

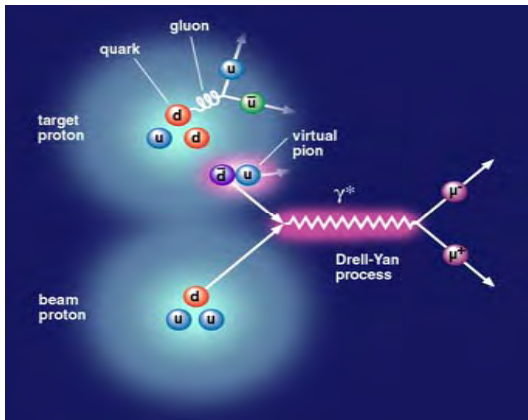


Exploring the nucleon structure with Drell-Yan

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Fermilab E866/Nusea Collaboration



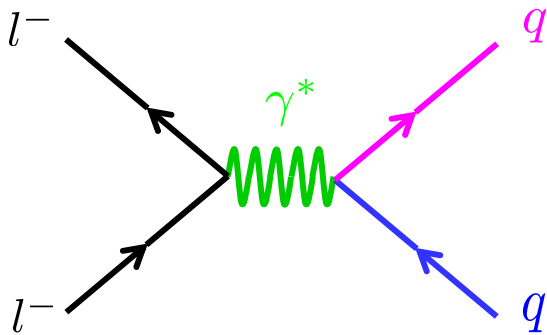
- ❖ \bar{d}/\bar{u} asymmetry
- ❖ nuclear dependence
- ❖ $\cos^2\phi$ asymmetry
- ❖ Projection with E906.



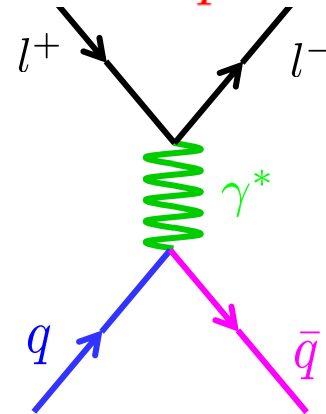
Drell-Yan & DIS



DIS : $ep \rightarrow e'X$



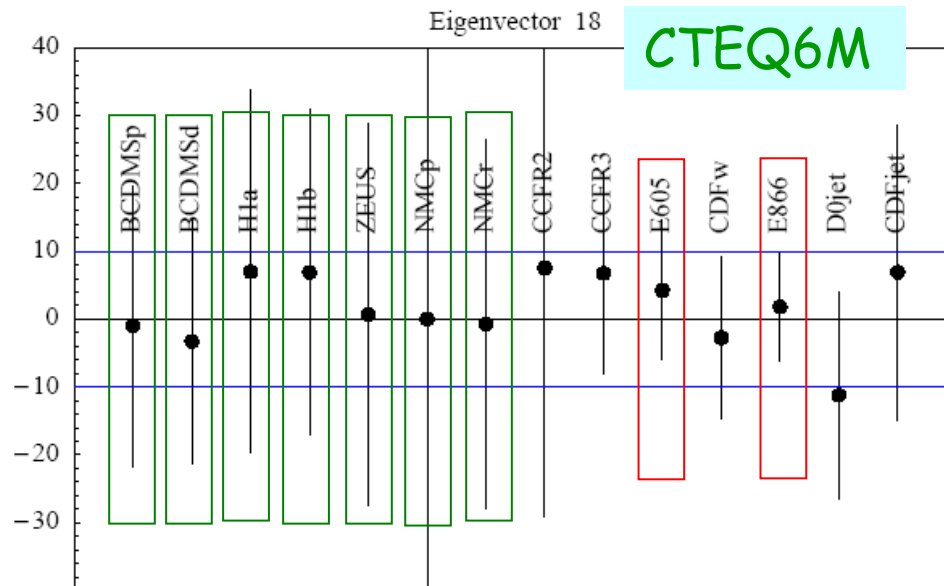
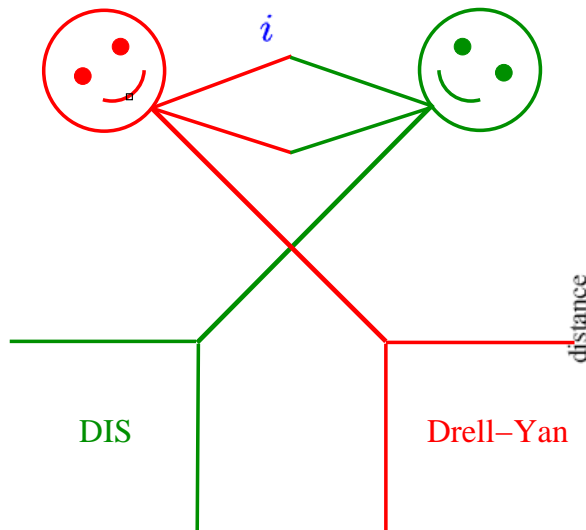
Drell - Yan : $pN \rightarrow \mu^+ \mu^- X$



T

$$\sigma_{DIS} \propto \sum_i e_i^2 [q_i(x_t) + \bar{q}_i(x_t)]$$

$$\sigma_{DY} \propto \sum_i e_i^2 [q_i(x_b)\bar{q}_i(x_t) + \bar{q}_i(x_b)q_i(x_t)]$$

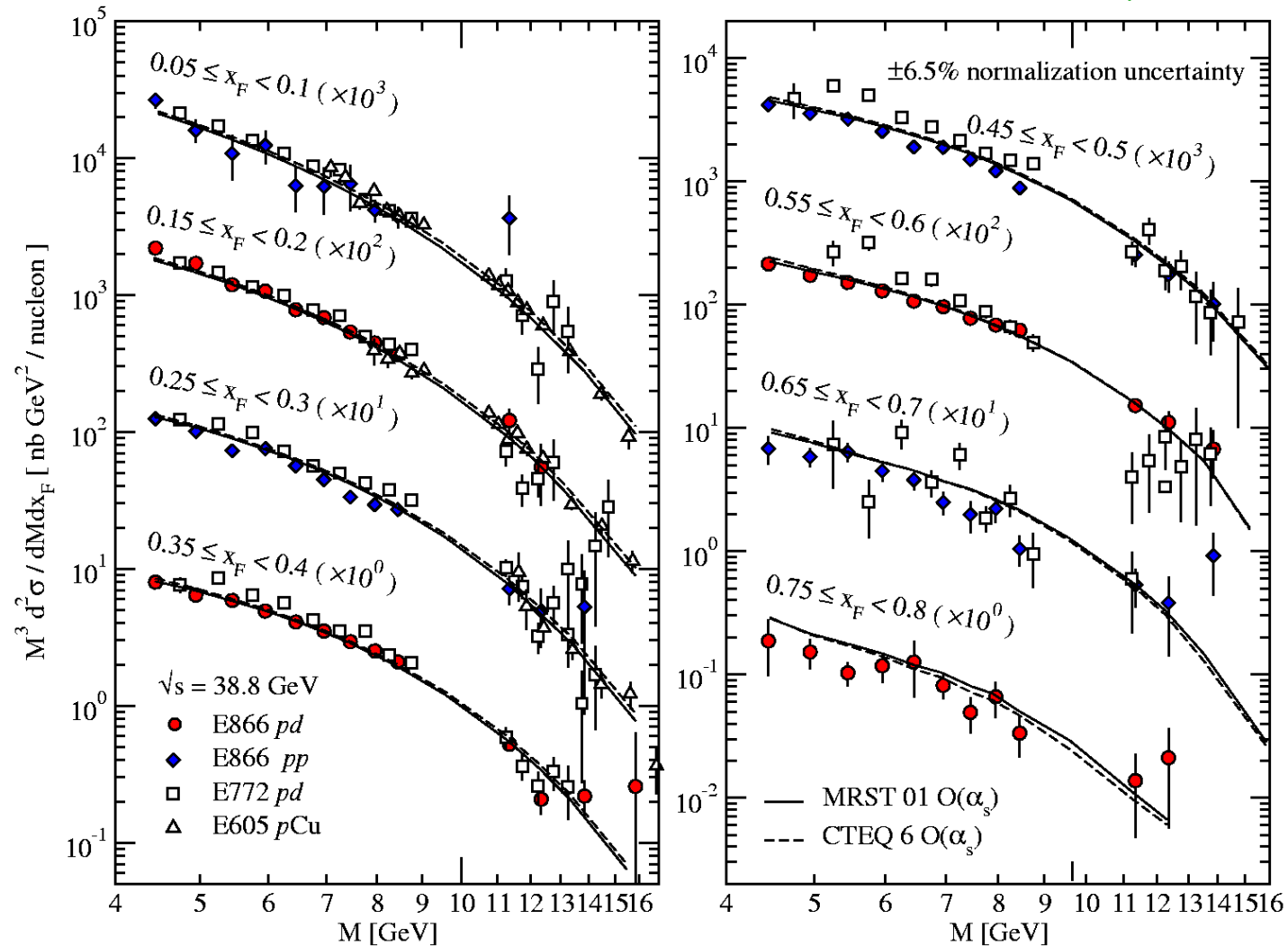




Drell-Yan Cross Sections & Global PDF



E866(J.C.Webb et al.), hep-ex/0302019.





Sea Asymmetry from DIS



$$F_2^p / x = \frac{4}{9}(u + \bar{u}) + \frac{1}{9}(d + \bar{d}) + \frac{1}{9}(s + \bar{s})$$

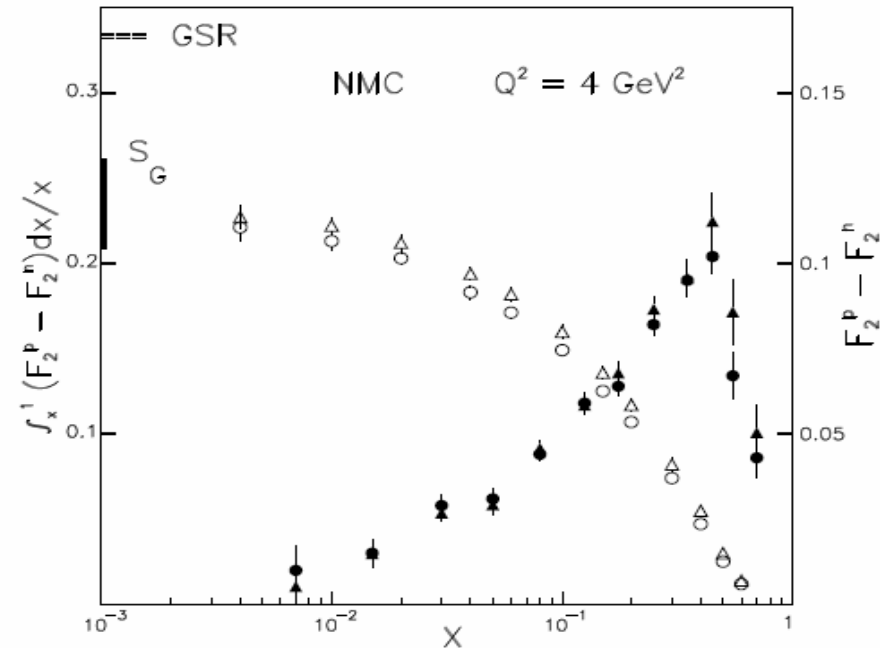
$$F_2^n / x = \frac{4}{9}(d + \bar{d}) + \frac{1}{9}(u + \bar{u}) + \frac{1}{9}(s + \bar{s})$$

$$[F_2^p - F_2^n] / x = \frac{1}{3}(u + \bar{u}) - \frac{1}{3}(d + \bar{d})$$

$$\text{proton}[uud] \Rightarrow \int_0^1 [u_v - d_v] dx = 1$$

$$\int_0^1 [\bar{d} - \bar{u}] dx = \frac{1 - 3 \int_0^1 [F_2^p - F_2^n] / x dx}{2} = \frac{1 - 3 * 0.235}{2} = 0.148 \neq 0$$

Gottfried Integral $S_G = 1/3$?



NMC at CERN: PRL66(1991)2712;PRD50(1994)R1

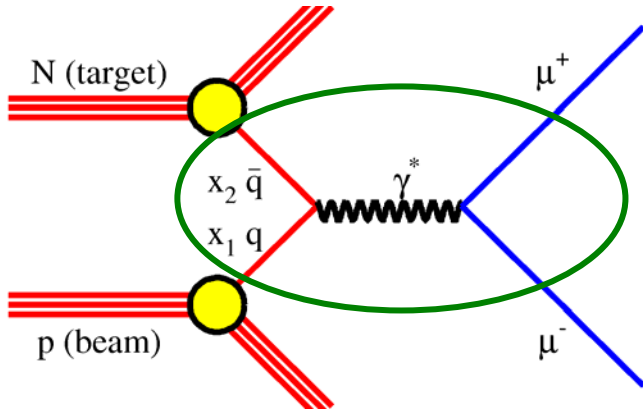


Sea Asymmetry from Drell-Yan



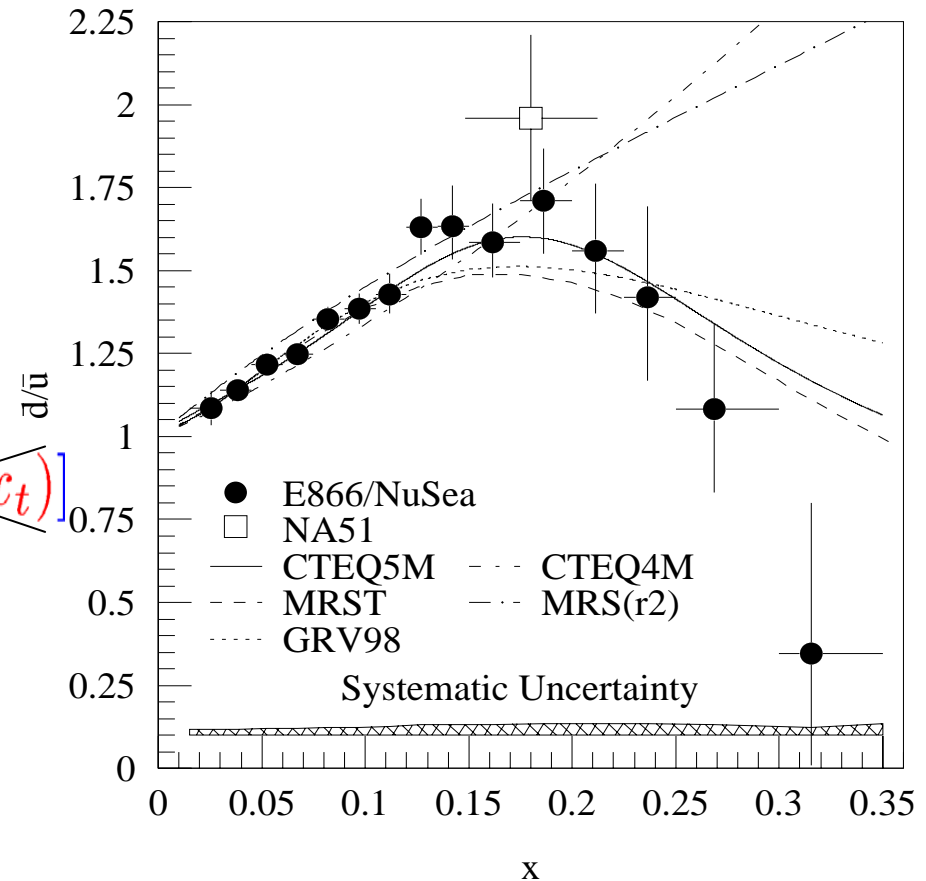
$$pN \rightarrow \mu^+ \mu^- X$$

Towell et al., Phys.Rev. D64 (2001) 052002



$$\sigma_{DY} \propto \sum_i e_i^2 [q_i(x_b) \bar{q}_i(x_t) + \bar{q}_i(x_b) q_i(x_t)]$$

$$\left. \frac{\sigma^{pd}}{2\sigma^{pp}} \right|_{x_b \gg x_t} \approx \frac{1}{2} \left[1 + \frac{\bar{d}(x_t)}{\bar{u}(x_t)} \right]$$

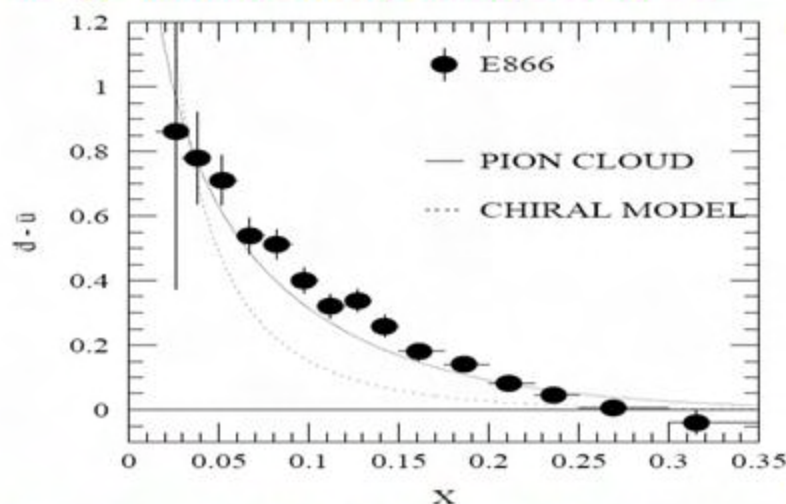




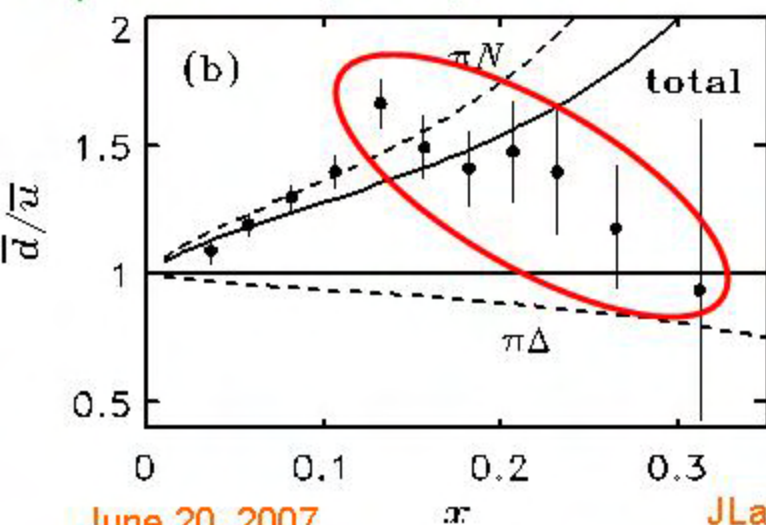
Sea Asymmetry from Models



Prog.Part.Nucl.Phys.47(2001)203

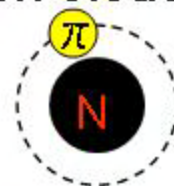


Phys.Rev.D59(1999)014033

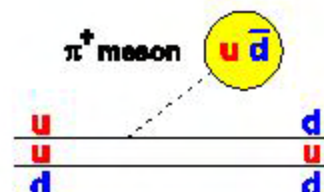


Pion cloud models: Bare nucleon + pion cloud

$$|p\rangle \rightarrow \sqrt{1-a-b}|p_0\rangle + \sqrt{a}\left(-\sqrt{\frac{1}{3}}|p_0\pi^0\rangle + \sqrt{\frac{2}{3}}|n_0\pi^+\rangle\right) + \sqrt{b}\left(-\sqrt{\frac{1}{2}}|\Delta_0^{++}\pi^-\rangle - \sqrt{\frac{1}{3}}|\Delta_0^+\pi^0\rangle + \sqrt{\frac{1}{6}}|\Delta_0^0\pi^+\rangle\right)$$



Chiral Models:



Constituent quarks + Goldstone Bosons

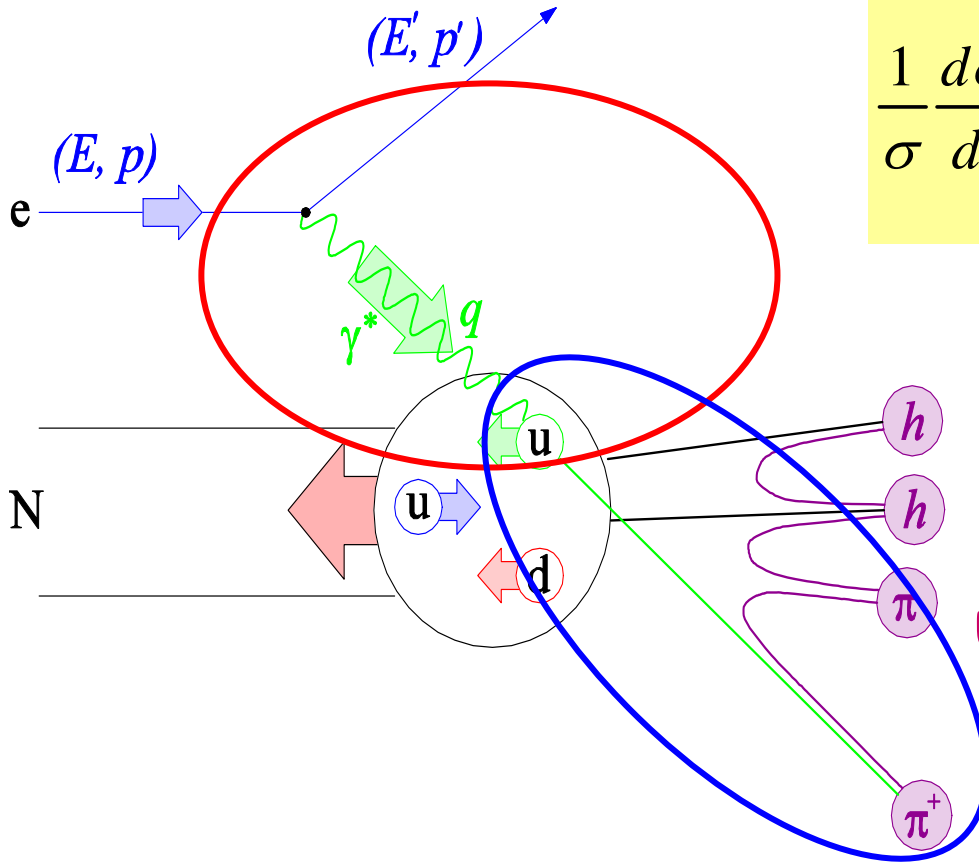
$$U \rightarrow (1 - \frac{3}{2}\alpha)U + \alpha\pi^+D + \frac{1}{2}\alpha\pi^0U$$

$$p \rightarrow 2U + D + \frac{7\alpha}{4}(u + \bar{u}) + \frac{11\alpha}{4}(d + \bar{d})$$

More: Instanton Models, Lattice Gauge Approach, Chiral-Quark Soliton Model



Flavor Decomposition in Semi-Inclusive DIS



$$\frac{1}{\sigma} \frac{d\sigma}{dz} (ep \rightarrow hX) = \frac{\sum_q e_q^2 f_q(x) D_q^h(z)}{\sum_q e_q^2 f_q(x)}$$

$f_q(x)$: parton distribution function

$D_q^h(z)$: fragmentation function
 $z = E_\pi / \nu$

Unfavored fragmentation \mathbf{D}^-
 $u \rightarrow \pi^- (d\bar{u})$ or $d \rightarrow \pi^+ (u\bar{d})$

Favored fragmentation \mathbf{D}^+
 $u \rightarrow \pi^+ (u\bar{d})$ or $d \rightarrow \pi^- (d\bar{u})$



Sea Asymmetry from HERMES SIDIS



Isospin Symmetry & Charge Conjugation Invariance:

$$D_{\pi}^{+} \equiv D_u^{\pi^{+}} = D_{\bar{u}}^{\pi^{-}} = D_{\bar{d}}^{\pi^{+}} = D_d^{\pi^{-}}$$

$$D_{\pi}^{-} \equiv D_{\bar{u}}^{\pi^{+}} = D_u^{\pi^{-}} = D_d^{\pi^{+}} = D_{\bar{d}}^{\pi^{-}}$$

$$Y_p^{\pi^{+}} \propto \frac{1}{9} [4u D_{\pi}^{+} + d D_{\pi}^{-} + 4\bar{u} D_{\pi}^{-} + \bar{d} D_{\pi}^{+} + s D_{\pi}^s + \bar{s} D_{\pi}^s]$$

$$Y_p^{\pi^{-}} \propto \frac{1}{9} [4u D_{\pi}^{-} + d D_{\pi}^{+} + 4\bar{u} D_{\pi}^{+} + \bar{d} D_{\pi}^{-} + s D_{\pi}^s + \bar{s} D_{\pi}^s]$$

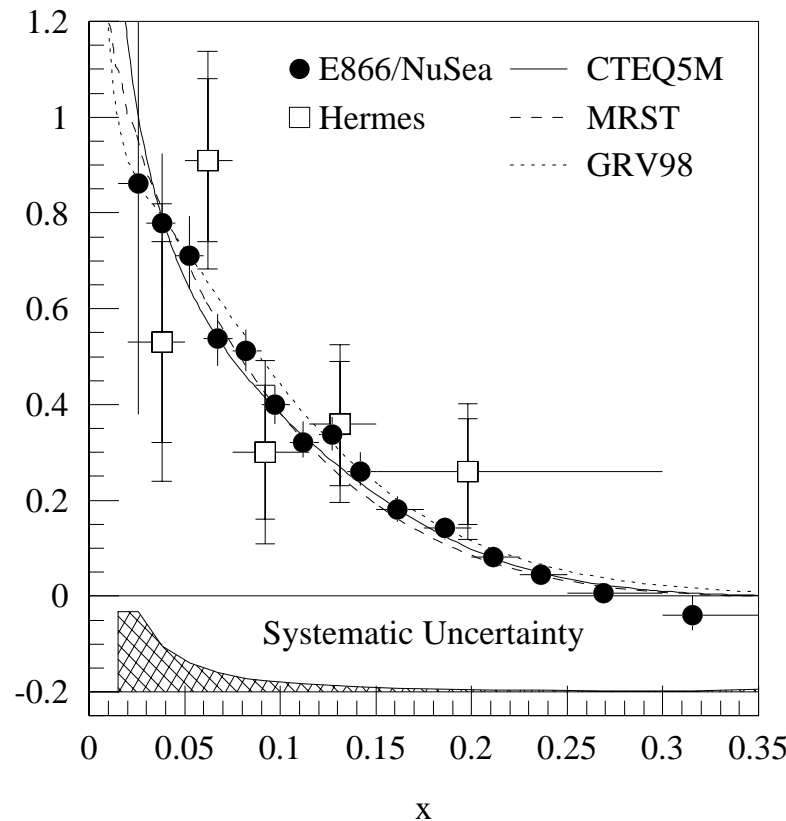
$$Y_n^{\pi^{+}} \propto \frac{1}{9} [4d D_{\pi}^{+} + u D_{\pi}^{-} + 4\bar{d} D_{\pi}^{-} + \bar{u} D_{\pi}^{+} + s D_{\pi}^s + \bar{s} D_{\pi}^s]$$

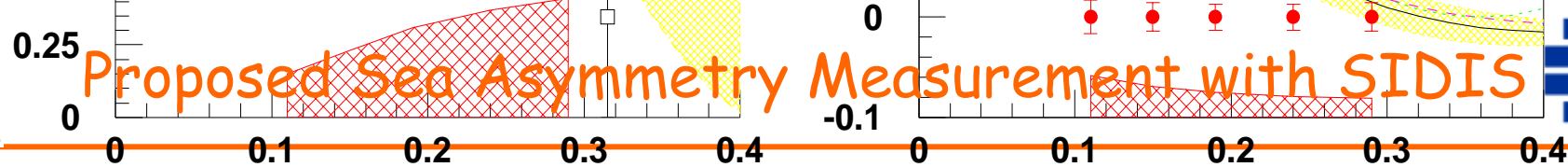
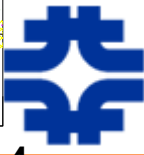
$$Y_n^{\pi^{-}} \propto \frac{1}{9} [4d D_{\pi}^{-} + u D_{\pi}^{+} + 4\bar{d} D_{\pi}^{+} + \bar{u} D_{\pi}^{-} + s D_{\pi}^s + \bar{s} D_{\pi}^s]$$

$$r(x, z) = \frac{Y_p^{\pi^{-}}(x, z) - Y_n^{\pi^{-}}(x, z)}{Y_p^{\pi^{+}}(x, z) - Y_n^{\pi^{+}}(x, z)}$$

$$J(z) = \frac{3}{5} \left(\frac{1 + D'(z)}{1 - D'(z)} \right), \quad D'(z) = \frac{D_{\pi}^{-}}{D_{\pi}^{+}} \approx \frac{4Y_d^{\pi^{-}} - Y_d^{\pi^{+}}}{4Y_d^{\pi^{+}} - Y_d^{\pi^{-}}}$$

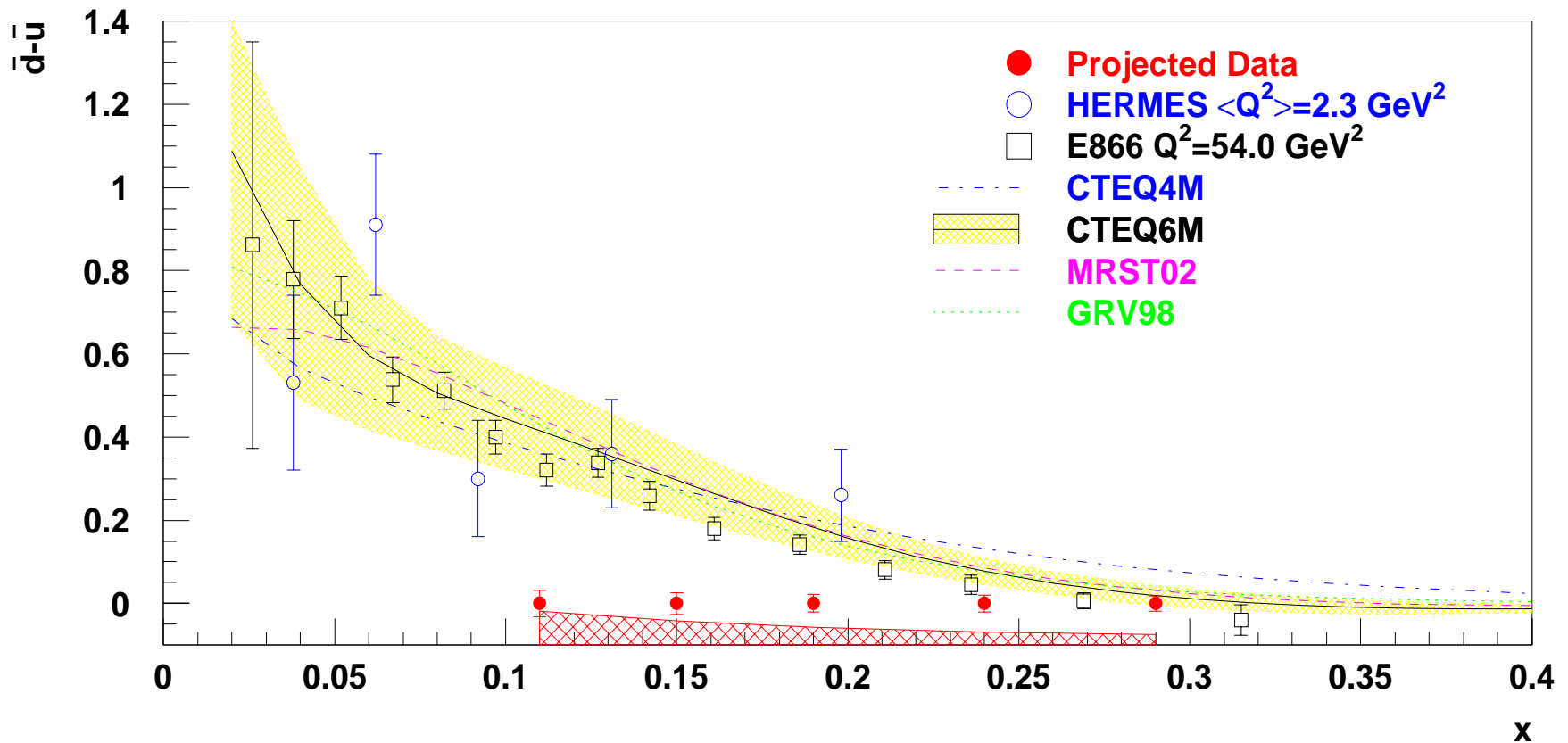
$$r_H \equiv \frac{J(z)[1 - r(x, z)] - [1 + r(x, z)]}{J(z)[1 - r(x, z)] + [1 + r(x, z)]} = \frac{\bar{d}(x) - \bar{u}(x)}{u(x) - d(x)}$$





Proposed Sea Asymmetry Measurement with SIDIS

L.Y.Zhu, J.P.Chen, X. Jiang and J.C. Peng, JLab Hall A PR04-114

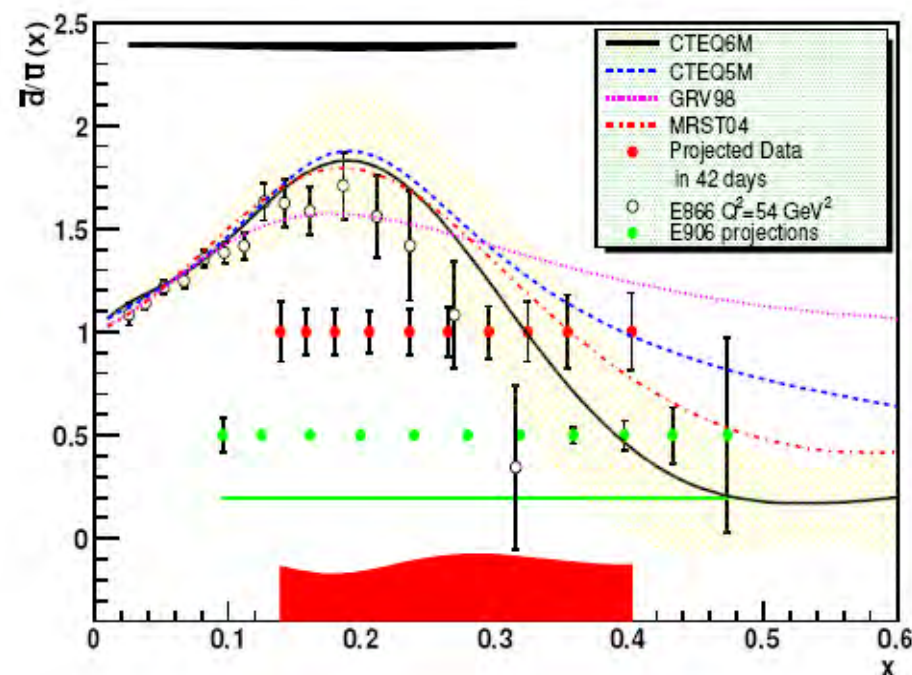
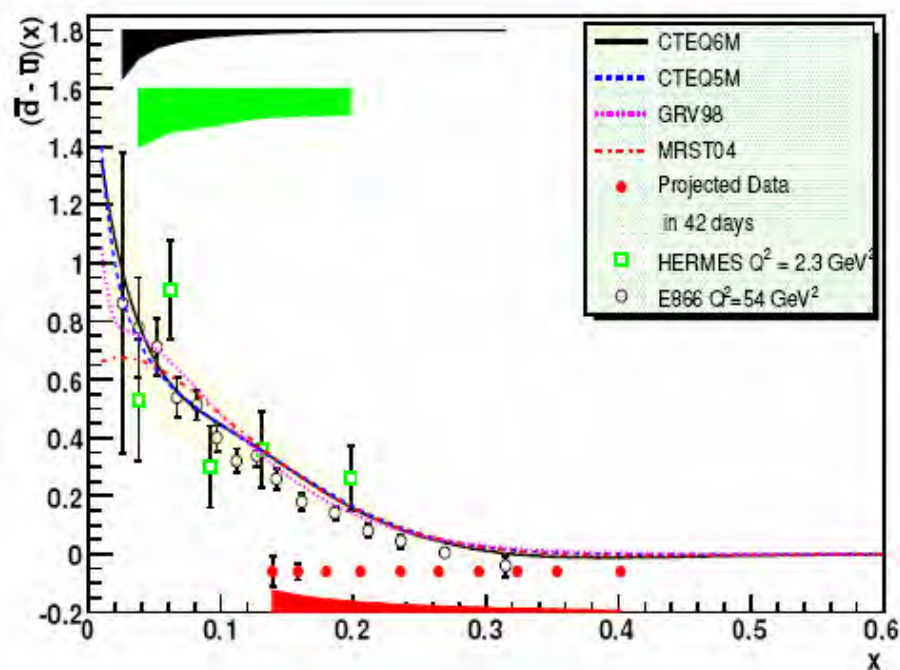




Sea Asymmetry Measurement with JLab Upgrade



H. Gao, A. Bruell, H. Mkrchyan, J.P. Chen & L.Y. Zhu, JLab Proposal PR12-06-111



The systematic uncertainty with SIDIS is large.



Sea Asymmetry with 120 GeV Proton Beam



Fermilab E866/NuSea

(M. Leitch)

- ❖ Data in 1996-1997
- ❖ 800 GeV proton beam

$$\sqrt{s} = 38.8 \text{ GeV}$$

$$\frac{d^2\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9x_1 x_2 s} \sum_i e_i^2 [q_{ti}(x_t)\bar{q}_{bi}(x_b) + \bar{q}_{ti}(x_t)q_{bi}(x_b)]$$



Fermilab E906

(D. Geesaman, P. E. Reimer)

- ❖ Data in 2009
- ❖ 120 GeV proton Beam

$$\sqrt{s} = 15.1 \text{ GeV}$$

$$\sigma_{J/\psi} = A \exp(-B \sqrt{M_{J/\psi}^2/s})$$

- ❖ Cross section scales as $1/s$
- ❖ Backgrounds, primarily J/ψ decays drop with s

E789, PRD52(1995)1507

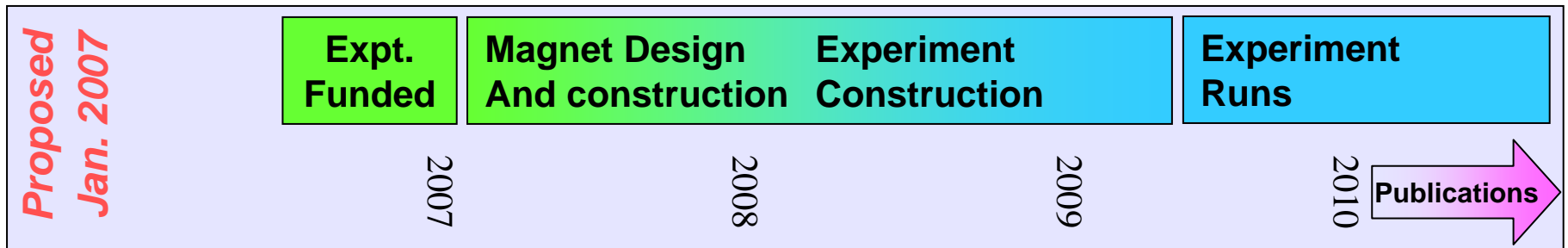
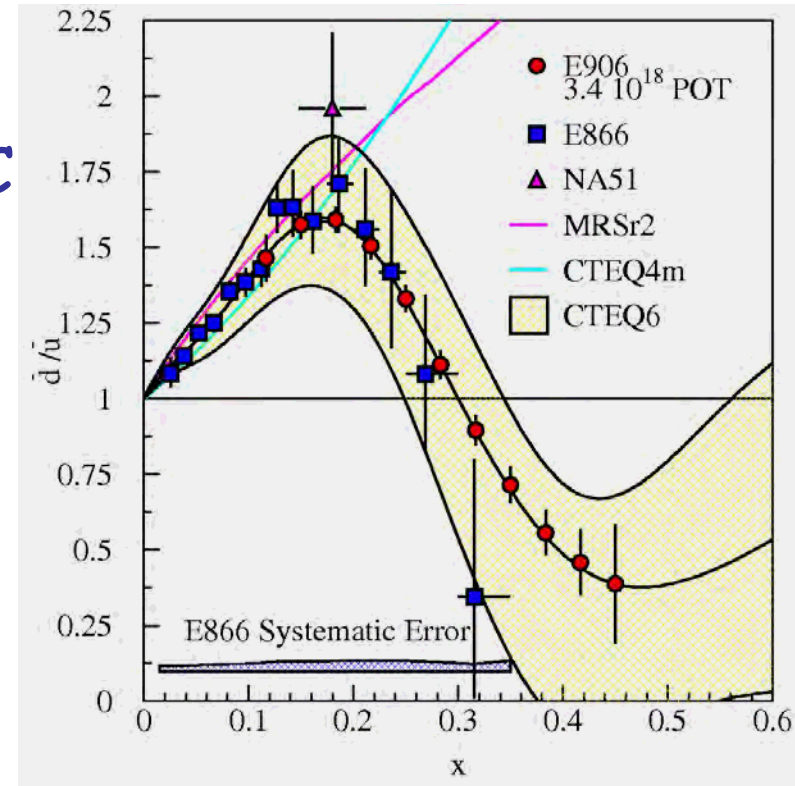
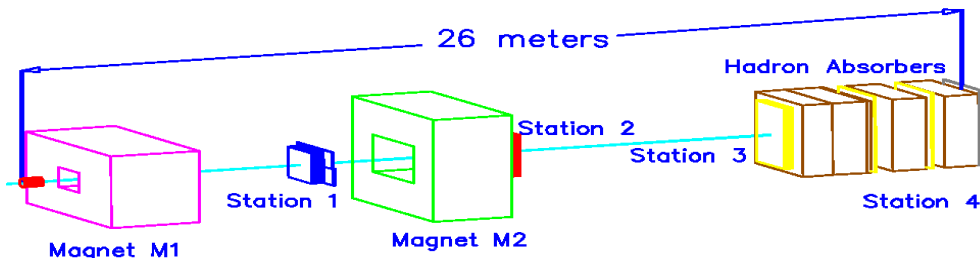
50× statistics!!



Fermilab E906 Drell-Yan timeline



- ❖ 2001: approved by Fermilab PAC
- ❖ 2006: reaffirmed by Fermilab PAC
- ❖ Funding request to DOE:
~\$2M primarily for the magnets.



June 20, 2007

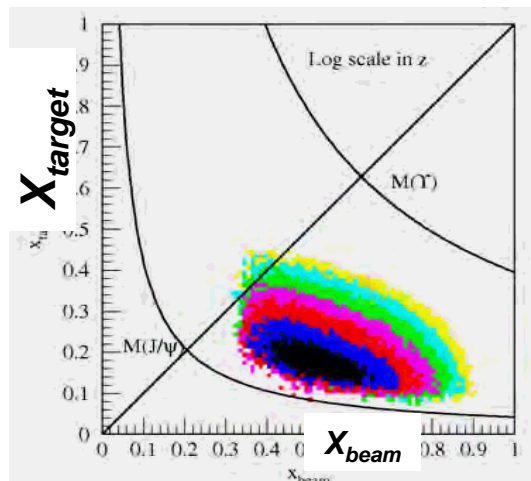
JLab User's Group Meeting



Impact of Drell-Yan on large- x PDFs



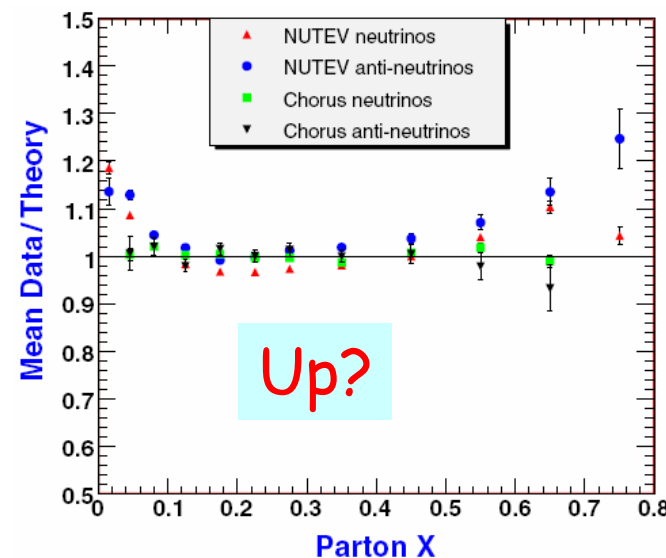
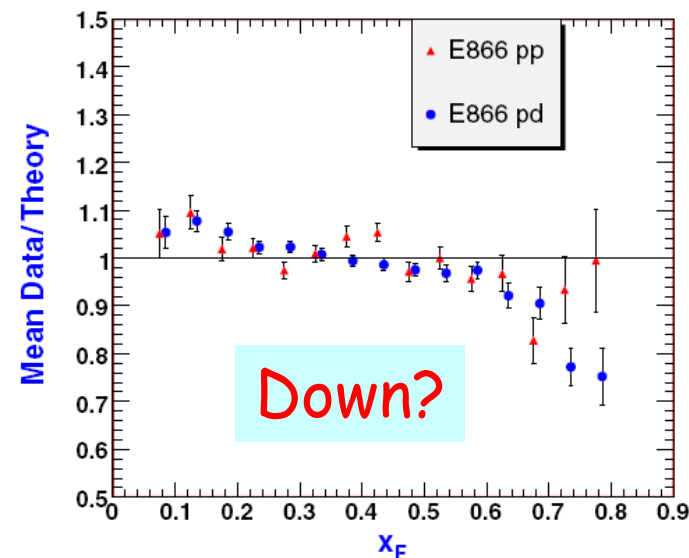
- ❖ E906 Drell-Yan will also probe the large- x region, like E866.

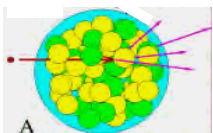


$$x_F = x_b - x_t$$

- ❖ There is some discrepancy between the E866 and NUTEV. Due to the nuclear corrections for NUTEV?

Owens *et al.*, PRD75(2007) 054030

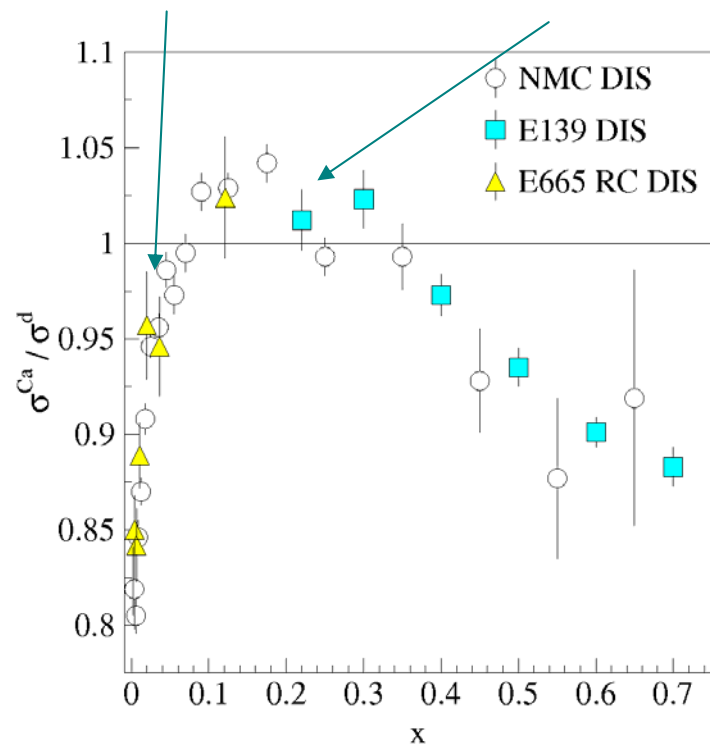




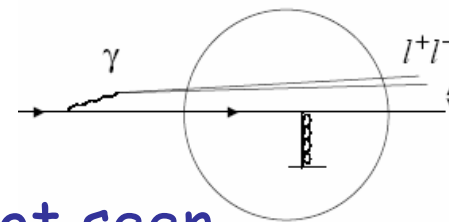
Nuclear Dependence/EMC Effect



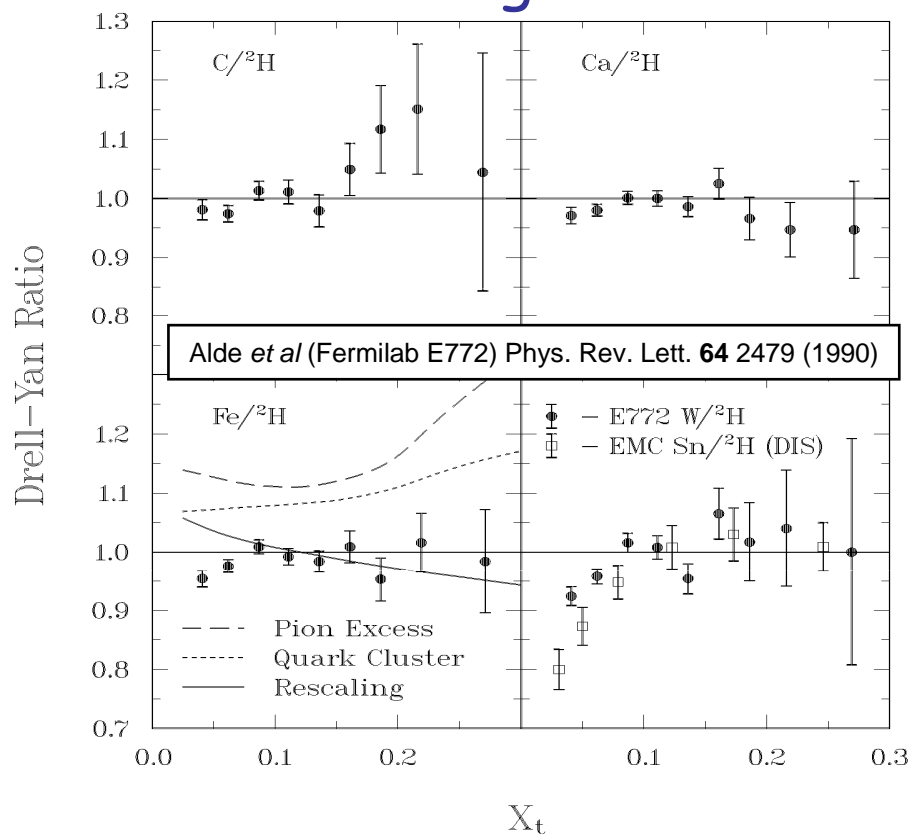
❖ DIS: shadowing, antishadowing,...



❖ E772 Drell-Yan: shadowing seen;



antishadowing not seen.

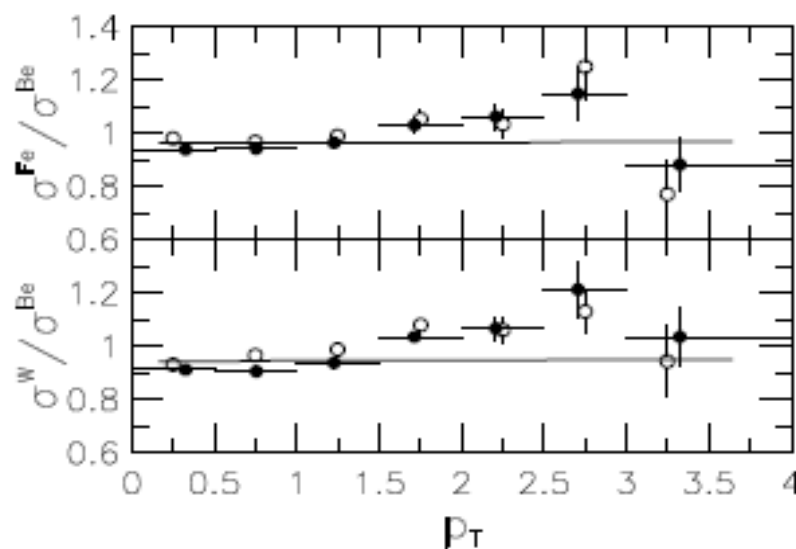
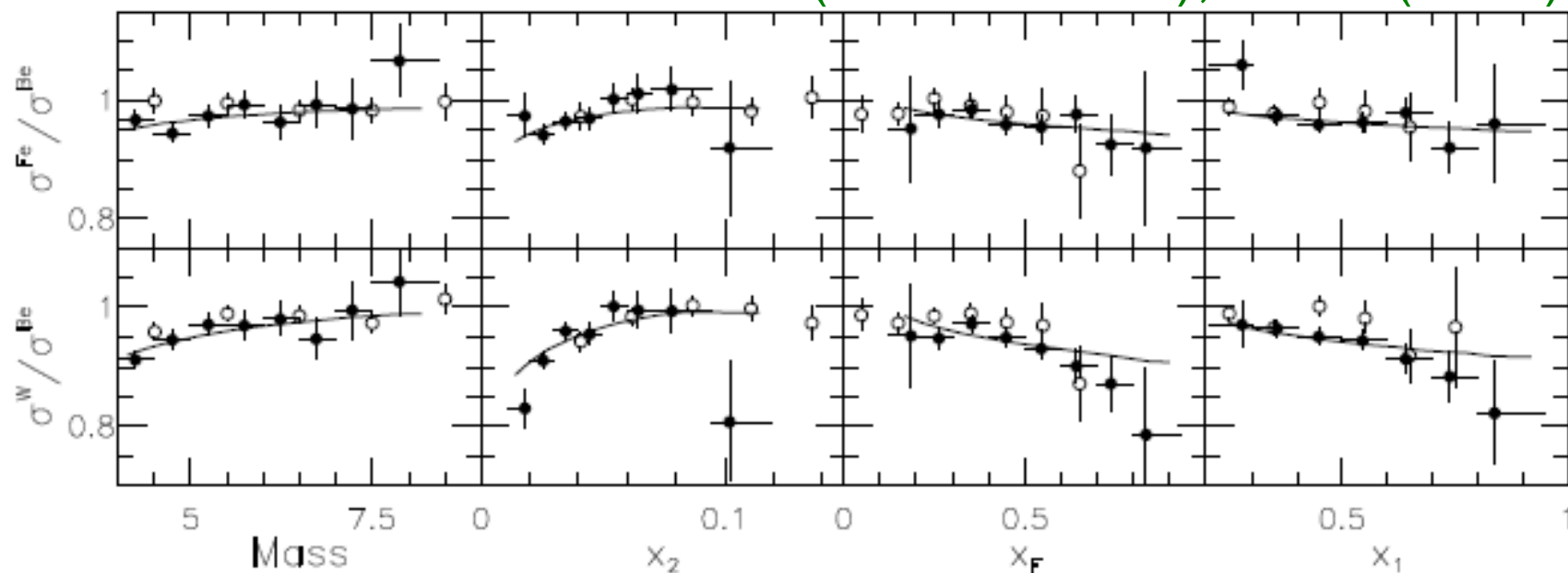




E866 Drell-Yan with Nuclear Targets



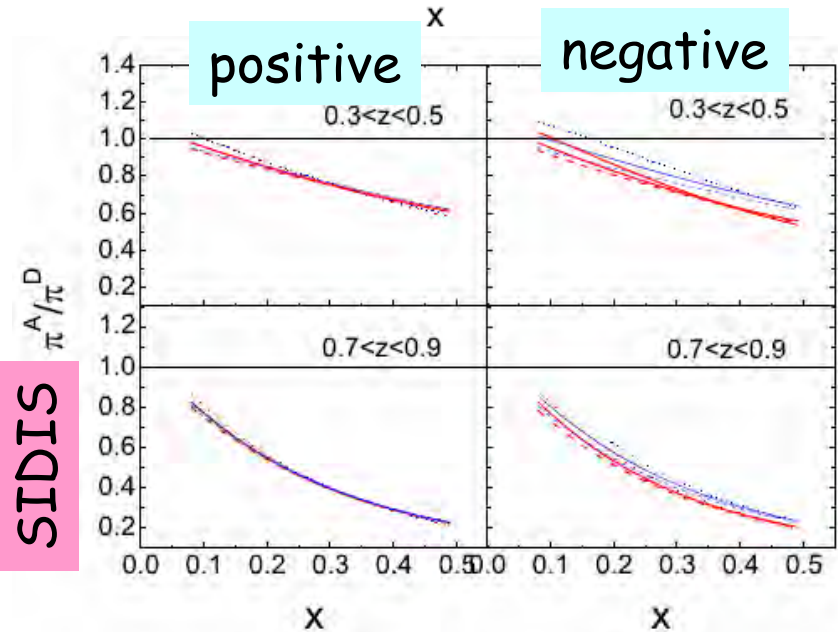
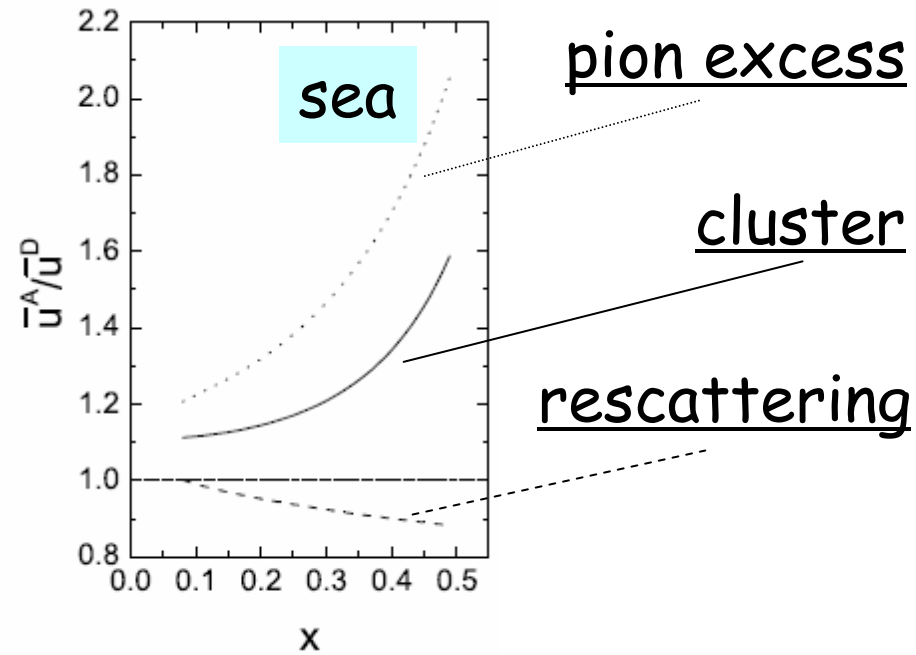
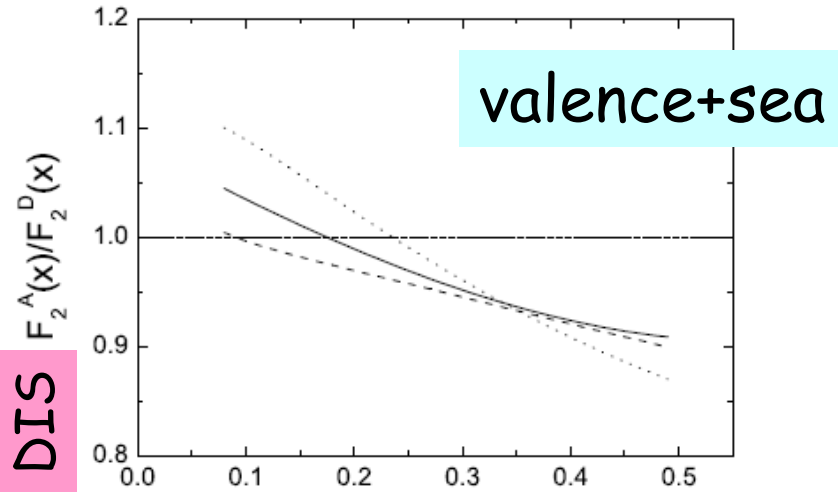
E866(Vasiliev *et al.*), PRL83(1999)2304



Compared with E772:
smaller x_2
bigger x_1, x_F



Models for EMC effect



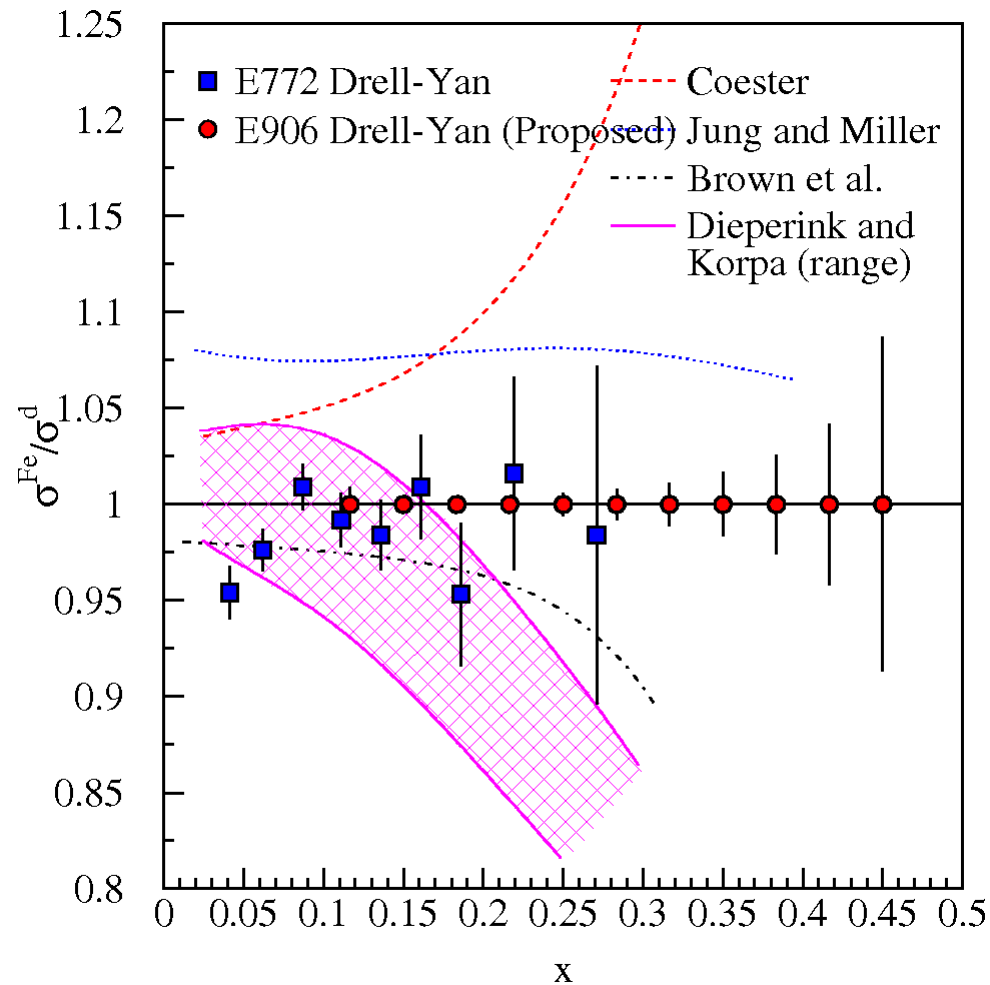
Lu & Ma, PRC74(2007)055202



E906 Drell-Yan with Nuclear Targets



- ❖ E906 will use three nuclear targets in addition to proton and deuterium.
- ❖ E906 will significantly reduce the statistical uncertainties while extend to higher x .
- ❖ The improvement will be essential to check different models.





Partonic Energy Loss



- ❖ The nuclear dependence is more sensitive to the partonic energy loss at low beam energy.

$$x_1^A = x_1^p + \alpha \frac{\langle L \rangle_A}{E_p}; \alpha = \frac{dE}{dx}$$

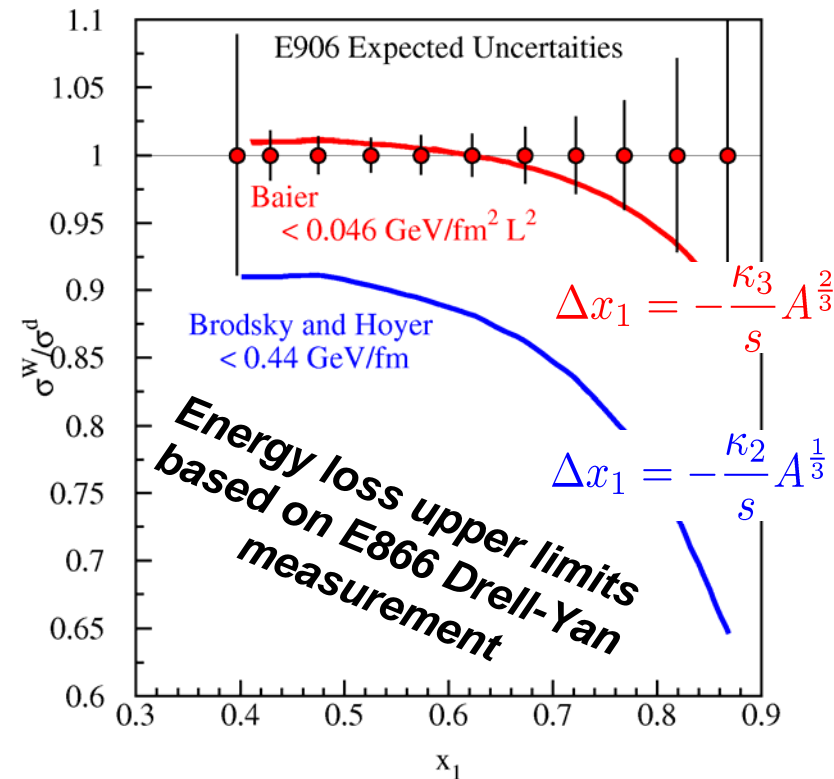
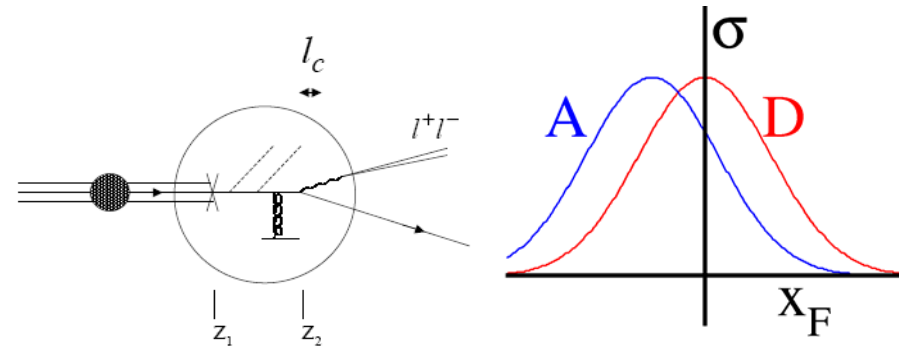
$$\frac{d\sigma_{p+A}^{DY}}{d\sigma_{p+D}^{DY}} \Big|_{x_1 \rightarrow 1, x_2} \sim 1 - \frac{2\alpha \langle L \rangle_A}{E_p (1 - x_1^p)}$$

Garvey & Peng, PRL90(2003)092302

- ❖ Difficulty with Drell-Yan: At fixed $x_F = x_1 - x_2$, the partonic energy loss at large x_1 is coupled with shadowing effect at very small x_2 . E866 only set the limit on energy loss.

E866(Vasiliev *et al.*), PRL83(1999)2304

- ❖ With good statistics, E906 allows us to select a region with small shadowing at $x_2 > 0.1$.

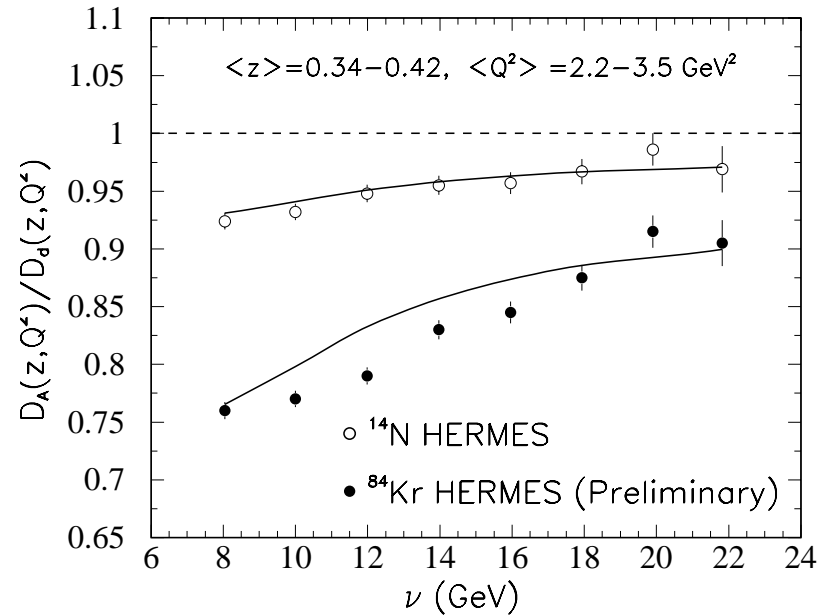
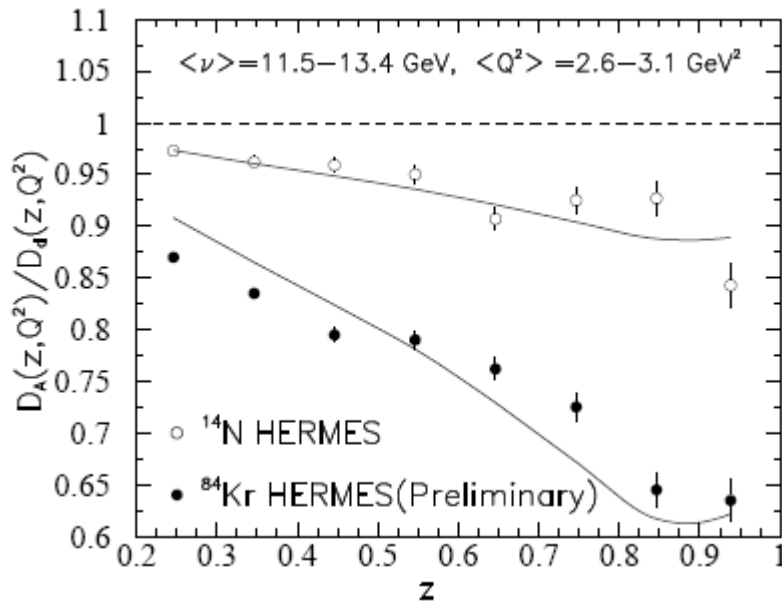




Energy Loss from SIDIS



❖ HERMES results: Wang & Wang, PRL89(2002) 162301



$dE/dx = 0.5$ GeV/fm

❖ The production time (related to the energy loss/ p_T broadening) as well as the hadron formation length will be measured with CLAS12.

W. Brooks *et al.*, JLab proposal PR12-06-117.

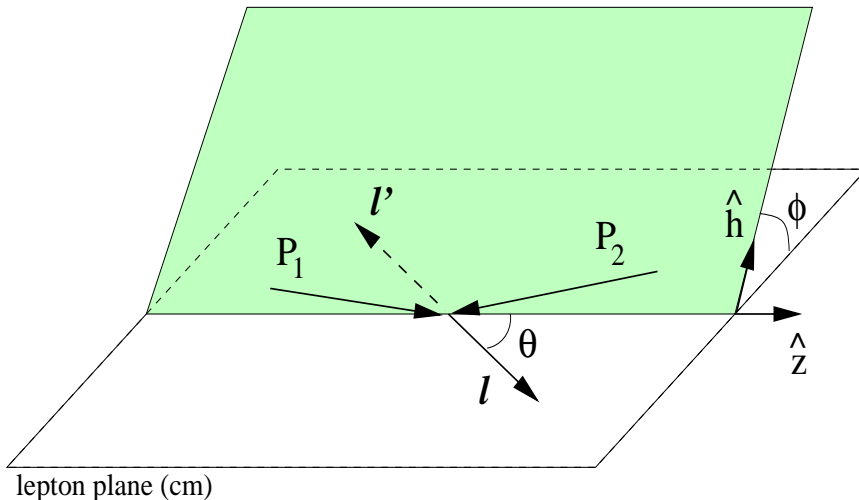


Angular Distribution in the Drell-Yan



$$pN \rightarrow \mu^+ \mu^- X$$

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left[1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right]$$



In the simple parton model:

(for massless quarks and θ measured relative to the annihilation axis)

$$\lambda=1 \text{ and } \mu=\nu=0$$

$$\frac{d\sigma}{d\Omega} \propto 1 + \cos^2 \theta$$

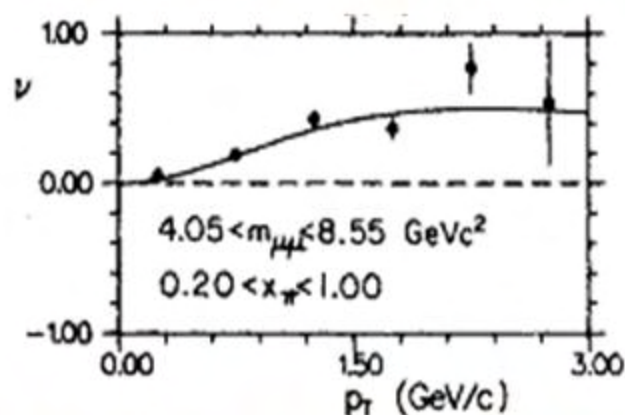


$\cos 2\varphi$ Distribution in the πW Drell-Yan

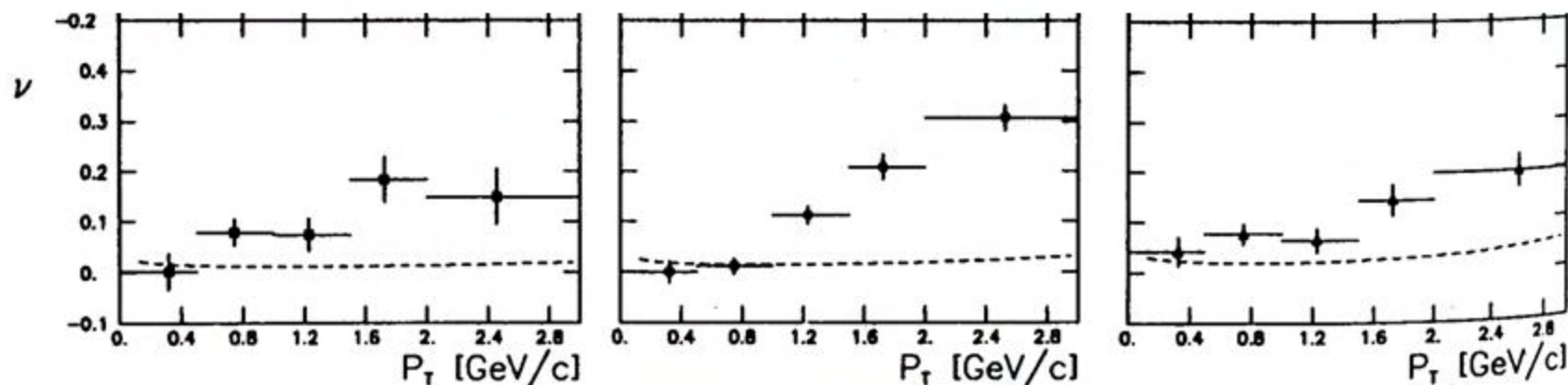


E615 at Fermilab: 252 GeV $\pi^- + W$

Conway et al., PRD39,92(1989)



NA10 at CERN: 140/194/286 GeV $\pi^- + W$ Z. Phys. C37, 545 (1988)



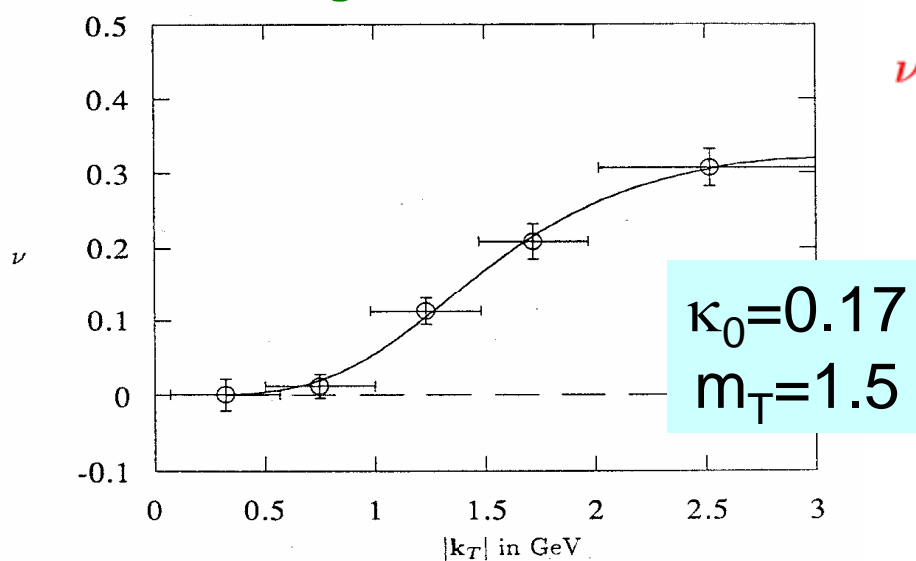


QCD Vacuum Effect



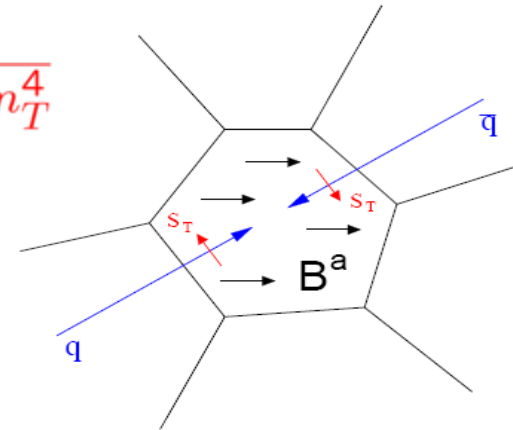
- ❖ The factorization-breaking spin correlation due to nontrivial QCD vacuum may fit the NA10 data at 194 GeV

Brandenburg, Nachtmann & Mirkes, Z. Phys. C60,697(1993)



$$\nu \approx 2\kappa = 2\kappa_0 \frac{p_T^4}{p_T^4 + m_T^4}$$

$$\lambda \approx 1; \mu \approx 0$$



- ❖ The helicity flip in the instanton-induced contribution may lead to nontrivial vacuum.

Boer, Brandenburg, Nachtmann & Utermann, EPC40,55(2005).

Brandenburg, Ringwald & Uermann, hep-ph/0605234

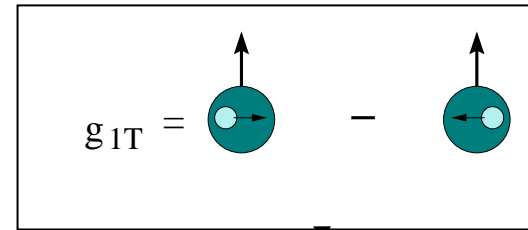
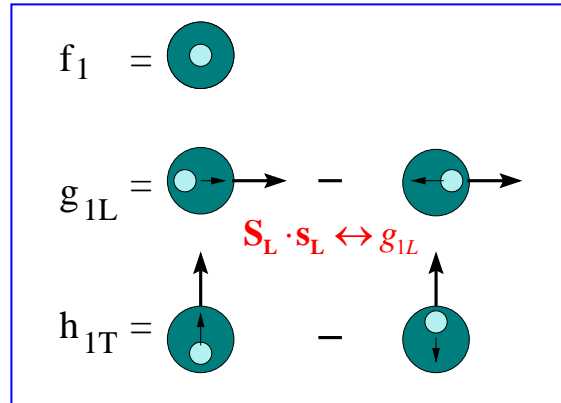
- ❖ This vacuum effect should be flavor blind.



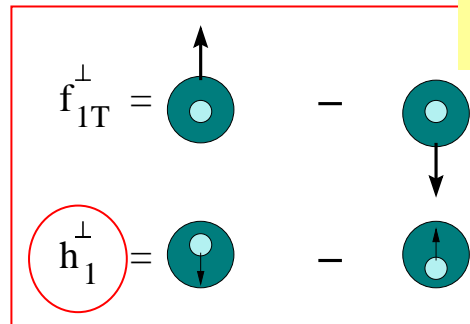
Leading-Twist Parton Distribution Functions



Survive k_T
integration



k_T - dependent,
T-odd



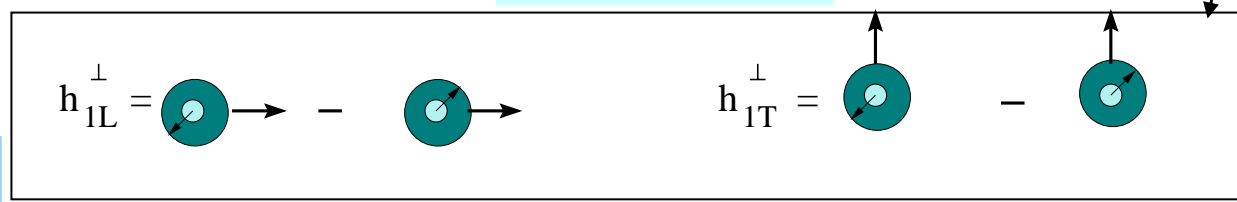
Sivers Function

$$S_T \cdot (\hat{p} \times k_T) \leftrightarrow f_{1T}^\perp$$

Boer-Mulders Function

$$s_T \cdot (\hat{p} \times k_T) \leftrightarrow h_1^\perp$$

k_T - dependent,
T-even



$$\left. \frac{h_1^\perp}{f_1} \right|_{x \rightarrow 1} \sim (1-x)$$

Brodsky & Yuan, hep-ph/0610236.



Boer-Mulders Function h_1^\perp

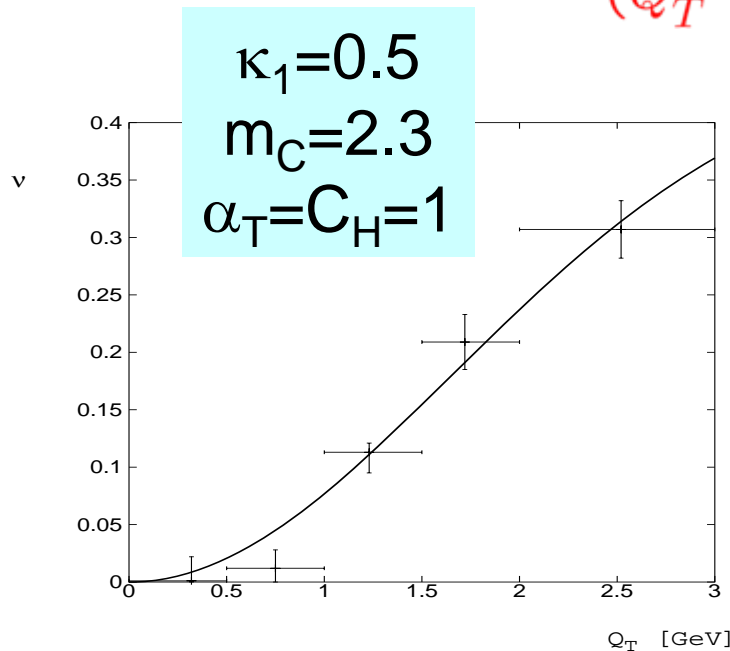


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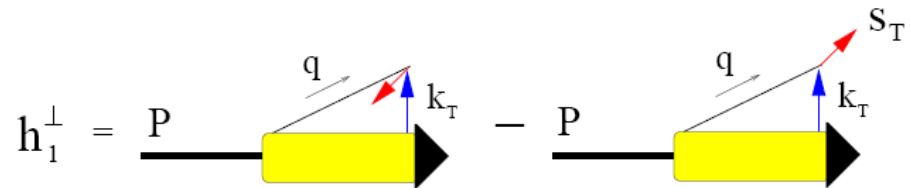
- ❖ An spin-correlation approach in terms of h_1^\perp can fit the NA10 data at 194 GeV. [Boer, PRD60,014012\(1999\)](#)

$$\nu = 2\kappa = 4\kappa_1 \frac{Q_T^2 M_C^2}{(Q_T^2 + 4M_C^2)^2}; \quad \lambda = 1; \mu = 0$$



$$\nu \propto h_1^\perp(x_1) \bar{h}_1^\perp(x_2)$$

$$h_1^\perp(x, k_T^2) = \frac{\alpha_T}{\pi} c_H \frac{M_C M_H}{k_T^2 + M_C^2} e^{-\alpha_T k_T^2} f_1(x)$$



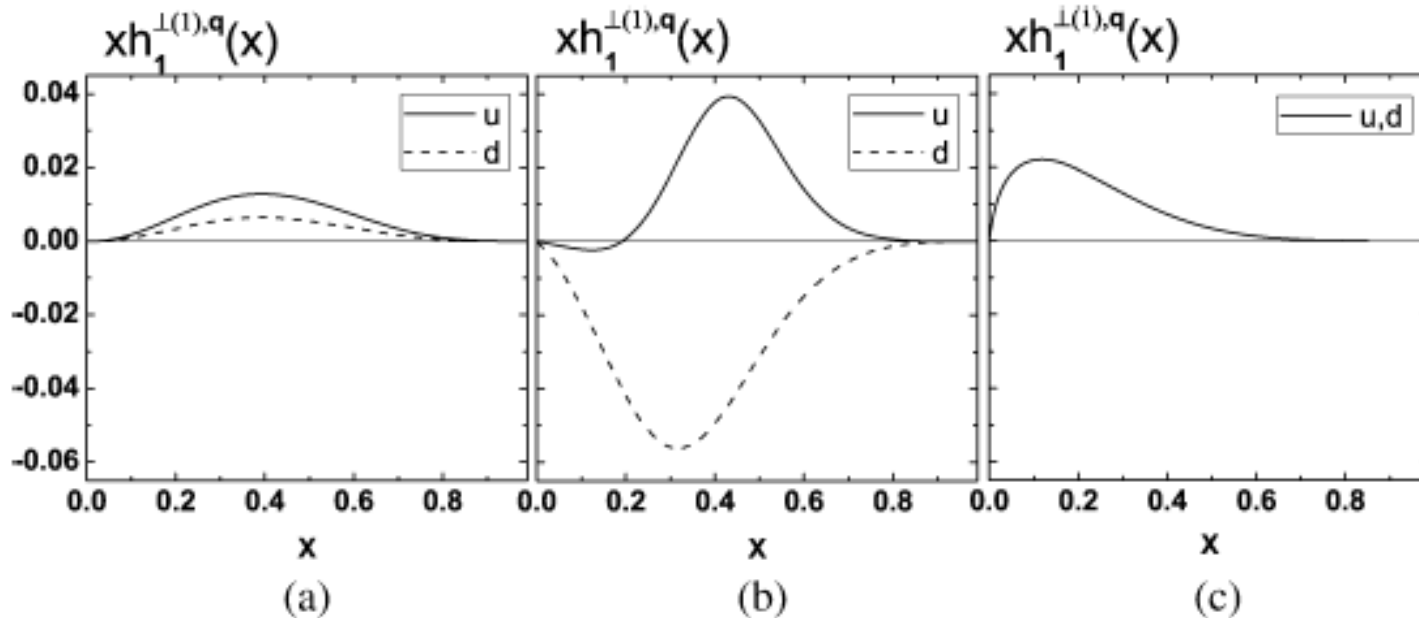
- ❖ On the base of quite general arguments, for $|q_T| \ll Q (=m_{\mu\mu})$, [Salvo, hep-ph/0407208](#). $\nu \propto |q_T|^2 / Q^2$



Boer-Mulders Functions from Models



Z. Lu, B.Q. Ma and I. Schmidt, Phys. Lett. B639(2006)494.



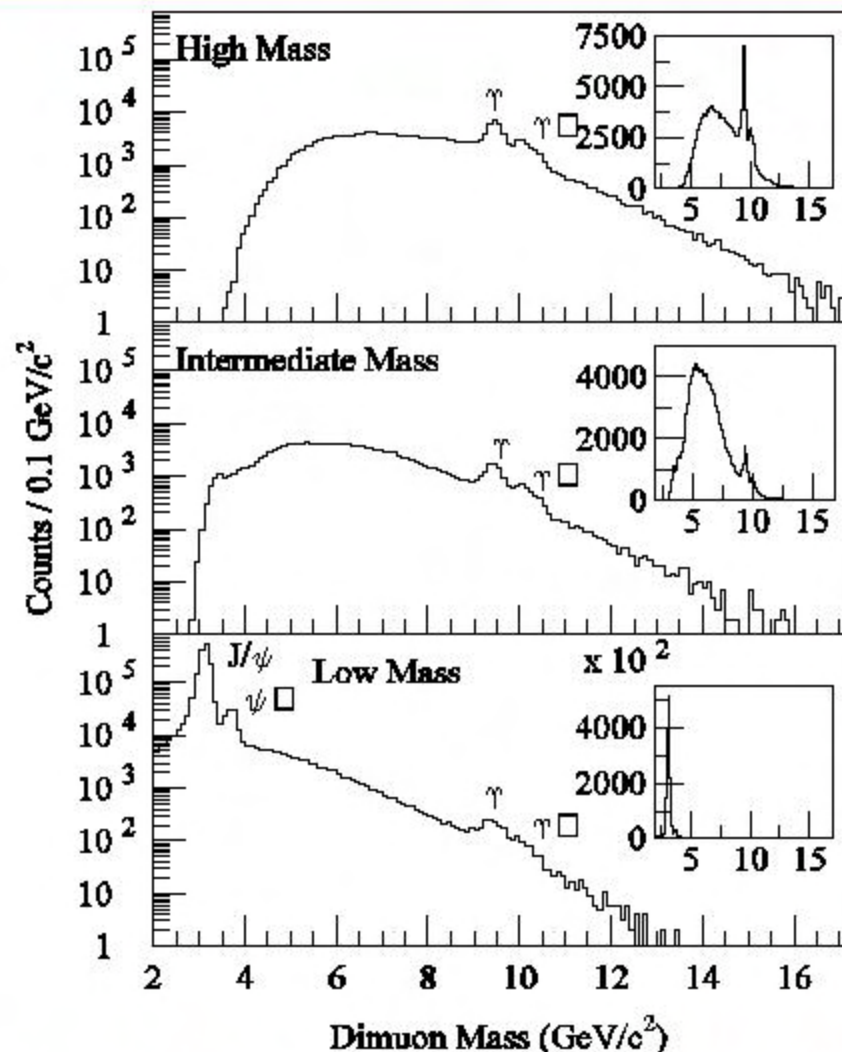
(a) MIT bag model: F. Yuan, Phys. Lett. B575,45(2003).

(b) Spectator model with axial-vector diquark: Bacchetta, Schaefer & Yang, Phys. Lett. B578,109(2004).

(c) Large- N_c limit, P.V. Pobylitsa, hep-ph/0301236



E866 Dimuon Mass Distribution



$$\sqrt{s} = 38.8 \text{ GeV}$$

Target: Proton, Deuterium

Data used for $\cos^2 \phi$ analysis:

High Mass: dset7-39k (+ polarity)
dset8-85k (+ polarity)
dset11-25k (- polarity)
Low Mass: dset5-68k (+ polarity)

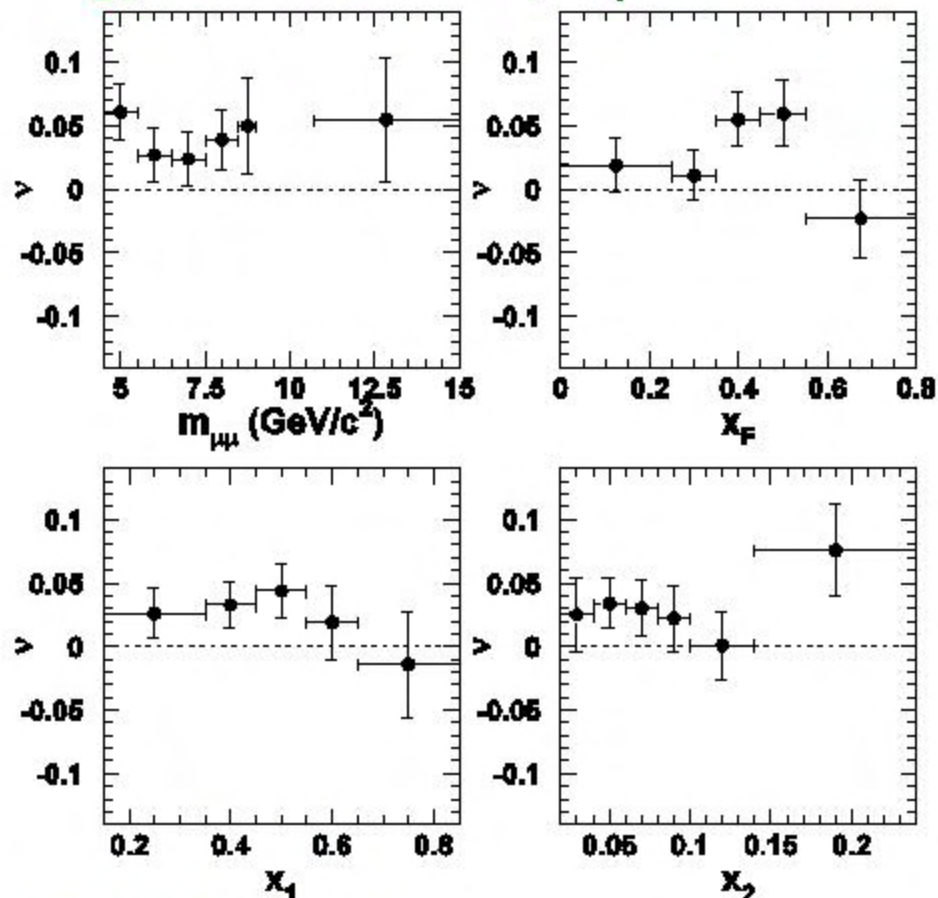
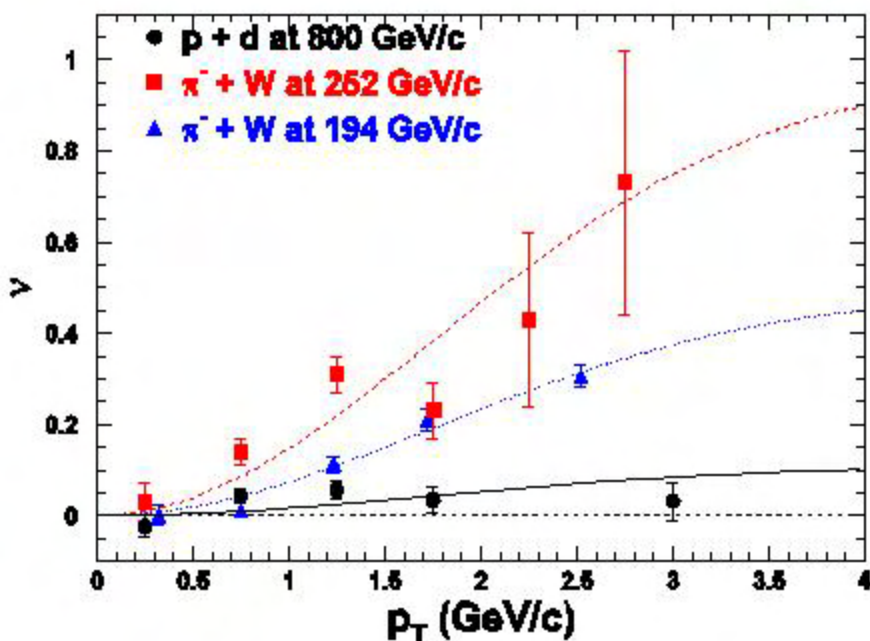
Towell et al., Phys.Rev. D64 (2001) 052002



Azimuthal Distribution from E866 pd Drell-Yan



L.Y. Zhu, J.C. Peng, P. Reimer et al., hep-ex/0609005.



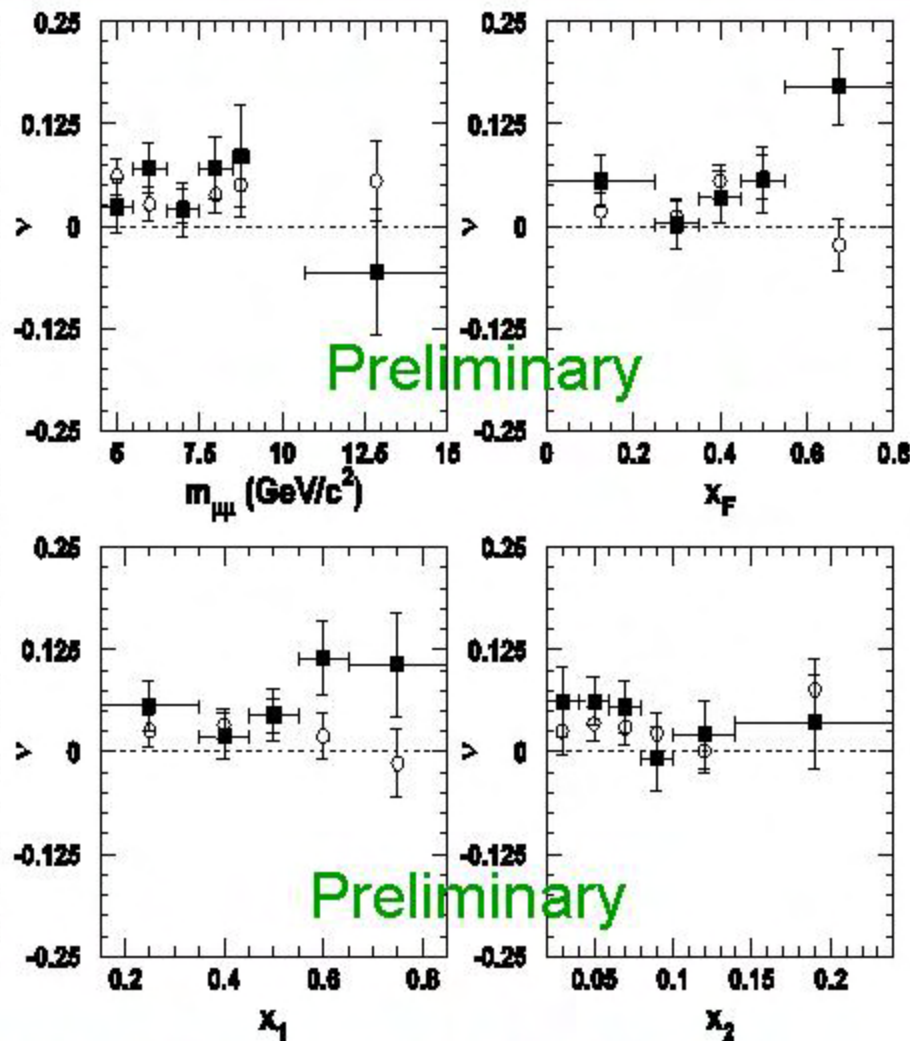
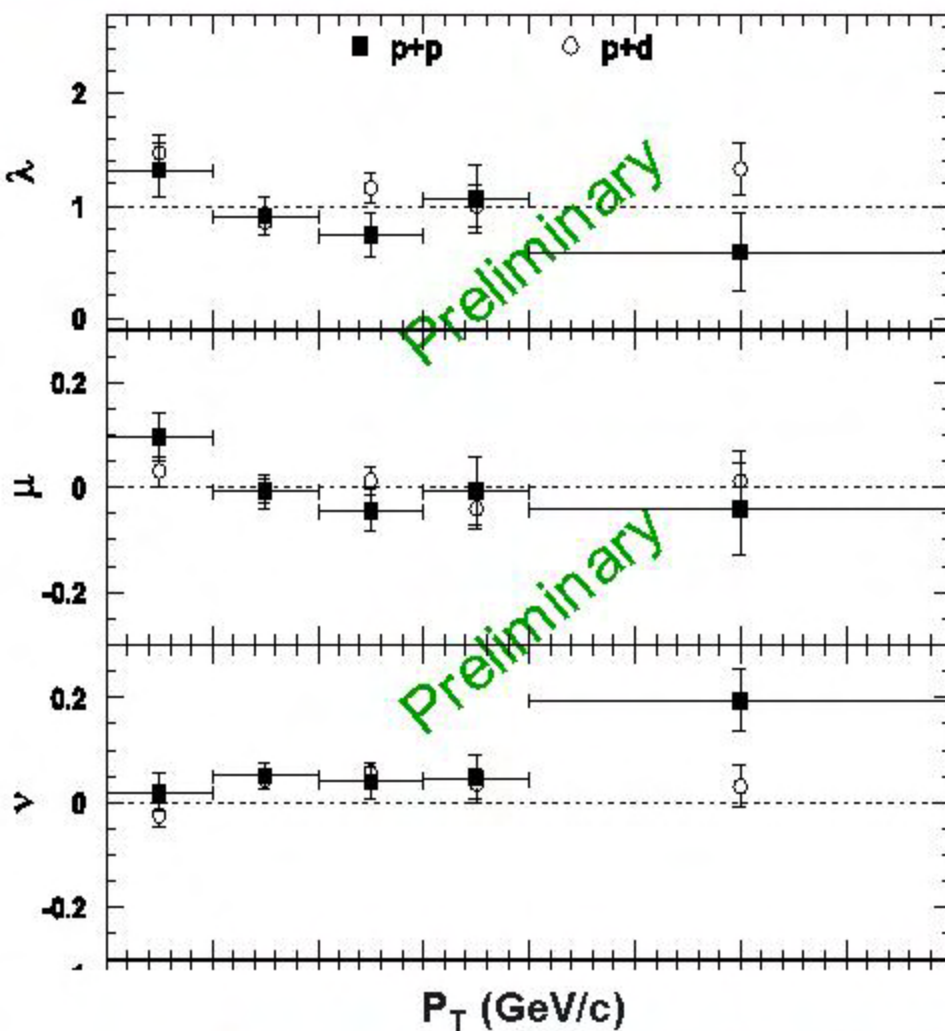
With Boer-Mulders function h_1^\perp :

$\nu(\pi^- W \rightarrow \mu^+ \mu^- X) \sim \text{valence } h_1^\perp(\pi) * \text{valence } h_1^\perp(p)$

$\nu(pd \rightarrow \mu^+ \mu^- X) \sim \text{valence } h_1^\perp(p) * \text{sea } h_1^\perp(p)$



Angular Distribution in E866 pp/pd Drell-Yan



The statistical uncertainties can be greatly improved in E906.



- ❖ The $\cos 2\phi$ distribution in SIDIS is related to the coupling of Boer-Mulders function and Collins fragmentation function. It is sensitive to valence Boer-Mulders function at large x .

$$\sigma_{UU}^{ep \rightarrow eh} \sim \sum_q C_{Cahn} f_1 \otimes D_1 + h_1^\perp \otimes H_1^\perp$$

H. Avakian, Z.-E. Meziani, K. Joo and B. Seitz,
JLab proposal PR12-06-112

- ❖ It will be very interesting to check

$$h_1^\perp(x, p_T^2)_{\text{SIDIS}} = -h_1^\perp(x, p_T^2)_{\text{DY}}$$

similar to that for Sivers function

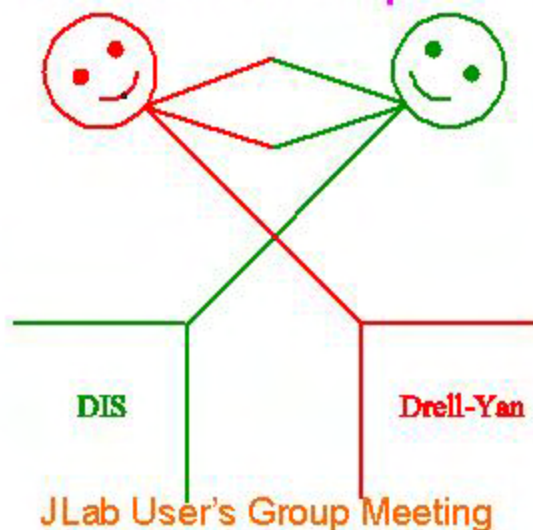
Collins, PLB536, 43(2002)



Summary

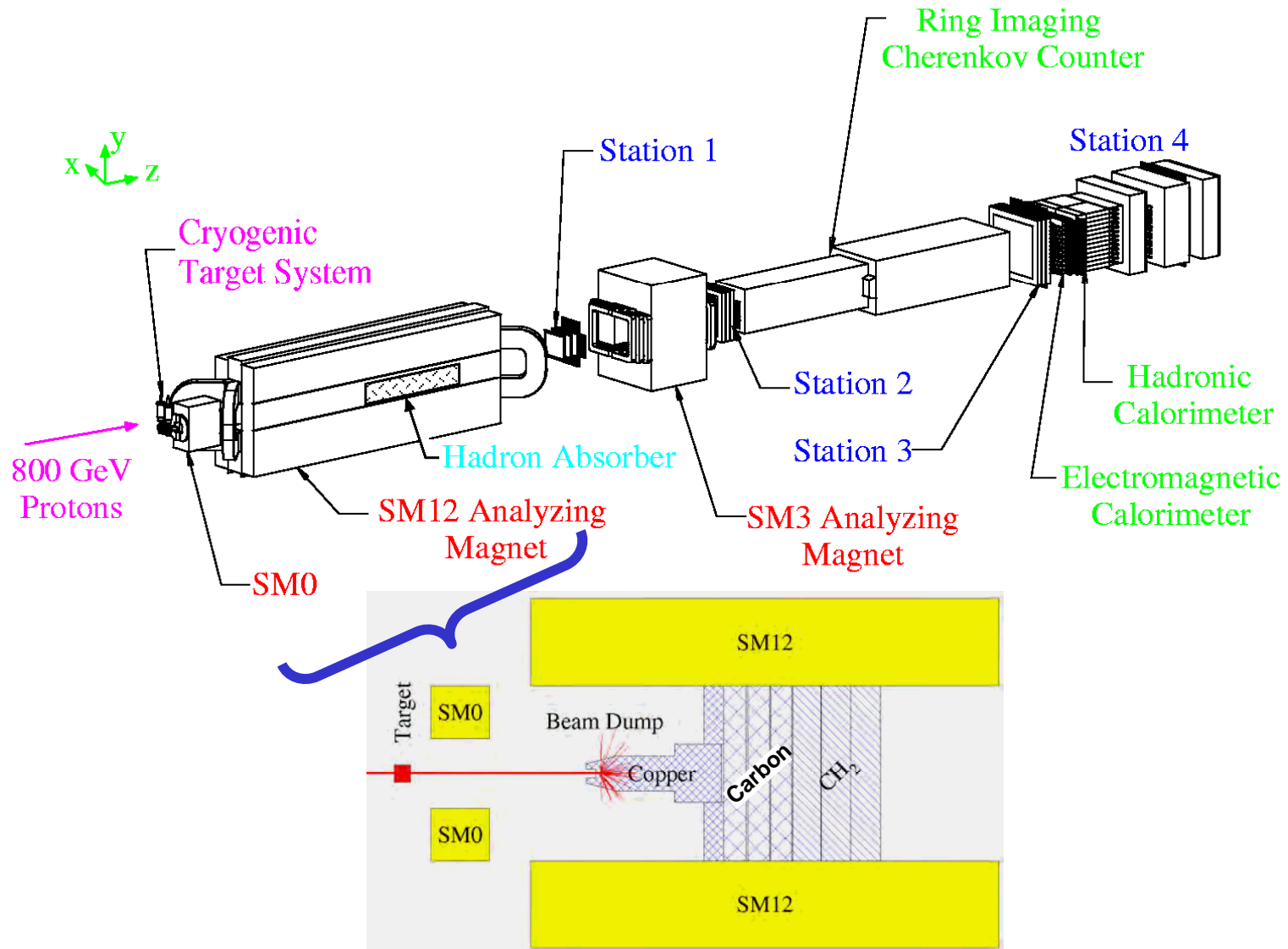


- ❖ The knowledge on sea quark distribution, nuclear effects and TMD PDFs are basically data driven. The Drell-Yan, complementary to DIS/SIDIS, is also a powerful tool to explore the proton structure.
- ❖ Drell-Yan including Fermilab E866 has produced a lot of interesting results. Fermilab E906 will extend the measurements to the large x region and further improve the statistics of the world unpolarized Drell-Yan data.



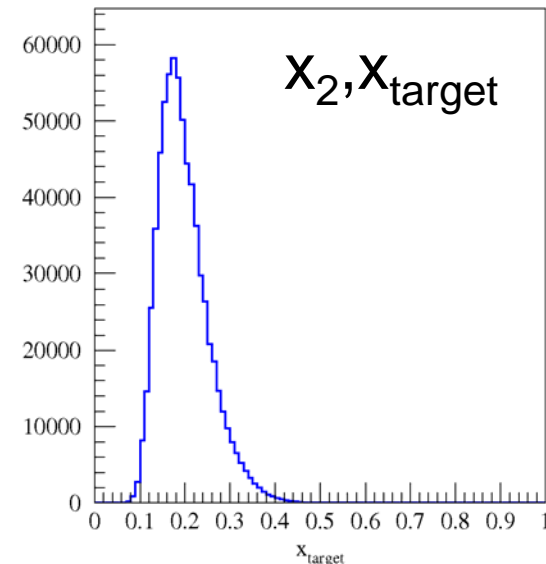
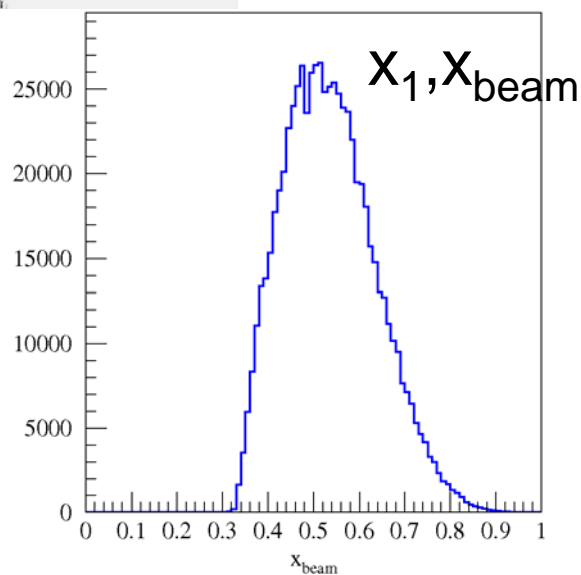
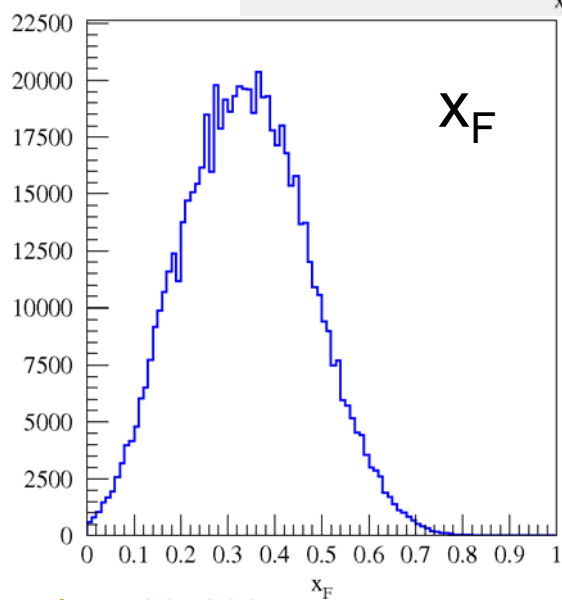
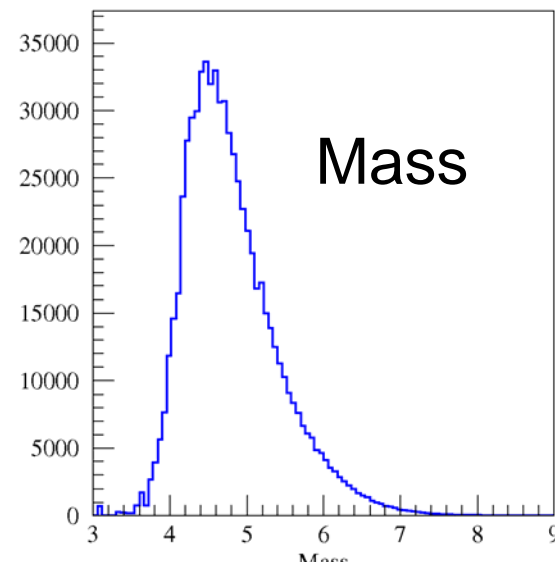
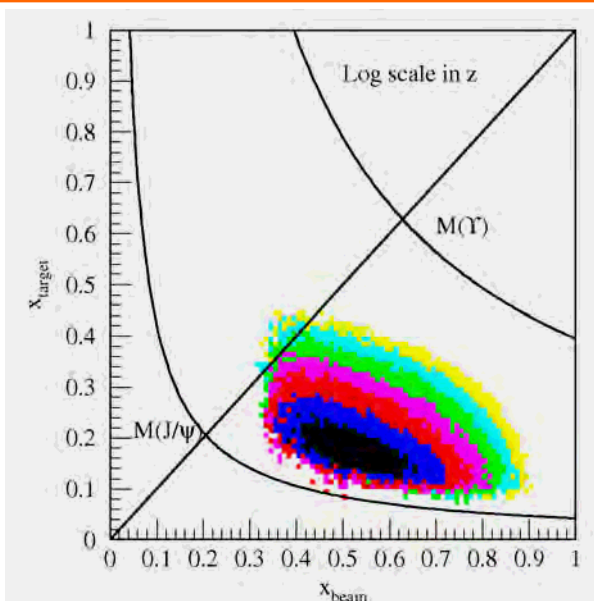


FNAL E866/NuSea Detector





E906 Drell-Yan Acceptance

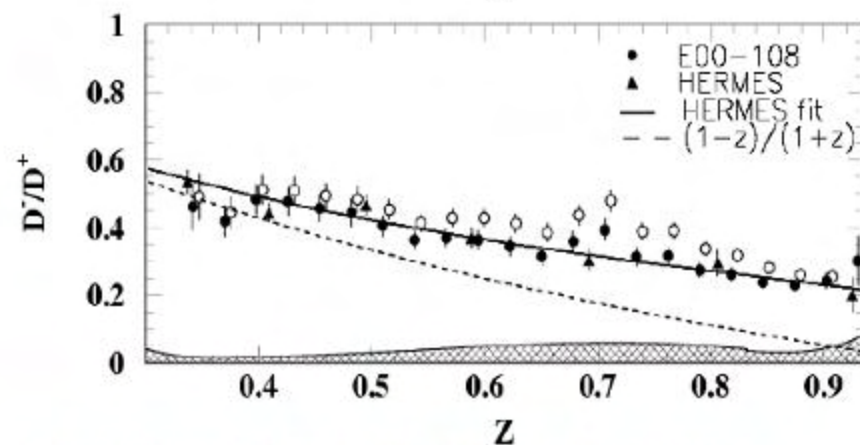
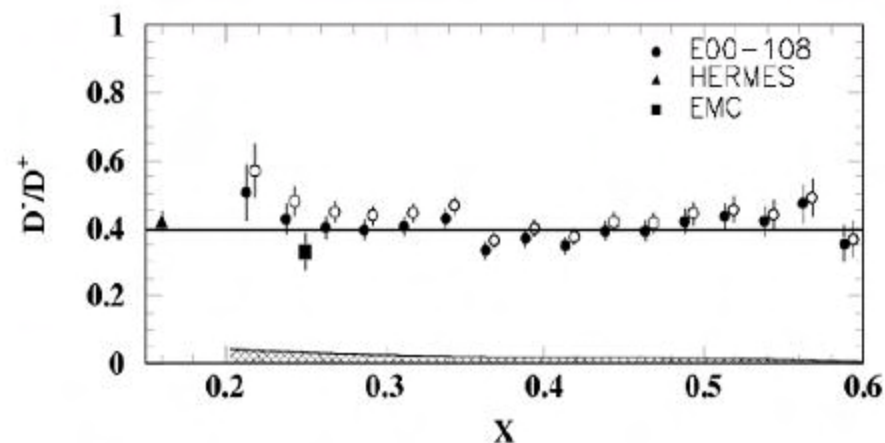
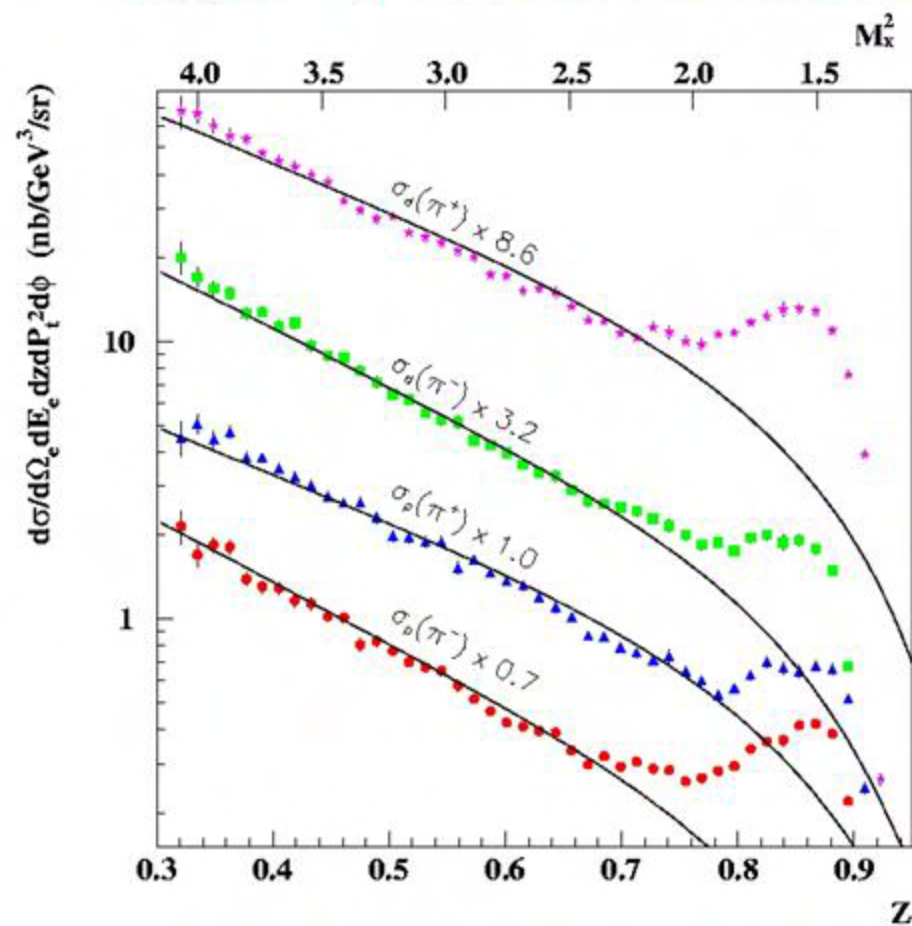




Factorization Check at JLab



$^1\text{H}(e,e'\pi^{+/-})$ from JLab Hall C E00-108: PRL98(2007) 022001 [hep-ph/0608214]



Data beyond Δ region are well described by LO SIDIS ansatz.



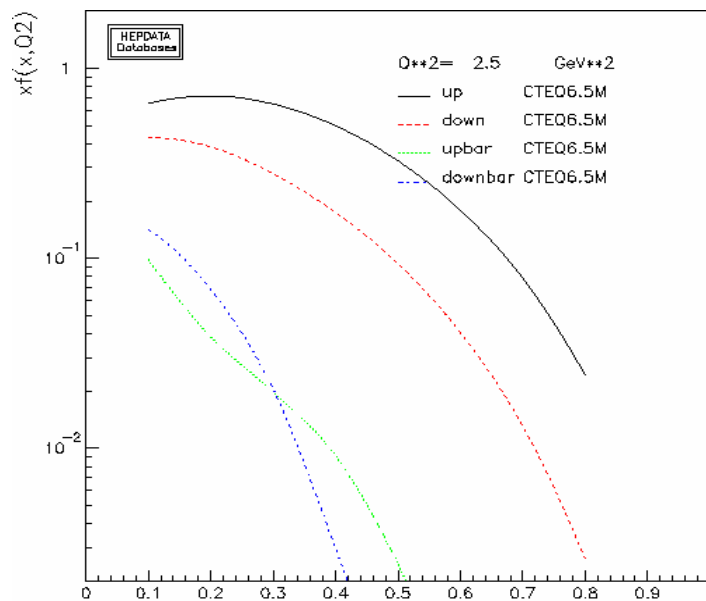
Drell-Yan & DIS Error Propagation



Drell – Yan : $pN \rightarrow \mu^+ \mu^- X$

$$\left. \frac{\sigma^{pd}}{2\sigma^{pp}} \right|_{x_b \gg x_t} \approx \frac{1}{2} \left[1 + \frac{\bar{d}(x_t)}{\bar{u}(x_t)} \right]$$

$$\bar{d} - \bar{u} = \left[\frac{\frac{\bar{d}}{\bar{u}} - 1}{\frac{\bar{d}}{\bar{u}} + 1} \right] (\bar{d} + \bar{u})$$



DIS : $ep \rightarrow e' X$

$$r(x, z) = \frac{Y_p^{\pi^-}(x, z) - Y_n^{\pi^-}(x, z)}{Y_p^{\pi^+}(x, z) - Y_n^{\pi^+}(x, z)}$$

$$J(z) = \frac{3}{5} \frac{1 + D'(z)}{1 - D'(z)}$$

$$\frac{J(z)[1 - r(x, z)] - [1 + r(x, z)]}{J(z)[1 - r(x, z)] + [1 + r(x, z)]} = \frac{\bar{d}(x) - \bar{u}(x)}{u(x) - d(x)}$$

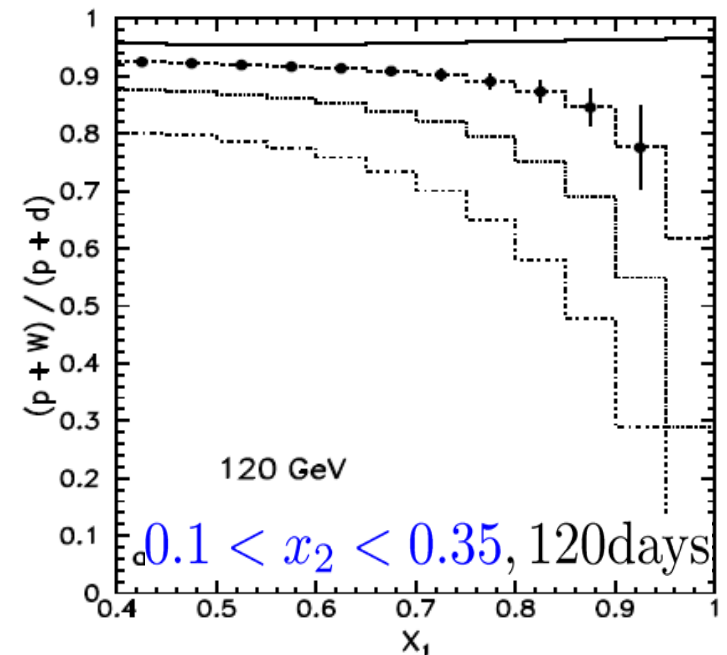
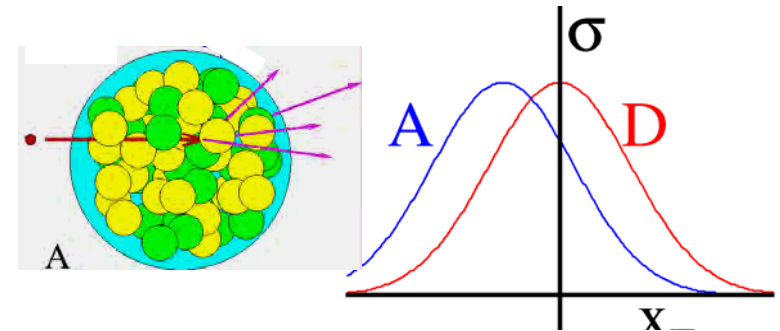
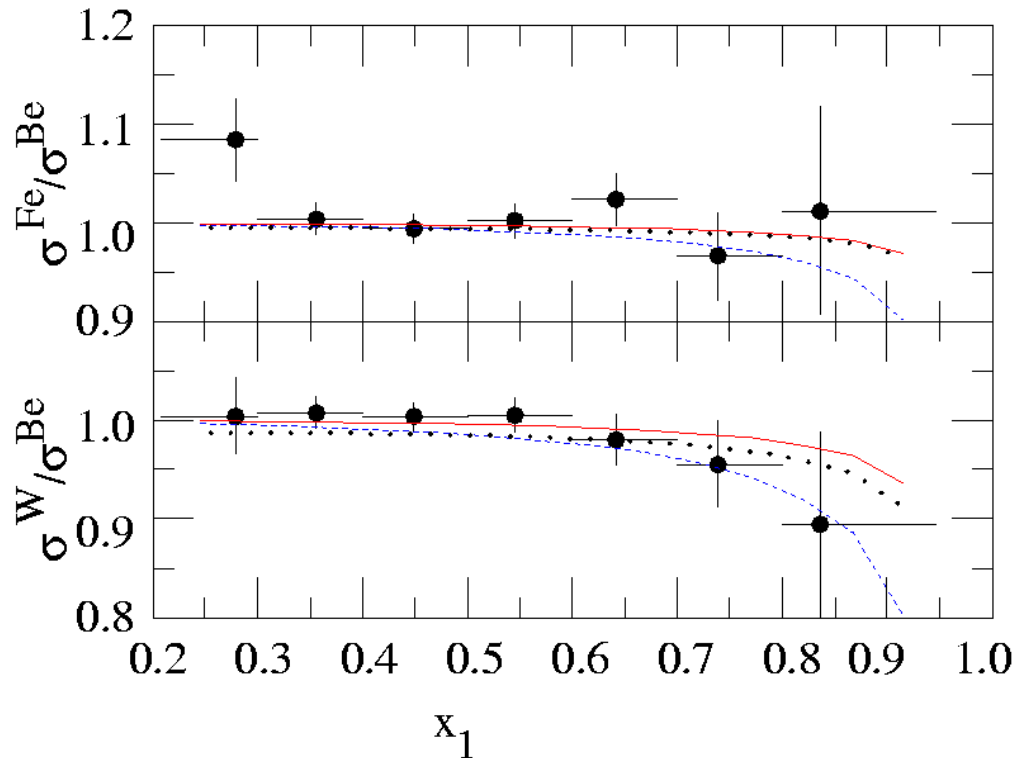
$$\bar{d} - \bar{u} = \left[\frac{\bar{d} - \bar{u}}{u - d} \right] (u - d)$$

$Q^2 = 2.5, \text{ CTEQ6.5M}$

	$d + \bar{u}$	u-d
x=0.2	0.54	1.65
x=0.3	0.13	1.23



Partonic Energy Loss from Drell-Yan



$dE/dx=0.1, 0.1, 0.25, 0.5$ GeV/fm
Garvey & Peng, PRL90(2003)092302



Nuclear Broadening of Transverse Momentum



- ❖ There is a factor of two difference between the new and old results on nuclear broadening of transverse momentum. The broadening effect is much bigger for resonances than that for Drell-Yan.

$$\Delta \langle p_T^2 \rangle \equiv \langle P_T^2 \rangle_A - \langle P_T^2 \rangle_N$$

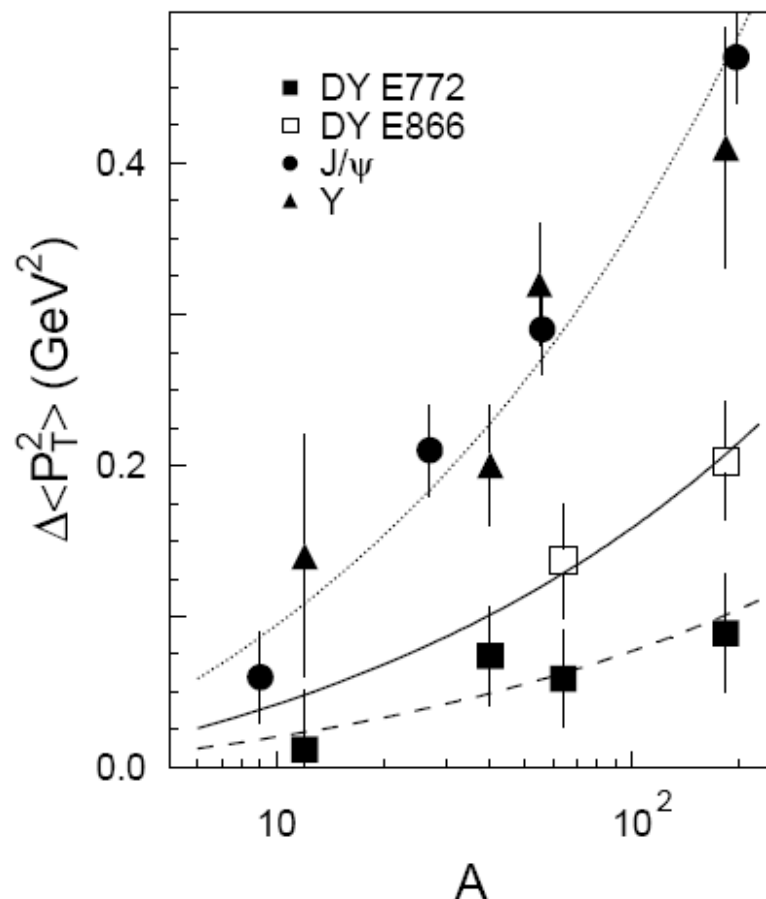
$$\Delta \langle p_T^2 \rangle = D[(A/2)^{1/3} - 1]$$

$$D(E772) = 0.029 \pm 0.008$$

$$D(E886) = 0.059 \pm 0.009$$

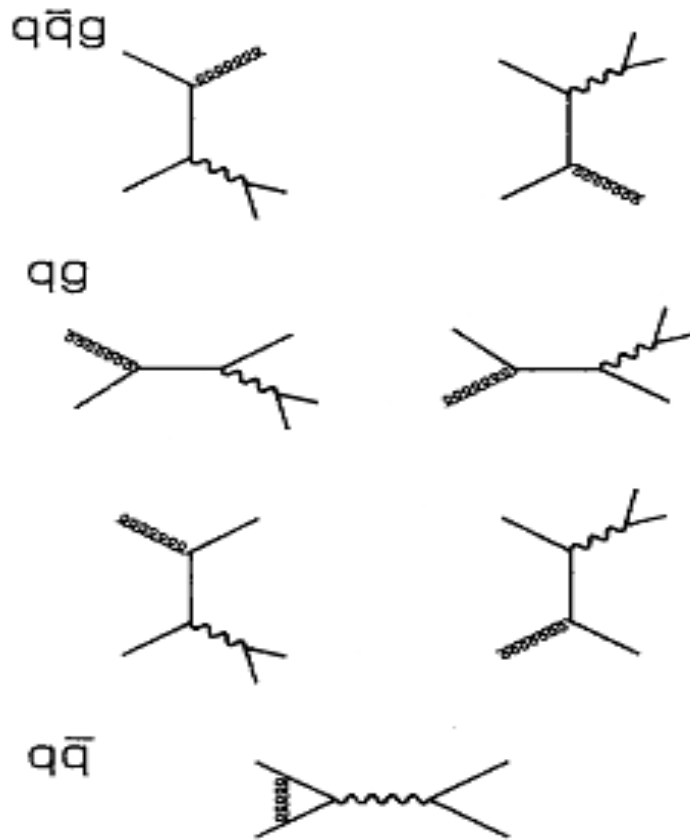
Johnson et al, PRC75(2007)035206

- ❖ Nuclear P_T broadening leads to medium-induced gluon radiation or energy loss.





First-order QCD Corrections to Drell-Yan



- Increase the overall cross section by a **K-factor** ~ 2 .

- The Lam-Tung relation still holds (in any reference frame for massless quarks), reflecting the spin-1/2 nature of the quarks. **Lam & Tung, PRD21,2712(1980)**

$$1 - \lambda - 2\nu = 0$$

(Analog to Callan-Gross relation in DIS)

- The NLO correction at $\mathcal{O}(\alpha_s^2)$ to the angular distribution is small. **Mirkes & Ohnemus, PRD51,4891(1995)**

Conway et al., PRD39,92(1989)

The QCD correction to the angular distribution is small.



Violation of the Lam-Tung Relation

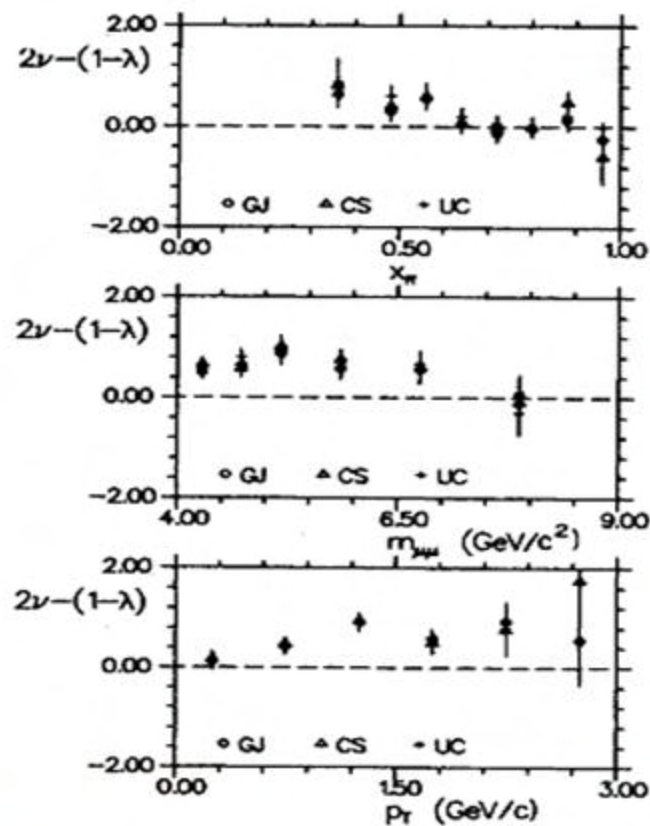


E615 at Fermilab: 252 GeV $\pi^- + W$

Conway et al., PRD39,92(1989)

NA10 at CERN:

140/194 GeV $\pi^- + W$, 286 GeV $\pi^- + W/d$
Z. Phys. C37, 545 (1988)



- The deviations from $1+\cos^2\theta$ due to the **soft-gluon resummation** are less than 5%.

Chiappatta & Bellac, ZPC32,521 (1986)

- The correction due to the **intrinsic transverse momenta** is estimated to be less than 0.05

Cleymans & Kuroda, PLB105,68(1981)

- Lam-Tung relation not affected by lowest order QCD correction even at small Q_T .

Boer & Wogelsang, hep-ph/0604177

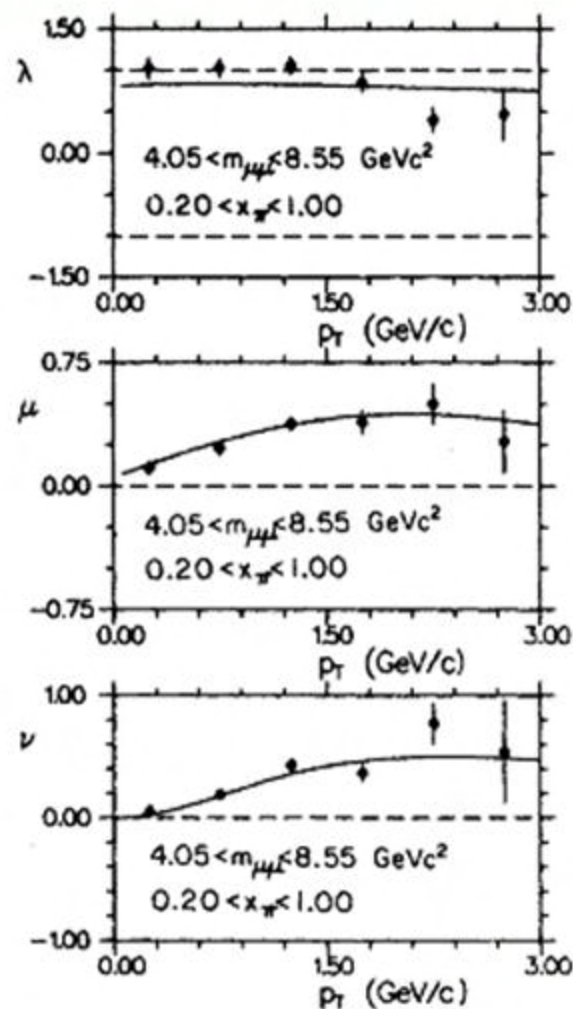
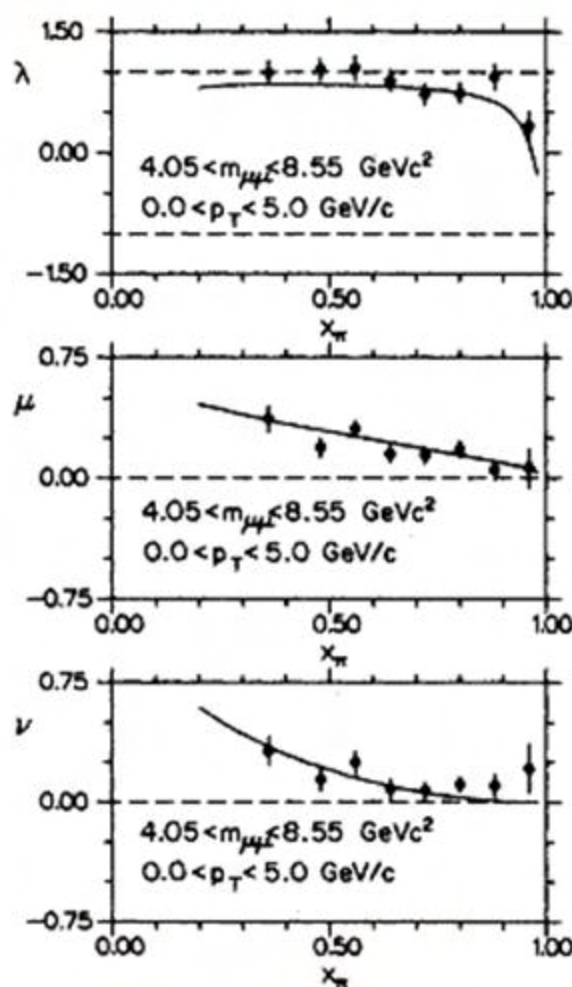
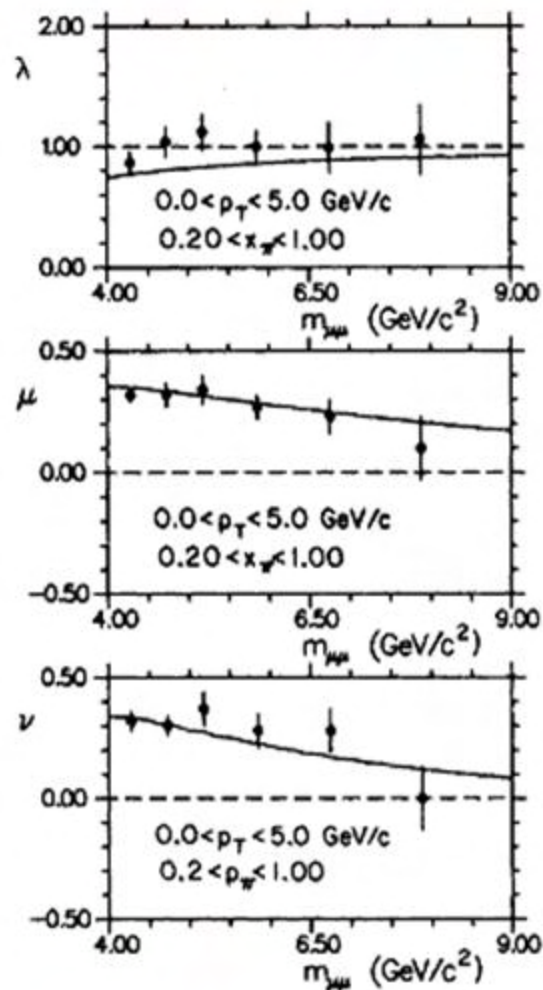


Angular Distribution in the πW Drell-Yan



E615 at Fermilab: 252 GeV $\pi^- + W$

Conway et al., PRD39,92(1989)





Nuclear Effect?



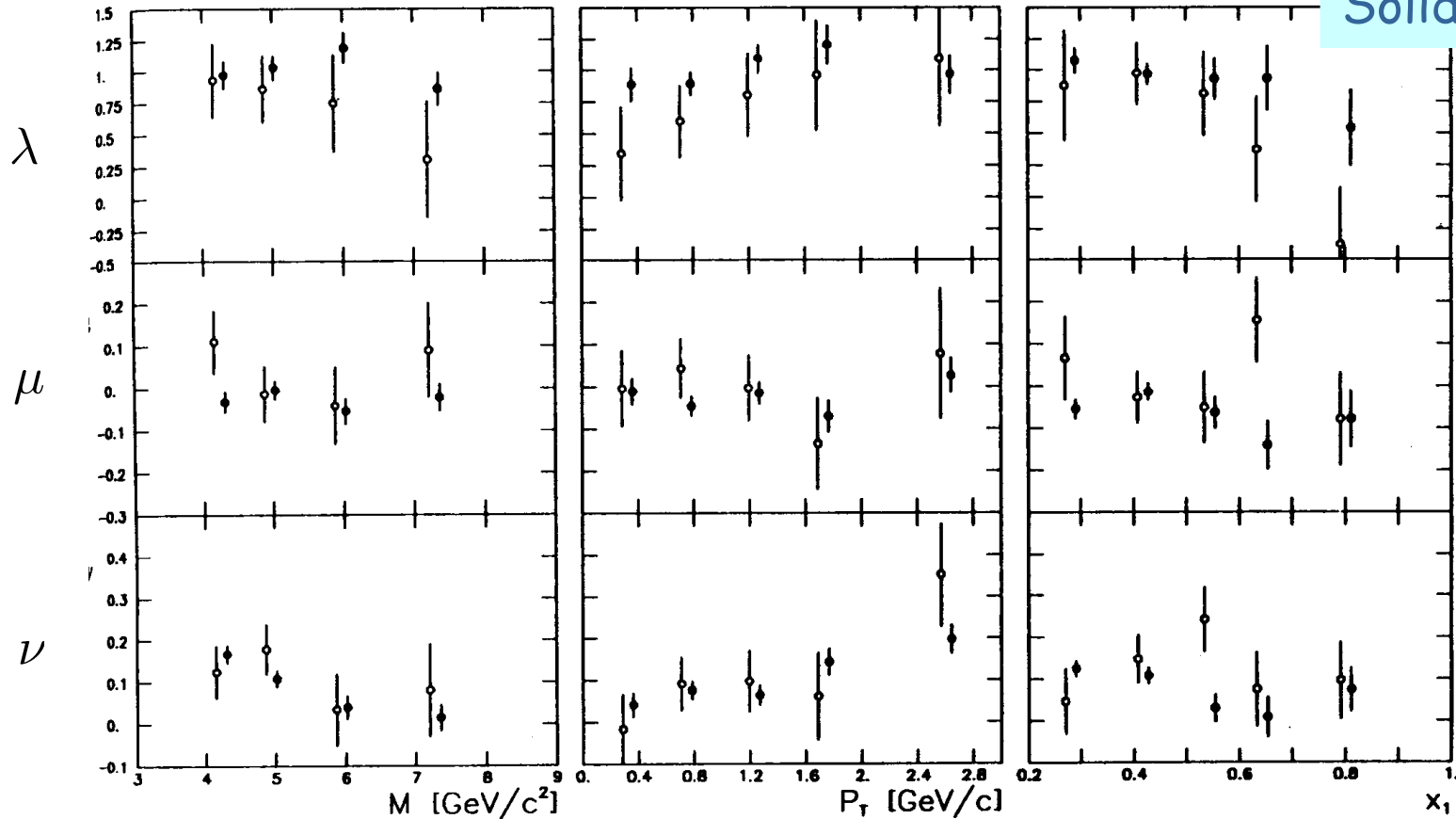
NA10 Z. Phys. C37, 545 (1988)

286 GeV/c

286 GeV/c

286 GeV/c

Open: Deuterium
Solid: Tungsten



Nuclear effect not likely to be the dominant contribution.



Boer-Mulders Function h_1^\perp in Spectator Model



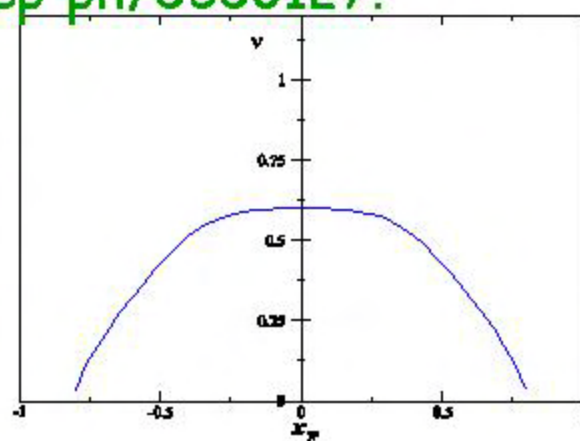
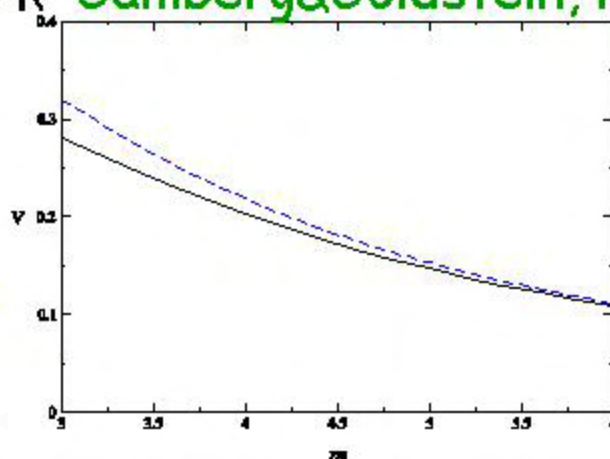
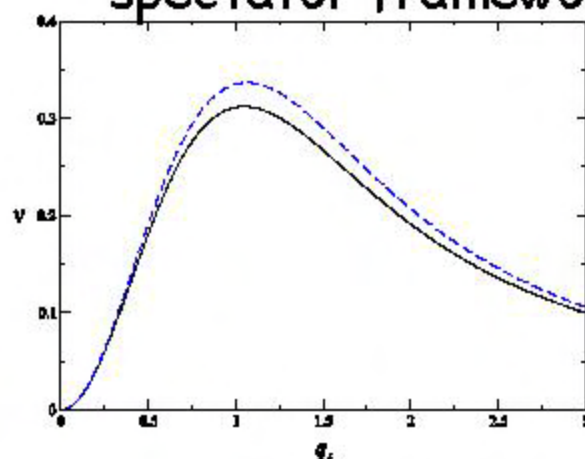
- ❖ Initial-state gluon interaction can produce nonzero h_1^\perp for the **proton** in the quark-scalar diquark model. **In this model,**

$$h_1^\perp = f_{1T}^\perp.$$

$$h_{1p}^\perp(x, k_\perp^2) = \frac{A_p(x)}{k_\perp^2 [k_\perp^2 + B_p(x)]} \ln \left[\frac{k_\perp^2 + B_p(x)}{B_p(x)} \right]$$

Boer, Brodsky & Hwang, PRD67,054003(2003).

- ❖ Twist 2 (as well as the kinematic twist 4) contribution in a parton-spectator framework **Gamberg & Goldstein, hep-ph/0506127.**



$$\nu_2 = \frac{\sum_a e_a^2 \mathcal{F} \left[(2\hat{h} \cdot k_\perp \cdot \hat{h} \cdot p_\perp - p_\perp \cdot k_\perp) h_1^\perp(x, k_\perp) \bar{h}_1^\perp(\bar{x}, p_\perp) / (M_1 M_2) \right]}{\sum_a e_a^2 \mathcal{F} [f_1(x, k_\perp) \bar{f}_1(\bar{x}, p_\perp)]}$$



Pion Boer-Mulders Function

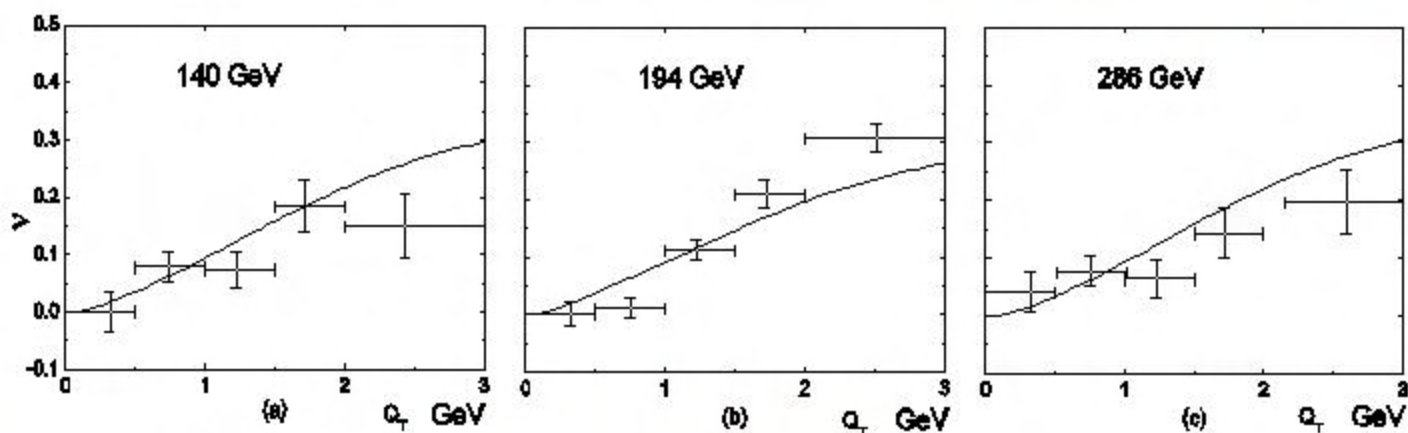


Final-state interaction with one gluon exchange can produce nonzero h_1^\perp for the **pion** in the quark-spectator-antiquark model **with constant coupling g_π** .

$$h_{1\pi}^\perp(x, k_\perp^2) = \frac{A_\pi(x)}{k_\perp^2 [k_\perp^2 + B_\pi(x)]} \ln \left[\frac{k_\perp^2 + B_\pi(x)}{B_\pi(x)} \right]$$

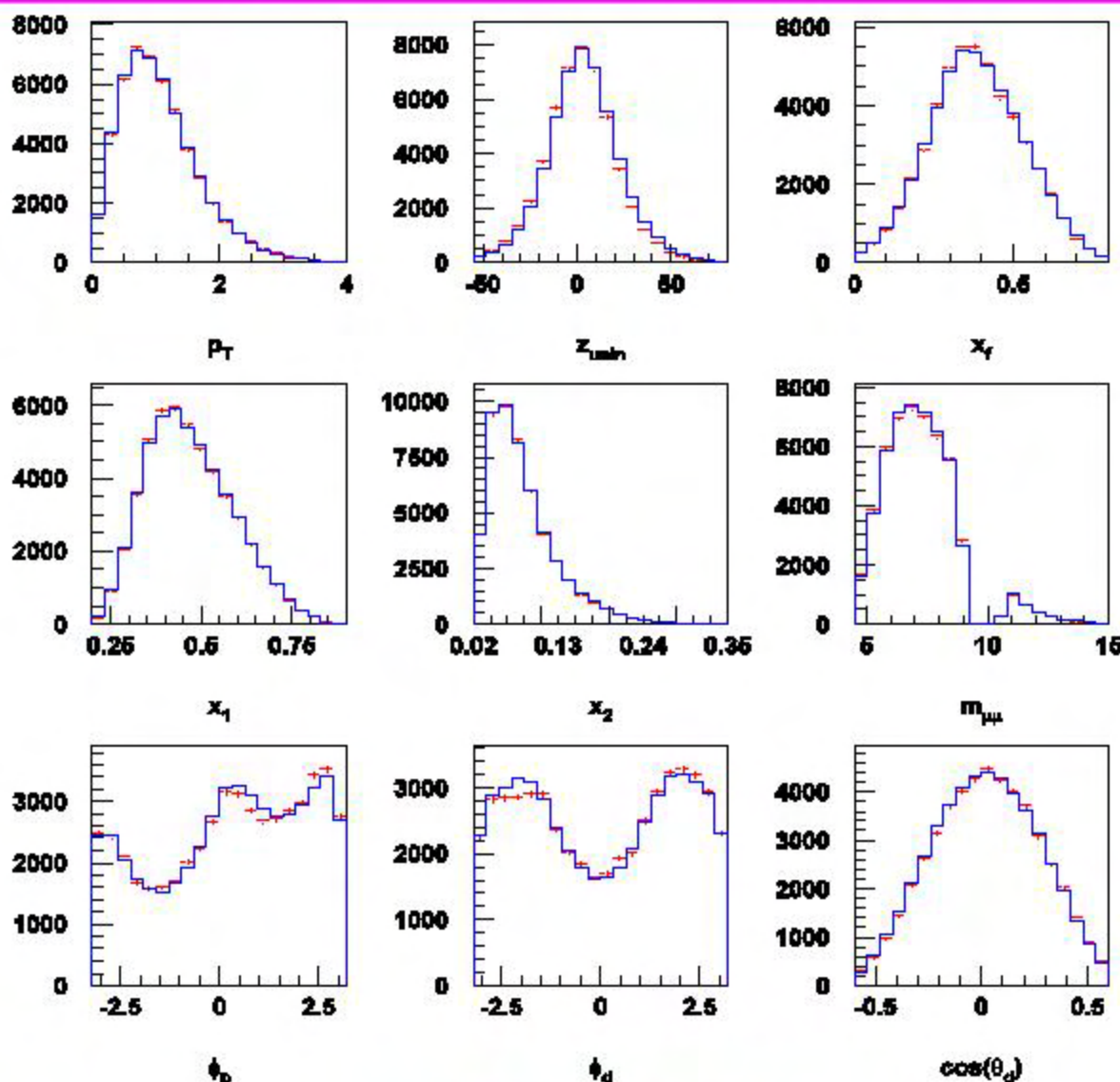
Lu&Ma, PRD70,094044(2004).

The quark-spectator-antiquark model with effective pion-quark-antiquark coupling as a dipole form factor **Lu & Ma, hep-ph/0504184**





Comparison of data and simulation



Blue: simulation
Red: data
(dset8)



Modeling Sea Boer-Mulders Functions



- ❖ Z. Lu, B.-Q. Ma and I. Schmidt, PRD75 (2007) 014026

Meson-baryon fluctuation model:

$$p \rightarrow n + \pi^+ ; p \rightarrow \Delta^{++} + \pi^-$$

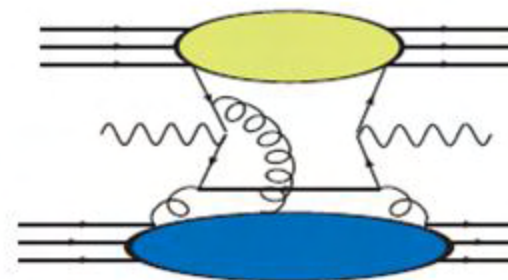
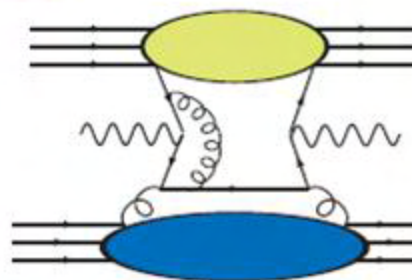
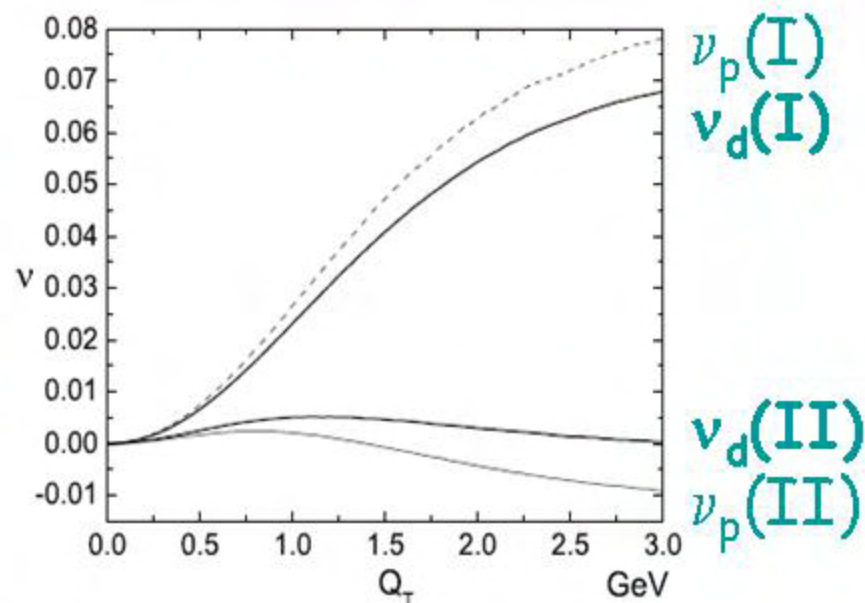
Predictions depend on the choice of valence Boer-Mulders functions:

I--scalar diquark;

II--scalar & vector diquark

- ❖ Group led by L. Gamburg and G. Goldstein

Two possible contributions to sea from the gauge link.



Probing the sea Boer-Mulders functions may constrain the valence ones.



$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega d\phi_S} \propto 1 + \lambda \cos^2 \theta + \sin^2 \theta \left[\frac{\nu}{2} \cos 2\phi + \rho |S_T| \sin(\phi + \phi_S) \right] + \dots$$

Assuming u-quark dominance

$$\rho = \frac{1}{2} \sqrt{\frac{\nu}{\nu_{max}}} \frac{h_1^\perp}{f_{1T}^\perp}$$

Burkardt Relation

$$\frac{f_{1T}^{\perp q}}{E} \approx \frac{h_1^{\perp q}}{2\tilde{H}_T + E_T}$$