

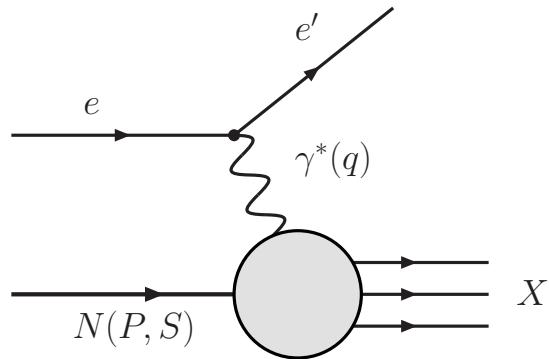
The Physics of TMDs

(A. Metz, Temple University, Philadelphia)

- Inclusive DIS and forward parton distributions (PDFs)
- Transverse momentum dependent parton distributions (TMDs) and 3-D nucleon structure
- Addressing TMDs in experiment
- Sign reversal of the Sivers function and TMD factorization
- TMDs at a future Electron Ion Collider
- Summary

Inclusive deep-inelastic scattering

- Process



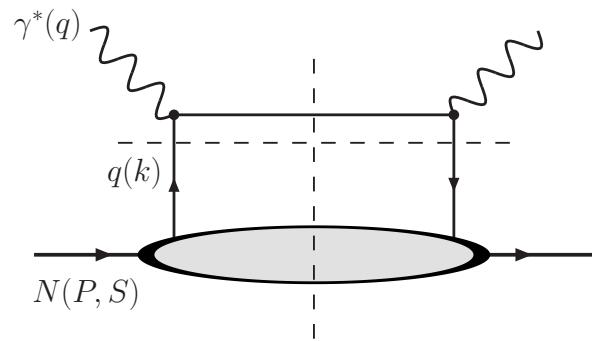
Variables

$$Q^2 = -q^2 > 0 \quad x = \frac{Q^2}{2P \cdot q} \quad \phi_S$$

- Optical theorem

$$\sigma_{\gamma^* N \rightarrow X} \propto \text{Im } A(\gamma^* N \rightarrow \gamma^* N, \vartheta = 0)$$

- Parton model

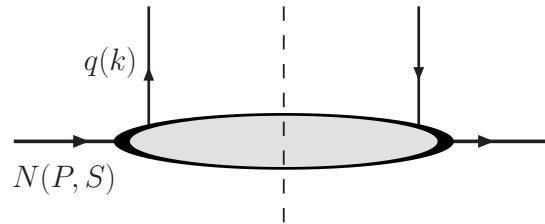


→ factorization

$$q(x, Q^2) \quad (f_1^q) \quad \Delta q(x, Q^2) \quad (g_1^q) \\ (g, \Delta g)$$

$$k^+ = \frac{1}{\sqrt{2}}(p^0 + p^3) = xP^+$$

Forward parton distributions (PDFs): field-theoretic definition



- Unpolarized PDF: unpolarized quarks in unpolarized nucleon

$$f_1^q(x) = \frac{1}{2} \int \frac{d\xi^-}{2\pi} e^{ik \cdot \xi} \langle P; S | \bar{\psi}^q(0) \gamma^+ \mathcal{W}_{PDF} \psi^q(\xi) | P; S \rangle \Big|_{\xi^+ = \xi_T = 0}$$

- Helicity PDF: long. polarized quarks in long. polarized nucleon

$$\lambda \langle | \bar{\psi}^q \gamma^+ \gamma_5 \psi^q | \rangle \sim \lambda \Lambda g_1^q(x) \rightarrow \text{spin-spin correlation}$$

- Transversity PDF: transv. polarized quarks in transv. polarized nucleon

$$s_T^i \langle | \bar{\psi}^q i \sigma^{i+} \gamma_5 \psi^q | \rangle \sim \vec{s}_T \cdot \vec{S}_T h_1^q(x) \rightarrow \text{spin-spin correlation}$$

- transversity decouples from inclusive DIS (chiral-odd)
- hard to measure! (\rightarrow talks by Sulkosky and Qian)

Transverse momentum dependent parton distributions (TMDs)

- TMD-correlator

$$\begin{aligned}\Phi^q &= \frac{1}{2} \int \frac{d\xi^-}{2\pi} \frac{d^2\vec{\xi}_T}{(2\pi)^2} e^{ik\cdot\xi} \langle P; S | \bar{\psi}^q(0) \gamma^+ \mathcal{W}_{TMD} \psi^q(\xi) | P; S \rangle \Big|_{\xi^+=0} \\ &= f_1^q(x, \vec{k}_T^2) + \frac{(\vec{S}_T \times \vec{k}_T) \cdot \hat{P}}{M} f_{1T}^{\perp q}(x, \vec{k}_T^2)\end{aligned}$$

- partonic nucleon structure beyond collinear approximation
→ 3-D structure in (x, \vec{k}_T) -space; important/major part of TMD physics
- complementary to 3-D structure in (x, \vec{b}_T) -space (→ GPDs)
- Sivers function f_{1T}^\perp describes strength of spin-orbit correlation (Sivers, 1989)
- spin asymmetry on the level of parton density
→ spin asymmetry in observables (e.g., Sivers SSA observed by HERMES, COMPASS, and JLab in semi-inclusive DIS)
- k_T compensated by hadronic scale (M) → no suppression !

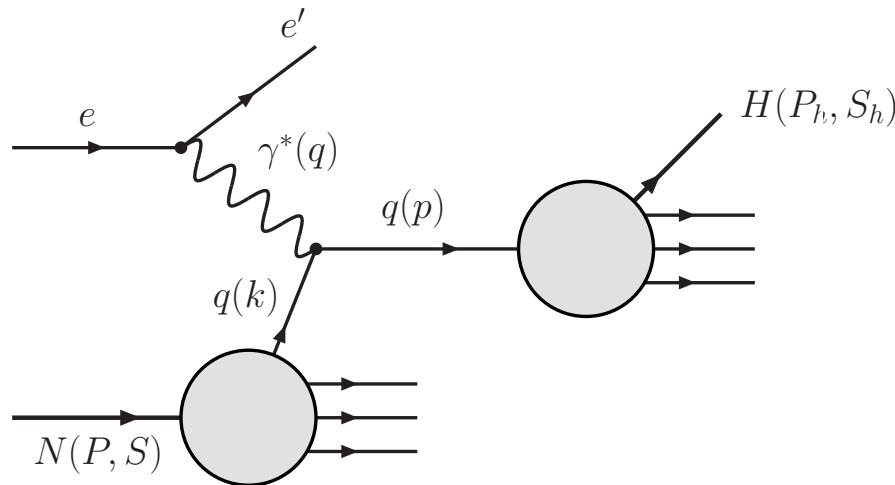
- Leading twist: overview

$$\begin{aligned}
 \langle | \bar{\psi}^q \gamma^+ \psi^q | \rangle &\sim f_1^q - \frac{\varepsilon_T^{ij} k_T^i S_T^j}{M} f_{1T}^{\perp q} \\
 \lambda \langle | \bar{\psi}^q \gamma^+ \gamma_5 \psi^q | \rangle &\sim \lambda \Lambda g_1^q + \frac{\lambda \vec{k}_T \cdot \vec{S}_T}{M} g_{1T}^q \\
 s_T^i \langle | \bar{\psi}^q i\sigma^{i+} \gamma_5 \psi^q | \rangle &\sim \vec{s}_T \cdot \vec{S}_T h_1^q + \frac{\Lambda \vec{k}_T \cdot \vec{s}_T}{M} h_{1L}^{\perp q} - \frac{\varepsilon_T^{ij} k_T^i s_T^j}{M} h_1^{\perp q} \\
 &\quad + \frac{1}{2M^2} \left(2 \vec{k}_T \cdot \vec{s}_T \vec{k}_T \cdot \vec{S}_T - \vec{k}_T^2 \vec{s}_T \cdot \vec{S}_T \right) h_{1T}^{\perp q}
 \end{aligned}$$

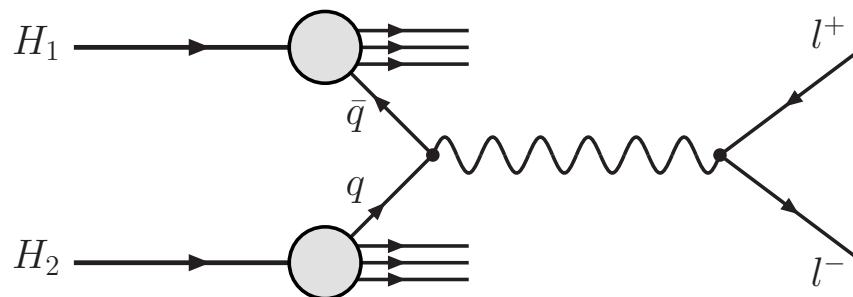
- 8 leading quark (and gluon) TMDs
- 2 (naive) T-odd TMDs: $f_{1T}^{\perp q}$, $h_1^{\perp q}$
- dipole pattern: $f_{1T}^{\perp q}$, $h_1^{\perp q}$, g_{1T}^q , $h_{1L}^{\perp q}$
- quadrupole pattern: $h_{1T}^{\perp q}$ (relation to quark OAM, She, Zhu, Ma, 2009)
- various model calculations and recent lattice calculation of TMDs
(Hägler, Musch, Negele, Schäfer, 2009, 2010)
- nontrivial (model-dependent) relations between TMDs and GPDs
(Burkardt, 2002, ... / Lu, Schmidt, 2006 / Meissner, Metz, Goeke, 2007, ...
Pasquini, Cazzaniga, Boffi, 2008 / Gamberg, Schlegel, 2009)
- no model-independent nontrivial relations between TMDs and GPDs
(Goeke, Meissner, Metz, Schlegel, 2008/09)

Addressing TMDs in experiment

- Semi-inclusive DIS $l N \rightarrow l' H X$

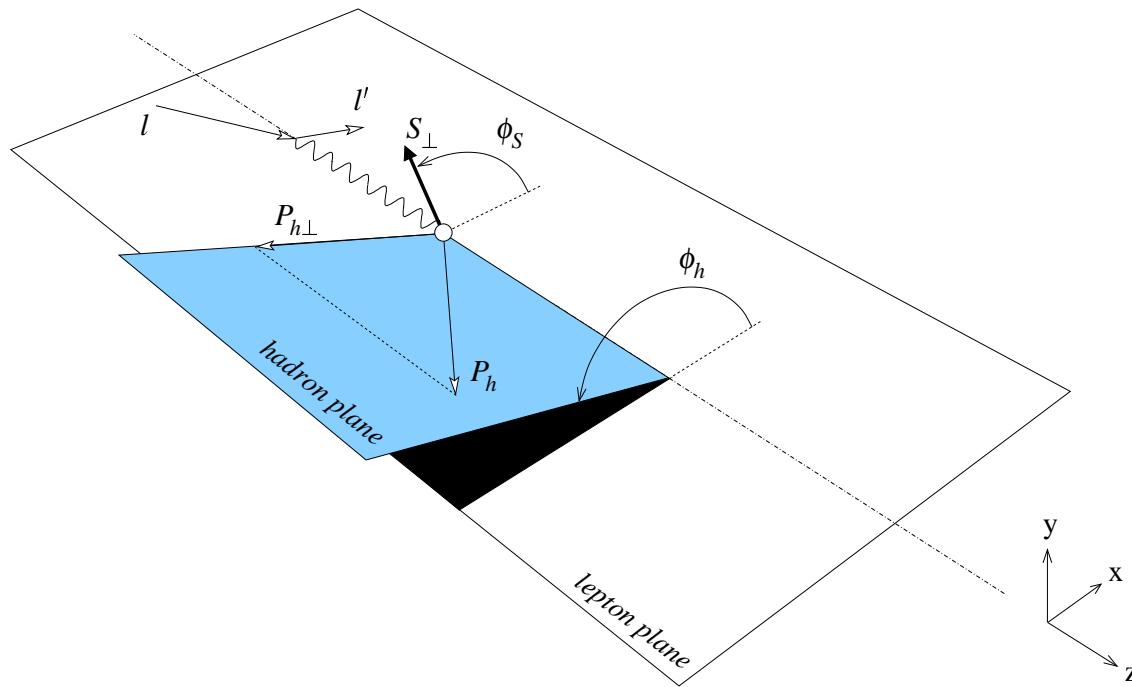


- Drell-Yan process $H_1 H_2 \rightarrow l^+ l^- X$



- Kinematics for semi-inclusive DIS

$$x \quad Q^2 \quad \phi_S \quad z \quad P_{h\perp} \quad \phi_h \quad \rightarrow \quad \text{multi-dimensional observables}$$



- σ integrated upon $P_{h\perp}$: collinear factorization
- σ differential in $P_{h\perp}$, $P_{h\perp} \sim Q$: collinear factorization
- σ differential in $P_{h\perp}$, $P_{h\perp} \ll Q$: TMD factorization
important: matching of collinear and TMD factorization for intermediate $P_{h\perp}$
(Ji, Qiu, Vogelsang, Yuan, 2006)

- (18) Structure functions $F_i(x, Q^2, z, P_{h\perp})$ for semi-inclusive DIS
(Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, 2006)

$$\begin{aligned}
 d^6\sigma \propto & \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} \right. \\
 & + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin \phi_h F_{LU}^{\sin \phi_h} \\
 & + \Lambda \left[\sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_h F_{UL}^{\sin \phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 & + \Lambda \lambda_e \left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_h F_{LL}^{\cos \phi_h} \right] \\
 & + |\vec{S}_T| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\
 & + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
 & \left. + \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_S F_{UT}^{\sin \phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] \\
 & + |\vec{S}_T| \lambda_e \left[\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \\
 & \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \}
 \end{aligned}$$

- Leading twist TMDs in semi-inclusive DIS

$$\sigma_{UU} : \quad f_1 \otimes D_1 \quad \cos(2\phi_h) \, h_1^\perp \otimes H_1^\perp$$

$$\sigma_{LL} : \quad g_1 \otimes D_1$$

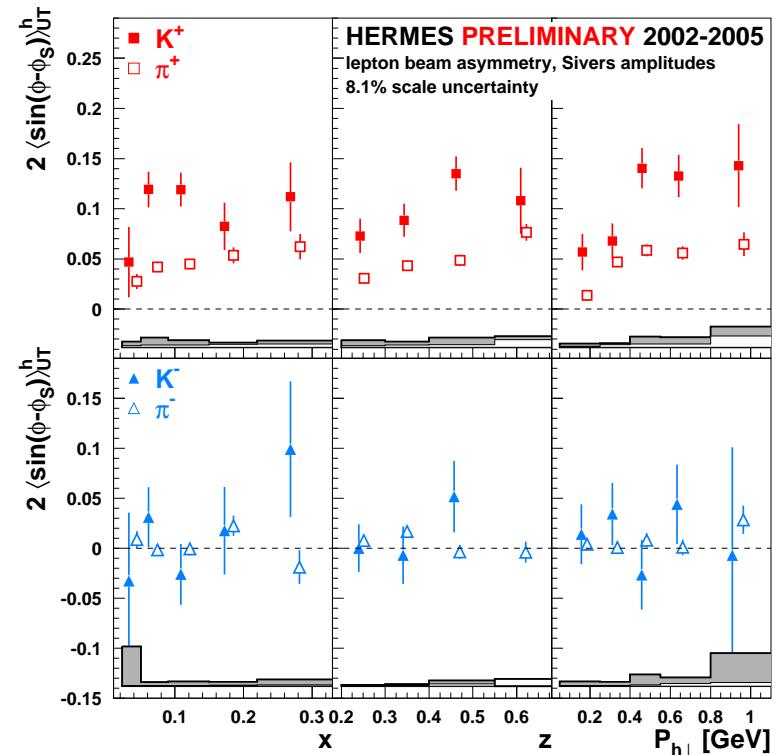
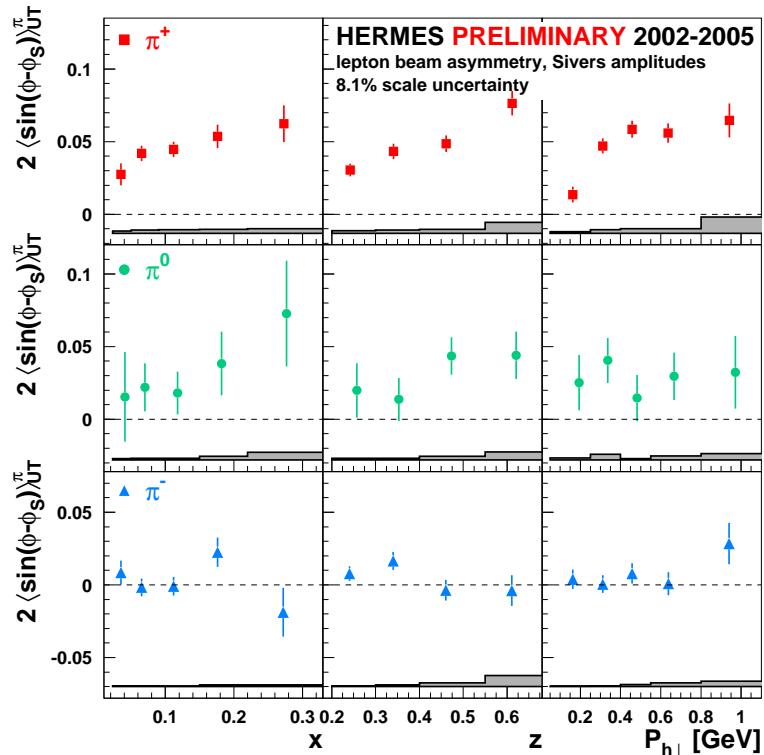
$$\sigma_{LT} : \quad \cos(\phi_h - \phi_S) \, g_{1T} \otimes D_1$$

$$\sigma_{UL} : \quad \sin(2\phi_h) \, h_{1L}^\perp \otimes H_1^\perp$$

$$\begin{aligned} \sigma_{UT} : \quad & \sin(\phi_h - \phi_S) \, f_{1T}^\perp \otimes D_1 & \sin(\phi_h + \phi_S) \, h_1 \otimes H_1^\perp \\ & \sin(3\phi_h - \phi_S) \, h_{1T}^\perp \otimes H_1^\perp \end{aligned}$$

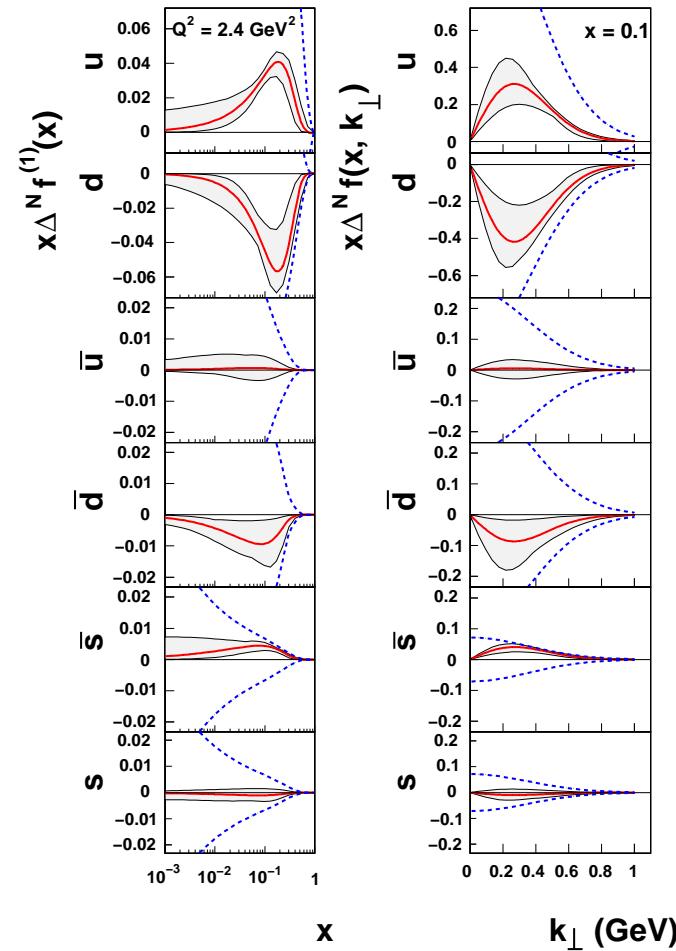
- H_1^\perp : Collins fragmentation function (Collins, 1992)
- complete experiment for TMDs possible
- various observables have been/will be studied at COMPASS, HERMES, JLab
- future Electron Ion Collider would be ideal for TMD-studies

- Sivers asymmetry for proton target (HERMES, 2007)



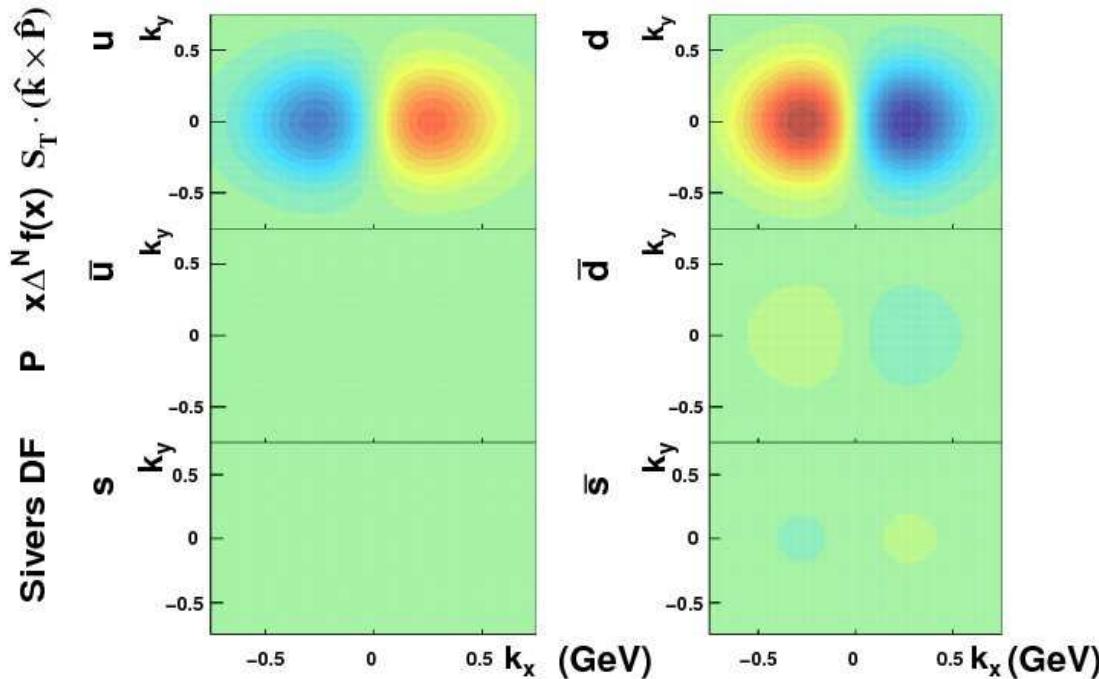
- related measurements/results from COMPASS and JLab

- Sivers function from data on semi-inclusive DIS (Anselmino et al, 2008)



- effects for u and d quark roughly equal in magnitude but opposite in sign, in agreement with large N_c -prediction (Pobylitsa, 2003)
- antiquark Sivers functions small, but effect for \bar{s} nonzero

- 3-D structure of the nucleon: dipole pattern due to Sivers effect $(x = 0.2)$



$$\frac{(\vec{S}_T \times \vec{k}_T) \cdot \hat{P}}{M} f_{1T}^{\perp q}(x, \vec{k}_T^2)$$

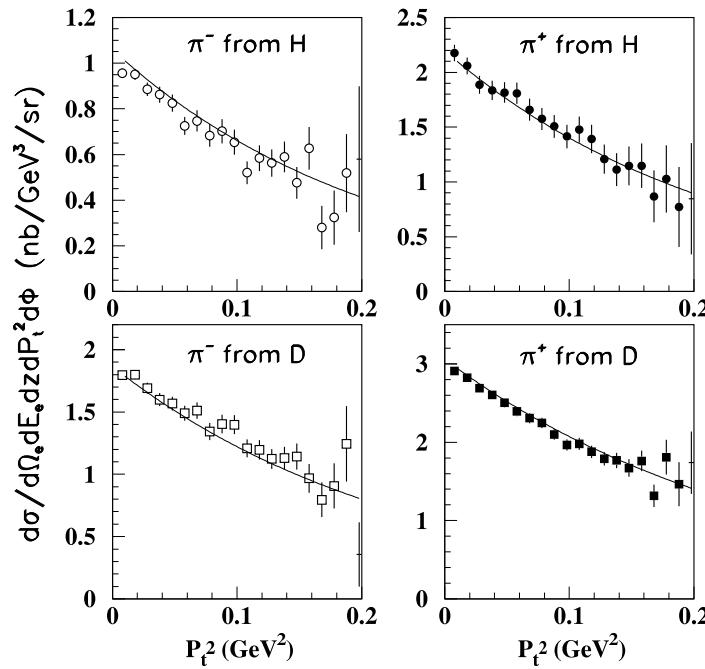
(Plot from Prokudin; red: positive effect, blue: negative effect)

- though model-dependent, plot generated from data !
- very fascinating aspect of TMD-field !

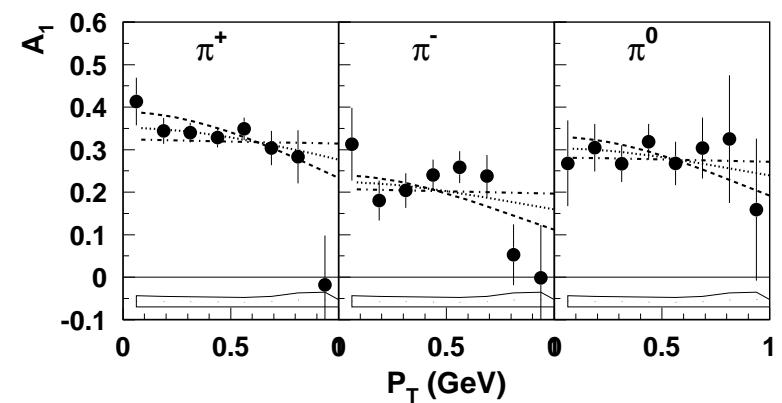
TMD studies at Jefferson Lab

- Some finalized projects at 6 GeV

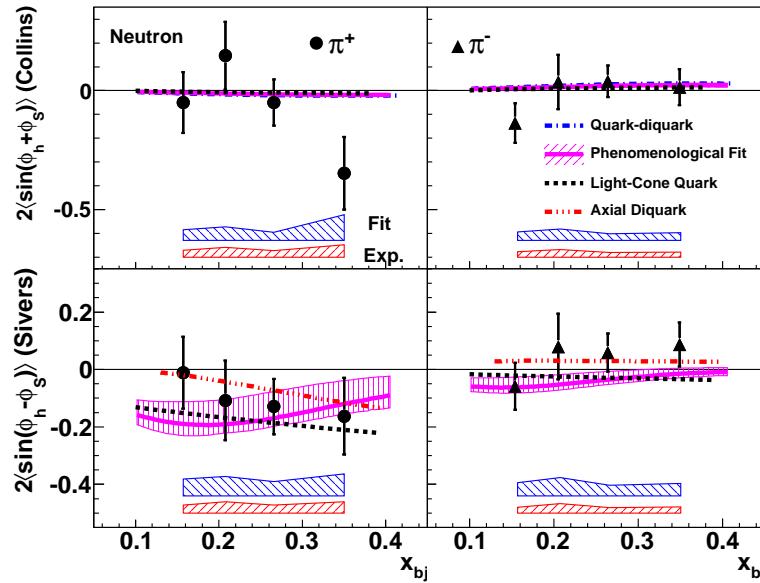
Hall C, 2007



Hall B, 2010



- measurement of $\sigma_{UU}(P_{h\perp})$
- information on k_T -dependence of $f_1^q(x, \vec{k}_T^2)$
- measurement of $\sigma_{LL}(P_{h\perp})$
- information on k_T -dependence of $g_1^q(x, \vec{k}_T^2)$
- data suggest different k_T -dependence for $f_1^q(x, \vec{k}_T^2)$ and $g_1^q(x, \vec{k}_T^2)$
- in agreement with lattice calculation (Musch et al, 2010)



- semi-inclusive DIS with transversely polarized 3He target
- Collins and Sivers effect for neutron target
- constraints on TMDs in region of larger x
- Extensive TMD program for JLab12
Many (approved) proposals / Gao, Gamberg et al, 2010

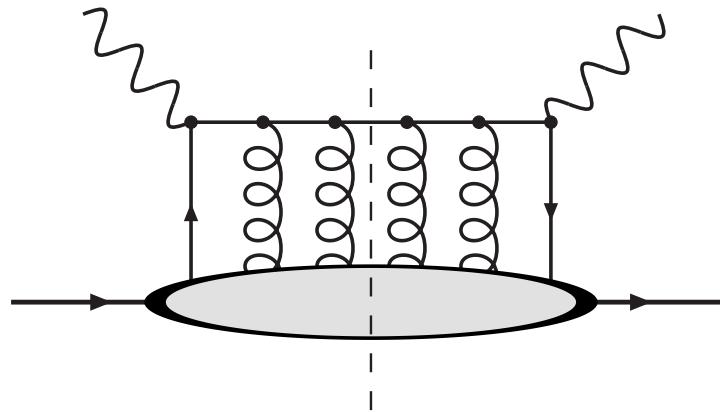
Color gauge invariance

- PDFs

$$\int d\xi^- e^{ik^+ \xi^-} \langle | \bar{\psi}(0) \Gamma \mathcal{W}_{PDF}(0; \xi^-) \psi(\xi^-) | \rangle$$

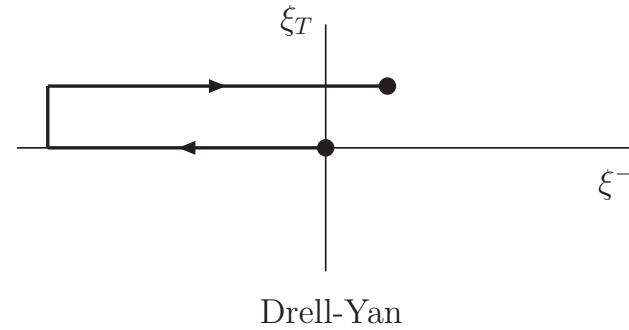
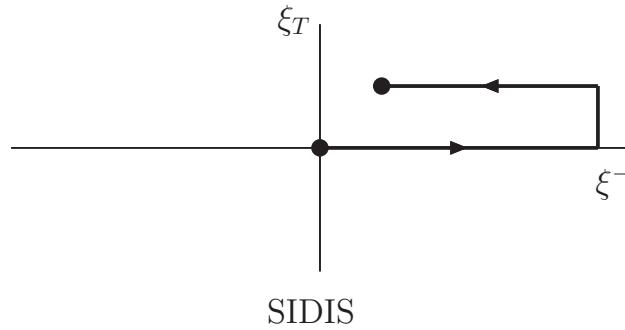
$$\mathcal{W}_{PDF}(0; \xi^-) = \mathcal{P} \exp \left(-ig \int_0^{\xi^-} d\eta^- A^+(0, \eta^-, \vec{0}_T) \right)$$

gauge-link generated by rescattering (FSI and/or ISI) (reduces to 1 for $A^+ = 0$)



- TMDs

$$\int d\xi^- d^2 \vec{\xi}_T e^{i(k^+ \xi^- - \vec{k}_T \cdot \vec{\xi}_T)} \langle | \bar{\psi}(0) \Gamma \mathcal{W}_{TMD}(0, \vec{0}_T; \xi^-, \vec{\xi}_T) \psi(\xi^-, \vec{\xi}_T) | \rangle$$



- $\mathcal{W}_{TMD}(0, \vec{0}_T; \xi^-, \vec{\xi}_T) = [0, \vec{0}_T; \infty, \vec{0}_T] \times [\infty, \vec{0}_T; \infty, \vec{\xi}_T] \times [\infty, \vec{\xi}_T; \xi^-, \vec{\xi}_T]$
(Belitsky, Ji, Yuan, 2002)
- T-odd TMDs (f_{1T}^\perp, h_1^\perp) nonzero only if Wilson-lines (final/initial state interactions) taken into account (unique for TMDs)
(Brodsky, Hwang, Schmidt, 2002 / Collins, 2002)
- different links for semi-inclusive DIS and Drell-Yan \rightarrow universality?
time-reversal: $f_{1T}^\perp|_{DY} = -f_{1T}^\perp|_{DIS}$ $h_1^\perp|_{DY} = -h_1^\perp|_{DIS}$ (unique for TMDs)
(Collins, 2002)

TMD factorization

- TMD factorization for

- $l N \rightarrow l' H X$
 - $e^+ e^- \rightarrow H_1 H_2 X$
 - $H_1 H_2 \rightarrow l^+ l^- X$

(Collins, Soper, 1981 / Collins, Soper, Sterman, 1985
Ji, Ma, Yuan, 2004 / Collins, Metz, 2004)

- Currently no TMD factorization beyond leading twist known

(Gamberg, Hwang, Metz, Schlegel, 2006 / Bacchetta, Boer, Diehl, Mulders, 2008)

- No TMD factorization for

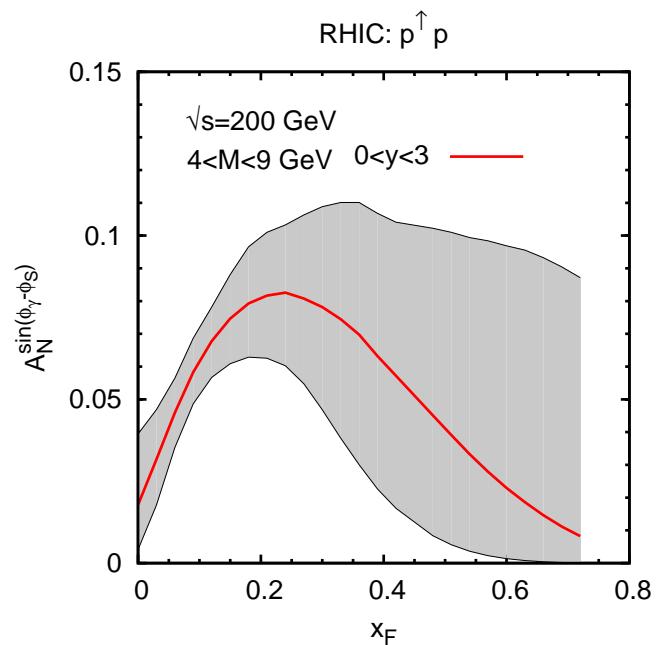
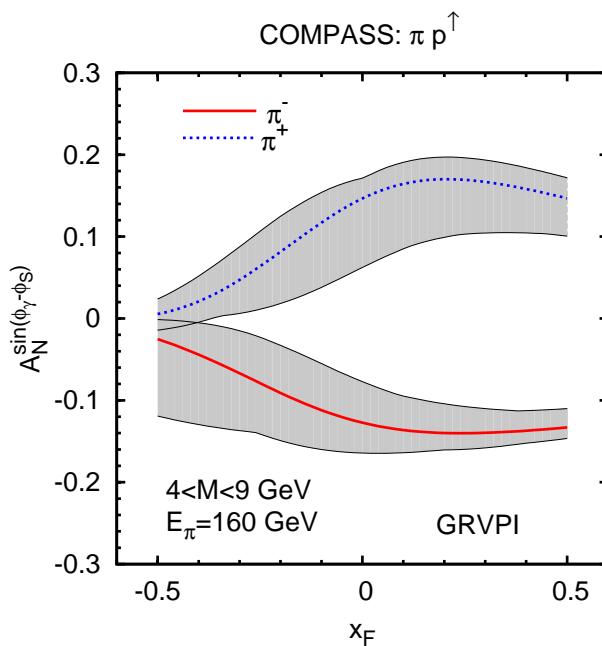
- $H_1 H_2 \rightarrow$ hadronic final states ('color entanglement')

(Rogers, Mulders, 2010 / etc.)

Sign reversal of the Sivers function

- Prospects for checking the sign reversal in Drell-Yan
 - potential labs: BNL, CERN, GSI, IHEP, JINR, J-PARC
 - first study: Efremov et al, 2004
 - more recent, comprehensive study: Anselmino et al, 2009

predictions for COMPASS and RHIC



→ promising prospects !

- What if sign reversal of f_{1T}^\perp is **not** confirmed by experiment ?
 - would not imply that QCD is wrong
 - would imply that SSAs not understood in QCD
 - problem with TMD-factorization
 - problem with resummation of large logarithms
 - resummation relevant if more than one scale present
 - CSS resummation in Drell-Yan (Collins, Soper, Sterman, 1985),
resum logarithms of the type
- has also implications for Fermilab and LHC physics
- might even imply problem with collinear factorization in hadronic collisions
(Collins, Drell-Yan workshop at BNL, May 2011)

$$\alpha_s^k \ln^{2k} \frac{q_T^2}{Q^2}$$

Moments of TMDs

- 3-parton qgq -correlator (twist-3)

$$\begin{aligned} \Phi_F^q(x, x') &\sim \int d\xi^- d\zeta^- e^{ik \cdot \xi} e^{i(k-k') \cdot \zeta} \\ &\times \langle P; S | \bar{\psi}^q(0) \Gamma F^{+i}(\zeta) \psi^q(\xi) | P; S \rangle \Big|_{\xi^+ = \xi_T = \zeta^+ = \zeta_T = 0} \end{aligned}$$

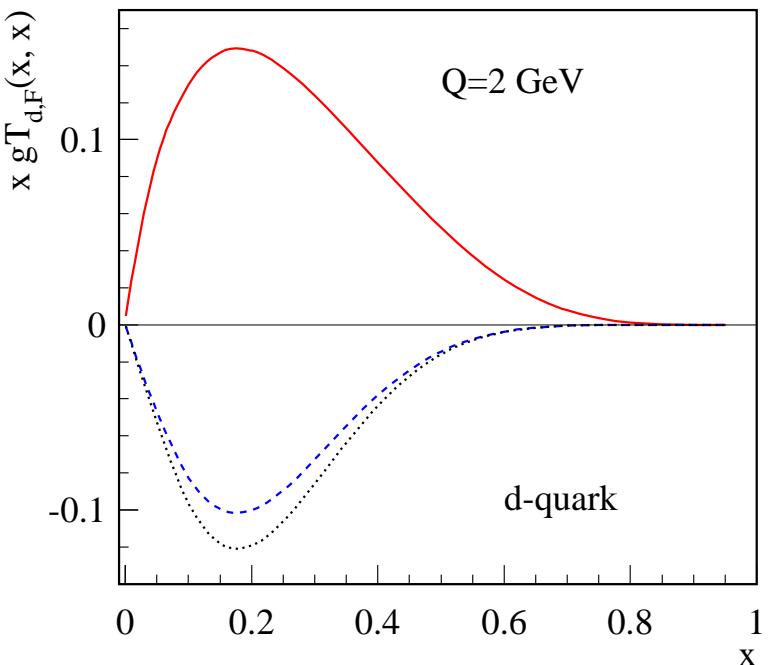
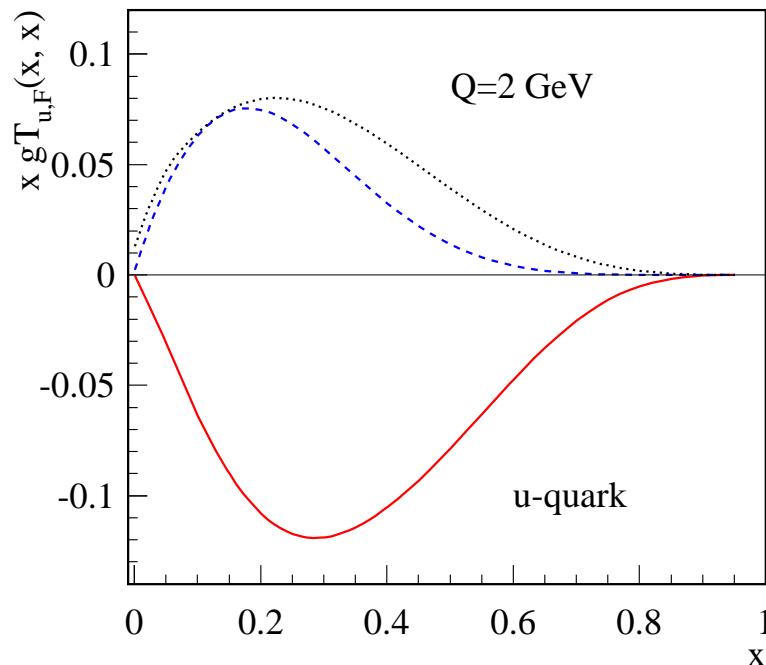
- 4 independent (leading) functions for $\Gamma = \{\gamma^+, \gamma^+ \gamma_5, i\sigma^{i+} \gamma_5\}$ (Jaffe, Ji, 1992)
- relevant for physics of DIS structure function g_2 (ongoing activity at JLab)
- Moments of TMDs (relevant for SSAs in, e.g., $pp^\uparrow \rightarrow \pi X$)
(Boer, Mulders, Pijlman, 2003 / Zhou, Yuan, Liang, 2008)

$$\begin{aligned} f_{1T}^{\perp(1)}(x) &\sim \int d^2 \vec{k}_T \vec{k}_T^2 f_{1T}^\perp(x, \vec{k}_T^2) \sim T_F(x, x) \\ h_1^{\perp(1)}(x) &\sim T_F^{(\sigma)}(x, x) \end{aligned}$$

- $T_F(x, x)$ represents ETQS matrix element
(Efremov, Teryaev, 1984 / Qiu, Sterman, 1991, ...)
- similarly for $g_{1T}^{(1)}$ and $h_{1L}^{\perp(1)}$

Sivers effect in $l p^\uparrow \rightarrow l H X$ and $p p^\uparrow \rightarrow H X$

- Comparison using $f_{1T}^{\perp(1)}(x) \sim T_F(x, x)$
 (Kang, Qiu, Vogelsang, Yuan, 2011)
 - f_{1T}^\perp from Anselmino et al 2005 /2008
 - T_F from Kouvaris, Qiu, Vogelsang, Yuan, 2006



- striking sign mismatch !
- Does f_{1T}^\perp have a node as function of k_T and/or as function of x ?
 (Boer, 2011 / Kang, Prokudin, 2011)

- Does a major part of the SSA in $p p^\dagger \rightarrow H X$ come from fragmentation side ?
 - seems unlikely (Kang, Yuan, Zhou, 2010)
 - measuring $p p^\dagger \rightarrow \text{jet } X$ would be very helpful !
- Do SSAs in both processes have completely different origin ?
- Alternative observables for studying relevant physics
 - transverse SSA in hadronic W-production: $p p^\dagger \rightarrow l \nu_l X$
 Brodsky, Hwang, Schmidt, 2002 / Schmidt, Soffer, 2003 /
 Kang, Qiu, 2009 / Metz, Zhou, 2010
 - correlation between hadron spin and orientation of lepton plane in Drell-Yan
 Hammon, Teryaev, Schäfer, 1996 / Boer, Mulders, Teryaev, 1997 /
 Boer, Mulders, 1999 / Boer, Qiu, 2001 / Ma, Wang, 2003 /
 Anikin, Teryaev, 2010 / Zhou, Metz, 2010

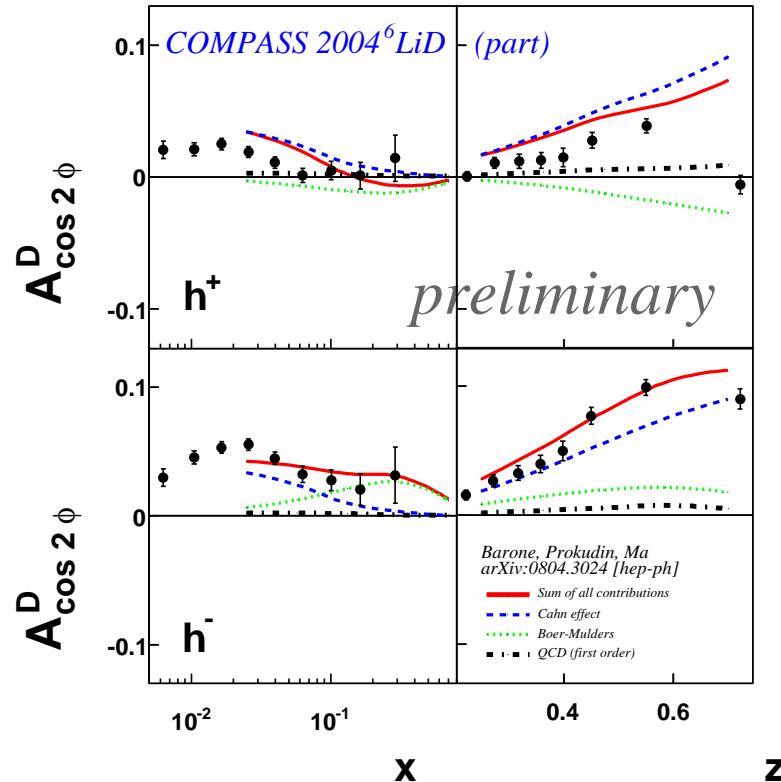
TMDs at a future Electron Ion Collider

See also: Anselmino et al, 2011, writeup of Duke workshop /
Boer, Diehl, Milner, Venugopalan, Vogelsang et al, to appear, writeup of INT program

- Main features
 - major benefit: higher energy and luminosity
 - multidimensional binning of observables
 - quantitative measurement of quark TMDs (suppression of higher twist)
 - first access to antiquark TMDs and (polarized) gluon TMDs
 - first serious study of moments of TMDs
 - study of saturation physics for TMDs (CGC-approach) (→ talk by Mueller)
 - systematic study of pQCD techniques for (polarized) SIDIS observables
(resummation/evolution)
 - etc

EIC would bring TMD field to new level

- Need for suppression of higher twist: $\cos(2\phi_h)$ modulation of SIDIS cross section
(COMPASS, 2009)



- contributions from
 - $\frac{P_{h\perp}^2}{M^2} h_1^\perp \otimes H_1^\perp$ (Boer, Mulders, 1997)
 - $\frac{P_{h\perp}^2}{Q^2} f_1 \otimes D_1$ kinematical higher twist effect (Cahn, 1978)
 - higher order collinear pQCD (plus resummation)

- Gluon TMDs
 - unpolarized gluon TMD $f_1^g(x, \vec{k}_T^2)$
 - * plays very important role for small x physics
 - * $f_1^g(x, \vec{k}_T^2)$ for large nucleus may be studied
 - linearly polarized gluons in unpolarized target, $h_1^{\perp g}(x, \vec{k}_T^2)$
 - * may be measured through production of heavy quark pair or of dijets
(Boer, Brodsky, Mulders, Pisano, 2010)
- The figure contains two Feynman diagrams. The left diagram shows a quark-gluon vertex where a quark line enters and a gluon line labeled $h_1^{\perp g}$ exits, interacting with a quark loop. The right diagram shows a similar vertex where a quark line enters and a gluon line labeled f_1^g exits, also interacting with a quark loop. Both diagrams include labels ±1 near the vertices.
- * may be very large in the small x saturation region
(Metz, Zhou, 2011)
 - so far, hardly anything known about gluon TMDs for polarized target

Summary

- Enormous progress in TMD field in last decade
- TMDs can be measured in, e.g., semi-inclusive DIS and Drell-Yan
- TMDs contain information about 3-D parton structure of the nucleon (in momentum space)
- (Some) TMDs intimately related to ISI/FSI, and underlying $SU(3)_c$ gauge symmetry of QCD
- Experimental check of sign reversal of Sivers effect is important issue in current hadronic physics
- More information on TMDs from existing facilities, JLab12, and future polarized Drell-Yan experiments
- Future Electron Ion Collider would be ideal for studying TMDs, and related physics through semi-inclusive processes in DIS