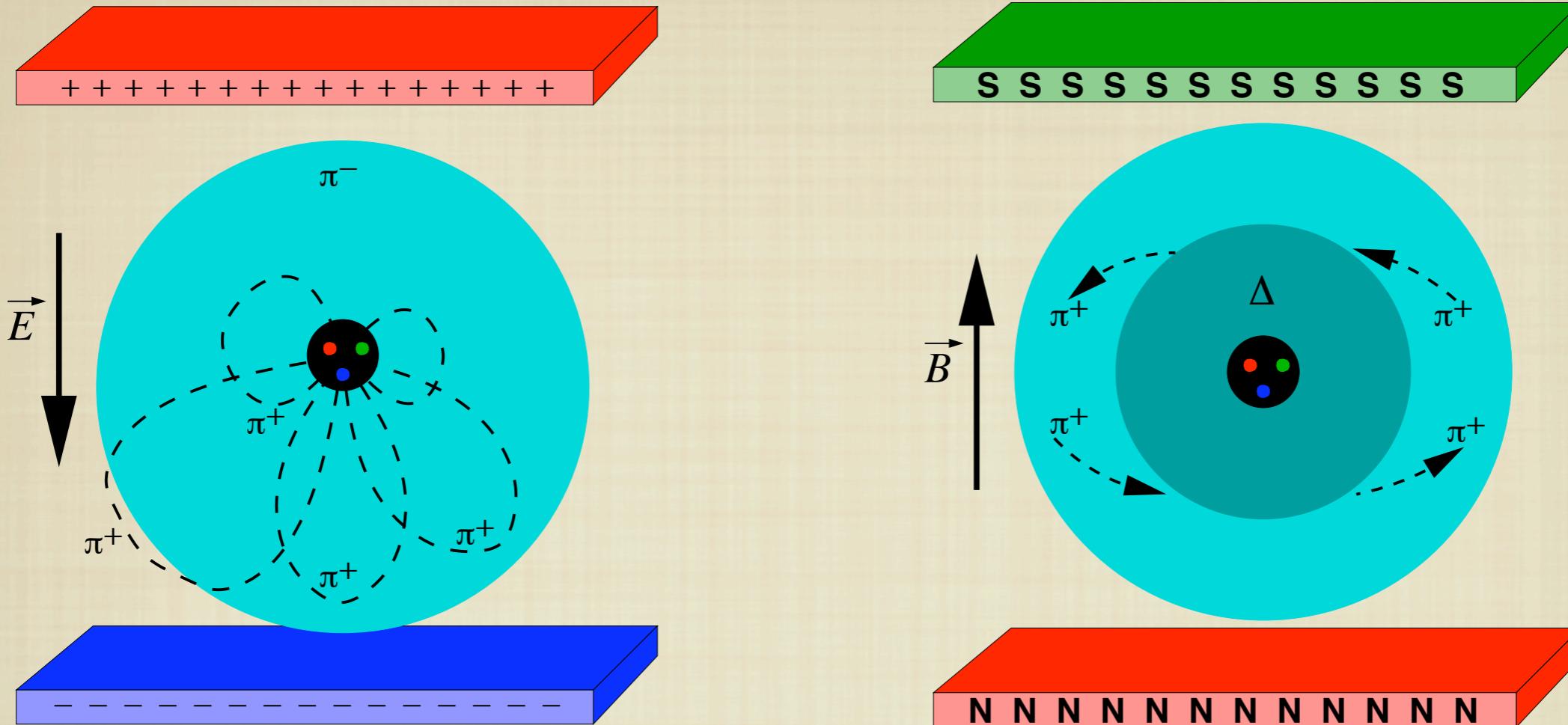


But two-body currents are large



Need systematic way to compute  $\gamma NN \rightarrow \gamma NN$  kernel

# Nucleon Polarizabilities



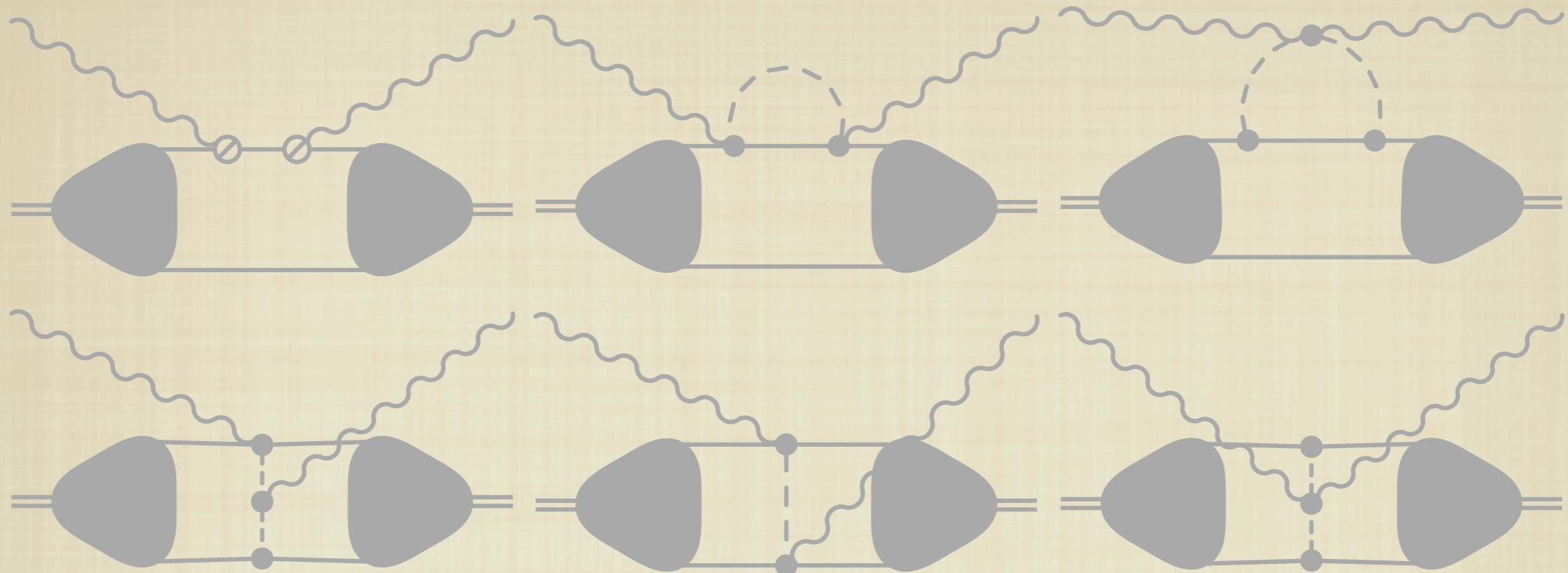
Pictures courtesy H. Griesshammer

$$H = -2\pi\alpha_E \mathbf{E}^2 - 2\pi\beta_M \mathbf{B}^2$$

$$\chi\text{PT } O(e^2 P) : \alpha_E^{(p)} = 10\beta_M^{(p)} = 12.5 \times 10^{-4} \text{ fm}^3$$

$$\alpha_E^{(p)} = \alpha_E^{(n)}; \quad \beta_M^{(p)} = \beta_M^{(n)}$$

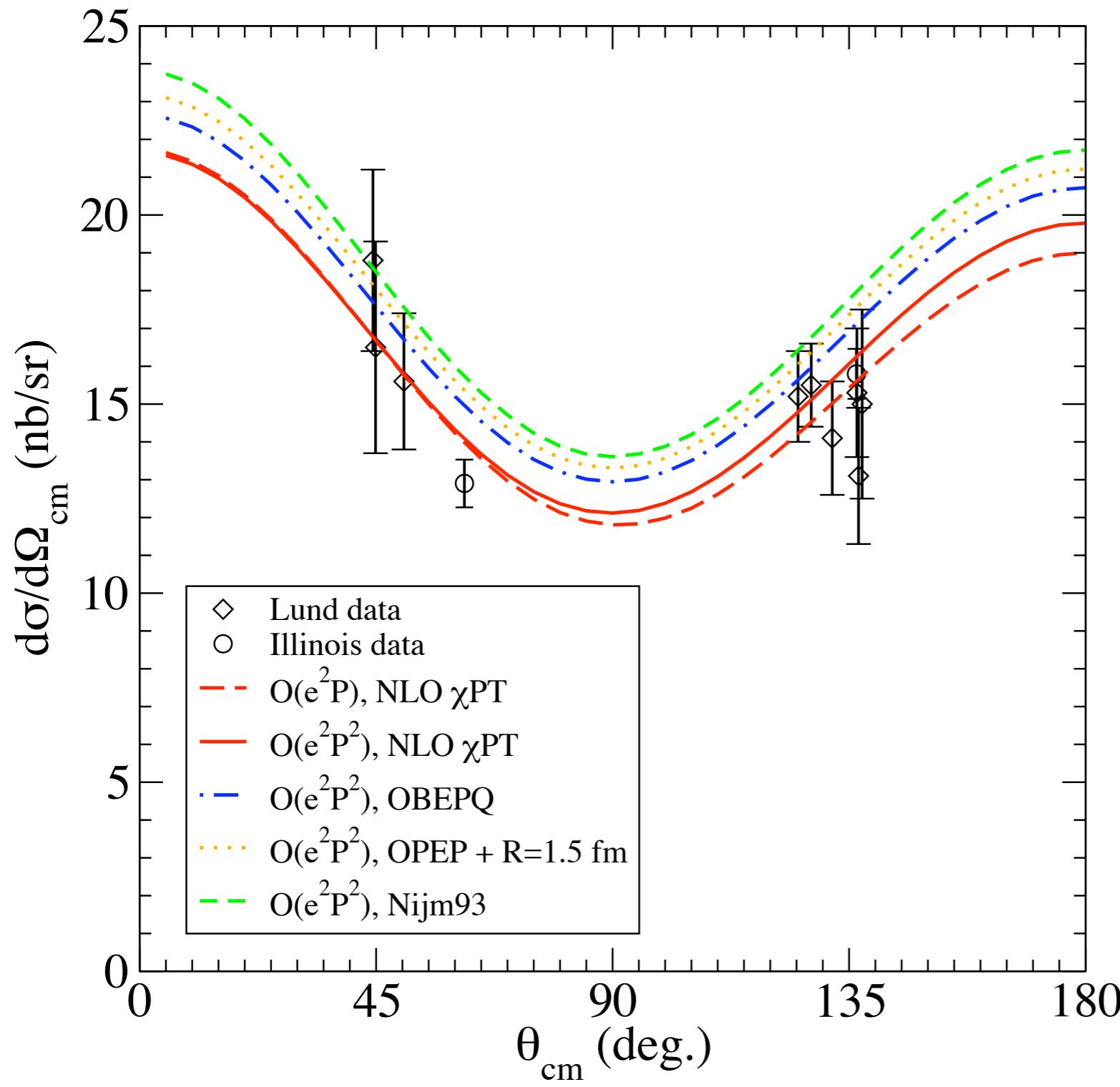
# $\gamma d$ scattering at $O(e^2 P)$ [NLO]



Beane, Malheiro, DP, van Kolck, Nucl. Phys. A (1999)

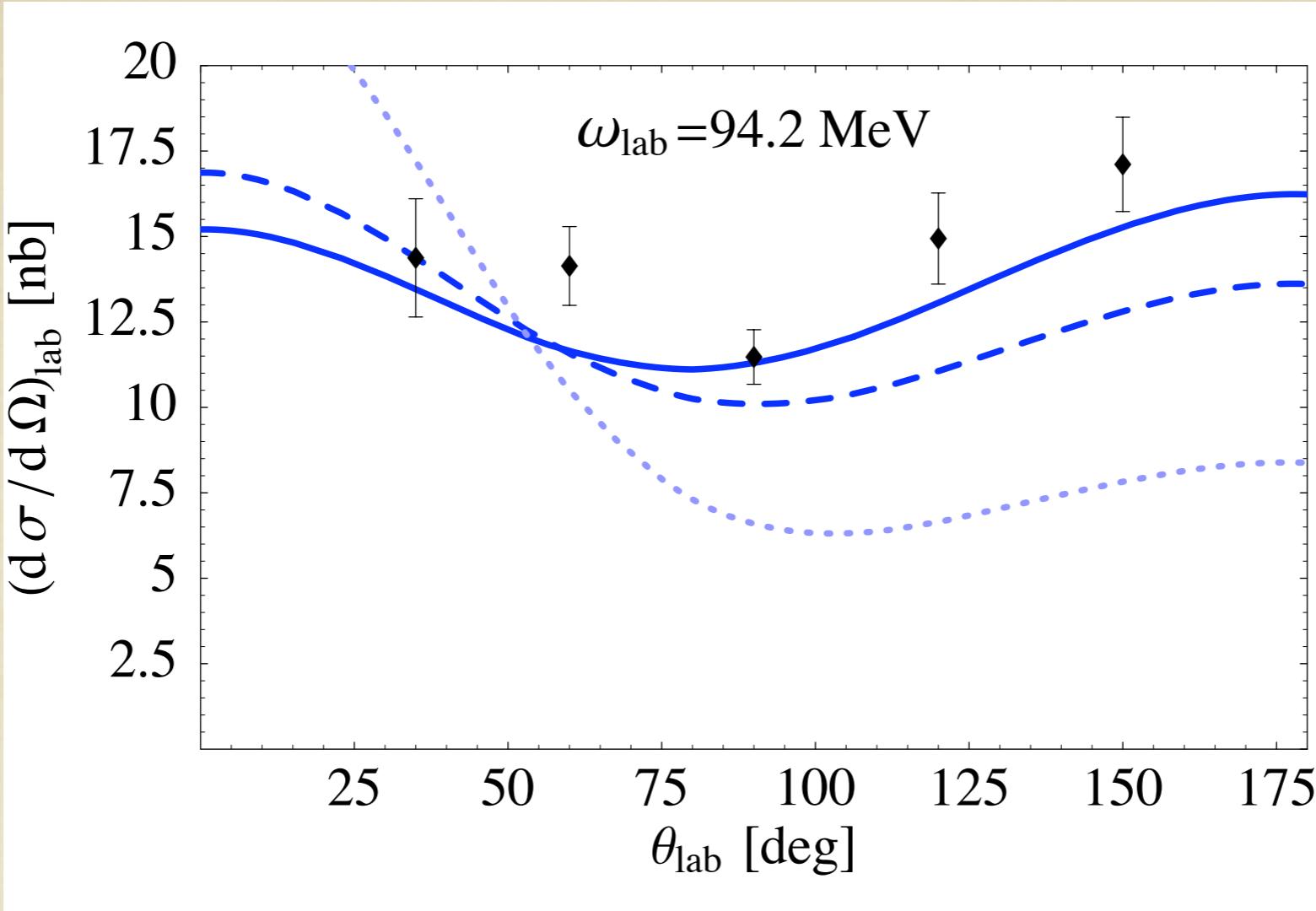
- No free parameters at  $O(e^2 P)$ : PREDICTION
- Polarizability and NN mechanisms on equal footing

# Results at $E_{\text{lab}}=66 \text{ MeV}$



- Good description; good convergence
- $O(e^2P^2)$ : two free parameters  
Beane, McGovern, Malheiro, Phillips, van Kolck (2004)
- Wave function dependence now understood  
Hildebrandt, Griesshammer, Hemmert (2005)
- Little sensitivity to polarizabilities here

# Results at E<sub>lab</sub>=95 MeV

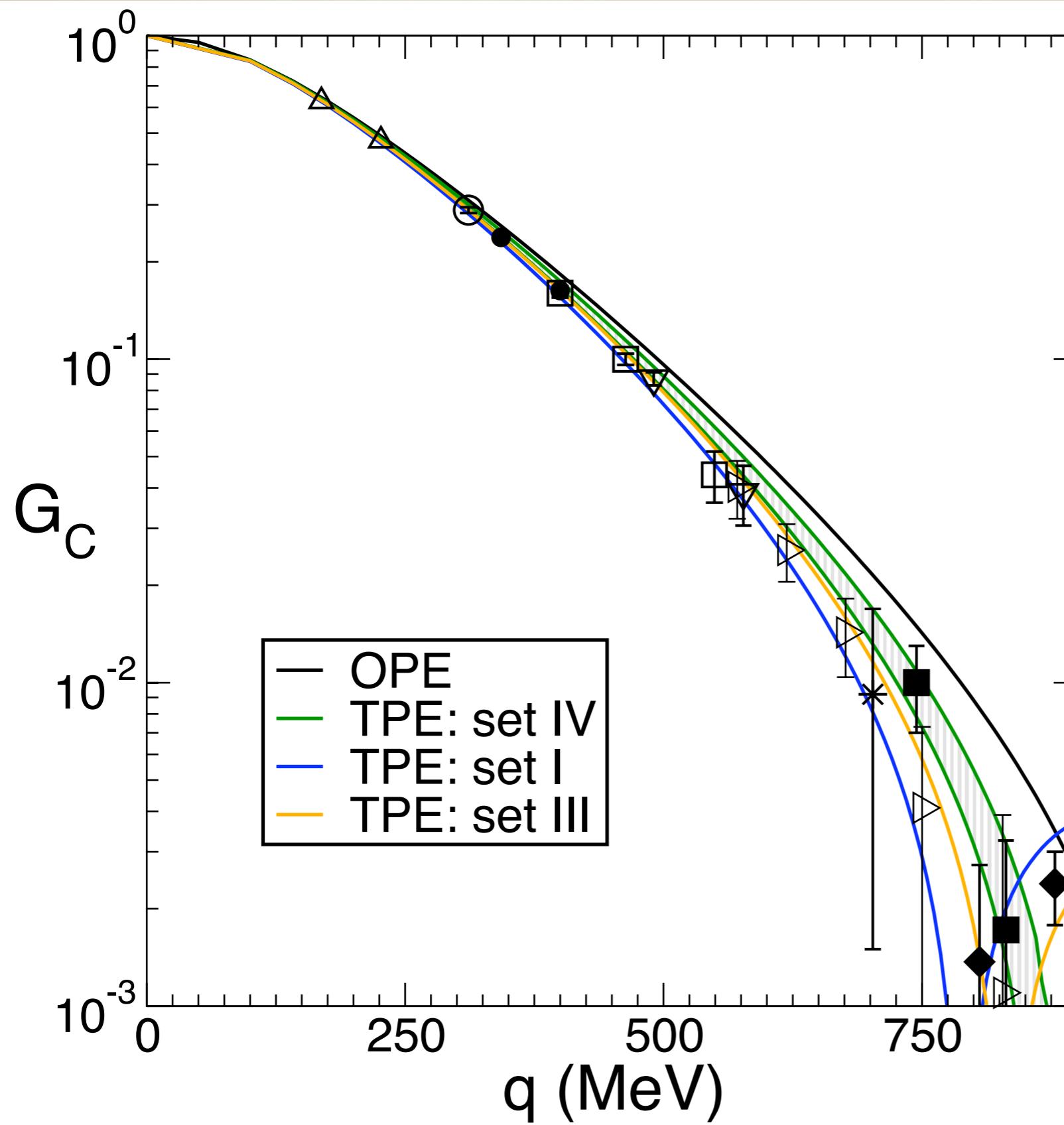


- $\Delta(1232)$  needed to describe backward-angle SAL data
- Prediction, assuming  $\alpha_E^{(p)} = \alpha_E^{(n)}$ ;  $\beta_M^{(p)} = \beta_M^{(n)}$
- Need better data for extraction: Compton@MAX-Lab, HIγS

# Conclusion

- Significant advances in ChiPT description of nuclear force over last ten years. Rigorous connection to QCD through lattice data now in sight.
- ChiPT used to reliably and systematically compute the interaction of low-energy photons with light nuclei
- New vigour to use of electromagnetic reactions as tool for investigating NN (and NNN) forces
- Light nuclei as effective neutron targets: extraction of neutron properties with reliable theoretical uncertainty

# Impact of two-pion exchange



- Impact on  $G_C$ 's minimum
- Sensitivity to  $\pi N$  LECs
- $G_Q$  largely insensitive

Pavon Valderrama, Ruiz  
Arriola, Nogga, DP (2007)