TMDs and Transverse Spin Structure of Hadrons

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Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
\[ H(x, 0, -\Delta^2_{\perp}) \rightarrow q(x, b_{\perp}) \]
\[ \tilde{H}(x, 0, -\Delta^2_{\perp}) \rightarrow \Delta q(x, b_{\perp}) \]
\[ E(x, 0, -\Delta^2_{\perp}) \rightarrow \perp \text{ distortion of PDFs when the target is } \perp \text{ polarized} \]
Chromodynamik lensing and \perp single-spin asymmetries (SSA)

\[
\begin{align*}
\text{transverse distortion of PDFs} & \quad \Rightarrow \quad \perp \text{ SSA in } \gamma N \rightarrow \pi + X \\
+ \text{ final state interactions} & \\
\end{align*}
\]
- Sivers
- Boer-Mulders
- Quark Gluon Correlations \( g_2(x) \) \( \rightarrow \perp \) force on quarks in DIS
- Summary

TMDs and Transverse Spin Structure of Hadrons – p.2/41
Generalized Parton Distributions (GPDs)

- **GPDs**: decomposition of form factors at a given value of $t$, w.r.t. the average momentum fraction $x = \frac{1}{2} (x_i + x_f)$ of the active quark

$$
\int dx H_q(x, \xi, t) = F_1^q(t) \quad \int dx \tilde{H}_q(x, \xi, t) = G_A^q(t)
$$

$$
\int dx E_q(x, \xi, t) = F_2^q(t) \quad \int dx \tilde{E}_q(x, \xi, t) = G_P^q(t),
$$

- $x_i$ and $x_f$ are the momentum fractions of the quark before and after the momentum transfer
- $2\xi = x_f - x_i$

- GPDs can be probed in deeply virtual Compton scattering (DVCS)

![Diagram showing DVCS process]
Generalized Parton Distributions (GPDs)

formal definition (unpol. quarks):

\[
\int \frac{dx^-}{2\pi} e^{ix^- p^+ x} \left\langle p' \left| \bar{q} \left( -\frac{x^-}{2} \right) \gamma^+ q \left( \frac{x^-}{2} \right) \right| p \right\rangle = H(x, \xi, \Delta^2) \bar{u}(p') \gamma^+ u(p)
\]

\[
+ E(x, \xi, \Delta^2) \bar{u}(p') \frac{i\sigma^{+\nu} \Delta_{\nu}}{2M} u(p)
\]

in the limit of vanishing \( t \) and \( \xi \), the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

\[
H_q(x, 0, 0) = q(x) \quad \tilde{H}_q(x, 0, 0) = \Delta q(x).
\]

DVCS amplitude

\[
A(\xi, t) \sim \int_{-1}^{1} \frac{dx}{x - \xi + i\varepsilon} \text{GPD}(x, \xi, t)
\]
### Form Factors vs. GPDs

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<th>Position Space</th>
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<td>( F(t) )</td>
<td>( \rho(\vec{r}) )</td>
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$q(x, b_\perp) = \text{impact parameter dependent PDF}$
Impact parameter dependent PDFs

- define \( \perp \) localized state [D.Soper, PRD 15, 1141 (1977)]

\[
|p^+, R_{\perp} = 0_{\perp}, \lambda\rangle \equiv \mathcal{N} \int d^2p_{\perp} |p^+, p_{\perp}, \lambda\rangle
\]

Note: \( \perp \) boosts in IMF form Galilean subgroup \( \Rightarrow \) this state has

\[
R_{\perp} \equiv \frac{1}{p_+} \int dx^- d^2x_{\perp} x_{\perp} T^{++}(x) = \sum_i x_i r_{i,\perp} = 0_{\perp}
\]

(cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

\[
q(x, b_{\perp}) \equiv \int \frac{dx^-}{4\pi} \langle p^+, R_{\perp} = 0_{\perp} | \bar{q}(-\frac{x^-}{2}, b_{\perp}) \gamma^+ q(\frac{x^-}{2}, b_{\perp}) | p^+, R_{\perp} = 0_{\perp} \rangle e^{ixp^+ x^-}
\]

\[
q(x, b_{\perp}) = \int \frac{d^2\Delta_{\perp}}{(2\pi)^2} e^{i\Delta_{\perp} \cdot b_{\perp}} H(x, 0, -\Delta_{\perp}^2),
\]

\[
\Delta q(x, b_{\perp}) = \int \frac{d^2\Delta_{\perp}}{(2\pi)^2} e^{i\Delta_{\perp} \cdot b_{\perp}} \tilde{H}(x, 0, -\Delta_{\perp}^2),
\]
Impact parameter dependent PDFs

- No relativistic corrections (Galilean subgroup!)

\[ \rightarrow \text{corollary: interpretation of 2d-FT of } F_1(Q^2) \text{ as charge density in transverse plane also free from relativistic corrections} \]

- \( q(x, b_\perp) \) has probabilistic interpretation as number density (\( \Delta q(x, b_\perp) \) as difference of number densities)

- Reference point for IPDs is transverse center of (longitudinal) momentum \( \mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{i,\perp} \)

\[ \rightarrow \text{for } x \to 1, \text{ active quark ‘becomes’ COM, and } q(x, b_\perp) \text{ must become very narrow (\( \delta \)-function like)} \]

\[ \rightarrow H(x, 0, -\Delta^2_\perp) \text{ must become } \Delta_\perp \text{ indep. as } x \to 1 \text{ (MB, 2000)} \]

\[ \rightarrow \text{consistent with lattice results for first few moments} \]

- Note that this does not necessarily imply that ‘hadron size’ goes to zero as \( x \to 1 \), as separation \( r_\perp \) between active quark and COM of spectators is related to impact parameter \( b_\perp \) via \( r_\perp = \frac{1}{1-x} b_\perp \).
$q(x, b_{⊥})$ for unpolarized proton

$x = \text{momentum fraction of the quark}$

$\vec{b} = \perp \text{position of the quark}$
Transversely Deformed Distributions and $E(x, 0, -\Delta_\perp^2)$


- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^-} x^- \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \uparrow \rangle = H(x, 0, -\Delta_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^-} x^- \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \downarrow \rangle = -\frac{\Delta_x i \Delta_y}{2M} E(x, 0, -\Delta_\perp^2).$$

- Consider nucleon polarized in $x$ direction (in IMF)

$$|X\rangle \equiv |p^+, R_\perp = 0_\perp, \uparrow \rangle + |p^+, R_\perp = 0_\perp, \downarrow \rangle.$$

$\rightarrow$ unpolarized quark distribution for this state:

$$q(x, b_\perp) = \mathcal{H}(x, b_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E(x, 0, -\Delta_\perp^2) e^{-ib_\perp \cdot \Delta_\perp}$$

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from $j^3$!

[X.Ji, PRL 91, 062001 (2003)]
Intuitive connection with $\vec{J}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($p_{\gamma^*}$ in $-\hat{z}$ direction)

$\rightarrow j^+$ larger than $j^0$ when quark current towards the $\gamma^*$; suppressed when away from $\gamma^*$

$\rightarrow$ For quarks with positive orbital angular momentum in $\hat{x}$-direction, $j^z$ is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side

$\rightarrow$ Details of $\perp$ deformation described by $E_q(x, 0, -\Delta_\perp^2)$

$\rightarrow$ not surprising that $E_q(x, 0, -\Delta_\perp^2)$ enters Ji relation!

$$\langle J_q^i \rangle = S^i \int dx \left[ H_q(x, 0, 0) + E_q(x, 0, 0) \right] x.$$
Transversely Deformed PDFs and $E(x, 0, -\Delta^2_\perp)$

- $q(x, b_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons!

- mean $\perp$ deformation of flavor $q$ ($\perp$ flavor dipole moment)

$$d^q_y \equiv \int dx \int d^2b_\perp q X(x, b_\perp)b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa^p_q}{2M}$$

with $\kappa^p_u/d \equiv F^{u/d}_2(0) = O(1-2) \Rightarrow d^q_y = O(0.2\text{fm})$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

$$E_u(x, 0, -\Delta^2_\perp) = \frac{\kappa^p_u}{2} H_u(x, 0, -\Delta^2_\perp)$$ $$E_d(x, 0, -\Delta^2_\perp) = \kappa^p_d H_d(x, 0, -\Delta^2_\perp)$$

with $\kappa^p_u = 2\kappa_p + \kappa_n = 1.673$ \quad $\kappa^p_d = 2\kappa_n + \kappa_p = -2.033$.

- Model too simple but illustrates that anticipated deformation is very significant since $\kappa_u$ and $\kappa_d$ known to be large!
p polarized in $+\hat{x}$ direction

lattice results (Hägler et al.)
SSAs in SIDIS ($\gamma + p \uparrow \rightarrow \pi^+ + X$)

- SIDIS = semi-inclusive DIS
- Single-Spin-Asymmetry (SSA) = left-right asymmetry in the X-section when only one spin is measured (e.g. target spin)
- example: nucleon transversely (relative to $e^-$ beam) polarized $\rightarrow$ left-right asymmetry of produced $\pi$-mesons relative to target pol.

- infer transverse momentum distribution $q(x, k_\perp)$ of quarks in target from transverse momentum distribution of produced $\pi$ (note: left-right asymmetry can also arise in ‘fragmentation’ process (Collins effect), but resulting asymmetry has different angular dependence...)

D}$

$q(x, k_\perp)$

$e'$
**Sivers:** distribution of unpol. quarks in ⊥ pol. proton

\[ f_{q/p^\uparrow}(x, k_\perp) = f_1^q(x, k_{2\perp}^2) - f_{1T}^q(x, k_{2\perp}^2) \frac{(\hat{P} \times k_\perp) \cdot S}{M} \]

- without FSI, \( f(x, k_\perp) = f(x, -k_\perp) \Rightarrow f_{1T}^q(x, k_{2\perp}^2) = 0 \)
- with FSI, \( f_{1T}^q(x, k_{2\perp}^2) \neq 0 \) (Brodsky, Hwang, Schmidt)

**Why interesting?**

- (like \( \kappa \)), Sivers requires matrix elements between wave function components that differ by one unit of OAM (Brodsky, Diehl, ..)
- probe for orbital angular momentum
- Sivers requires nontrivial final state interaction phases
- learn about FSI
**GPD ↔ SSA (Sivers)**

- **Example:** \( \gamma p \rightarrow \pi X \)

- \( u, d \) distributions in \( \perp \) polarized proton have left-right asymmetry in \( \perp \) position space (T-even!); sign “determined” by \( \kappa_u \) & \( \kappa_d \)

- Attractive FSI deflects active quark towards the center of momentum

- FSI translates position space distortion (before the quark is knocked out) in \( +\hat{y} \)-direction into momentum asymmetry that favors \( -\hat{y} \) direction

- Correlation between sign of \( \kappa_q^p \) and sign of SSA: \( f_{1T}^{\perp q} \sim -\kappa_q^p \)

- \( f_{1T}^{\perp q} \sim -\kappa_q^p \) confirmed by HERMES data (also consistent with COMPASS deuteron data \( f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0 \)
Consider quark in ground state polarized out of the plane

\[ \text{expect counterclockwise net current } \vec{j} \text{ associated with the magnetization density in this state} \]

\[ \text{virtual photon ‘sees’ enhancement of quarks (polarized out of plane) at the top, i.e.} \]

\[ \text{virtual photon ‘sees’ enhancement of quarks with polarization up (down) on the left (right) side of the hadron} \]
Transversity Distribution in Unpolarized Target
IPDs on the lattice (Hägler et al.)

- lowest moment of distribution $q(x, b_\perp)$ for unpol. quarks in $\perp$ pol. proton (left) and of $\perp$ pol. quarks in unpol. proton (right):
Boer-Mulders Function

- SIDIS: attractive FSI expected to convert position space asymmetry into momentum space asymmetry
  - e.g. quarks at negative $b_x$ with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

  (qualitative) connection between Boer-Mulders function $h_1^\perp(x, k_\perp)$ and the chirally odd GPD $\bar{E}_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, k_\perp)$ and the GPD $E$.

- Boer-Mulders: distribution of $\perp$ pol. quarks in unpol. proton

$$f_{q^\uparrow/p}(x, k_\perp) = \frac{1}{2} \left[ f_1^q(x, k_\perp^2) - h_1^{q\perp}(x, k_\perp^2) \frac{(\mathbf{P} \times k_\perp) \cdot S_q}{M} \right]$$

- $h_1^{q\perp}(x, k_\perp^2)$ can be probed in Drell-Yan (RHIC, J-PARC, GSI) and tagged SIDIS (JLab, EIC), using Collins-fragmentation
probing BM function in tagged SIDIS

- how do you measure the transversity distribution of quarks without measuring the transversity of a quark?
- consider semi-inclusive pion production off unpolarized target
- spin-orbit correlations in target wave function provide correlation between (primordial) quark transversity and impact parameter
- (attractive) FSI provides correlation between quark spin and \( \perp \) quark momentum \( \Rightarrow \) BM function
- Collins effect: left-right asymmetry of \( \pi \) distribution in fragmentation of \( \perp \) polarized quark \( \Rightarrow \) ‘tag’ quark spin
- \( \cos(2\phi) \) modulation of \( \pi \) distribution relative to lepton scattering plane
- \( \cos(2\phi) \) asymmetry proportional to: Collins \( \times \) BM
probing BM function in tagged SIDIS

Primordial Quark Transversity Distribution

→ $\perp$ quark pol.
**⊥ polarization and $\gamma^*$ absorption**

- QED: when the $\gamma^*$ scatters off $\bot$ polarized quark, the $\bot$ polarization gets modified
  - gets reduced in size
  - gets tilted symmetrically w.r.t. normal of the scattering plane

---

[Diagram showing the polarization before and after $\gamma^*$ absorption]
probing BM function in tagged SIDIS

Primordial Quark Transversity Distribution

→ $\perp$ quark pol.
probing BM function in tagged SIDIS

Quark Transversity Distribution after $\gamma^*$ absorption

$\rightarrow$ $\perp$ quark pol.

quark transversity component in lepton scattering plane flips
probing BM function in tagged SIDIS

\[ \bot \text{ momentum due to FSI} \]

\[ \rightarrow \bot \text{ quark pol.} \]

\[ \downarrow k^q_\bot \text{ due to FSI} \]

lepton scattering plane

on average, FSI deflects quarks towards the center
Collins effect

- When a $\perp$ polarized struck quark fragments, the structure of jet is sensitive to polarization of quark
- distribution of hadrons relative to $\perp$ polarization direction may be left-right asymmetric
- asymmetry parameterized by Collins fragmentation function
- Artru model:
  - struck quark forms pion with $\bar{q}$ from $q\bar{q}$ pair with $^3P_0$ ‘vacuum’ quantum numbers
  - pion ‘inherits’ OAM in direction of $\perp$ spin of struck quark
  - produced pion preferentially moves to left when looking into direction of motion of fragmenting quark with spin up
- Artru model confirmed by HERMES experiment
- more precise determination of Collins function under way (KEK)
probing BM function in tagged SIDIS

\[ k_\perp \text{ due to Collins} \]
\[ k^q_\perp \text{ due to FSI} \]

\[ \perp \text{ momentum due to Collins} \]

lepton scattering plane

SSA of \( \pi \) in jet emanating from \( \perp \) pol. \( q \)
probing BM function in tagged SIDIS

\[ k_\perp \text{ due to Collins} \]
\[ k_\perp^q \text{ due to FSI} \]
\[
\begin{align*}
\text{net } k_\perp^q \\
\text{lepton scattering plane}
\end{align*}
\]

\[ \text{in this example, enhancement of pions with } \perp \text{ momenta } \perp \text{ to lepton plane} \]
probing BM function in tagged SIDIS

\[ \text{net } k'_\perp (\text{FSI + Collins}) \]

\[ \downarrow \text{net } k^q_\perp \]

lepton scattering plane

\[ \leftrightarrow \text{expect enhancement of pions with } \perp \text{ momenta } \perp \text{ to lepton plane} \]
HERMES p-data shows significant Sivers

COMPASS (higher $Q^2$, lower $x$) p-data no significant Sivers

higher twist?

evolution?

- gluon dressing changes color of active quark
- evolution destroys long distance color correlation

(expect ‘chromodynamic lensing’ mechanism for Sivers to disappear at high $Q^2$/low $x$)

suggests rapid evolution of SSAs
Quark-Gluon Correlations (Introduction)

- (longitudinally) polarized polarized DIS at leading twist \( \rightarrow \) ‘polarized quark distribution’ \( g_1^q(x) = q^\uparrow(x) + \bar{q}^\uparrow(x) - q^\downarrow(x) - \bar{q}^\downarrow(x) \)

- \( \frac{1}{Q^2} \)-corrections to X-section involve ‘higher-twist’ distribution functions, such as \( g_2(x) \)

\[
\sigma_{TT} \propto g_1 - \frac{2 M x}{\nu} g_2
\]

- \( g_2(x) \) involves quark-gluon correlations and does not have a parton interpretation as difference between number densities

- for \( \perp \) polarized target, \( g_1 \) and \( g_2 \) contribute equally to \( \sigma_{LT} \)

\[
\sigma_{LT} \propto g_T \equiv g_1 + g_2
\]

\( \rightarrow \) ‘clean’ separation between higher order corrections to leading twist (\( g_1 \)) and higher twist effects (\( g_2 \))

- what can one learn from \( g_2 \)?
Quark-Gluon Correlations (QCD analysis)

- $g_2(x) = g_2^{WW}(x) + \bar{g}_2(x)$, with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$
- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^2 + 2Sx} \langle P, S \mid \bar{q}(0)gG^+(0)\gamma^+q(0) \rangle P, S \rangle$$

- $\sqrt{2}G^{+y} \equiv G^{0y} + G^{zy} = -E^y + B^x$
- matrix elements of $\bar{q}B^x\gamma^+q$ and $\bar{q}E^y\gamma^+q$ are sometimes called color-electric and magnetic polarizabilities
  
  $$2M^2 \vec{S}^{\chi_E} = \langle P, S \mid \vec{j}_a \times \vec{E}_a \rangle P, S \rangle \quad \text{&} \quad 2M^2 \vec{S}^{\chi_B} = \langle P, S \mid j_0^a \vec{B}_a \rangle P, S \rangle$$

with $d_2 = \frac{1}{4} (\chi_E + 2\chi_M)$ — but these names are misleading!
\[ \bar{g}_2(x) \] involves quark-gluon correlations, e.g.

\[ \int dxx^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^2 S^x} \left\langle P, S \left| \bar{q}(0) g G^{+y} (0) \gamma^+ q(0) \right| P, S \right\rangle \]

**QED:** \[ \bar{q}(0) e F^{+y} (0) \gamma^+ q(0) \] correlator between quark density \( \bar{q} \gamma^+ q \) and (\( \hat{y} \)-component of the) Lorentz-force

\[ F^y = e \left[ \vec{E} + \vec{v} \times \vec{B} \right]^y = e (E^y - B^x) = -e \left( F^{0y} + F^{zy} \right) = -e \sqrt{2} F^{+y}. \]

for charged particle moving with \( \vec{v} = (0, 0, -1) \) in the \(-\hat{z}\) direction

\[ \leftrightarrow \] matrix element of \( \bar{q}(0) e F^{+y} (0) \gamma^+ q(0) \) yields \( \gamma^+ \) density (density relevant for DIS in Bj limit!) weighted with the Lorentz force that a charged particle with \( \vec{v} = (0, 0, -1) \) would experience at that point

\[ \leftrightarrow \] \( d_2 \) a measure for the color Lorentz force acting on the struck quark in SIDIS in the instant after being hit by the virtual photon

\[ \left\langle F^y (0) \right\rangle = -M^2 d_2 \quad \text{(rest frame; } S^x = 1) \]
Interpretation of $d_2$ with the transverse FSI force in DIS also consistent with $\langle k^y_\perp \rangle \equiv \int_0^1 dx \int d^2 k_\perp k^2_\perp f_{1T}^+(x, k^2_\perp)$ in SIDIS (Qiu, Sterman)

$$\langle k^y_\perp \rangle = -\frac{1}{2p^+} \left< P, S \left| \bar{q}(0) \int_0^\infty dx^- gG^{+y}(x^-) \gamma^+ q(0) \right| P, S \right>$$

semi-classical interpretation: average $k_\perp$ in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity

matrix element defining $d_2$ same as the integrand (for $x^- = 0$) in the QS-integral:

$$\langle k^y_\perp \rangle = \int_0^\infty dt F^y(t) \quad \text{(use } dx^- = \sqrt{2}dt\text{)}$$

$\leftarrow$ first integration point $\rightarrow$ $F^y(0)$

$\leftarrow$ (transverse) force at the begin of the trajectory, i.e. at the moment after absorbing the virtual photon
Quark-Gluon Correlations (Interpretation)

- $x^2$-moment of twist-4 polarized PDF $g_3(x)$

$$\int dx x^2 g_3(x) \sim \left\langle P, S \left| \bar{q}(0) g \tilde{G}^{\mu\nu}(0) \gamma_\nu q(0) \right| P, S \right\rangle \sim f_2$$

- different linear combination $f_2 = \chi_E - \chi_B$ of $\chi_E$ and $\chi_M$

- combine with $d_2 \Rightarrow$ disentangle electric and magnetic force

What should one expect (sign)?

- $\kappa_q^p \rightarrow$ signs of deformation ($u/d$ quarks in $\pm \hat{y}$ direction for proton polarized in $+\hat{x}$ direction $\rightarrow$ expect force in $\mp \hat{y}$

- $d_2$ positive/negative for $u/d$ quarks in proton

- large $N_C$: $d_2^u/p = -d_2^d/p$

- consistent with $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$

- lattice (Göckeler et al.): $d_2^u \approx 0.010$ and $d_2^d \approx -0.0056$

- $(M^2 \approx 5 \frac{\text{GeV}}{f_m} \quad \left\langle F_u^y(0) \right\rangle \approx -50 \frac{\text{MeV}}{f_m} \quad \left\langle F_d^y(0) \right\rangle \approx 28 \frac{\text{MeV}}{f_m}$

- $x^2$-moment of chirally odd twist-3 PDF $e(x) \rightarrow$ transverse force on transversely polarized quark in unpolarized target ($\leftrightarrow$ Boer-Mulders $h_{1T}^\perp$)
Summary

- GPDs $\leftrightarrow^{FT}$ IPDs (impact parameter dependent PDFs)
- $E(x, 0, -\Delta^2_\perp) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- $\kappa^{q/p} \Rightarrow$ sign of deformation
- attractive FSI $\Rightarrow f_{1T}^{u} < 0 \& f_{1T}^{d} > 0$
- Interpretation of $M^2d_2 \equiv 3M^2 \int dx x^2 \tilde{g}_2(x)$ as $\perp$ force on active quark in DIS in the instant after being struck by the virtual photon

\[ \langle F^y(0) \rangle = -M^2 d_2 \quad \text{(rest frame; } S^x = 1) \]

- In combination with measurements of $f_2$
  - color-electric/magnetic force $\frac{M^2}{4} \chi_E$ and $\frac{M^2}{2} \chi_M$
- $\kappa^{q/p} \Rightarrow \perp$ deformation $\Rightarrow d_{2}^{u/p} > 0 \& d_{2}^{d/p} < 0$ (attractive FSI)
- combine measurement of $d_2$ with that of $f_{1T}^{1}$ $\Rightarrow$ range of FSI
- $x^2$-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ transverse force on transversely polarized quark in unpolarized target ($\leftrightarrow$ Boer-Mulders $h^{1}_1$)
distribution of \( \perp \) polarized quarks in unpol. target described by chirally odd GPD \( \vec{E}_T^q = 2\vec{H}_T^q + E_T^q \)

\( \leftrightarrow \) attractive FSI \( \Rightarrow \) measurement of \( h_1^q \) (DY,SIDIS) provides information on \( \vec{E}_T^q \) and hence on spin-orbit correlations

expect:

\[
h_1^q, q < 0 \quad \quad |h_1^q, q| > |f_{1T}^q|
\]

\( x^2 \)-moment of chirally odd twist-3 PDF \( e(x) \) \( \rightarrow \) transverse force on transversely polarized quark in unpolarized target \( \rightarrow \) Boer-Mulders

\[ f_{1T}^{\perp}(x, k_\perp)^{DY} = -f_{1T}^{\perp}(x, k_\perp)^{SIDIS} \]

a) time reversal: FSI $\leftrightarrow$ ISI

SIDIS: compare FSI for ‘red’ $q$ that is being knocked out with ISI for an anti-red $\bar{q}$ that is about to annihilate that bound $q$

$\leftrightarrow$ FSI for knocked out $q$ is attractive

DY: nucleon is color singlet $\rightarrow$ when to-be-annihilated $q$ is ‘red’, the spectators must be anti-red

$\leftrightarrow$ ISI with spectators is repulsive
What is a Polarizability?

- Polarizability is the relative tendency of a charge distribution, like the electron cloud of an atom or molecule, to be distorted from its normal shape by an external electric field, which may be caused by the presence of a nearby ion or dipole (Wikipedia).

- It may be consistent with this original use of the term to enlarge the definition to encompass all observables that describe the ease with which a system can be distorted in response to an applied field or force.

- Suppose one enlarges this definition to encompass ‘how the color electric and magnetic field responds to the spin of the nucleon’.

  - many other observables also become ‘polarizabilities’, e.g.:  
    - \( \Delta q \), as is describes how the quark spin responds to the spin of the nucleon.
    - \( \vec{\mu}_N \), as it describes how the magnetic field of the nucleon responds to the spin of the nucleon.
    - \( \vec{L}_q \), as it describes how the quark orbital angular momentum responds to the spin of the nucleon.
    - as well as many other ‘static’ properties of the nucleon.
Sivers Mechanism in $A^+ = 0$ gauge

- Gauge link along light-cone trivial in light-cone gauge

$$U_{[0,\infty]} = P \exp \left( i g \int_0^\infty d\eta^- A^+ (\eta) \right) = 1$$

← Puzzle: Sivers asymmetry seems to vanish in LC gauge (time-reversal invariance)!

- X.Ji: fully gauge invariant definition for $P(x, k_\perp)$ requires additional gauge link at $x^- = \infty$

$$f(x, k_\perp) = \int \frac{dy^- d^2y_\perp}{16\pi^3} e^{-ip^+ y^- + i k_\perp \cdot y_\perp}$$

$$\times \langle p, s | \bar{q}(y) \gamma^+ U_{[y^-,y_\perp;\infty^- ,y_\perp]} U_{[\infty^- ,0_\perp;\infty^- ,0_\perp]} U_{[\infty^- ,0_\perp;0^- ,0_\perp]} q(0) | p, s \rangle.$$