Does QCD predict light hybrid mesons?

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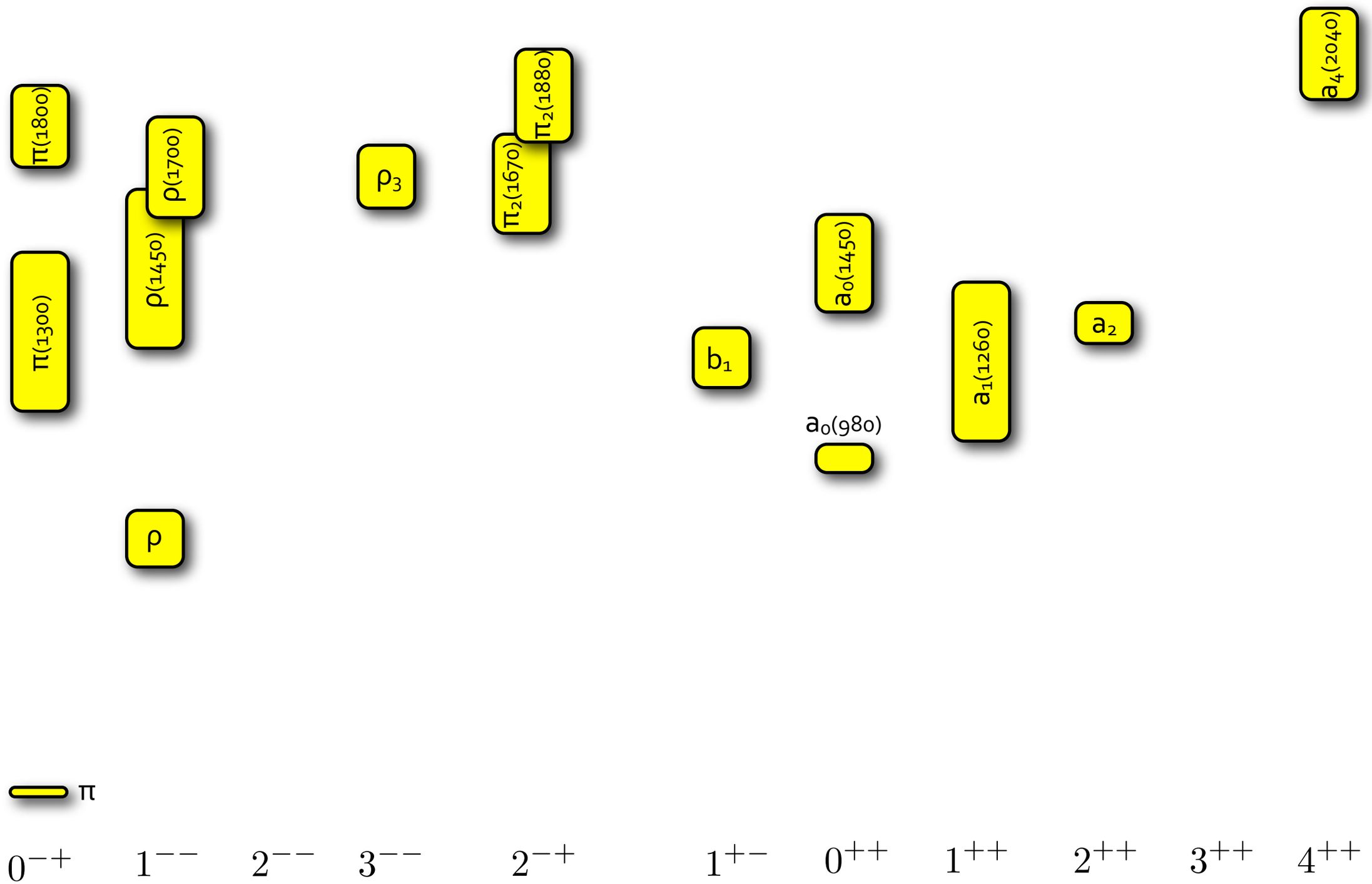
for the **Hadron Spectrum Collaboration**

JLab Users' Group Workshop

constituent quark model

(historical) empirical motivation



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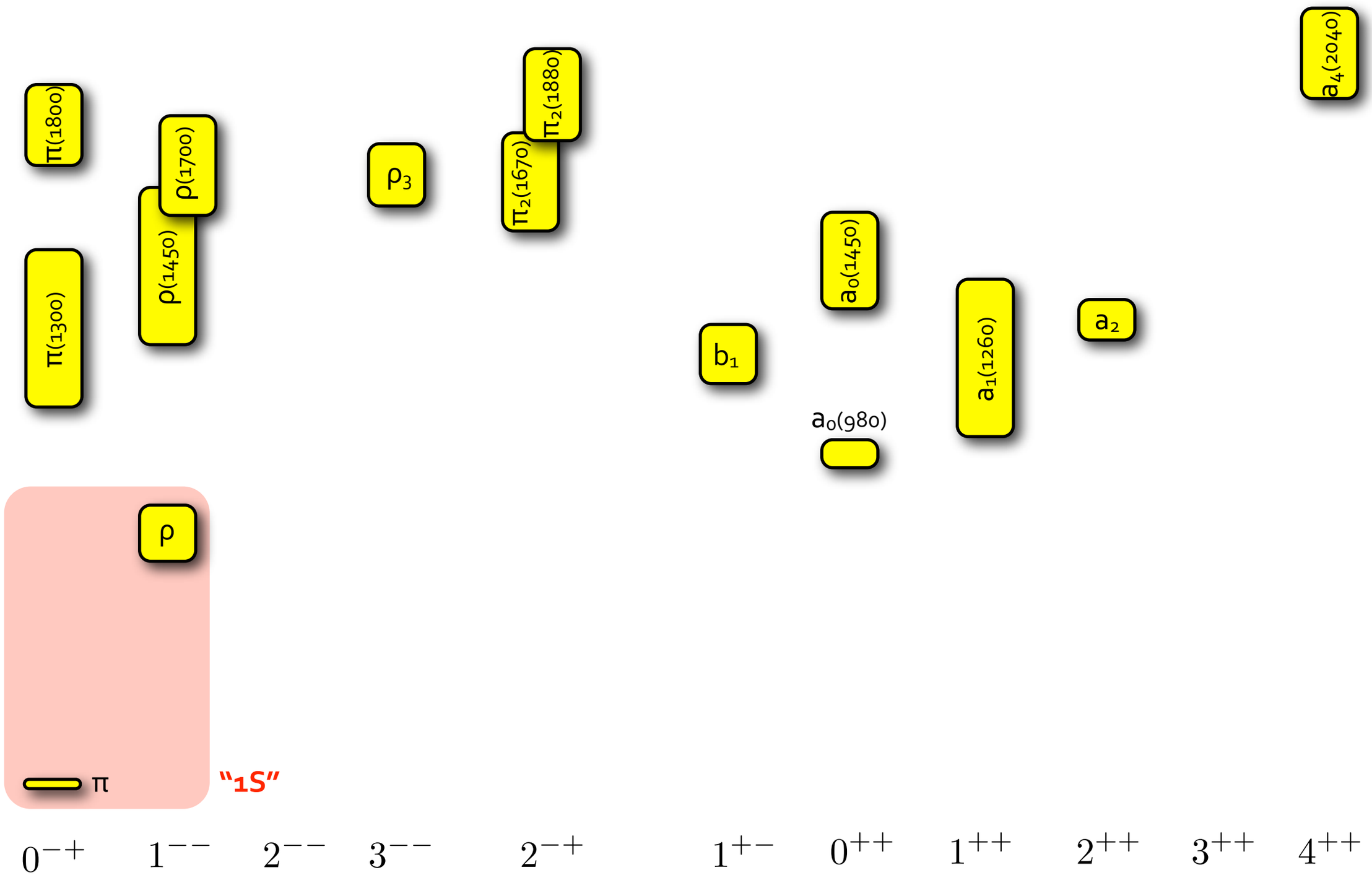

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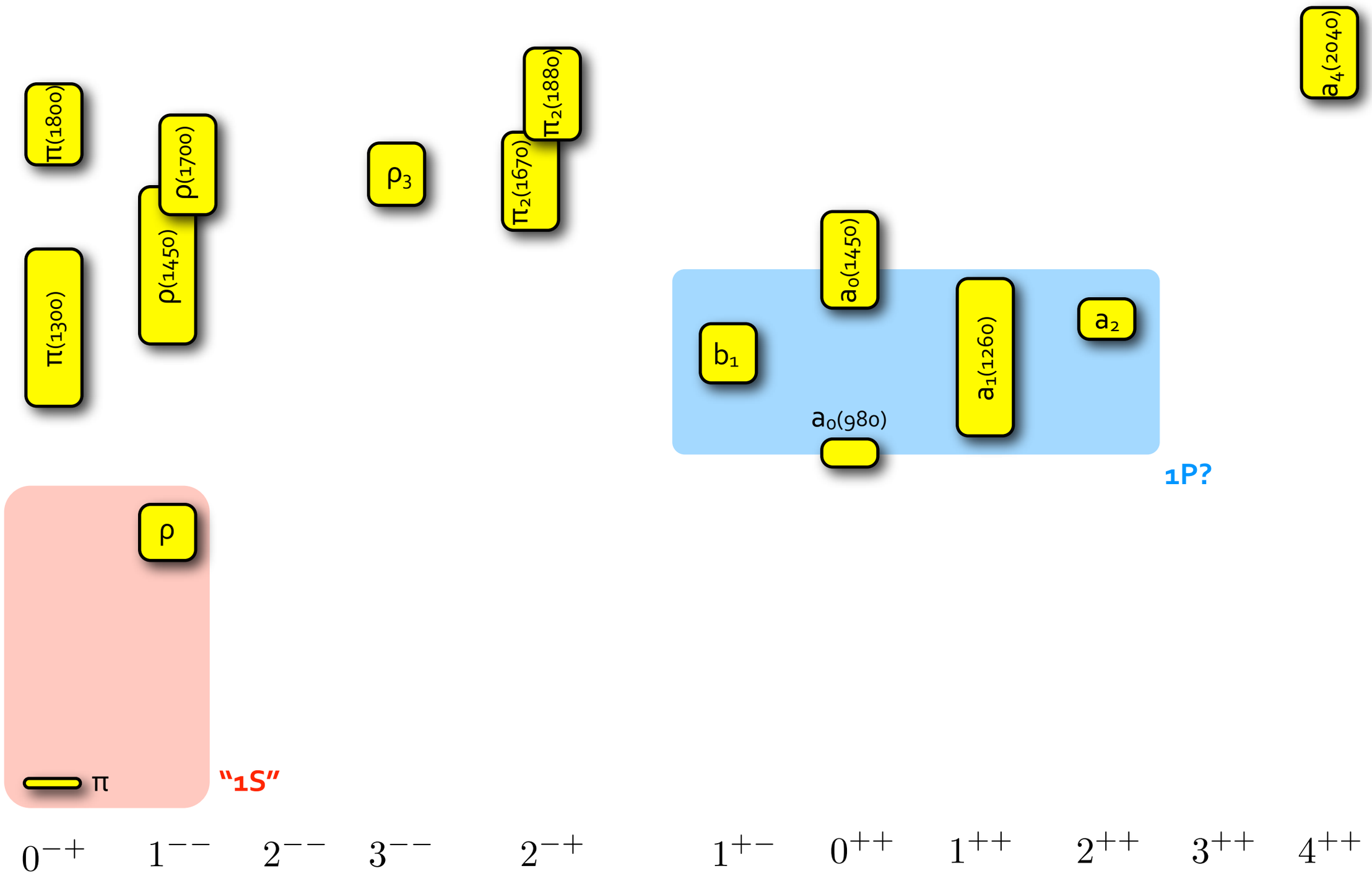

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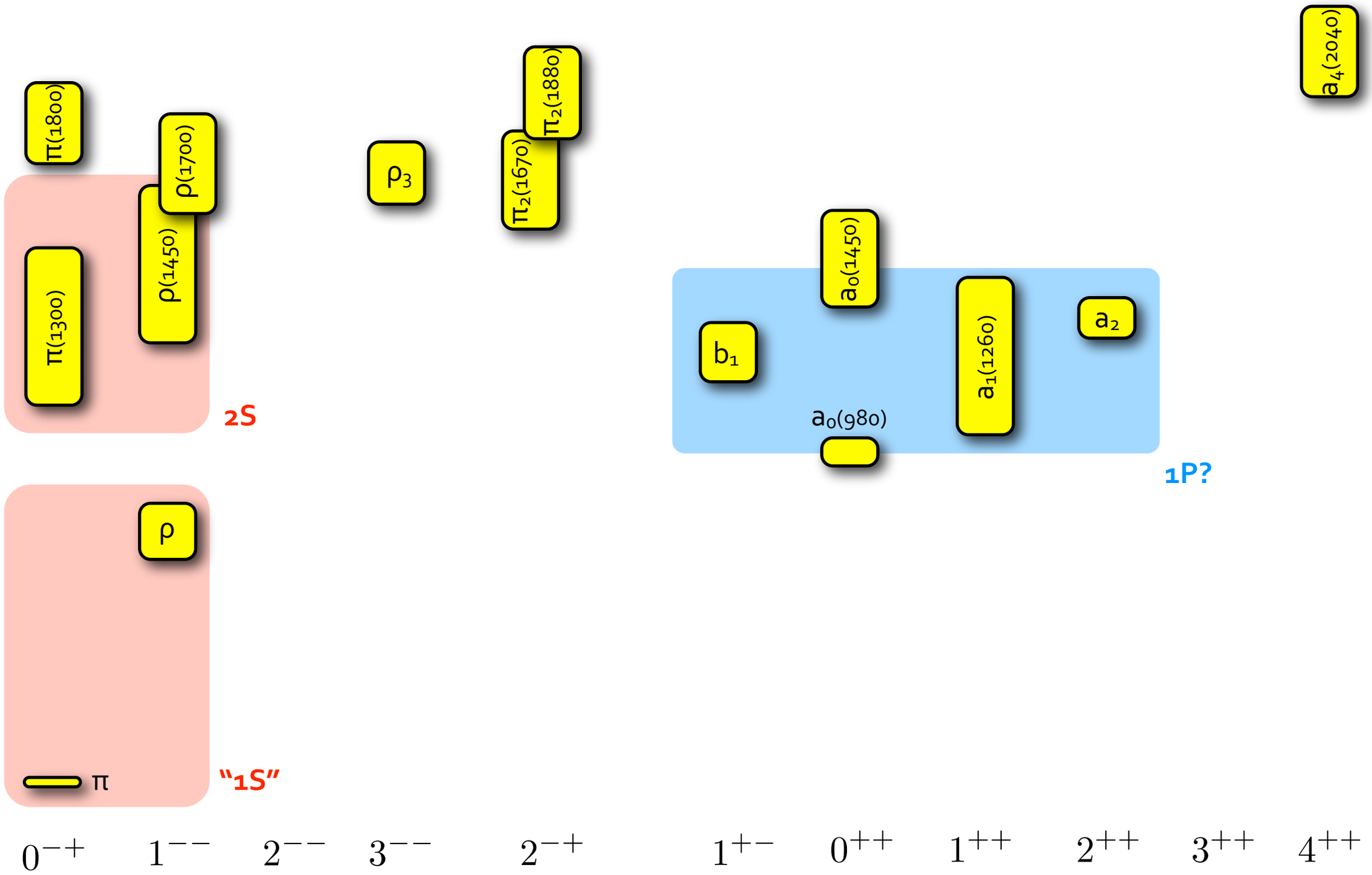

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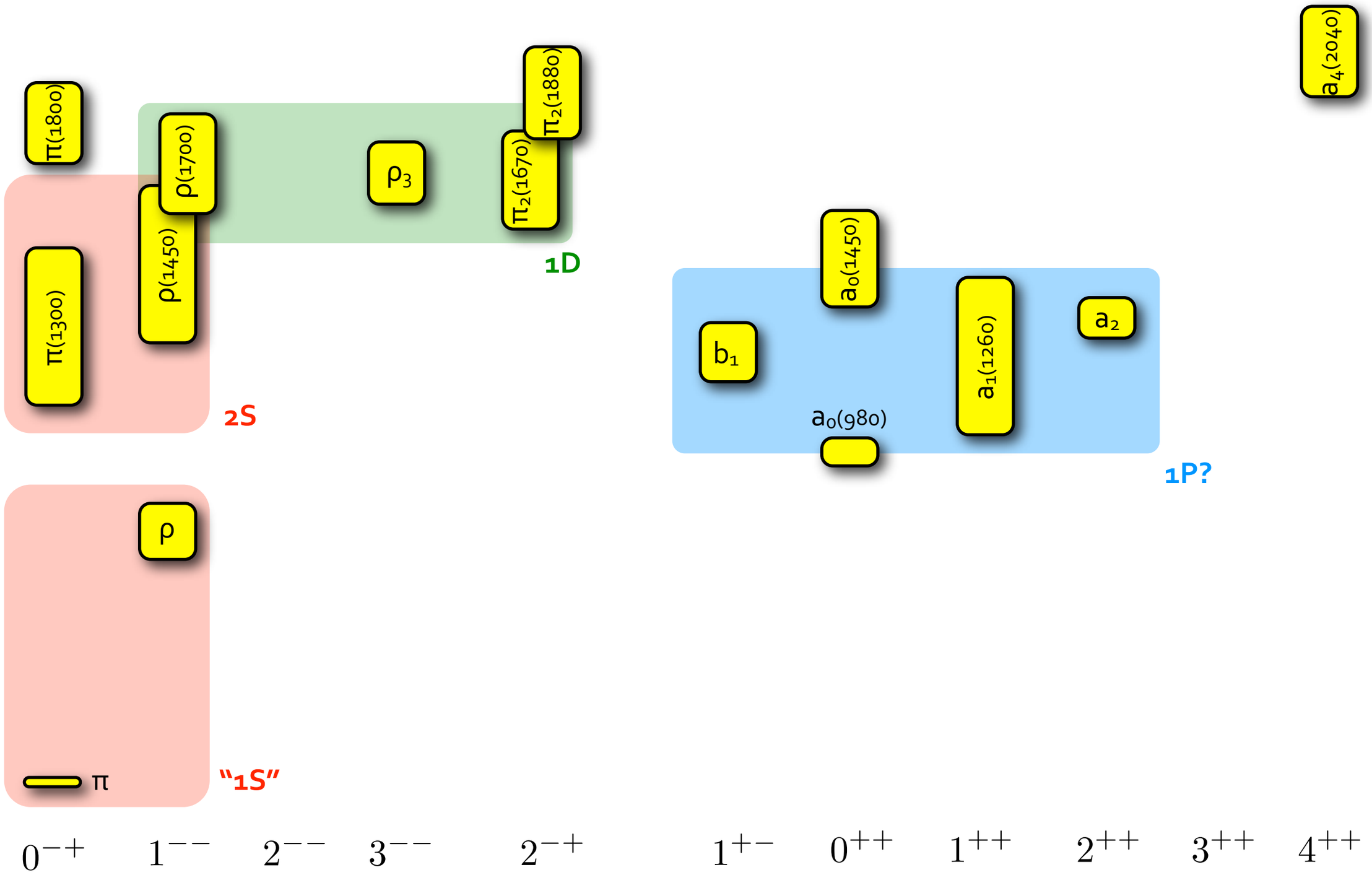

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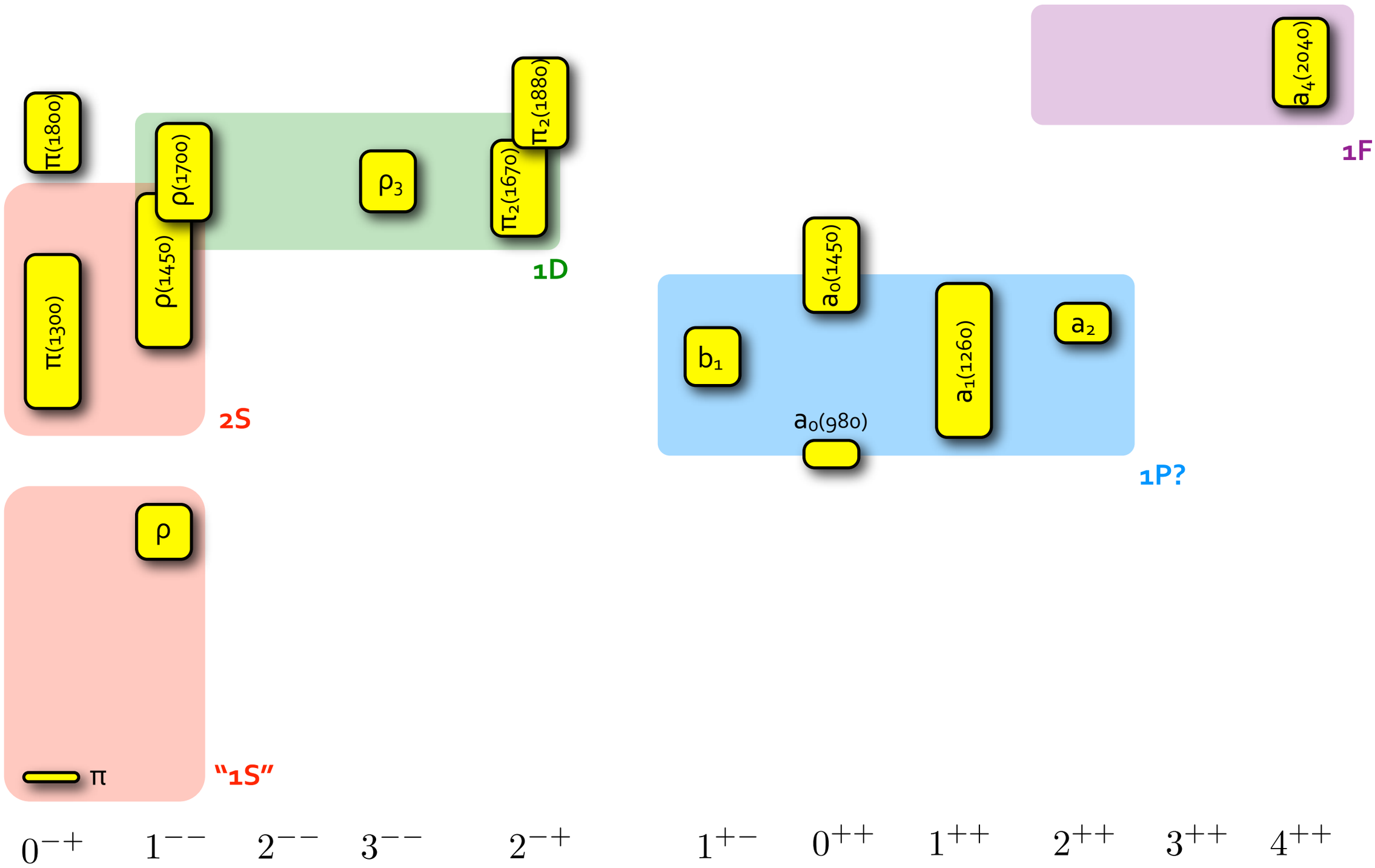

hybrid mesons - beyond the quark model

observed meson state flavor & J^{PC} systematics suggest $q\bar{q}$

$$q\bar{q}[S,L] \rightarrow (J = L \otimes S)^{P=(-1)^{L+1}, C=(-1)^{L+S}}$$

"constituent quarks"

$$\begin{array}{cccc} \cdot & 0^{-+} & 0^{++} & \cdot \\ 1^{--} & \cdot & 1^{++} & 1^{+-} \\ 2^{--} & 2^{-+} & 2^{++} & \cdot \\ & & \vdots & \end{array}$$

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exotic quantum numbers

$$0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$$

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but what if excited gluonic fields play a rôle - a *hybrid* meson, $q\bar{q}G$?

possibly exotic J^{PC} & extra 'non-exotic' states

must be 'heavier' or 'harder to produce' ?

hybrid mesons - models

tubes, bags & heavy glue

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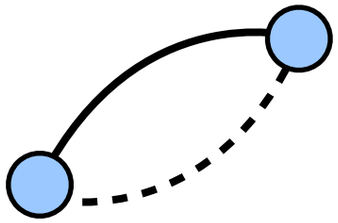
flux-tube model



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roughly degenerate

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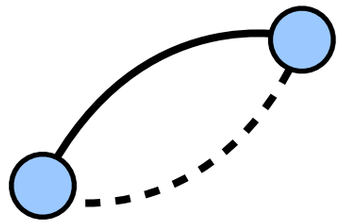

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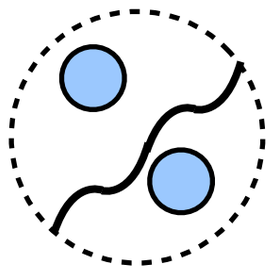
bag model



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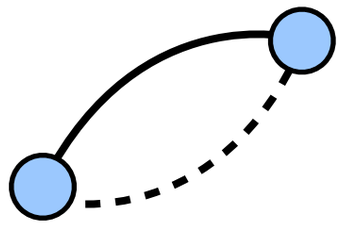

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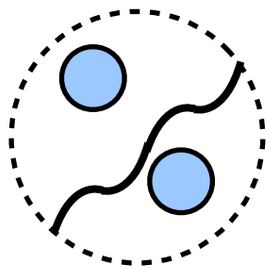

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massive quasi-gluons

Coulomb gauge transverse (no 3-body)

1^{--} "S-wave"

0^{++}, 1^{++}, 2^{++}, 1^{+-}

$\mathbf{0^{--}}$, $\mathbf{1^{-+}}$, ...

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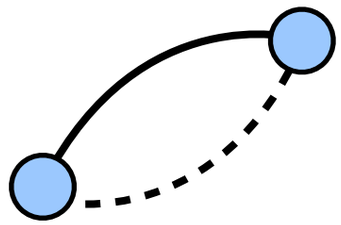

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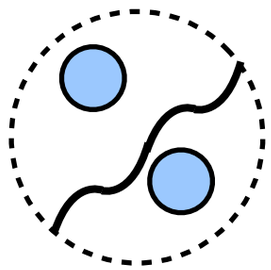

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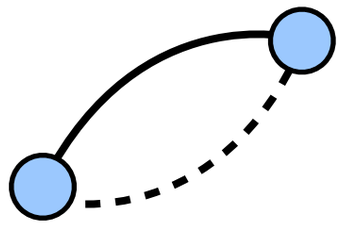

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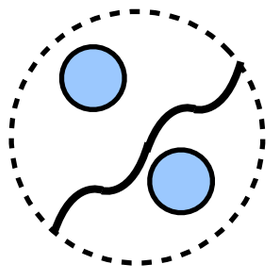

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$\mathbf{0^{+-}}$, $(\mathbf{2^{+-}})^2$, ...

without much data - nothing much to constrain them ...

spectrum from lattice QCD

two-point correlator

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

e.g. $\mathcal{O}(t) = \sum_{\vec{x}} \left(\bar{\psi} \gamma^5 \psi \right)_{\vec{x},t}$

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two-point correlator **matrix**

$$\begin{pmatrix} C_{11}(t) & C_{12}(t) & C_{13}(t) & \\ C_{21}(t) & C_{22}(t) & C_{23}(t) & \cdots \\ C_{31}(t) & C_{32}(t) & C_{33}(t) & \\ & \vdots & & \ddots \end{pmatrix}$$

e.g. 1^{--}

$$\bar{\psi} \gamma^i \psi$$

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$$c_{ijk} \bar{\psi} \Gamma \overleftrightarrow{D}^j \overleftrightarrow{D}^k \psi$$

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solve by using a 'Rayleigh-Ritz'-style variational approach - "diagonalize the matrix"

each state comes from an orthogonal combination of $\mathcal{O}_i$

optimal operator : $\Omega^{(\mathfrak{n})} = \sum_i v_i^{(\mathfrak{n})} \mathcal{O}_i$

mesonic operator basis

fermion bilinears with up to three covariant derivatives - project into good J^{PC}

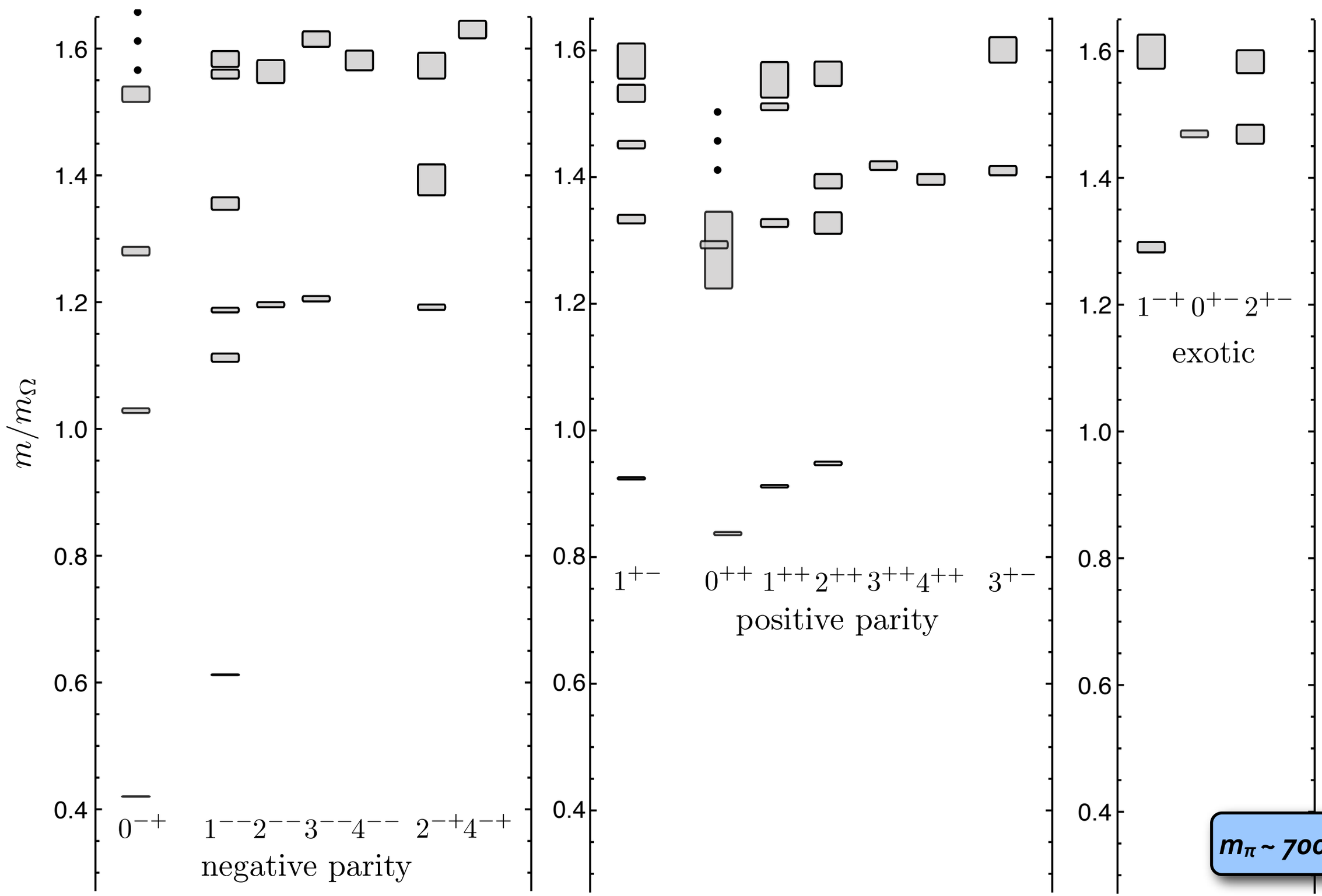
$$\bar{\psi}\Gamma\overleftrightarrow{D}\ldots\psi$$

actually also have to project into irreducible representations of the cubic symmetry group

	# of operators							
~J=0	A_1^{++}	13	A_1^{+-}	5	A_1^{-+}	12	A_1^{--}	6
~J=1	T_1^{++}	22	T_1^{+-}	22	T_1^{-+}	18	T_1^{--}	26
~J=2	T_2^{++}	22	T_2^{+-}	14	T_2^{-+}	18	T_2^{--}	18
~J=2	E^{++}	17	E^{+-}	9	E^{-+}	14	E^{--}	12
~J=3	A_2^{++}	5	A_2^{+-}	5	A_2^{-+}	4	A_2^{--}	6

this is by far the largest operator set ever used in a calculation like this

isovector spectrum



$m_\pi \sim 700$ MeV

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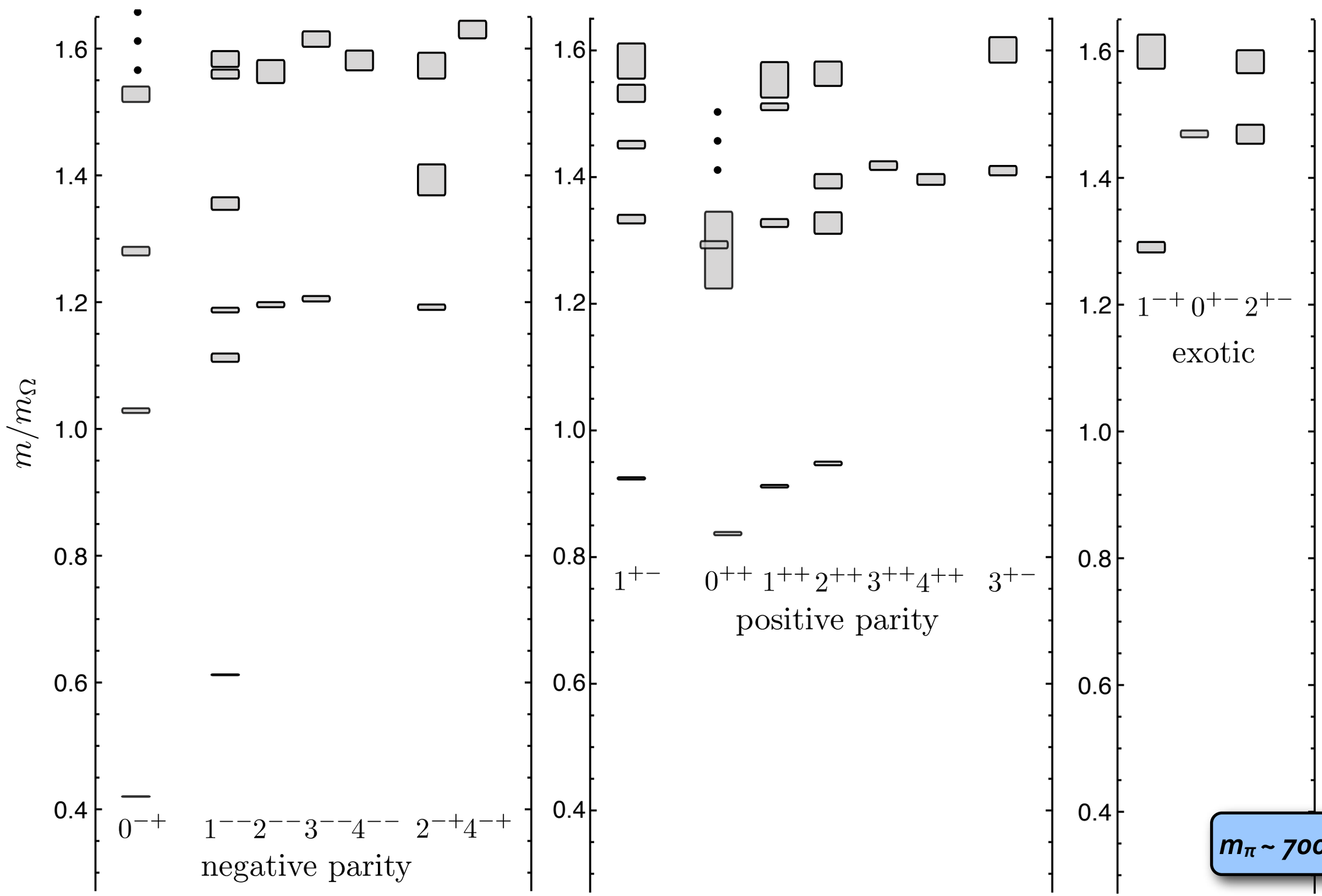

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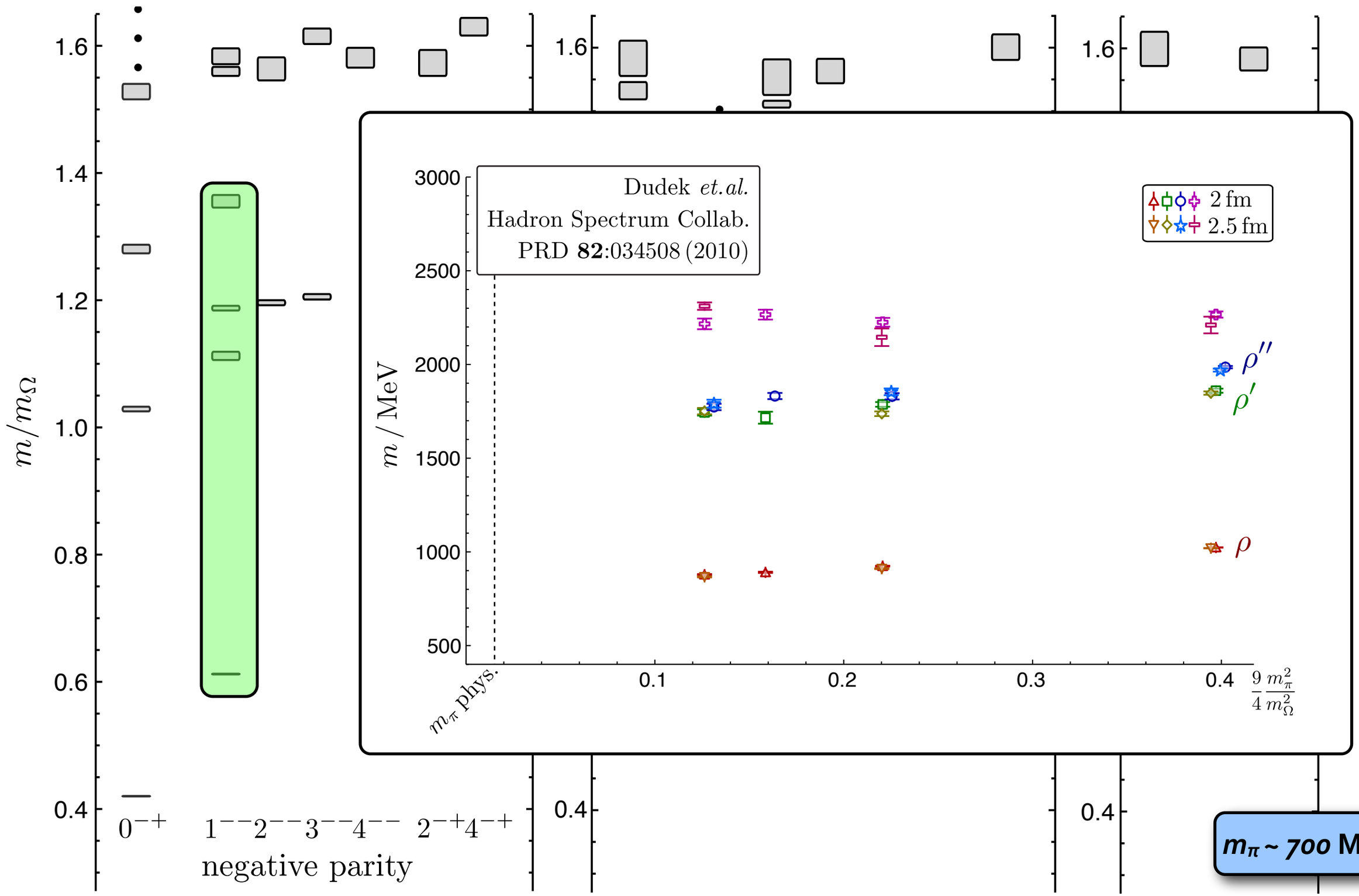

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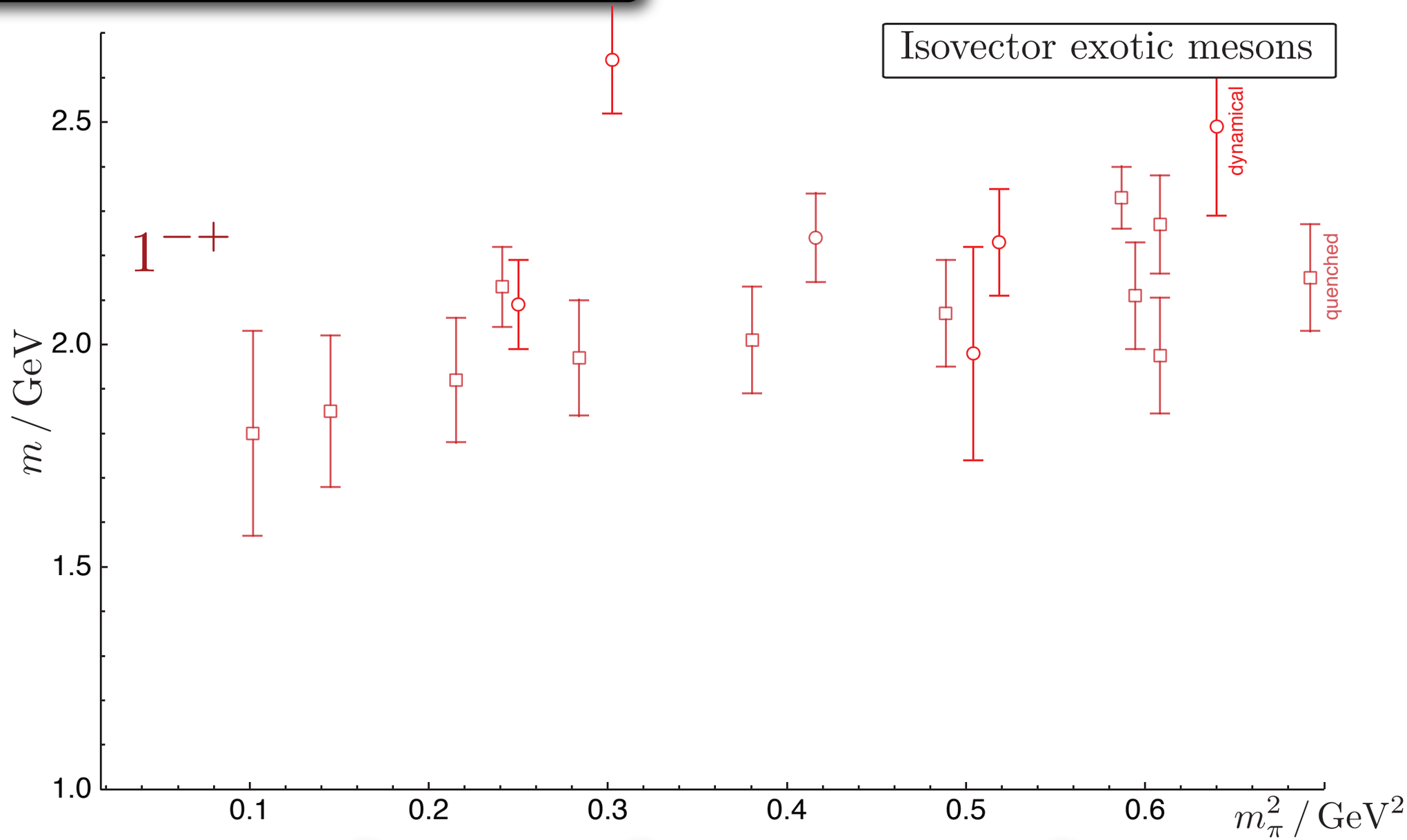
exotic J^{PC} - before

lattice QCD exotic meson spectrum circa 2006



m_π ~ 300 MeV

m_π ~ 500 MeV

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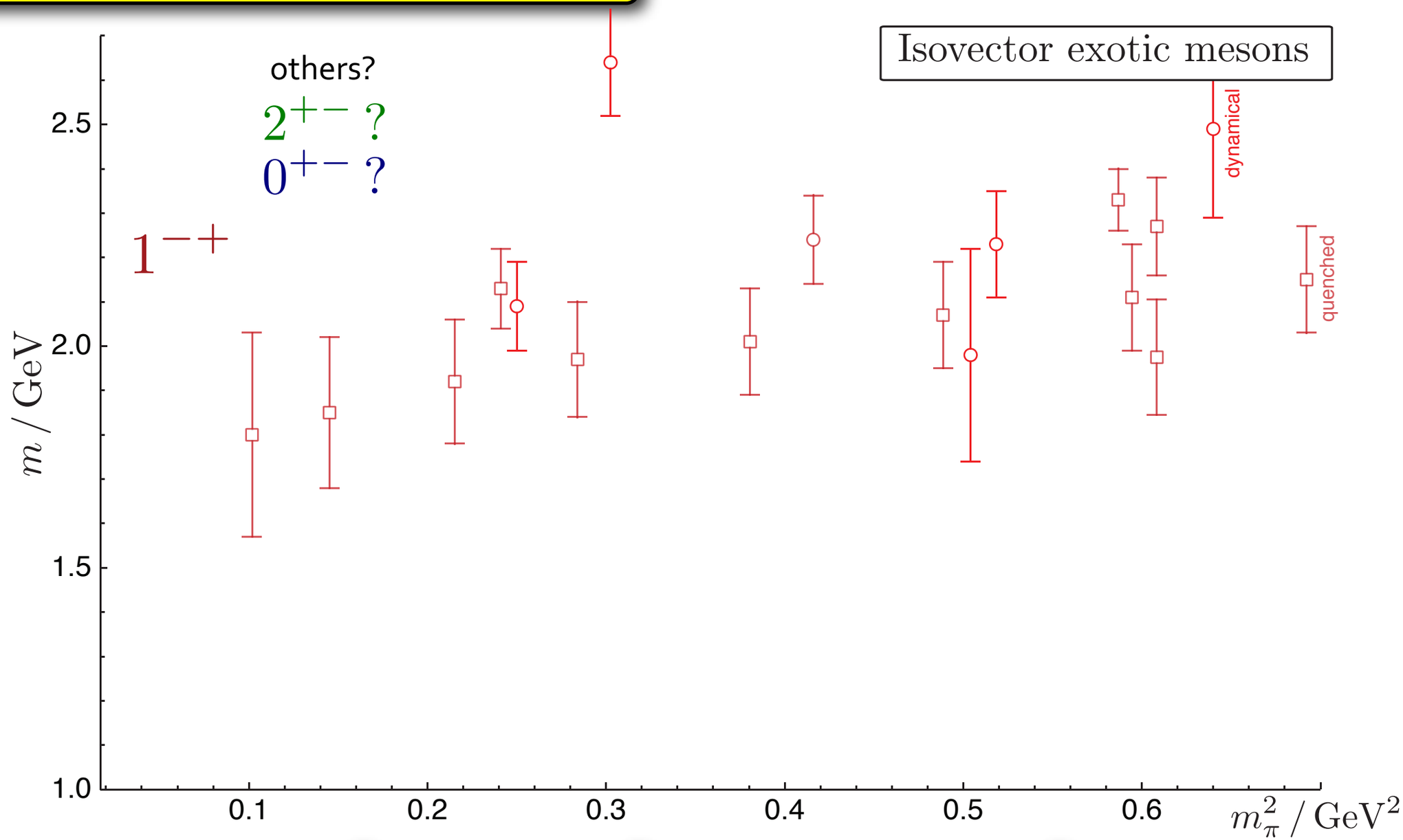

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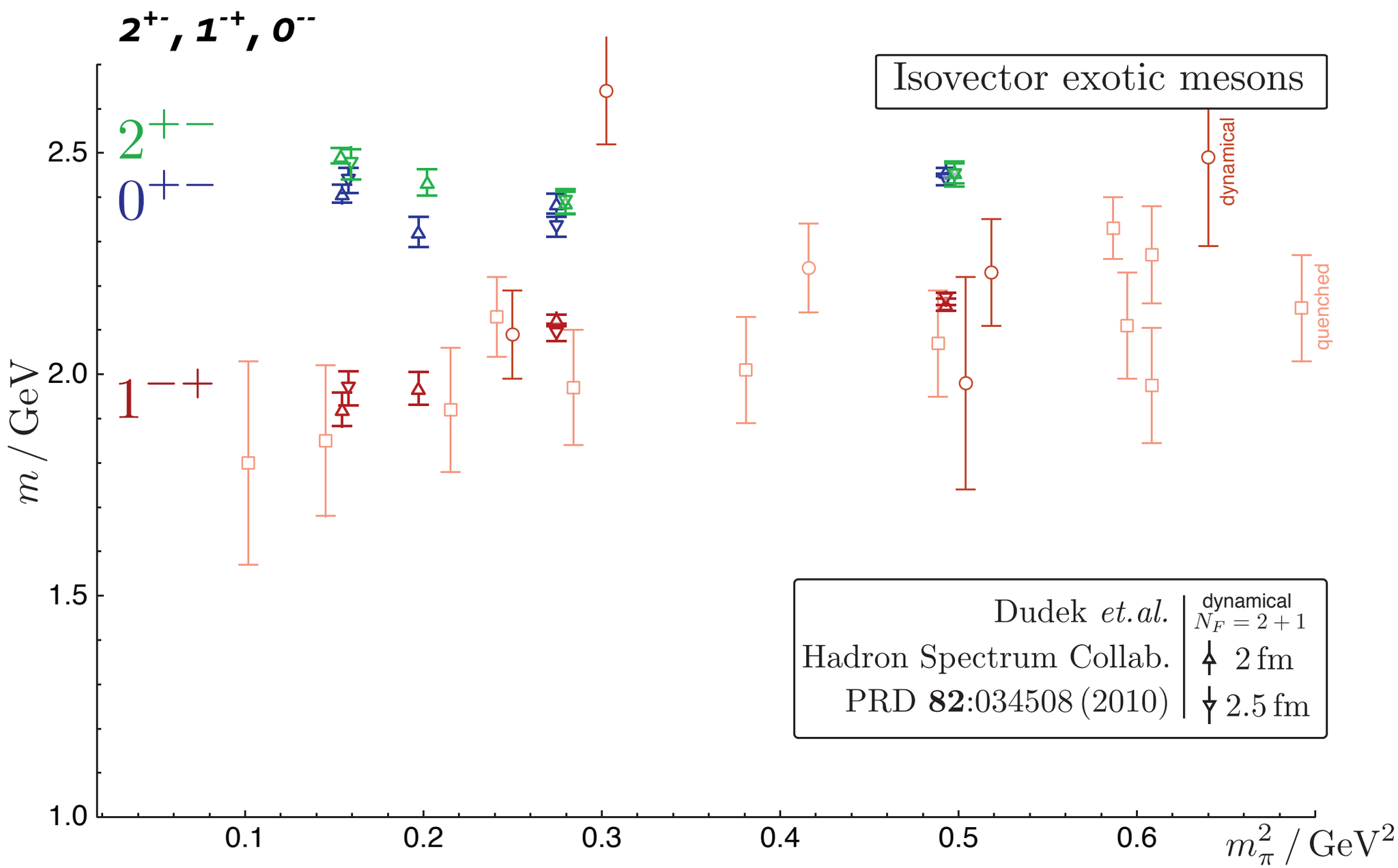

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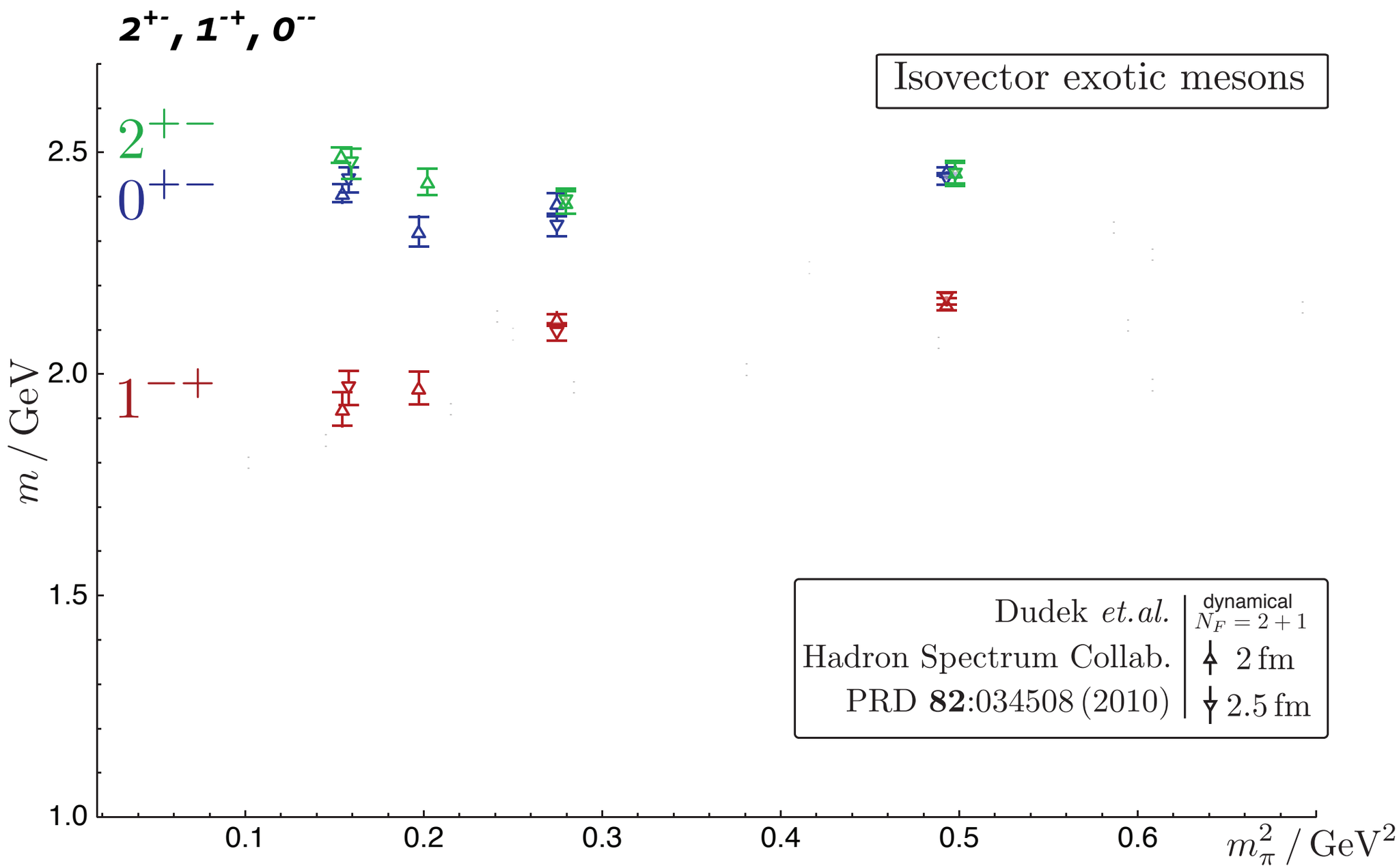
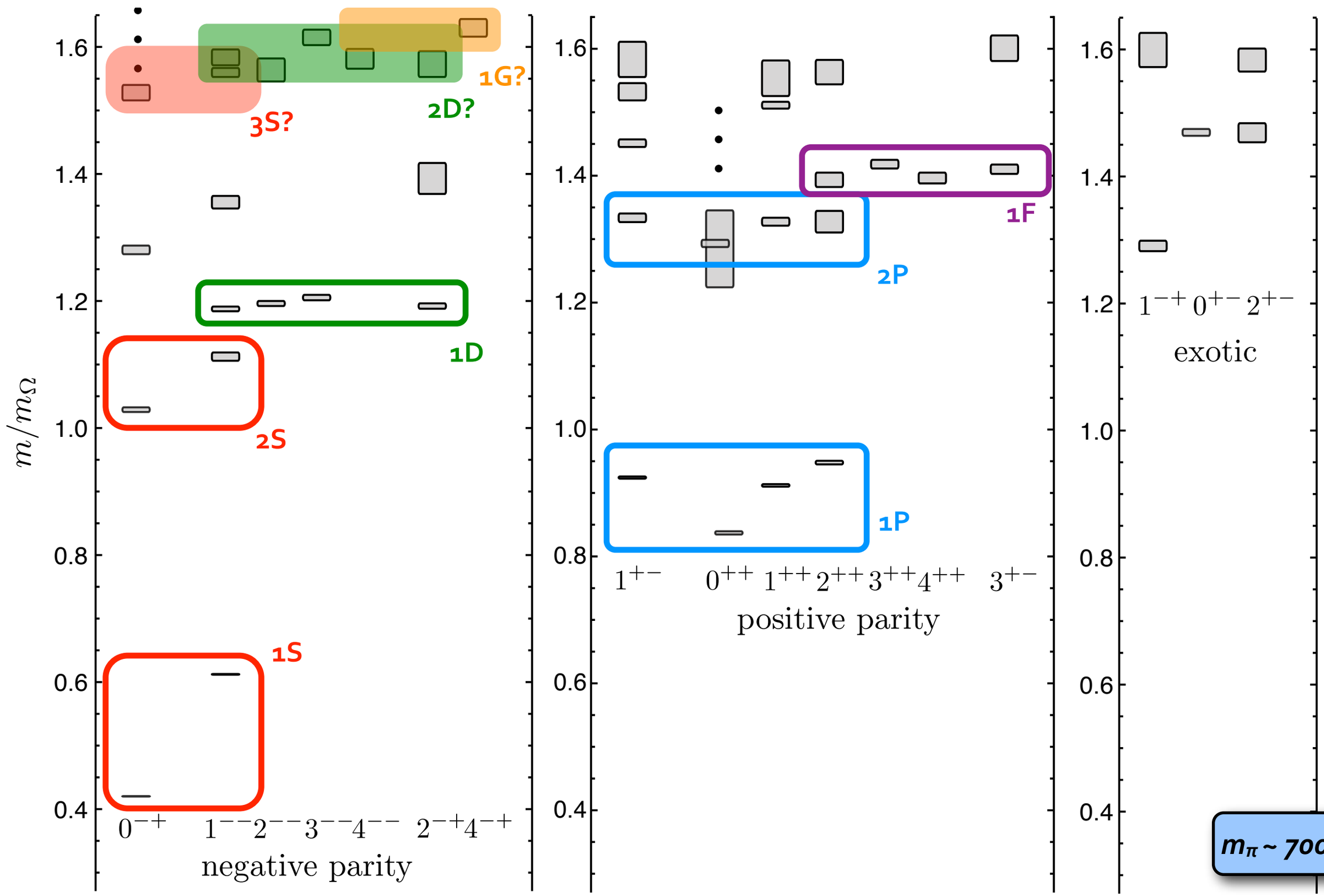


understanding & interpreting ?

systematics of a $q\bar{q}$ pair



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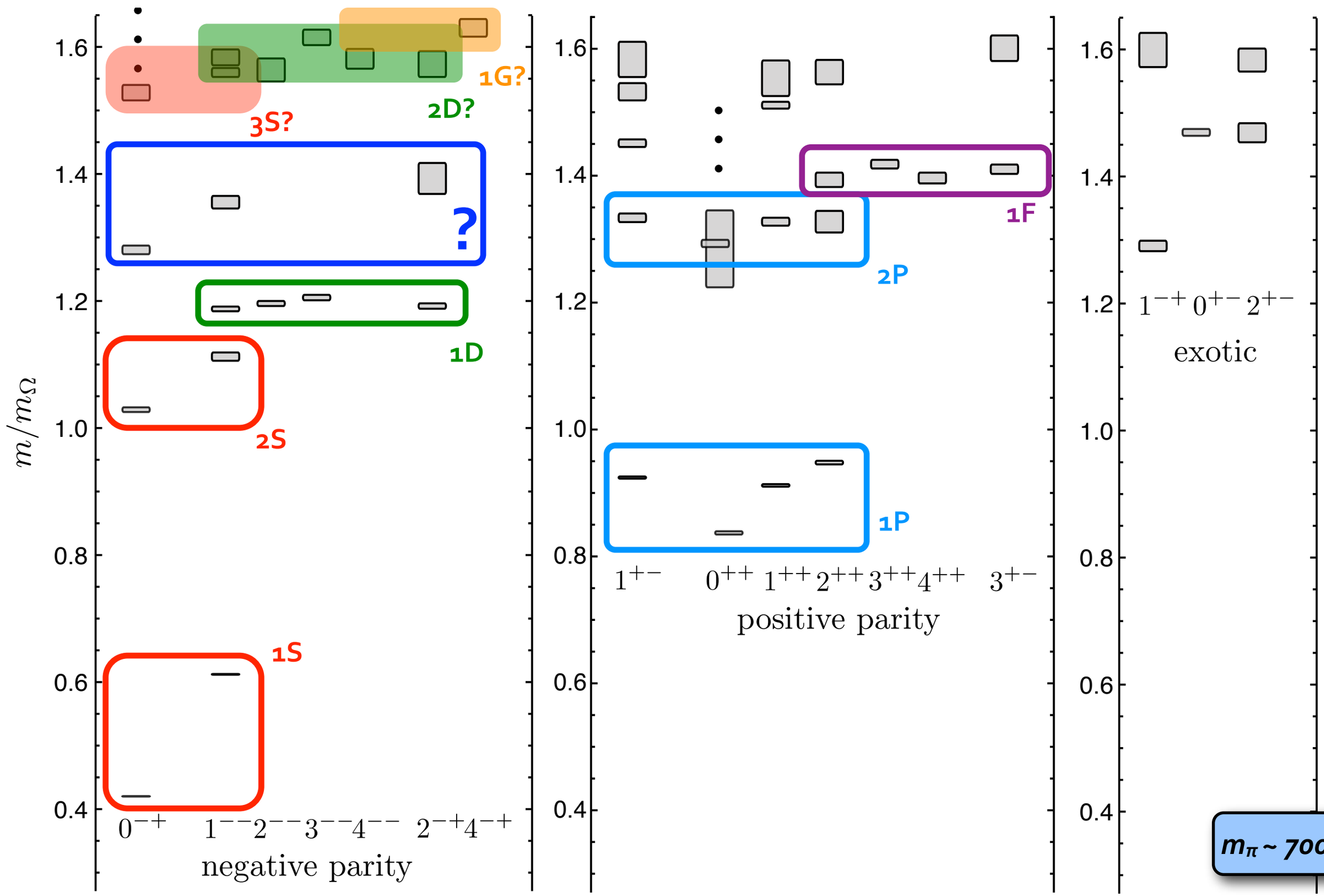

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try (model-dependent) analysis of matrix elements

$$Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n} | \mathcal{O}_i | 0 \rangle$$

1^{--}

$\rho \qquad \gamma_i \qquad \xrightarrow[D \to \partial]{} \; {}^3S_1$

$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1} \qquad D^{[2]}_{J=2} \equiv \langle 1, m_1; 1, m_2 | 2, m \rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2} \qquad \xrightarrow[D \to \partial]{} \; {}^3D_1$

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$\sim [D, D] \sim F$

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(spin-singlet)

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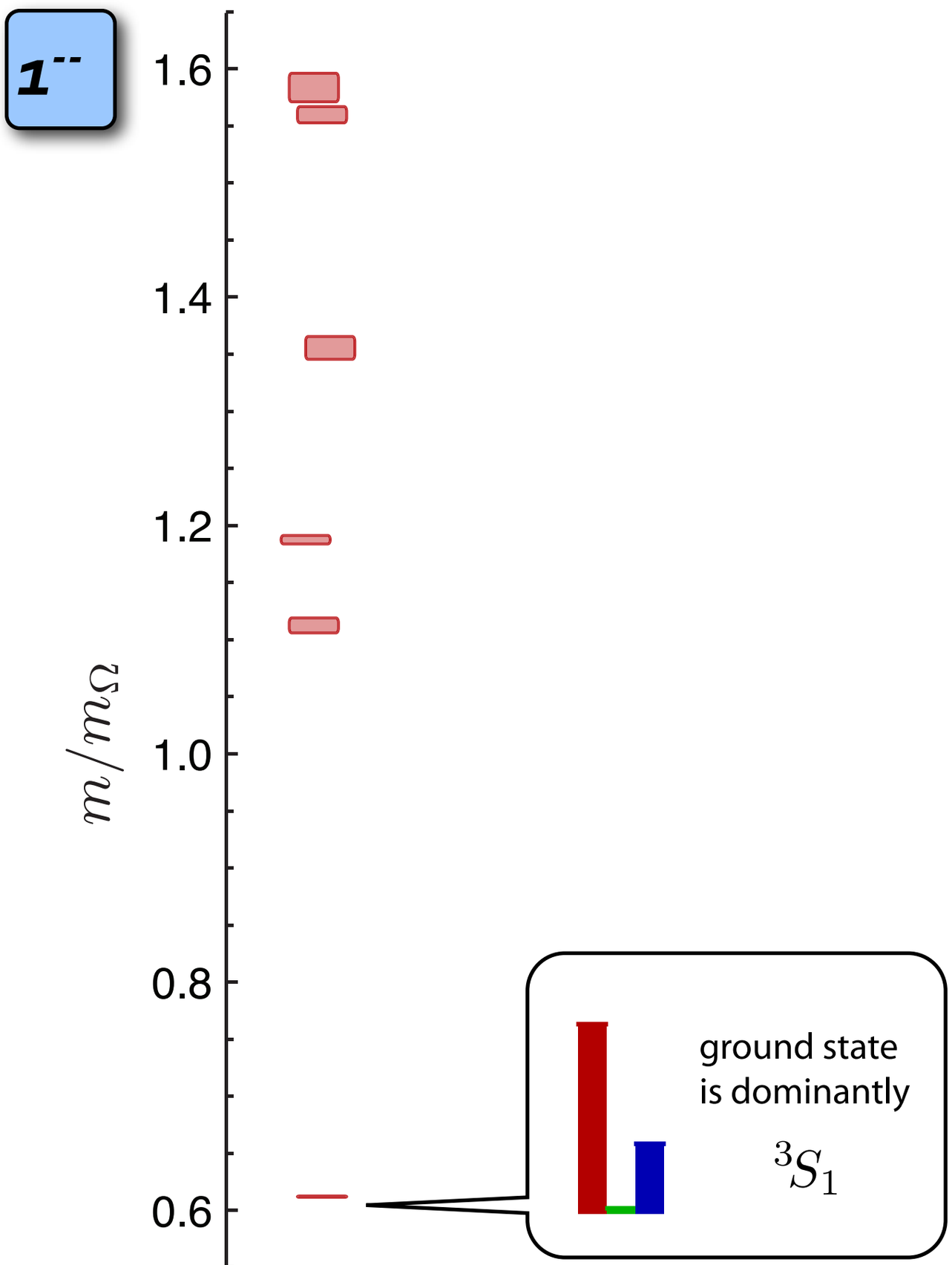

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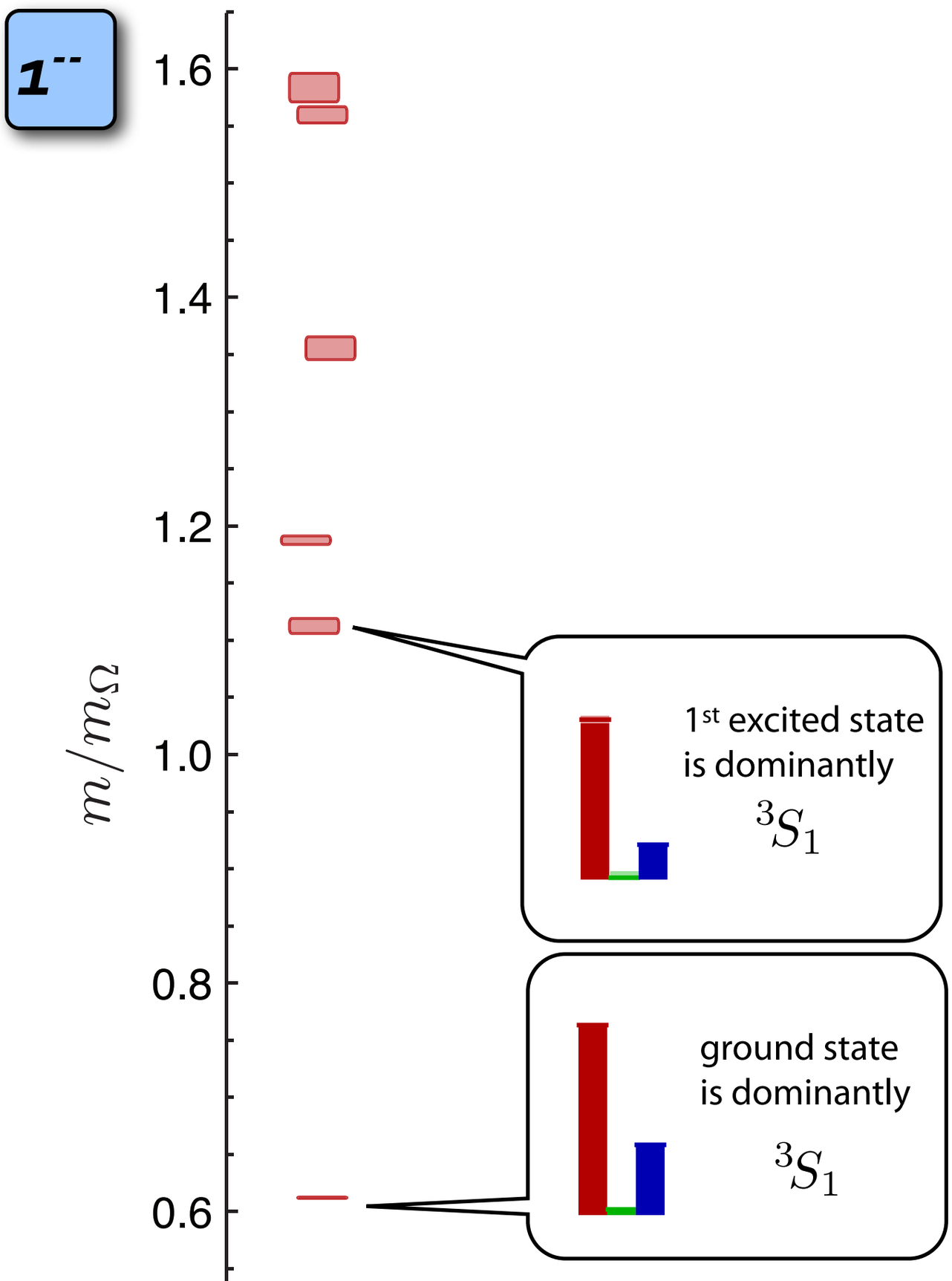

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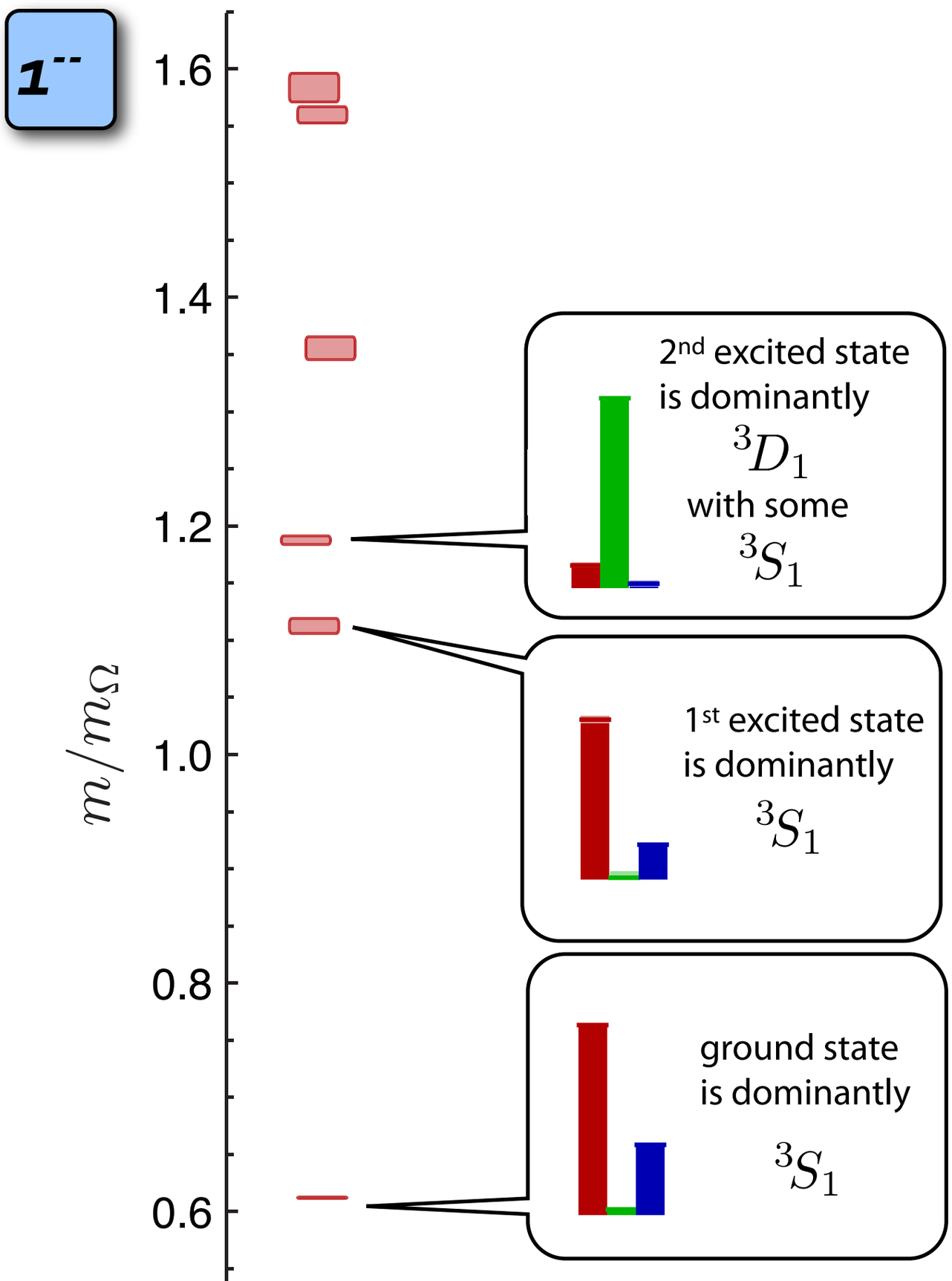

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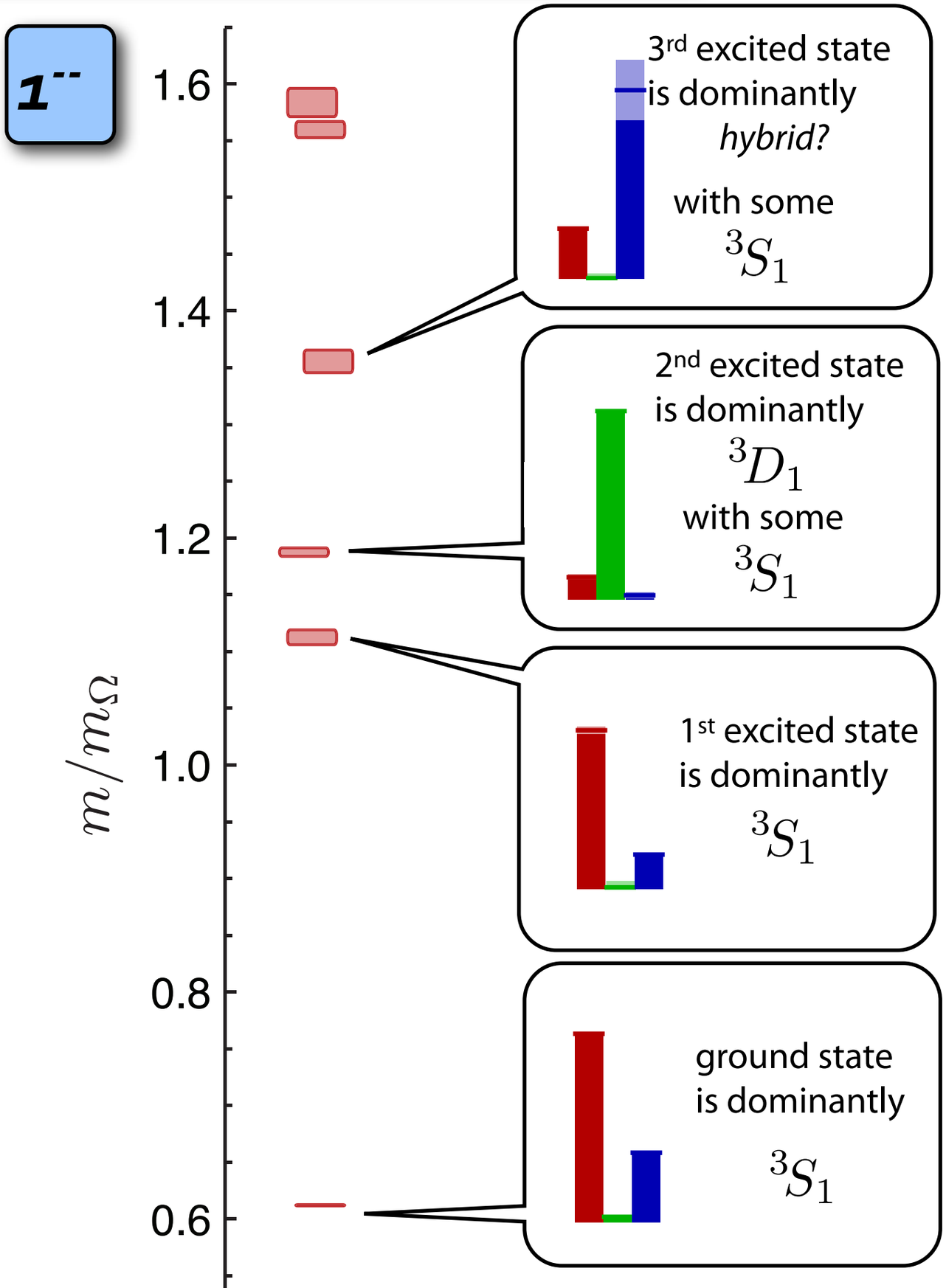

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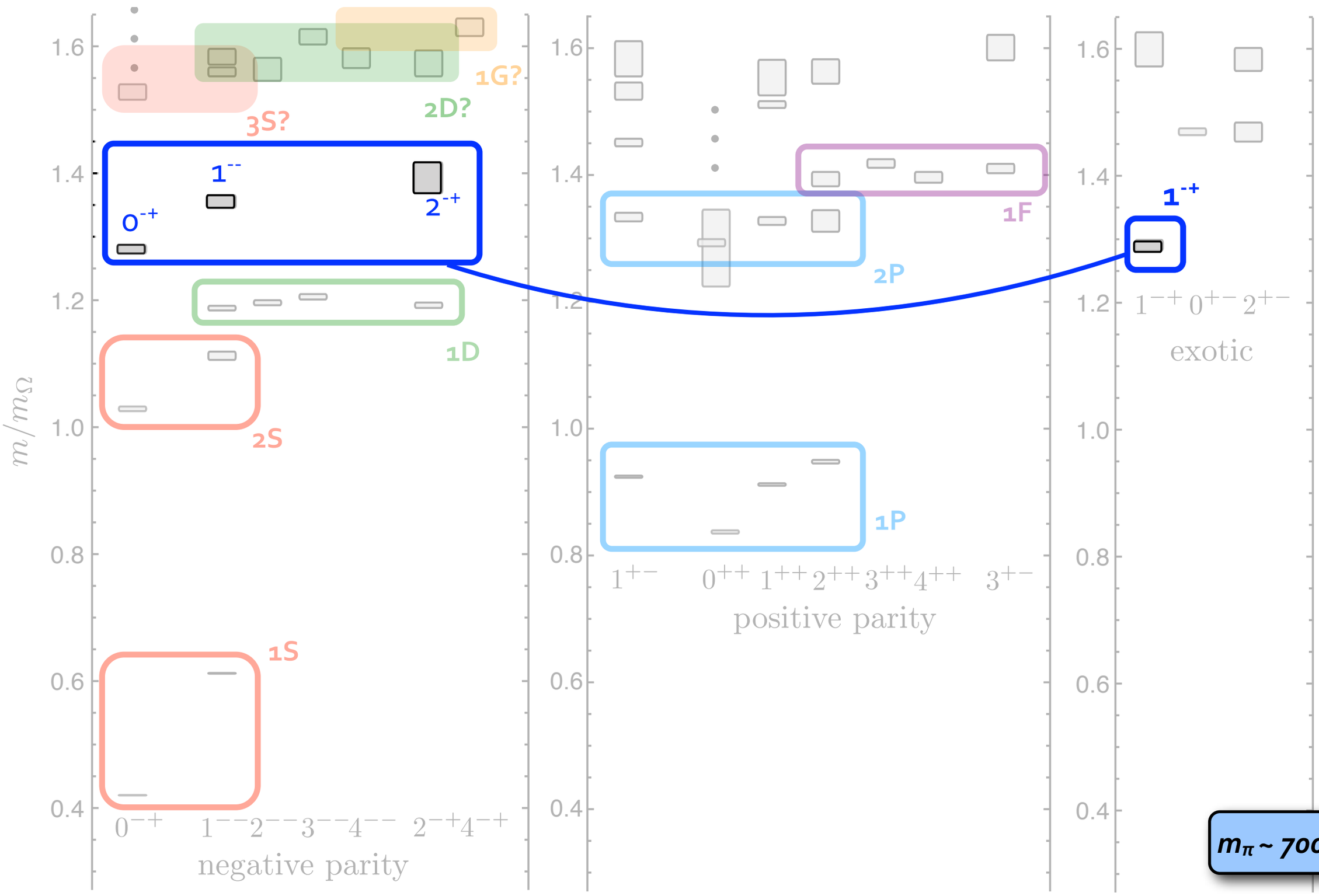
the lightest hybrid supermultiplet ?



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Does QCD predict light hybrid mesons?

YES

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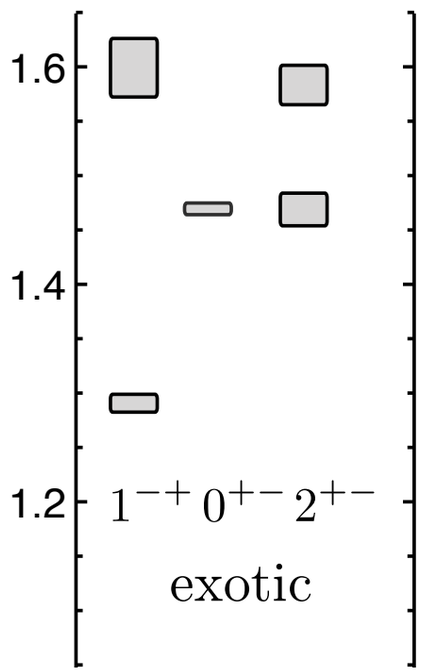
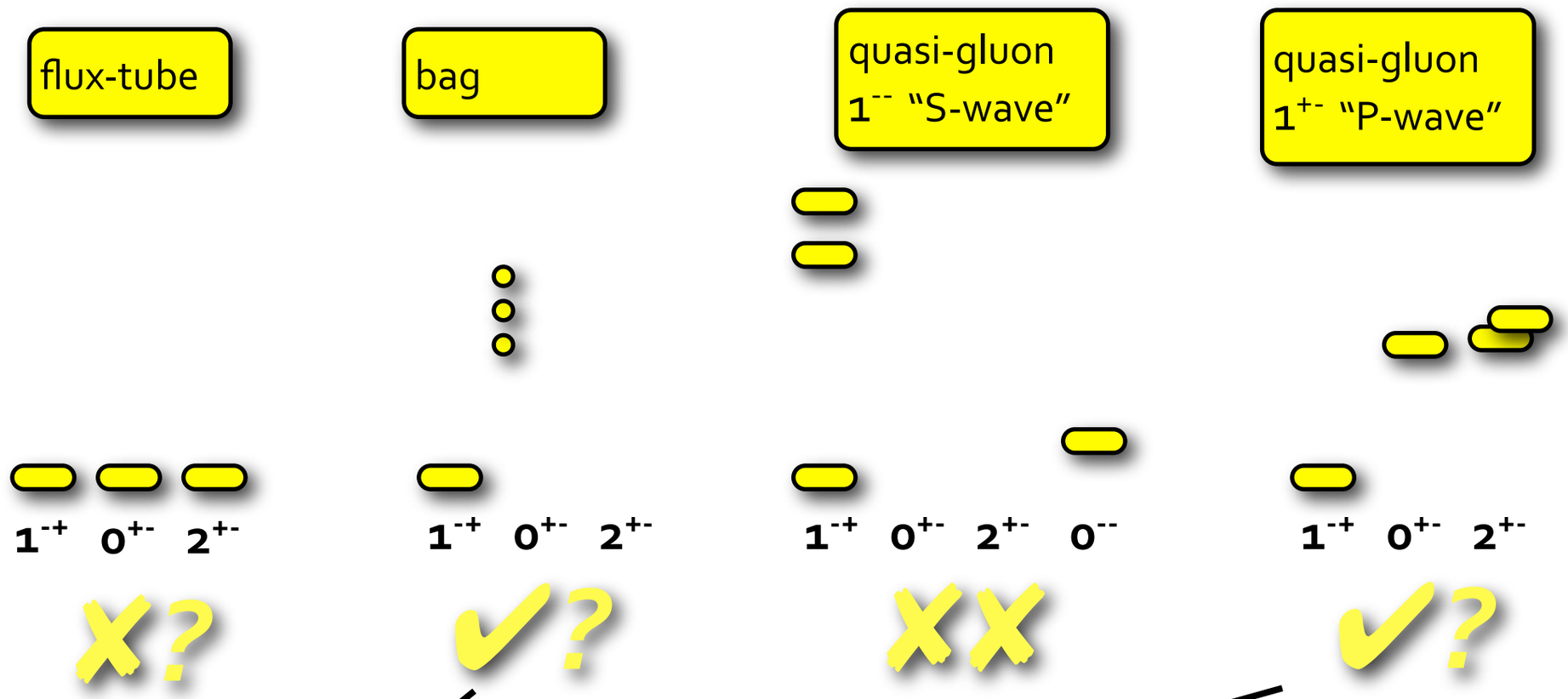
YES *

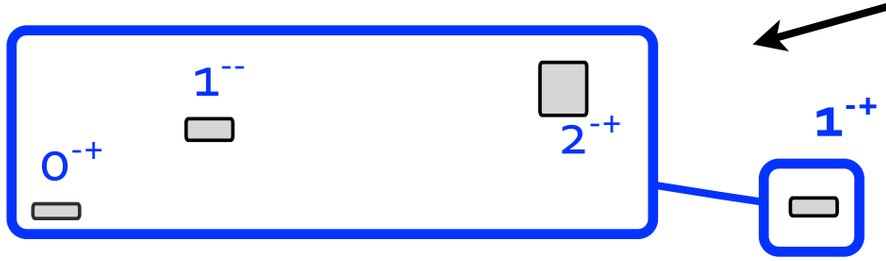
* Light hybrid mesons may be observed only for heavy quark masses, risk of large hadronic widths has not been determined. The Hadron Spectrum Collaboration offers no guarantee that they can be produced in photoproduction.

models in light of what we've seen

exotic state degeneracy pattern







have the 'right' **complete** lightest hybrid supermultiplet

addressing the small print

* **Light hybrid mesons may be observed only for heavy quark masses,** risk of large hadronic widths has not been determined. The Hadron Spectrum Collaboration offers no guarantee that they can be produced in photoproduction.

calculate at lighter quark masses

increased computational cost

just a matter of time ...

addressing the small print

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observing states as decaying resonances

scattering of composite objects
in non-perturbative field theory !

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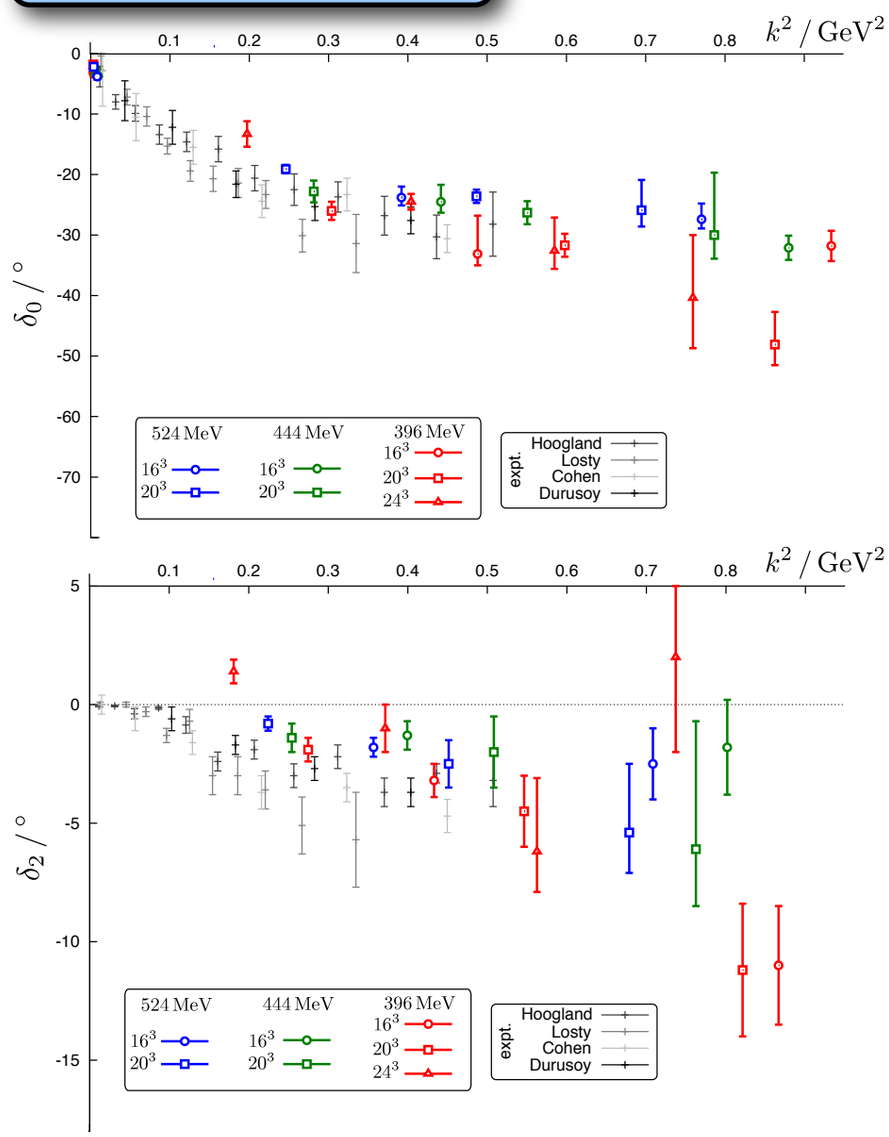
Isospin=2 ππ scattering



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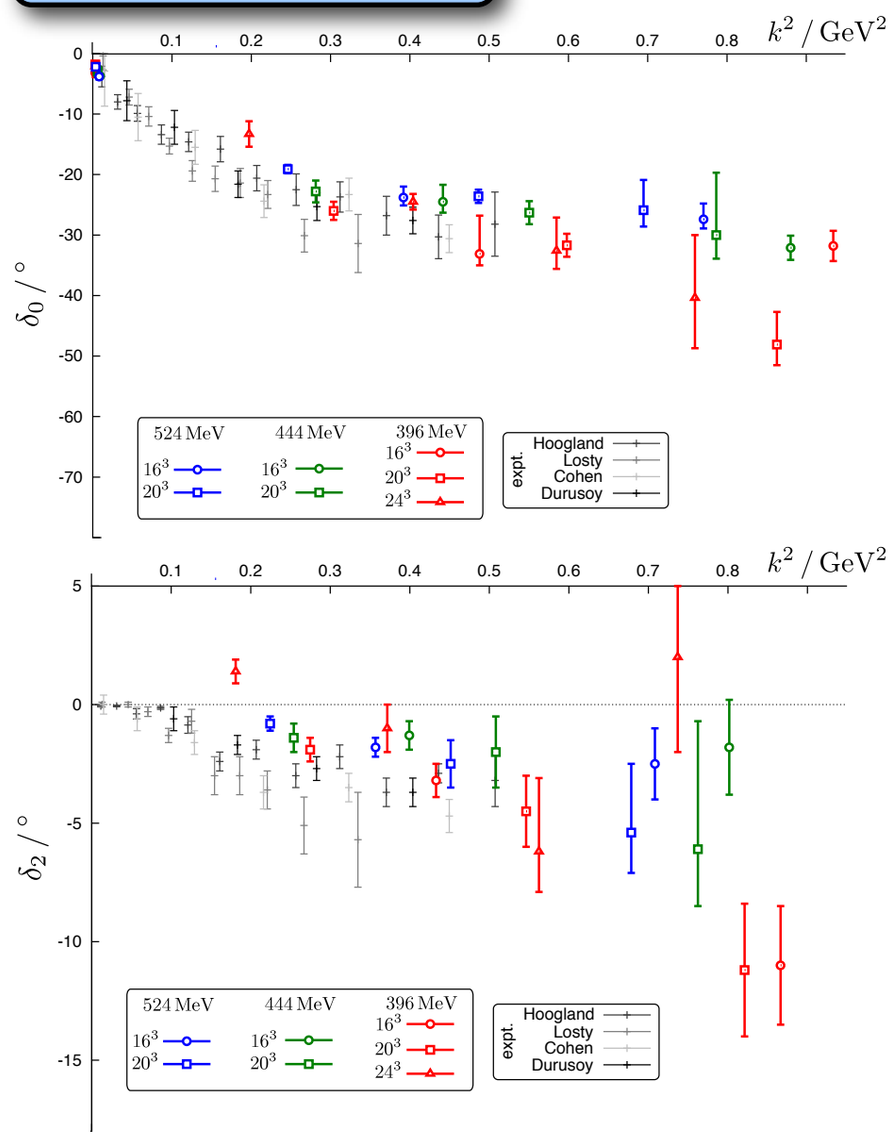
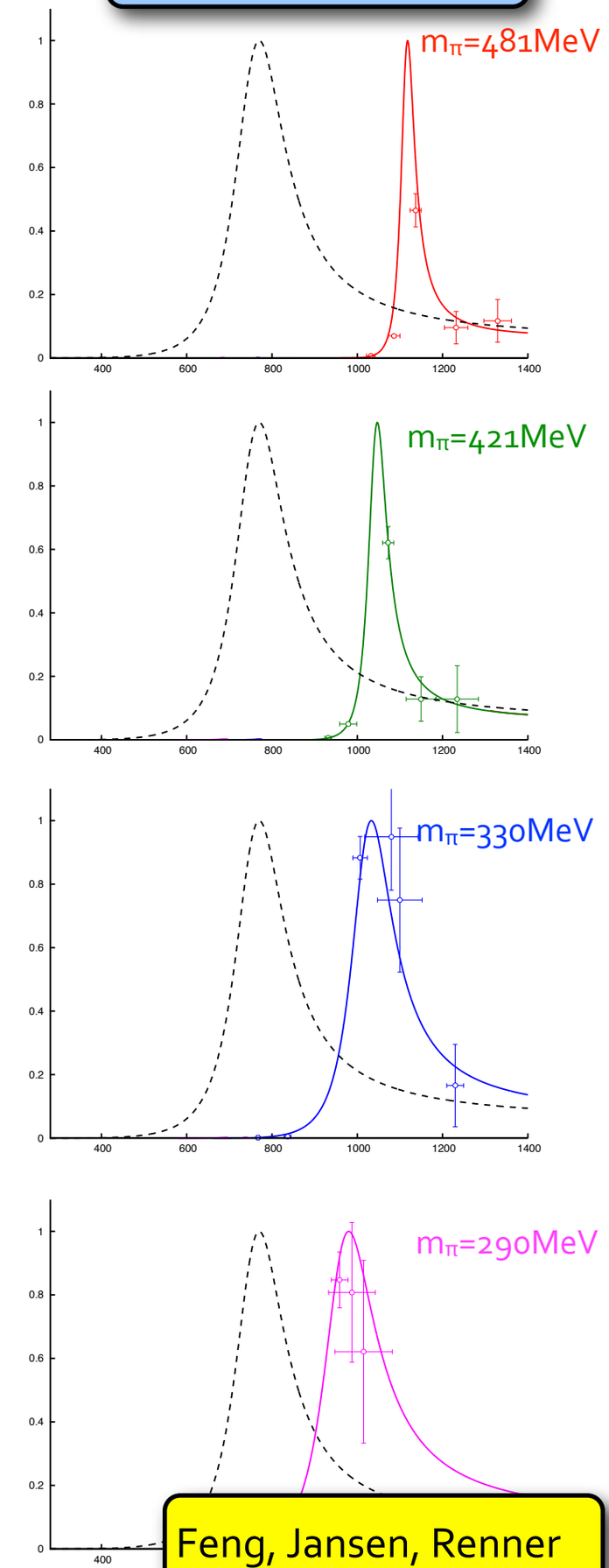


Isospin=1 ππ scattering



Feng, Jansen, Renner

addressing the small print

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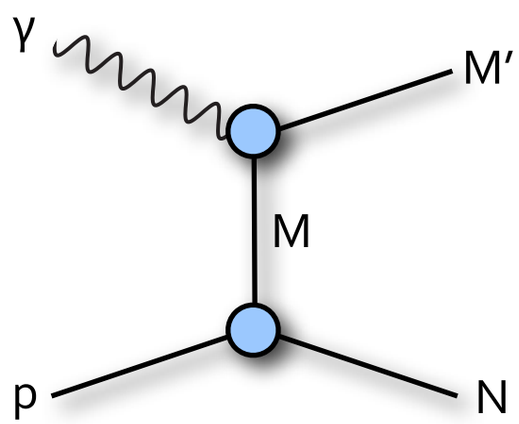
determining coupling to photons is relatively straighforward

production modeling



t-channel meson exchange

addressing the small print

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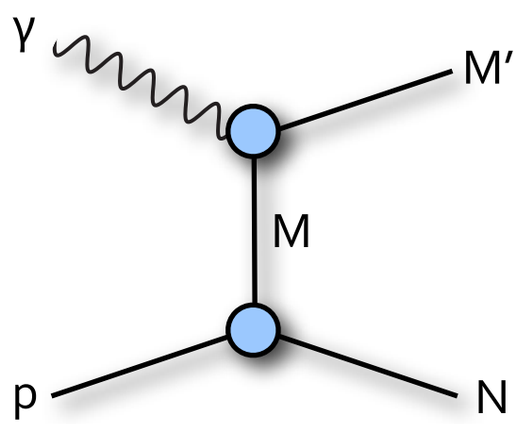

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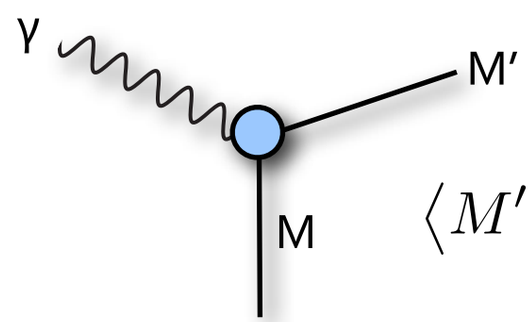

$$\langle M' | \bar{\psi} \gamma^{\mu} \psi | M \rangle$$

photocoupling / transition form-factor

lattice calculation of this

addressing the small print

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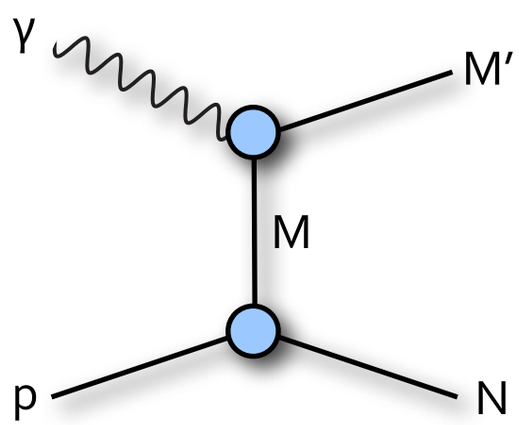

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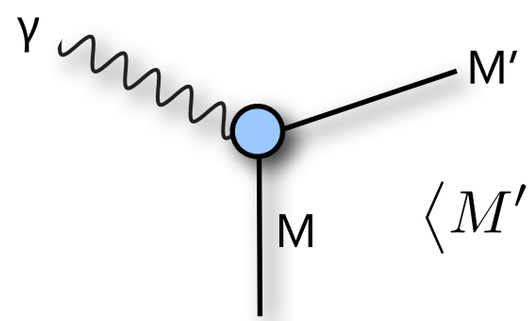
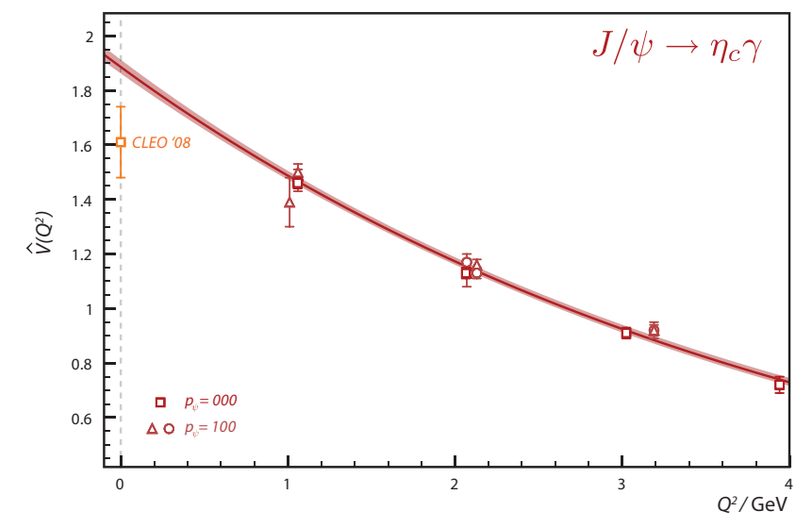


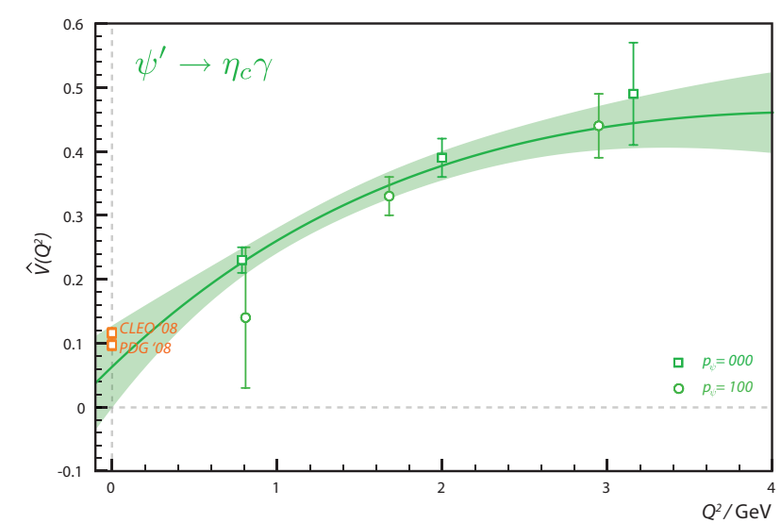
$$\langle M' | \bar{\psi} \gamma^{\mu} \psi | M \rangle$$

photocoupling / transition form-factor

lattice calculation of this

e.g. in charmonium





Does QCD predict light hybrid mesons?

Jozef Dudek
Old Dominion University & Jefferson Lab

Robert Edwards (JLab)
Balint Joo (JLab)
David Richards (JLab)
Christopher Thomas (JLab)
Mike Peardon (Trinity Coll., Dublin)

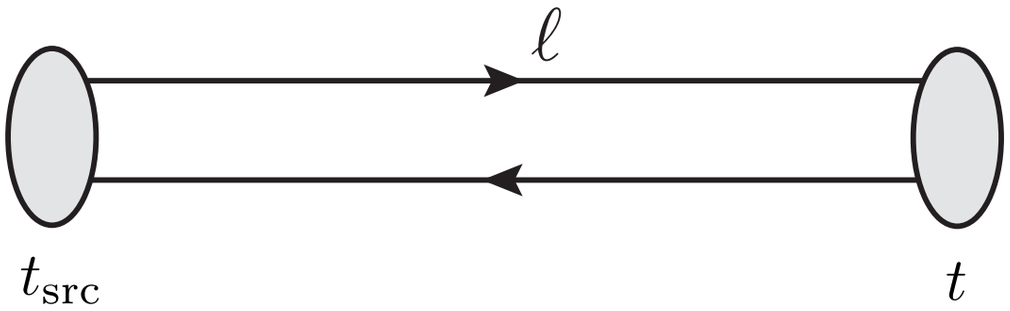
for the **Hadron Spectrum Collaboration**

"Isoscalar meson spectroscopy from lattice QCD" - arXiv:1102.4299 [hep-lat] (2011) (PRD in press)
"The phase-shift of isospin-2 ππ scattering from lattice QCD" - PRD.83.071504 (2011)
"Toward the excited meson spectrum of dynamical QCD" - PRD.82.034508 (2010)
"Highly excited and exotic meson spectrum from dynamical lattice QCD" - PRL.103.262001 (2009)
"A novel quark-field creation operator construction for hadronic physics in lattice QCD" - PRD.80.054506 (2009)

isoscalar mesons

isovector correlator :

$$C_{ij}^{[I=1]}(t, t_{\rm src}) =$$



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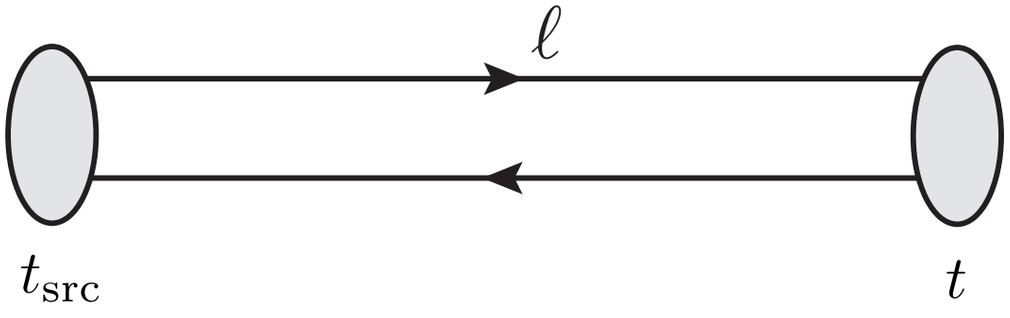

isoscalar (with just light quarks) correlator :

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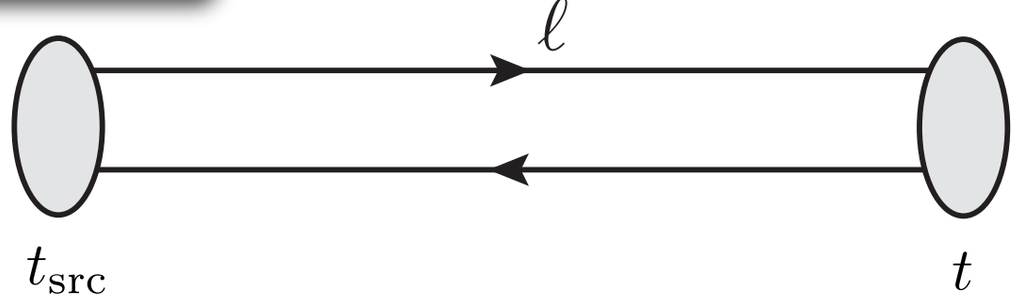

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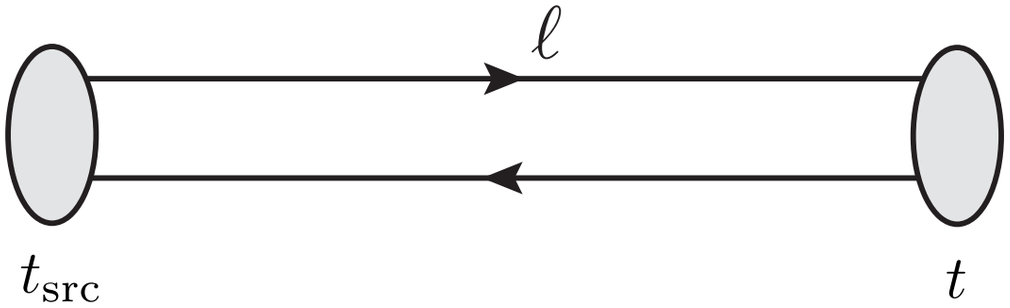

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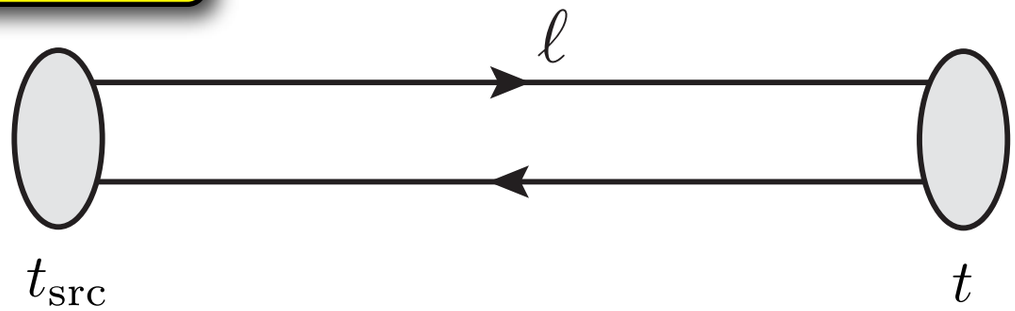
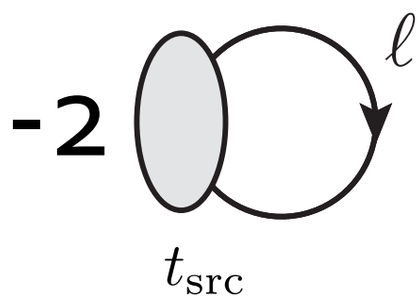
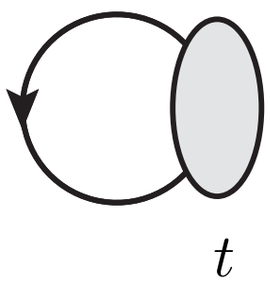






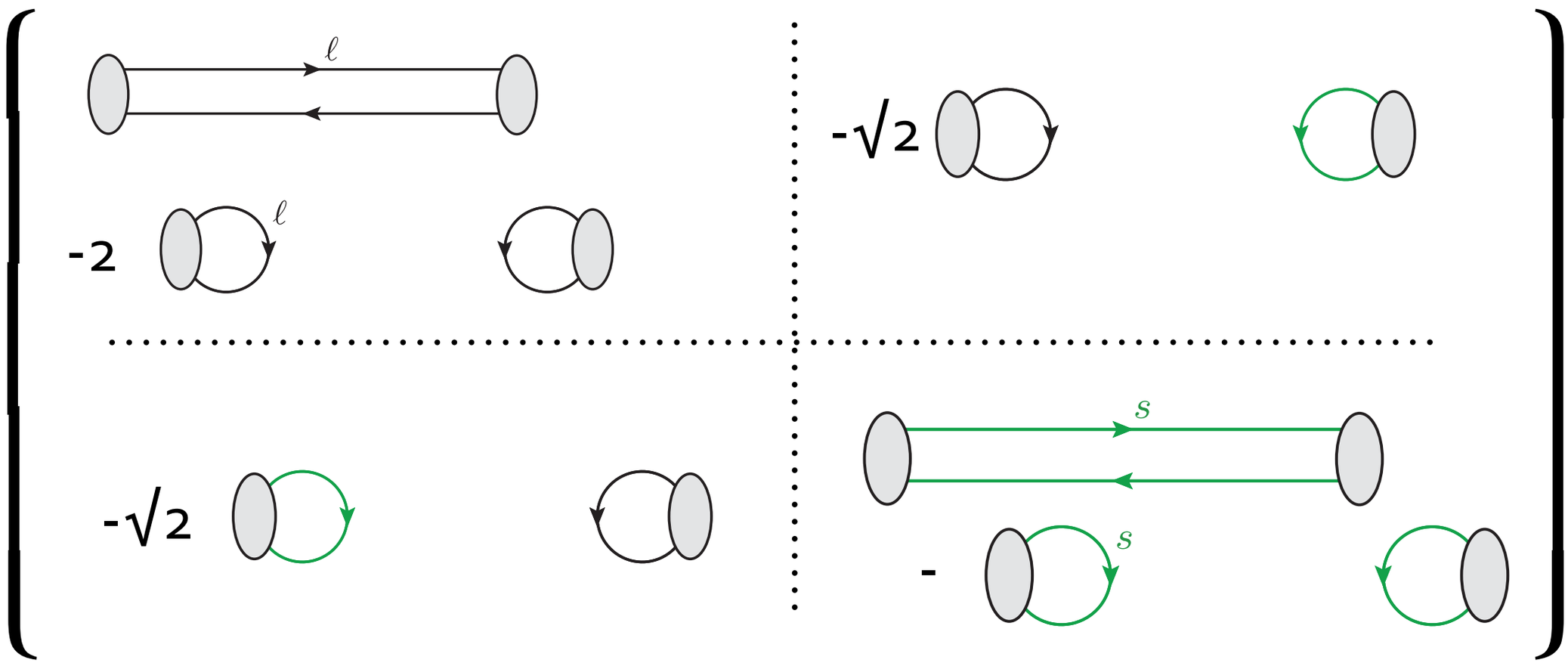
noisy

need inversion from all ***t***

GPUs ideal for the 'grunt' work

isoscalar mesons

isoscalar (with light & strange) :



diagonalising gives the $\ell\bar{\ell}$, $s\bar{s}$ mixing

isoscalar spectrum

very few results due to the difficulty of calculation

C. Michael et al (UKQCD, 2001) [heavy quarks, 2-flavour theory]
found f_1/a_1 , b_1/h_1 , ρ/ω splittings consistent with zero

ETMC (2009) [2-flavour, extrap. to phys. quark mass]
ρ/ω splitting of 27(10) MeV

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RBC/UKQCD (2010)

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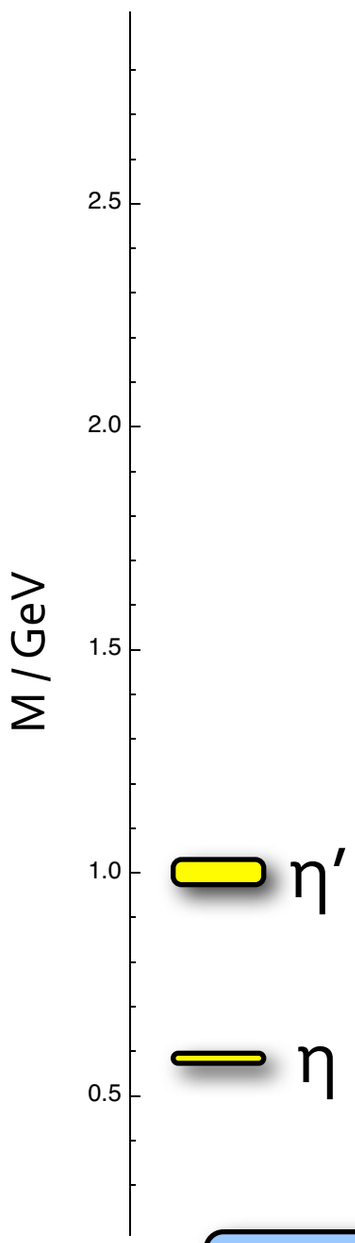
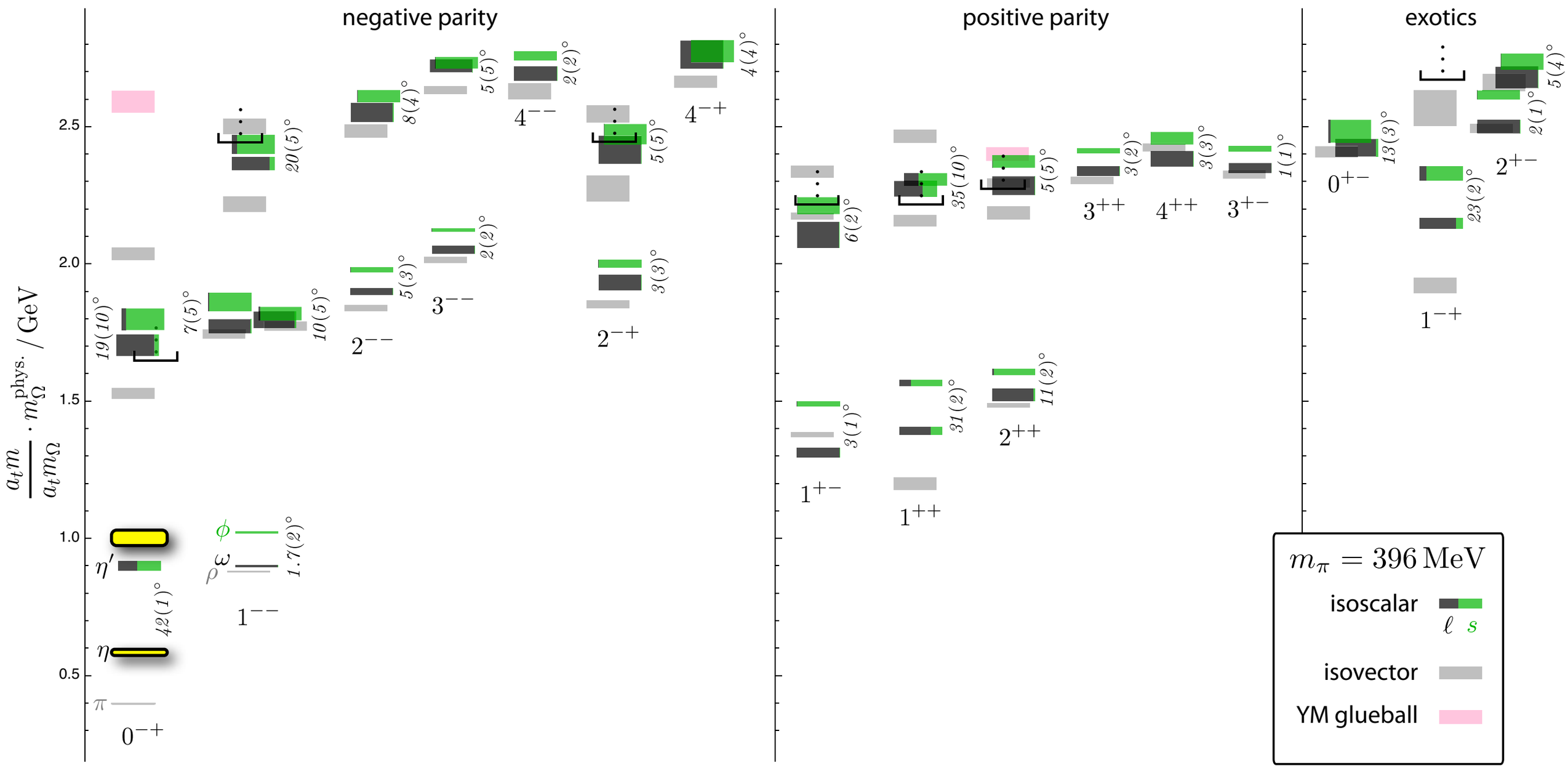


$m_\pi \sim 420$ MeV

isoscalar spectrum



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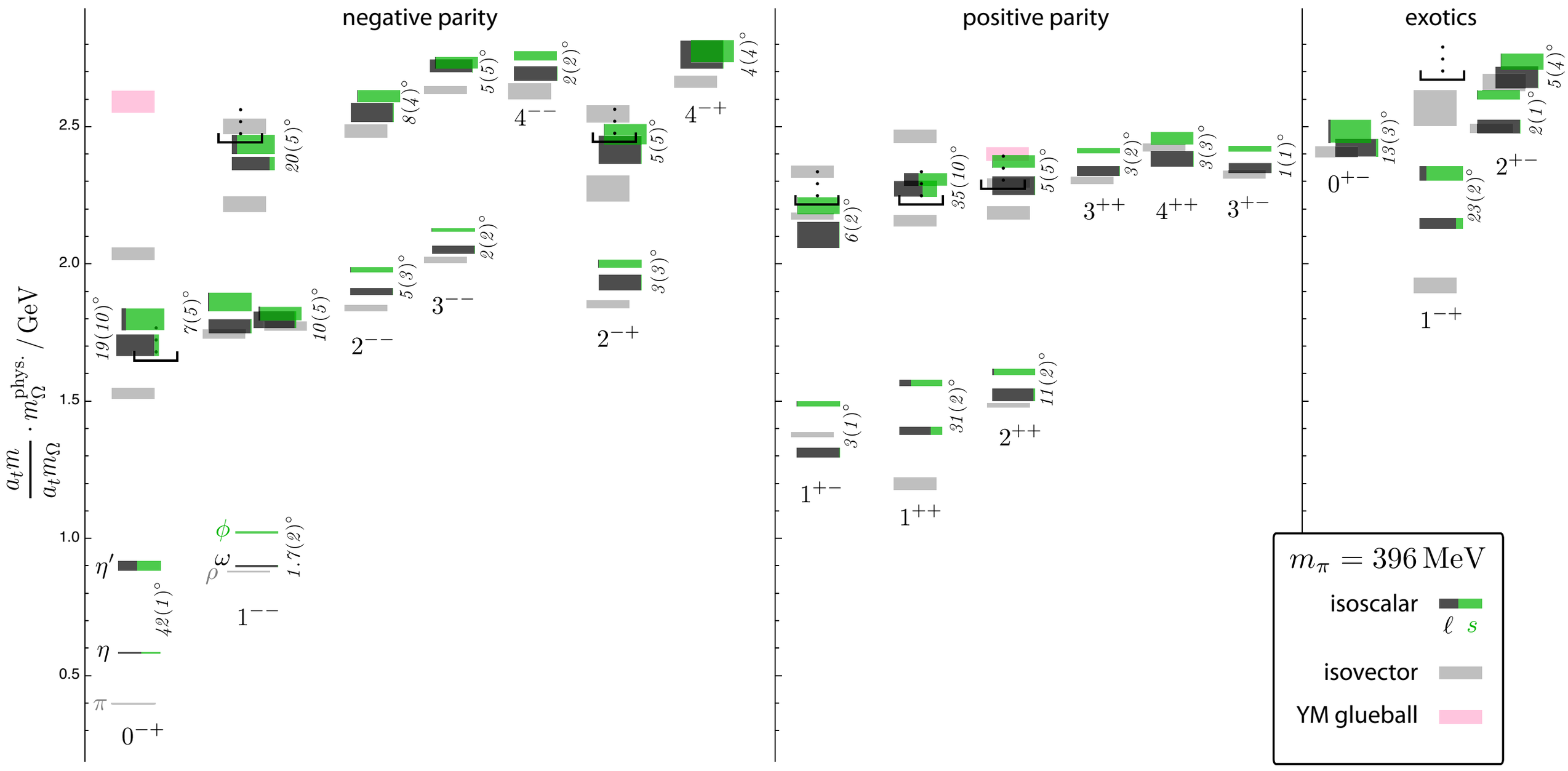

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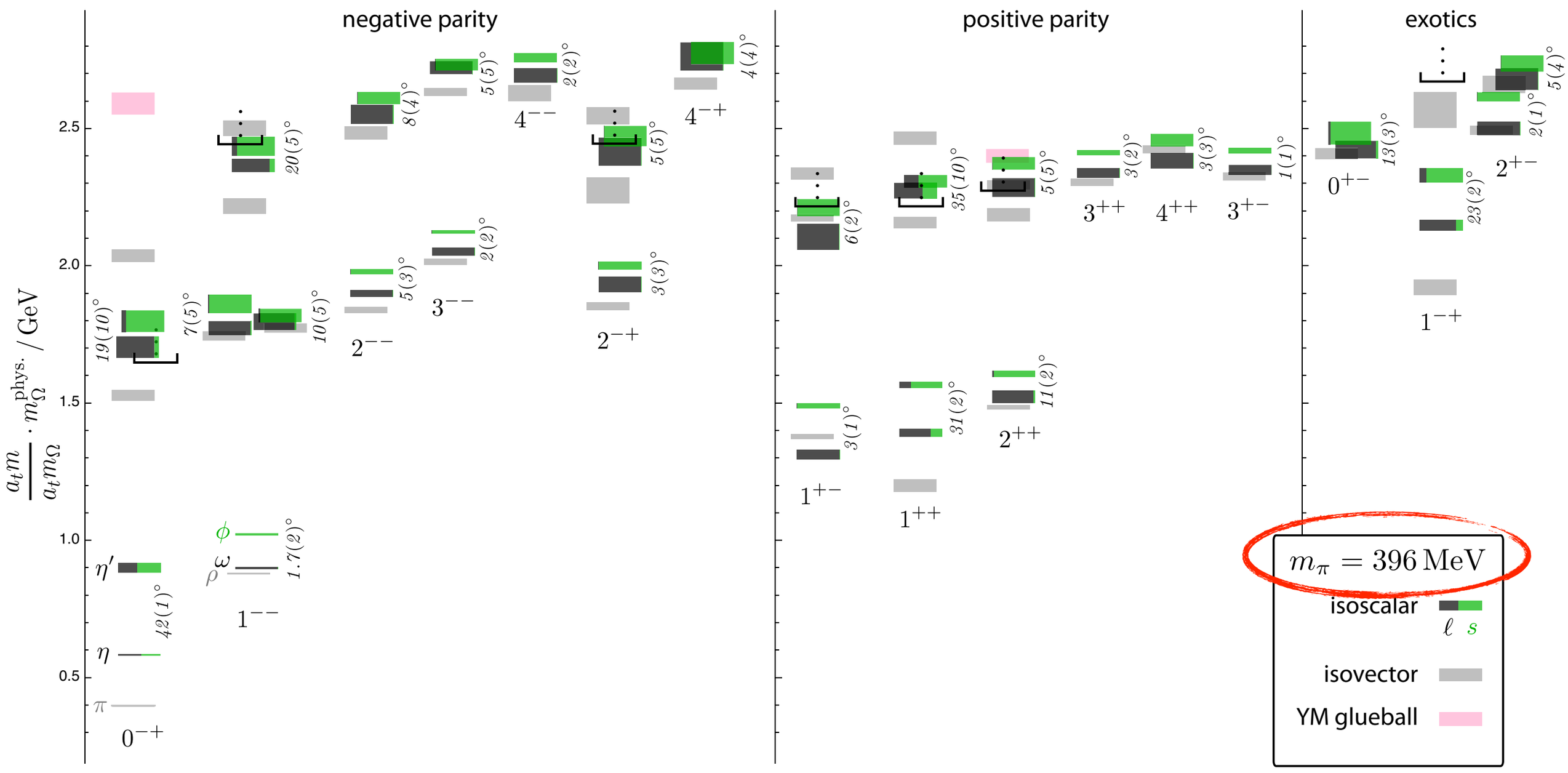
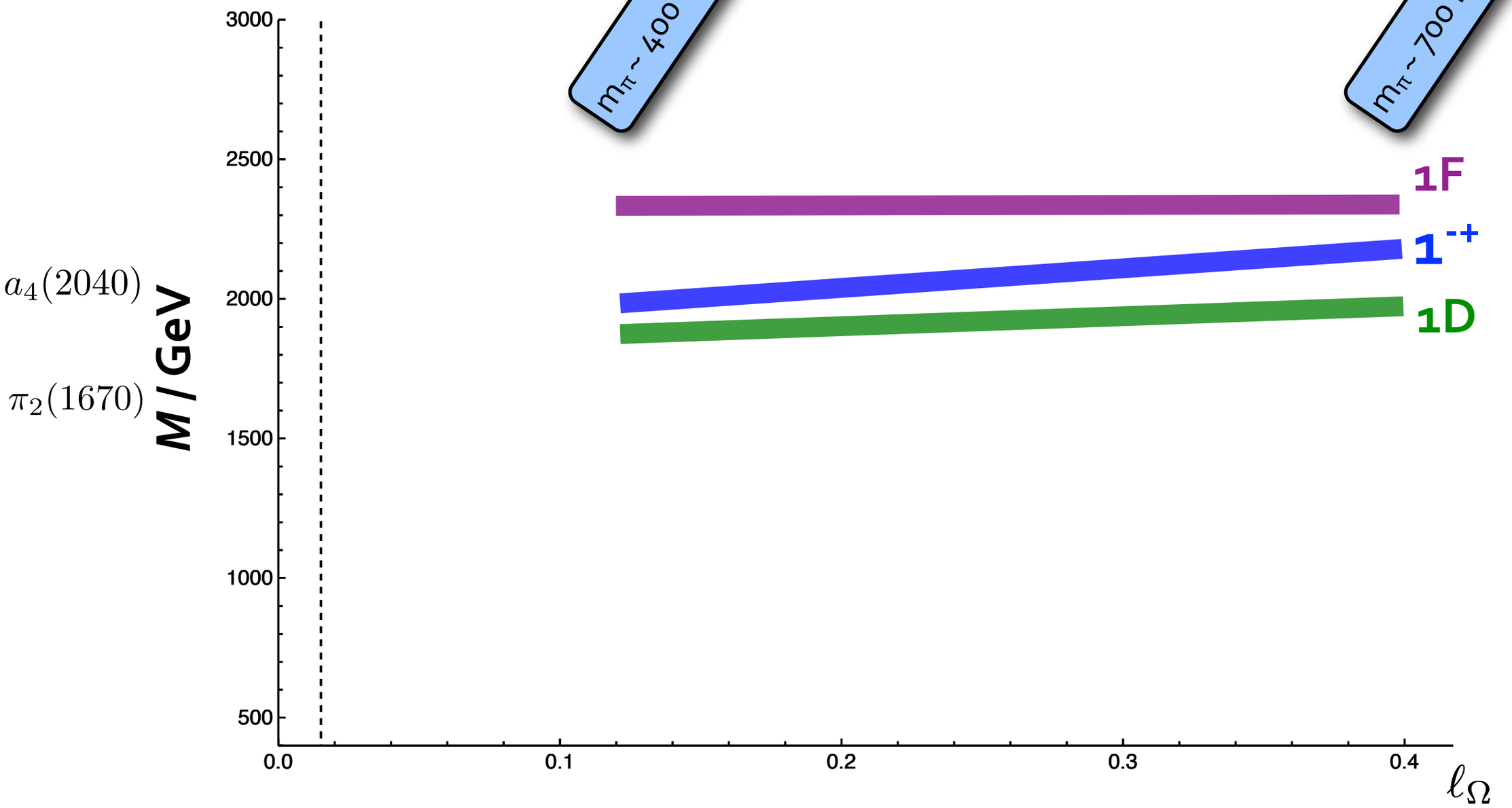


level ordering ?



real* resonances

**complex*

but we can't be satisfied with this ...

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the spectrum should not be this simple

excited states should be **resonances**

enhancements in the meson-meson scattering continuum

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in finite volume only **discrete meson-meson** states

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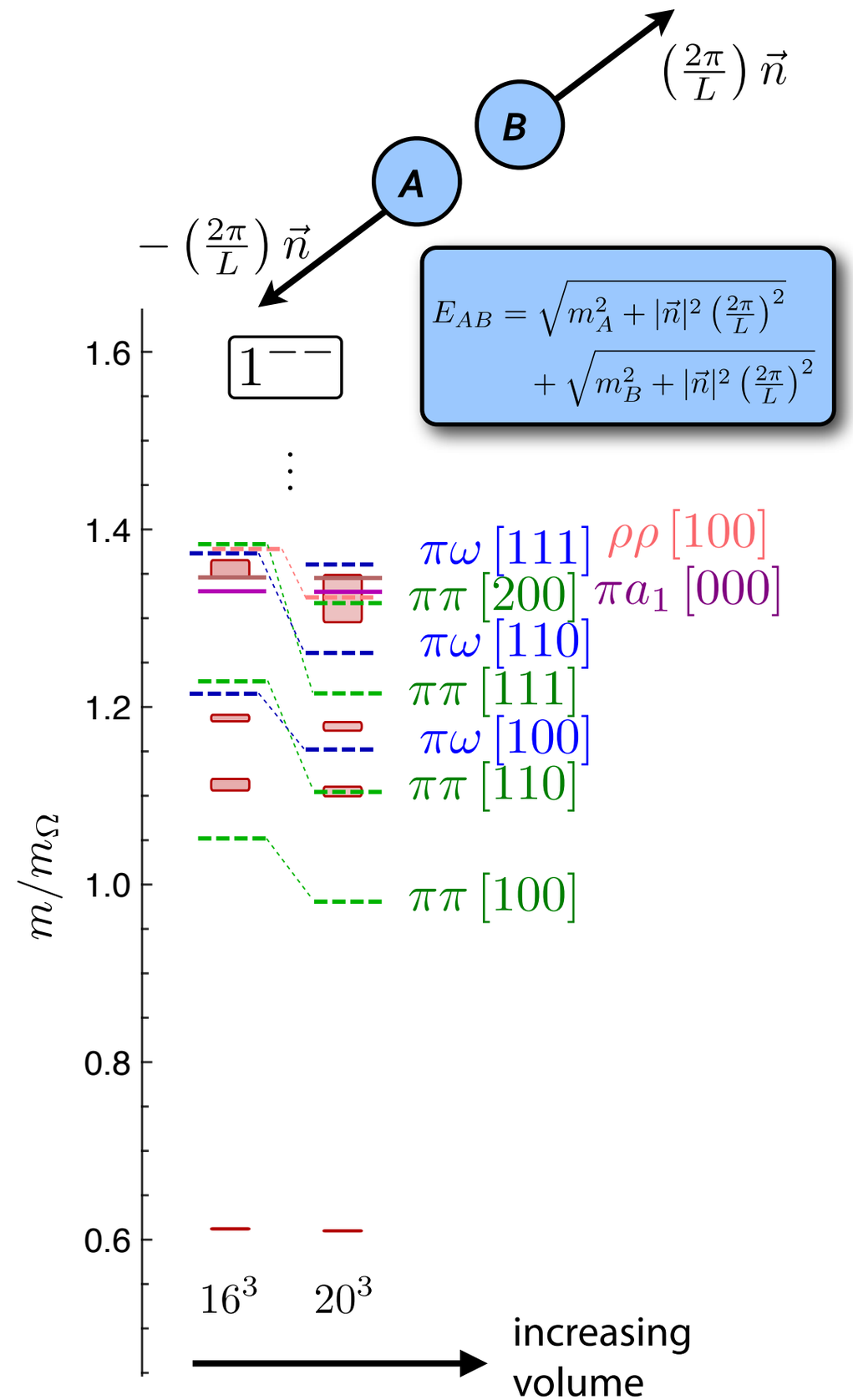

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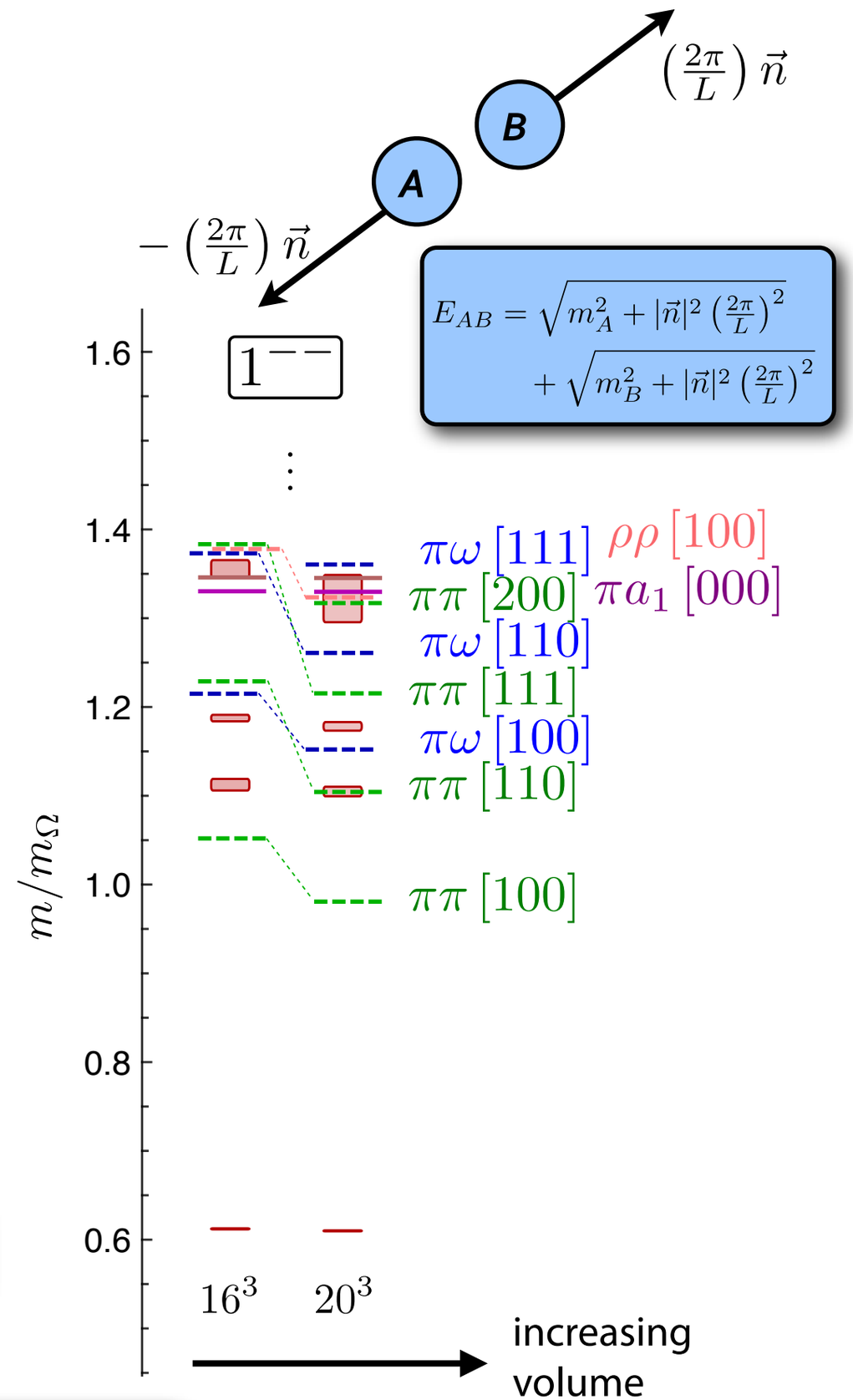


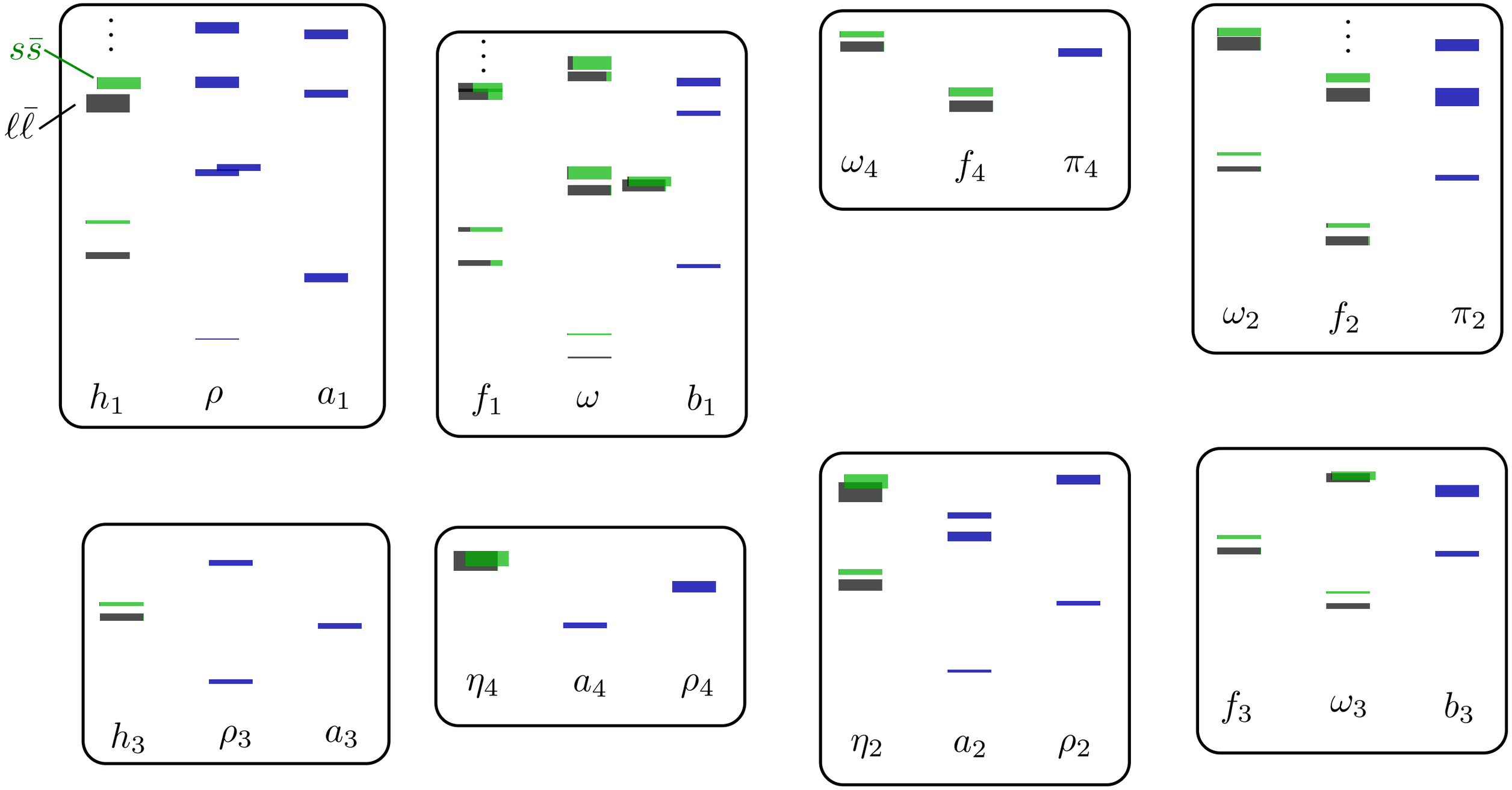
our operators closely resemble single hadrons ...

$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \psi$

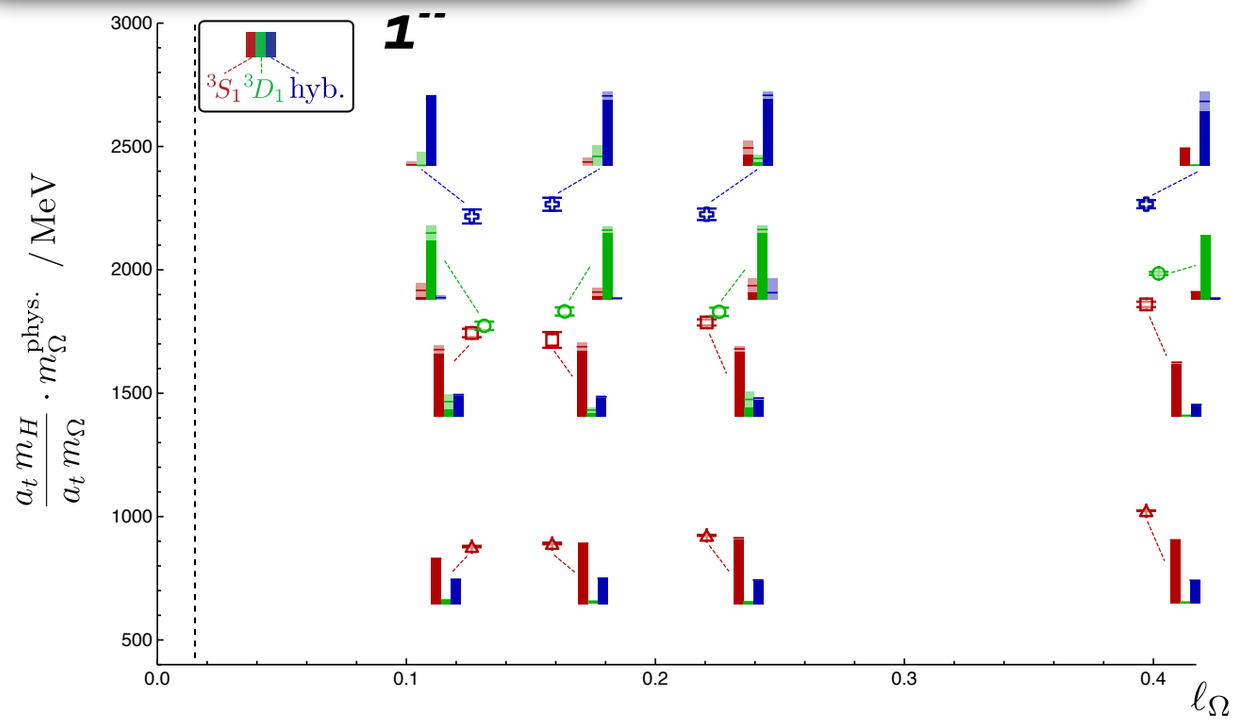
and not meson-meson pairs ~

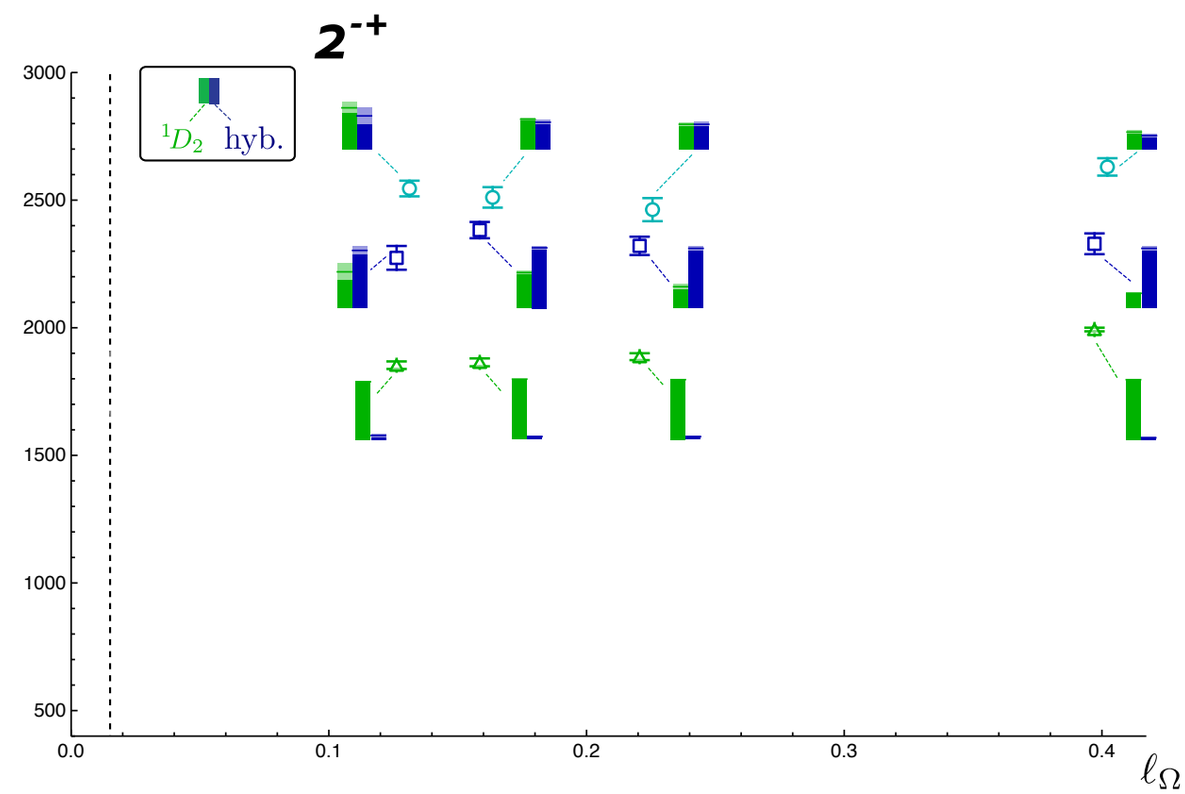
$\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \left(\bar{\psi}\Gamma\psi\right)_{\vec{x}} \cdot \sum_{\vec{y}} e^{i(-\vec{p})\cdot\vec{y}} \left(\bar{\psi}\Gamma\psi\right)_{\vec{y}}$

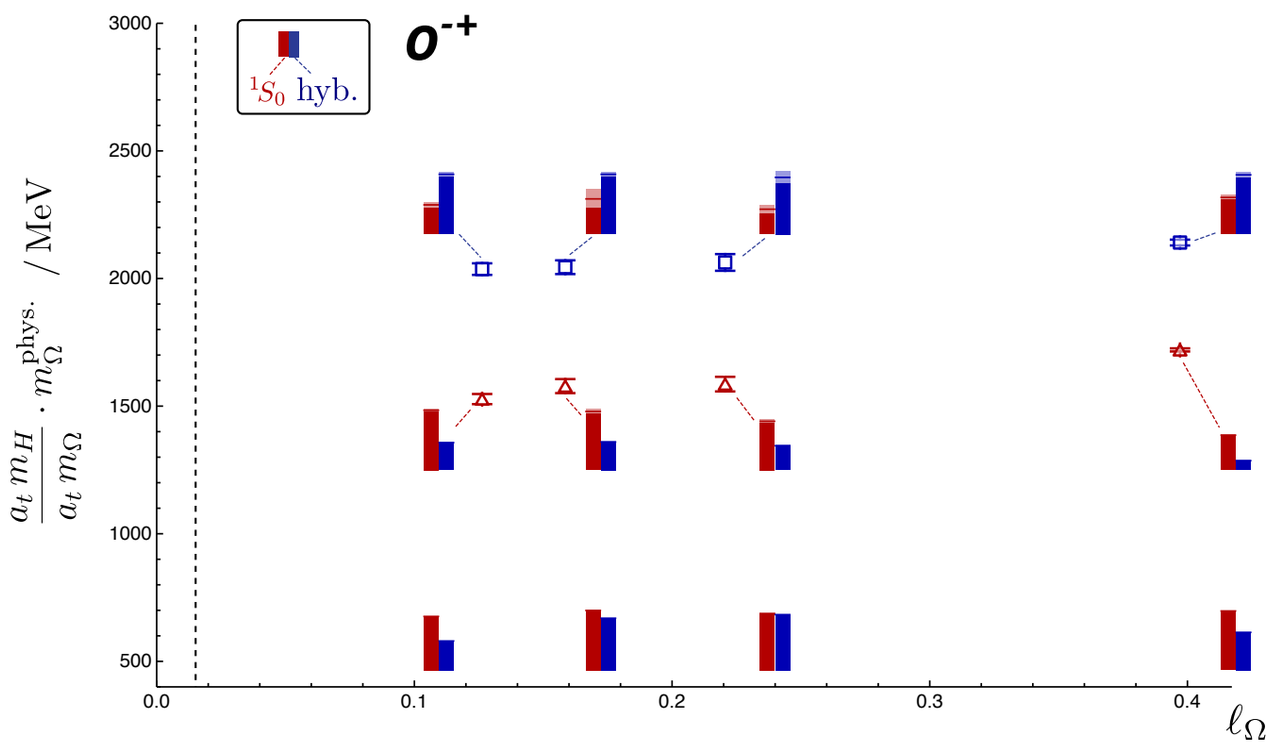
parity doubling & chiral symmetry restoration ?



overlaps with decreasing quark mass







cubic complications ...

integer spin not a good quantum number

restricted rotational symmetry of a cube

A_1	$0, 4 \ldots$
T_1	$1, 3, 4 \ldots$
T_2	$2, 3, 4 \ldots$
E	$2, 4 \ldots$
A_2	$3 \ldots$

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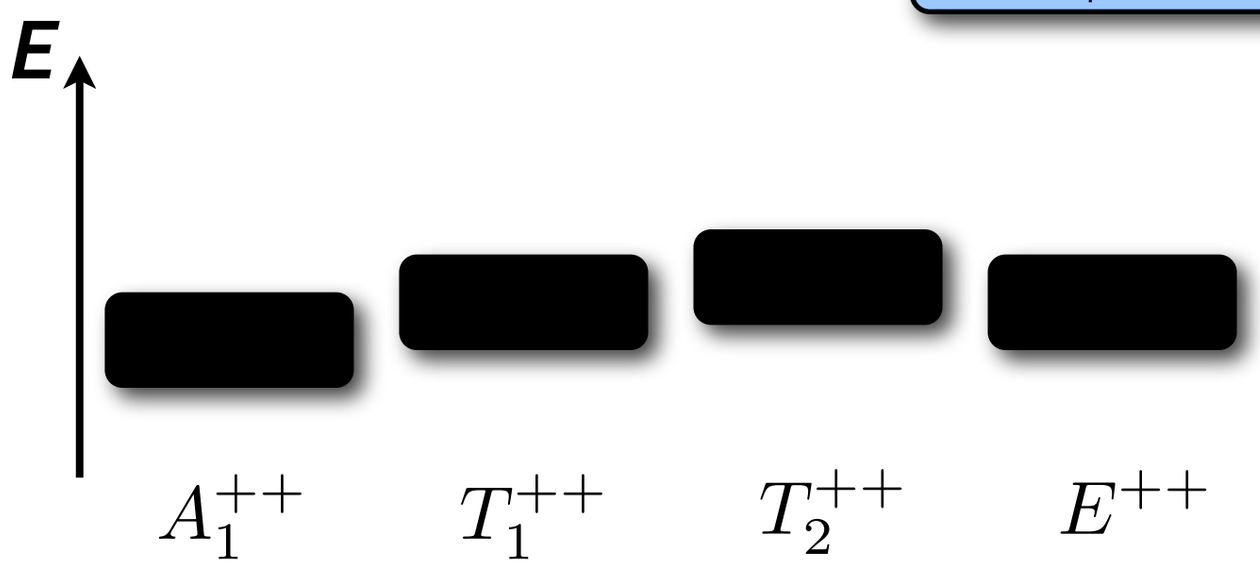

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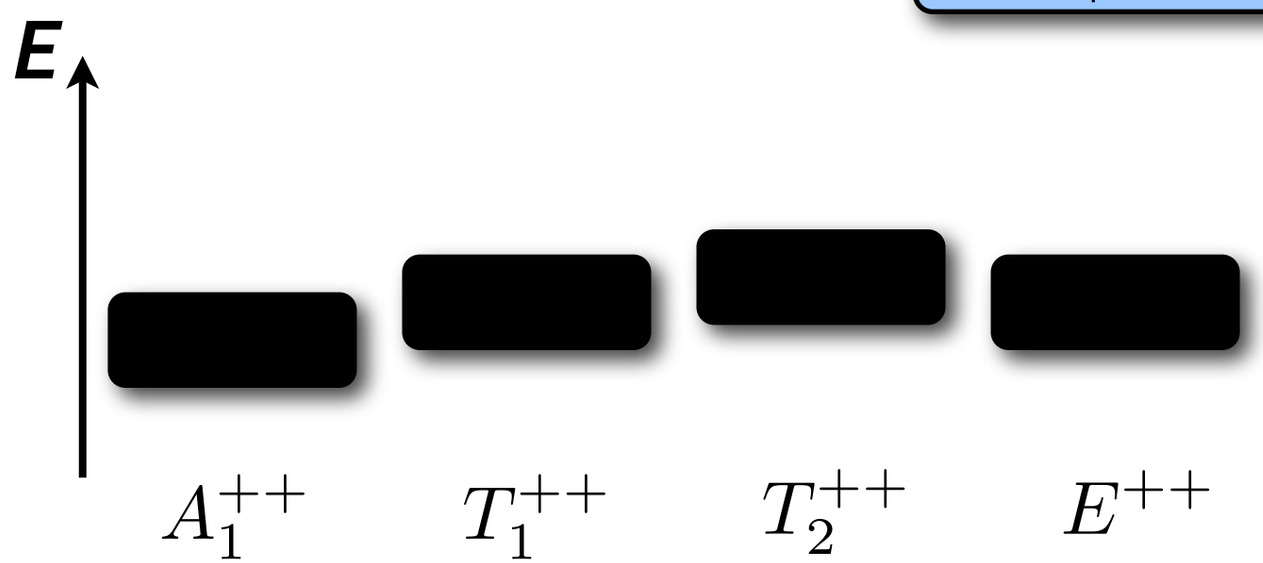

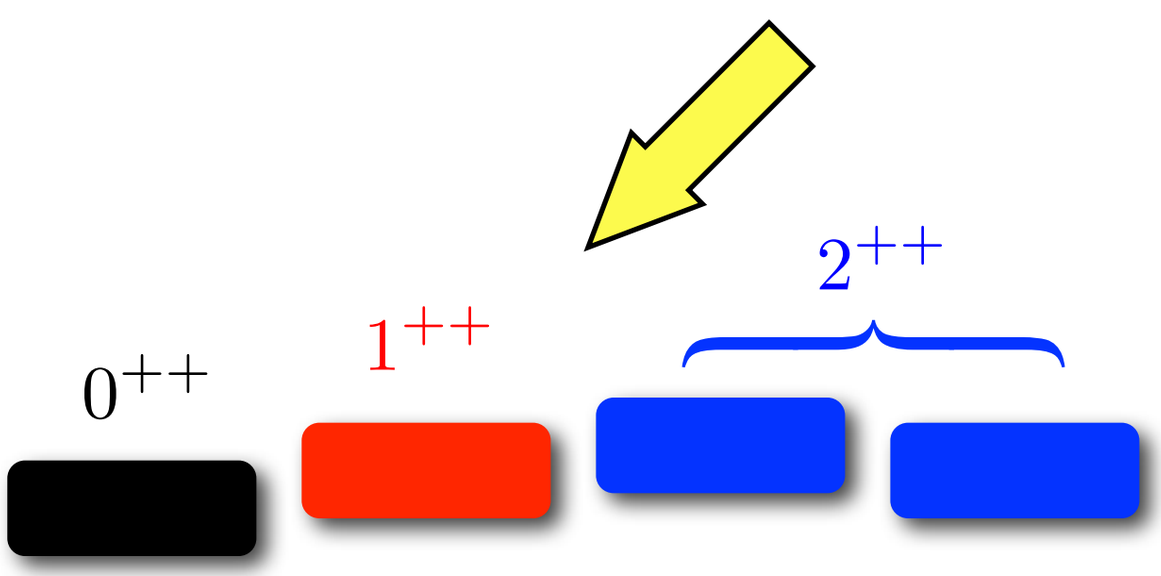

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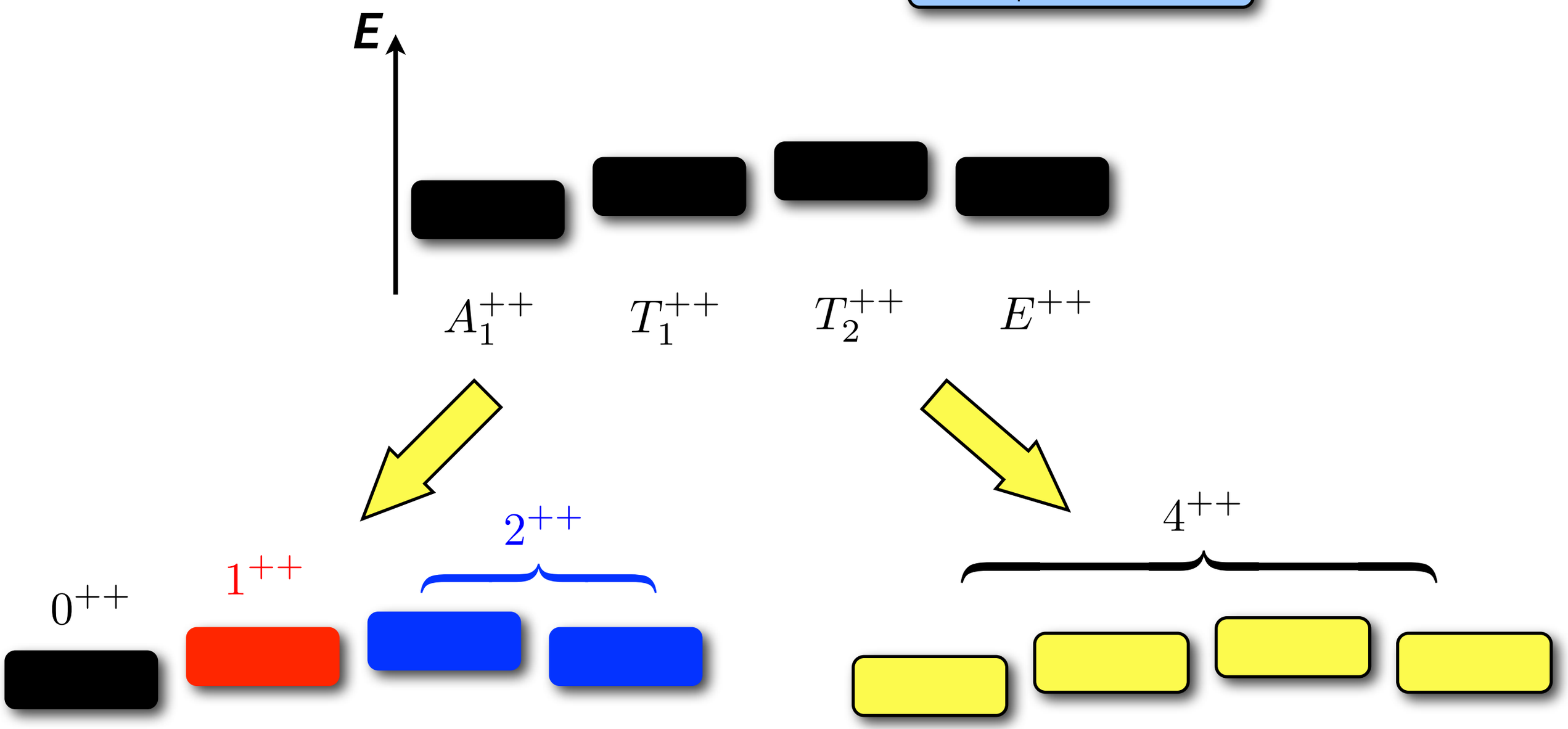


cubic complications ...

'solved' by careful operator construction

construct operators of definite ***J*** in the continuum $\mathcal{O}^{JM}$

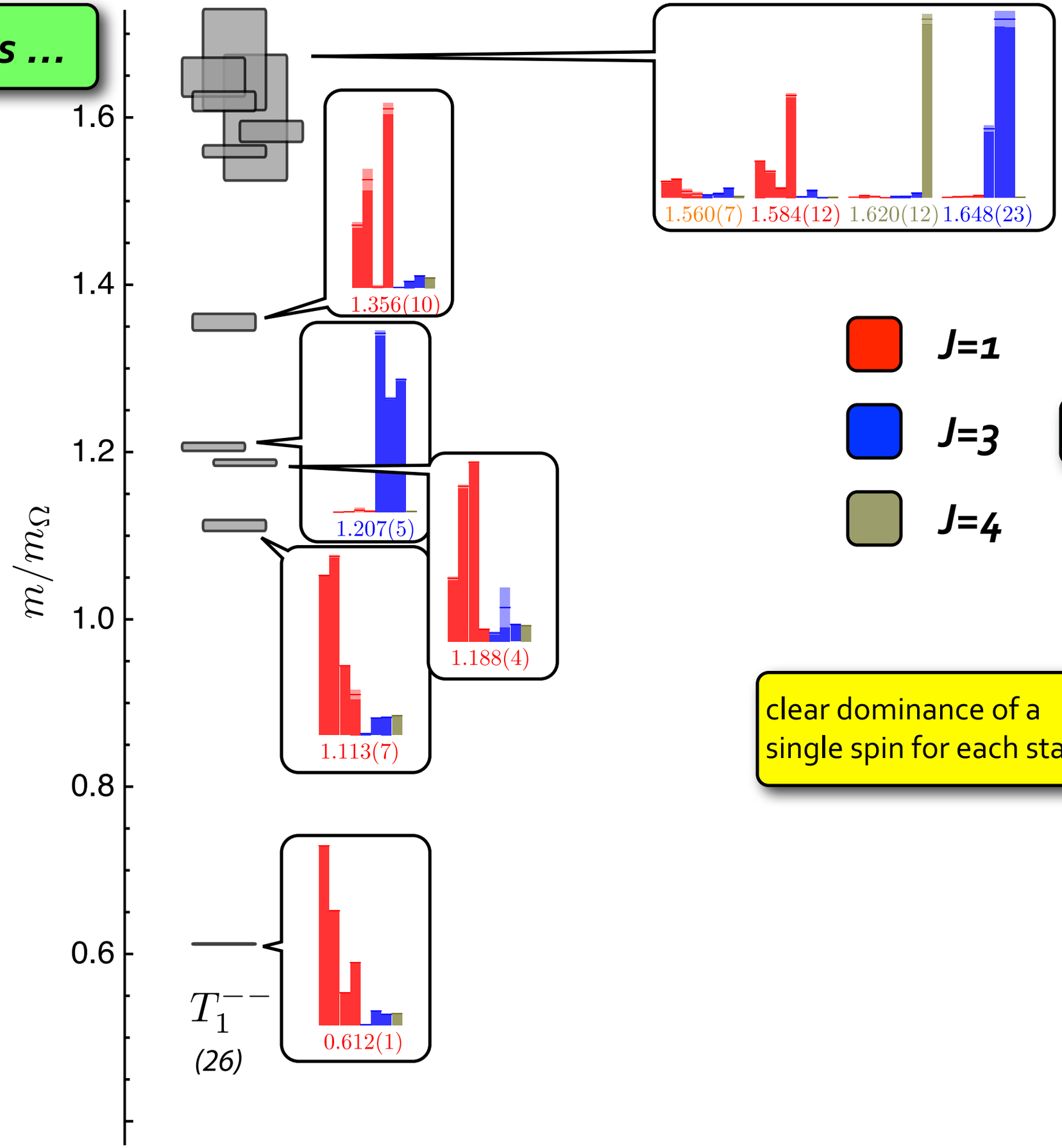
"subduce" into the cubic group irreps $\mathcal{O}^{[J]}_{\Lambda,\lambda} \equiv \sum_M \mathcal{S}^{JM}_{\Lambda,\lambda} \cdot \mathcal{O}^{JM}$

and then $\langle \mathfrak{n}(J) | \mathcal{O}^{[J']}_{\Lambda} | 0 \rangle \approx Z^{[J]}_{\mathfrak{n}} \cdot \delta_{J'J}$ if the rotational symmetry is "restored"

operators respect cubic symmetry, but are 'preconditioned' to be ***J***-diagonal

... but does it work in practice ?

cubic complications ...



J=1

J=3

J=4

$\langle n | \mathcal{O}_{T_1} | 0 \rangle$

clear dominance of a single spin for each state

APS April meeting 2011