Extraction of the Compton Form Factor $\mathcal{H}$ from recent DVCS measurements at JLab

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1. Preliminary analysis

2. Fitting strategies

3. Results

(Generalized) Parton Distributions.
From Deep Inelastic Scattering to Deeply Virtual Compton Scattering.

DIS
\[ e^- \rightarrow e^- \]
\[ \gamma, Q^2 \]
\[ p \rightarrow X \]

Parton Distributions
\[ \gamma, Q^2 \]
nucleon

Extraction of \( \mathcal{H} \) from DVCS

Preliminary analysis

About GPDs
Leading twist
Selected data
GV formalism
Assumptions

Fitting strategies
Local fits
Global fit

Results
\( \text{Im} \mathcal{H} \) and \( \text{Re} \mathcal{H} \)

Discussion

Conclusions
(Generalized) Parton Distributions.
From Deep Inelastic Scattering to Deeply Virtual Compton Scattering.

- Correlation of the **longitudinal momentum** and the **transverse position** of the struck quark.
(Generalized) Parton Distributions.
From Deep Inelastic Scattering to Deeply Virtual Compton Scattering.

- Correlation of the **longitudinal momentum** and the **transverse position** of the struck quark.
- **3-dimensional** description of the nucleon.
- Insights on:
  - spin structure,
  - energy-momentum structure.

\[
\begin{align*}
H(x, \xi, t) & \\
E(x, \xi, t) & \\
\tilde{H}(x, \xi, t) & \\
\tilde{E}(x, \xi, t) & \\
\end{align*}
\]
Compton Form Factors.

At leading twist, DVCS cross sections are described by 4 complex functions.

**Example:** GPD $H$

$$
\mathcal{H} = \int_{-1}^{+1} dx \, H(x, \xi, t) \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right)
$$

Integration yields **real** and **imaginary** parts to $H$:

$$
\text{Re}\mathcal{H} = \mathcal{P} \int_{-1}^{+1} dx \, H(x, \xi, t) \left( \frac{1}{\xi - x} - \frac{1}{\xi + x} \right)
$$

$$
\text{Im}\mathcal{H} = \pi \left( H(\xi, \xi, t) - H(-\xi, \xi, t) \right)
$$

Relation between $\text{Im}\mathcal{H}$ and $\text{Re}\mathcal{H}$ **weakly constrained** by dispersion relations.
Selected JLab data: recent DVCS measurements.
Fine kinematic binning and large kinematic coverage.

**Hall A: Helicity-dependent and independent cross sections**

- 12 bins: 1 value of $x_B$, 3 values of $Q^2$ and 4 values of $t$.
- Each kinematic bin contains 24 $\phi$-bins.
- Statistical uncertainties:
  - helicity-dependent: at least 20 %
  - helicity-independent: $\sim$ 5 %

**Hall B: Beam Spin Asymmetries**

- 62 bins: 5 value of $x_B$, 4 values of $Q^2$ and 5 values of $t$.
- Each kinematic bin contains (at most) 12 $\phi$-bins.
- Statistical uncertainties: $\sim$ 25 %
Analytic $ep \rightarrow ep\gamma$ cross sections.
Interference between Bethe-Heitler and VCS processes treated exactly.

Example: DVCS helicity-dependent cross section at twist 2

- **BMK formalism**:

$$C_1 \sin \phi \Im \left( \mathcal{H} + \frac{x_B}{2-x_B} \left(1 + \frac{F_2}{F_1}\right) \mathcal{\tilde{H}} - \frac{t}{4M^2} \frac{F_2}{F_1} \mathcal{E} \right)$$

A.V. Belitsky, D. Mueller and A. Kirchner

- **GV formalism**:

$$C_2 \sin \phi \Im \left( \mathcal{H} + c_\mathcal{E} \mathcal{E} + c_\mathcal{\tilde{H}} \mathcal{\tilde{H}} + c_\mathcal{\tilde{E}} \mathcal{\tilde{E}} \right)$$

P.A.M. Guichon and M. Vanderhaeghen, unpublished
Analytic $ep \to ep\gamma$ cross sections.
Interference between Bethe-Heitler and VCS processes treated exactly.

Example : DVCS helicity-dependent cross section at twist 2

- BMK formalism: coefficients do not depend on $Q^2$

$$C_1 \sin \phi \text{ Im} \left( \mathcal{H} + \frac{x_B}{2-x_B} \left(1 + \frac{F_2}{F_1}\right) \tilde{\mathcal{H}} - \frac{t}{4M^2} \frac{F_2}{F_1} \mathcal{E} \right)$$

A.V. Belitsky, D. Mueller and A. Kirchner

- GV formalism: coefficients depend on $Q^2$

$$C_2 \sin \phi \text{ Im} \left( \mathcal{H} + \frac{c\mathcal{E}}{20 \%} \mathcal{E} + \frac{c\tilde{\mathcal{H}}}{20 \%} \tilde{\mathcal{H}} + \frac{c\tilde{\mathcal{E}}}{30 \%} \tilde{\mathcal{E}} \right)$$

P.A.M. Guichon and M. Vanderhaeghen, unpublished
Main assumptions.
Expectation: extraction of $\mathcal{H}$ with $\geq 40\%$ total uncertainty.

- **Twist 2 accuracy**
  - Early $Q^2$-scaling was observed in Hall A.
    
    C. Muñoz Camacho et al.
  
  - Test of twist 3 contribution left for future work.

- **$H$-dominance**
  - Dramatically decreases the number of degrees of freedom in the fits.
  - Theoretical expectations: systematic error between 20 and 50%.
  - Systematic error $\lesssim 25\%$ from direct test of hypothesis with VGG model.
Local fits.
Fits on each kinematic bin to twist 2 expressions.

- Keep bins with \( \frac{|t|}{Q^2} < \frac{1}{2} \).
- Low model dependence \((H\text{-dominance, twist 2})\).
- But fits may still be underconstrained.
- **Estimation** of systematic errors caused by \(H\text{-dominance hypothesis}\) by fitting data with subdominant GPDs set to 0 or to their VGG value.
Global fit.
Fit to a parametrization from the dual model.

- DVCS cross sections depend on singlet combination $H_+$:

\[ H_+(x, \xi, t, Q^2) = H(x, \xi, t, Q^2) - H(-x, \xi, t, Q^2) \]

- Dual model parametrization of $H_+$:

\[
2 \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left( 1 - \frac{x^2}{\xi^2} \right) \left( 1 - \frac{x^2}{\xi^2} \right) C_{2n+1}^3 \left( \frac{x}{\xi} \right) P_{2l} \left( \frac{1}{\xi} \right)
\]
Global fit.
Fit to a parametrization from the dual model.

- DVCS cross sections depend on singlet combination $H_+$:
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  \]
  Legendre polynomial

Dual model parametrization of $H_+$:

\[
2 \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right) C_{2n+1}^{\frac{3}{2}} \left(\frac{x}{\xi}\right) P_{2l} \left(\frac{1}{\xi}\right)
\]

Legendre polynomial
Global fit.
Fit to a parametrization from the dual model.

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$$H_+(x, \xi, t, Q^2) = H(x, \xi, t, Q^2) - H(-x, \xi, t, Q^2)$$

- Dual model parametrization of $H_+$:

$$2 \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right) C_{2n+1}^{3} \left(\frac{x}{\xi}\right) P_{2l} \left(\frac{1}{\xi}\right)$$

Gegenbauer polynomial
Global fit.
Fit to a parametrization from the dual model.

- DVCS cross sections depend on singlet combination $H_+$:
  $$H_+(x, \xi, t, Q^2) = H(x, \xi, t, Q^2) - H(-x, \xi, t, Q^2)$$

- Dual model parametrization of $H_+$:
  $$2 \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left( 1 - \frac{x^2}{\xi^2} \right) \left( 1 - \frac{x^2}{\xi^2} \right) C_{2n+1}^{3/2} \left( \frac{x}{\xi} \right) P_{2l} \left( \frac{1}{\xi} \right)$$
  
  Support:
  \[ \text{Resummed} \]
Global fit.
Fit to a parametrization from the dual model.

- DVCS cross sections depend on singlet combination $H_+$:
  $$H_+(x, \xi, t, Q^2) = H(x, \xi, t, Q^2) - H(-x, \xi, t, Q^2)$$

- Dual model parametrization of $H_+$:
  $$2 \sum_{n=0}^{\infty} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right) C_{2n+1}^{\frac{3}{2}} \left(\frac{x}{\xi}\right) P_{2l} \left(\frac{1}{\xi}\right)$$

Model $t$-dep.

with $B_{nl}(t, Q^2) = \left(\ln \frac{Q^2_0}{\Lambda^2} / \ln \frac{Q^2}{\Lambda^2}\right)^{\frac{\gamma p}{\beta_0}} B_{nl}(t, Q^2_0)$. 
Global fit.
Fit to a parametrization from the dual model.

- DVCS cross sections depend on singlet combination $H_+$:
  \[ H_+(x, \xi, t, Q^2) = H(x, \xi, t, Q^2) - H(-x, \xi, t, Q^2) \]

- Dual model parametrization of $H_+$:
  \[
  2 \sum_{n=0}^{N} \sum_{l=0}^{n+1} B_{nl}(t, Q^2) \theta \left( 1 - \frac{x^2}{\xi^2} \right) \left( 1 - \frac{x^2}{\xi^2} \right) C_{2n+1}^3 \left( \frac{x}{\xi} \right) P_{2l} \left( \frac{1}{\xi} \right)
  \]

  with $B_{nl}(t, Q^2) = \left( \ln \frac{Q_0^2}{\Lambda^2} / \ln \frac{Q^2}{\Lambda^2} \right)^{\gamma p / \beta_0} \frac{a_{nl}}{1 + b_{nl}(t-t_0)^2}$.

- Non-trivial correlation between $x$ and $t$.
- $a_{nl}$ and $b_{nl}$ are fitted. $t_0$ is chosen prior to the fits.
Global fit.
Iterative fitting procedure and systematic uncertainties.

- Keep bins with \( \frac{|t|}{Q^2} < \frac{1}{2} \) (1001 \( \phi \)-bins fitted).

- \( \frac{N(N+3)}{2} \) fitted coefficients for a given truncation \( N \).
  - Restrict to low values of \( N \).
  - 10, 18 and 28-parameter fits for \( N = 2, 3 \) and 4.
  - **Estimation** of the **truncation error** by comparison of the results of these 3 fits.

- Iterative fitting procedure to handle large number of parameters.

- **Estimation** of systematic errors caused by **\( H \)-dominance hypothesis** by fitting data with subdominant GPDs set to 0 or to their VGG value.

- **Purpose**: smooth parametrization of data. **No extrapolation** outside the domain of the fit.
Effect of the truncation of the series.
Hall B data.

- 3 global fits qualitatively similar:
  \[
  N \quad \chi^2/d.o.f.
  \]
  \[
  \begin{array}{ll}
    2 & 1.73 \\
    3 & 1.61 \\
    4 & 1.78 \\
  \end{array}
  \]

- No differences on Hall A data (next slide).
- \(N=2\) fails to reproduce BSAs at small \(\xi\).
- \(N=3\) always good and close to local fits.
- \(N=4\) is uncontrolled at large \(\xi\).
Effect of the truncation of the series.
Hall A data.

![Graph showing the effect of truncation of the series for different values of t (GeV^2). The graph includes data points and curves for local fits and a global fit, with N=2, N=3, and N=4.](image)
**ImH** on Hall B kinematics. 

$Q^2$-dependence.

- **Compatibe results of local and global fits**:
  - **Strong consistency check**.
  - **Local fits**: fluctuations but model-independent.
  - **Global fit**: no fluctuations but truncation effect.

- **Realistic estimation of systematic uncertainties**:
  - Comparable accuracy from local and global fits.
  - Accuracy in agreement with expectations.

- **Restricted kinematic region suitable for GPD-analysis.**
Large fluctuations in $Re\mathcal{H}$ from local fits. Global fit is smoother.

Unreliable extraction of $Im\mathcal{H}$ or $Re\mathcal{H}$ at large $\xi$.

$Re\mathcal{H}$ weakly constrained.
ImH on Hall A kinematics.

$t$-dependence.

- Good agreement between results of local and global fits but...
- Discrepancy seems to be larger at small $|t|$!
- Sizeable scaling deviation for $t = -0.17$ GeV$^2$.
- Noticeable deviations if

$$
\xi = x_B \frac{1 + \frac{t}{2Q^2}}{2 - x_B + \frac{x_B t}{Q^2}} \rightarrow \frac{x_B}{2 - x_B}
$$

- Call for a twist 3 analysis!
Im$\mathcal{H}$ and Re$\mathcal{H}$ on Hall A kinematics. $t$-dependence.
Comparison with previous studies (Hall A data). Significant deviations to an extraction with the BMK formalism.

- Agreement with model-independent approach at twist 2.
Comparison to the VGG model.
Similar $x_B$-dependence but loss of information during the extraction.
Conclusions.
Extraction of GPDs from measurements is a challenge to phenomenology.

- JLab DVCS measurements are already a strong constraint on phenomenological interpretation and will remain a reference in the near future.

- $\text{Im}\mathcal{H}$ extracted with 20 to 50% accuracy on a wide kinematic range.

- Realistic first estimation of systematic errors.

- Plausible early $Q^2$-scaling but twist 3 study necessary.

- Working without $H$-dominance hypothesis?

- More generally, a fitting strategy allowing an extrapolation outside the domain of the extraction is still missing.