Excited baryon spectrum using Lattice QCD

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JLab Users Group Meeting 2011

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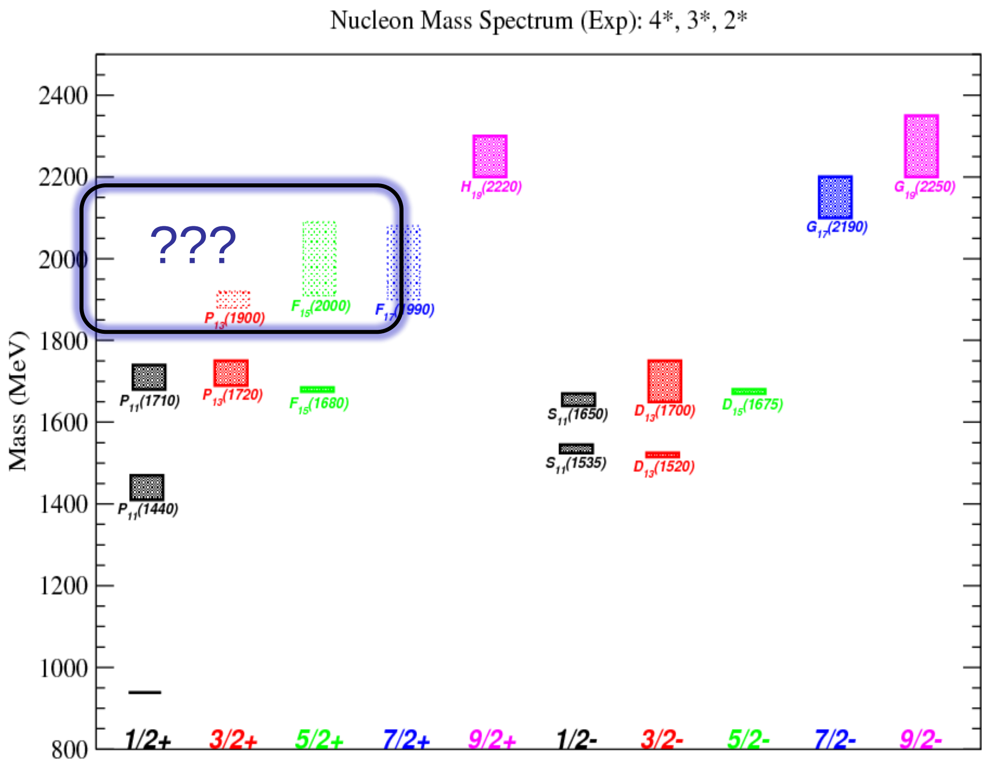
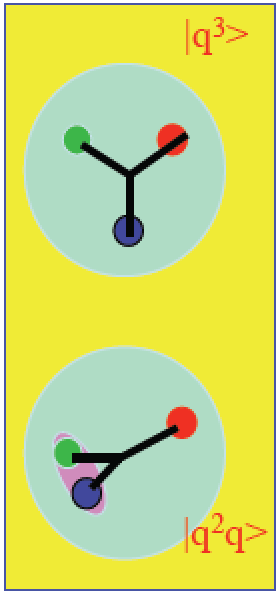
Auspices of the Hadron Spectrum Collaboration

Baryon Spectrum

Where are the "Missing Resonances"?

- What are collective modes?
- Is there "freezing" of degrees of freedom?
- What is the structure of the states?





Nucleon spectrum

PDG uncertainty on B-W mass

Spectrum from variational method

Two-point correlator

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0|\Phi_i(0)|\mathfrak{n}\rangle\langle\mathfrak{n}|\Phi_j^\dagger(0)|0\rangle$$

$$Z_i^{\mathfrak{n}} \equiv \langle\mathfrak{n}|\, \Phi_i^\dagger \,|0\rangle$$

Matrix of correlators

$$C(t) = \begin{pmatrix} \langle 0|\Phi_1(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \langle 0|\Phi_2(t)\Phi_1^\dagger(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2^\dagger(0)|0\rangle & \cdots \\ \vdots & & \ddots \end{pmatrix}$$

"Rayleigh-Ritz method"
Diagonalize:
- eigenvalues $\rightarrow$ spectrum
- eigenvectors $\rightarrow$ spectral "overlaps" Z_i^n

Each state optimal combination of Φ_i

$$\Omega^{(\mathfrak{n})} = \sum_i v_i^{(\mathfrak{n})} \Phi_i$$

Benefit: orthogonality for near degenerate states

Baryon operators

Construction : permutations of 3 objects

- Symmetric:
  - •e.g., uud+udu+duu
- Antisymmetric:
  - •e.g., uud-udu+duu-…
- Mixed: (antisymmetric & symmetric)
  - •e.g., udu - duu & 2duu - udu - uud

Multiplication rules:
- Symmetric Antisymmetric → Antisymmetric
- Mixed Mixed → Symmetric⊕Antisymmetric⊕Mixed
- ….

Color antisymmetric → Require **Space** [**Flavor** **Spin**] symmetric

Space: couple covariant derivatives onto single-site spinors - build any J,M

$$\Phi^{JM} \leftarrow (CGC's)_{i,j,k} \left[\vec{D}\right]_i \left[\vec{D}\right]_j [\Psi]_k$$

$$J \leftarrow 1 \quad 1 \quad S$$

Classify operators by permutation symmetries:
- Leads to rich structure

1104.5152

Baryon operator basis

3-quark operators & up to two covariant derivatives: some J^P

$$\left(\left[\text{Flavor} \quad \text{Dirac} \right] \quad \text{Space}_{\text{symmetry}} \right)^{J^P}$$

Spatial symmetry classification:

e.g., Nucleons: $N\ {}^{2S+1}L_{\pi}\ J^P$

J^P	#ops	E.g., spatial symmetries	
$J=1/2^-$	24	$N\ {}^2P_M\ \frac{1}{2}^-$	$N\ {}^4P_M\ \frac{1}{2}^-$
$J=3/2^-$	28	$N\ {}^2P_M\ 3/2^-$	$N\ {}^4P_M\ 3/2^-$
$J=5/2^-$	16	$N\ {}^4P_M\ 5/2^-$	
$J=1/2^+$	24	$N\ {}^2S_S\ \frac{1}{2}^+$ $N\ {}^2S_M\ \frac{1}{2}^+$	$N\ {}^4D_M\frac{1}{2}^+$ $N\ {}^2P_A\ \frac{1}{2}^+$
$J=3/2^+$	28	$N\ {}^2D_S 3/2^+$ $N\ {}^2D_M 3/2^+$ $N\ {}^2P_A\ 3/2^+$	$N\ {}^4S_M 3/2^+$ $N\ {}^4D_M 3/2^+$
$J=5/2^+$	16	$N\ {}^2D_S 5/2^+$ $N\ {}^2D_M 5/2^+$	$N\ {}^4D_M 5/2^+$
$J=7/2^+$	4	$N\ {}^4D_M 7/2^+$	

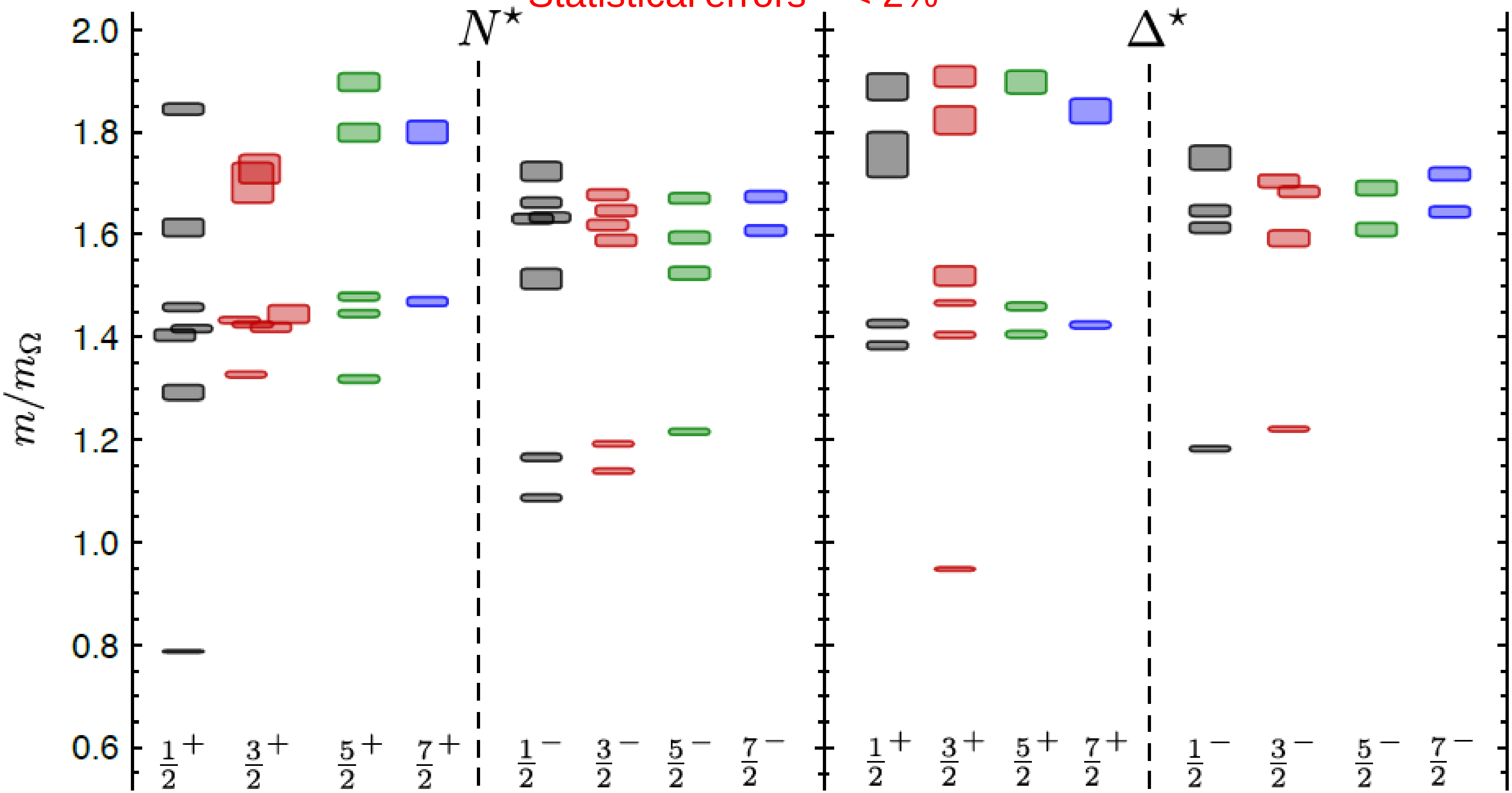
By far the largest operator basis ever used for such calculations

Spin identified Nucleon & Delta spectrum

$m_\pi \sim 520$MeV

arXiv:1104.5152

Statistical errors < 2%



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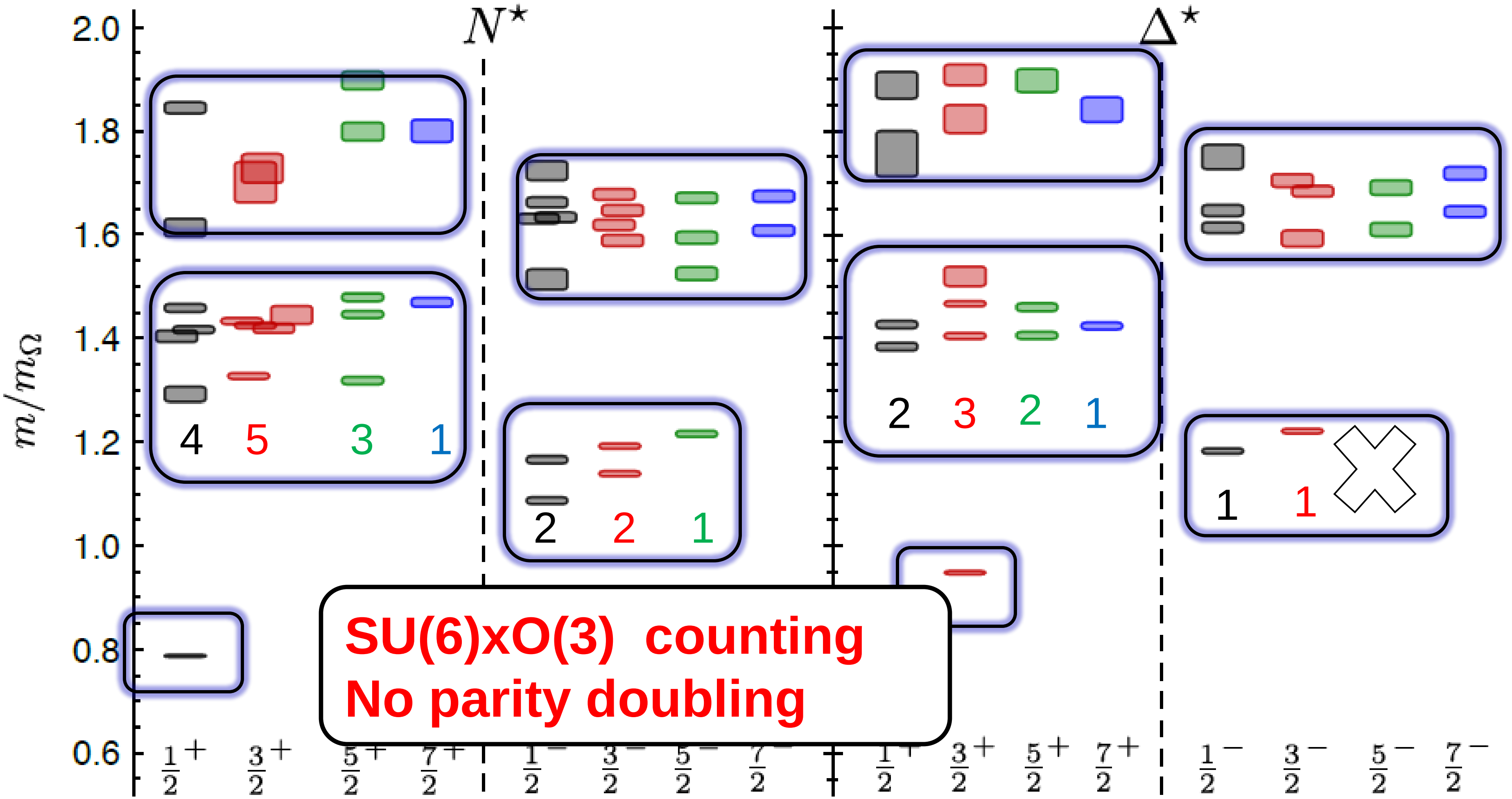


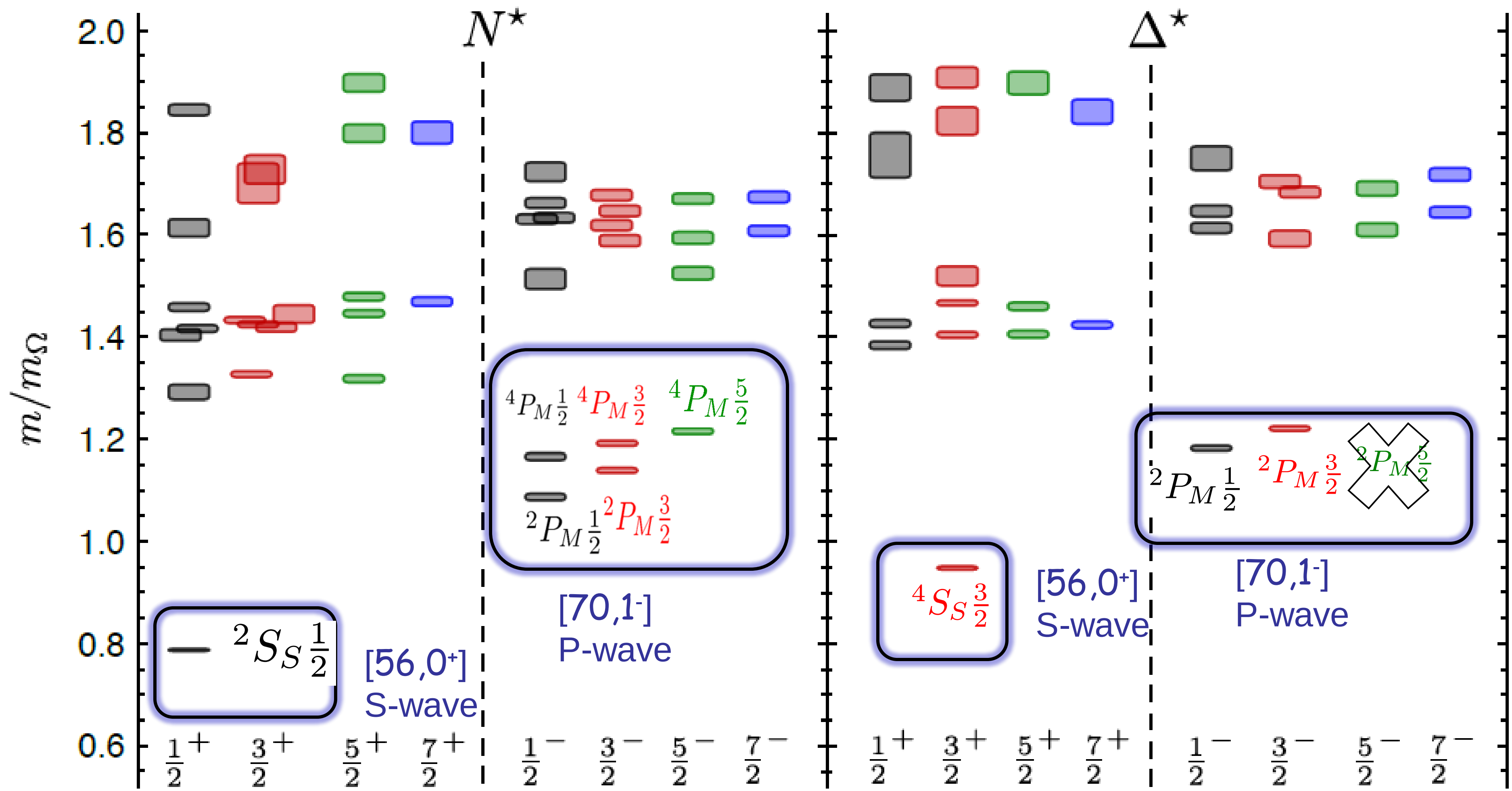
SU(6)xO(3) counting
No parity doubling

Spin identified Nucleon & Delta spectrum

Discern structure: spectral overlaps

arXiv:1104.5152

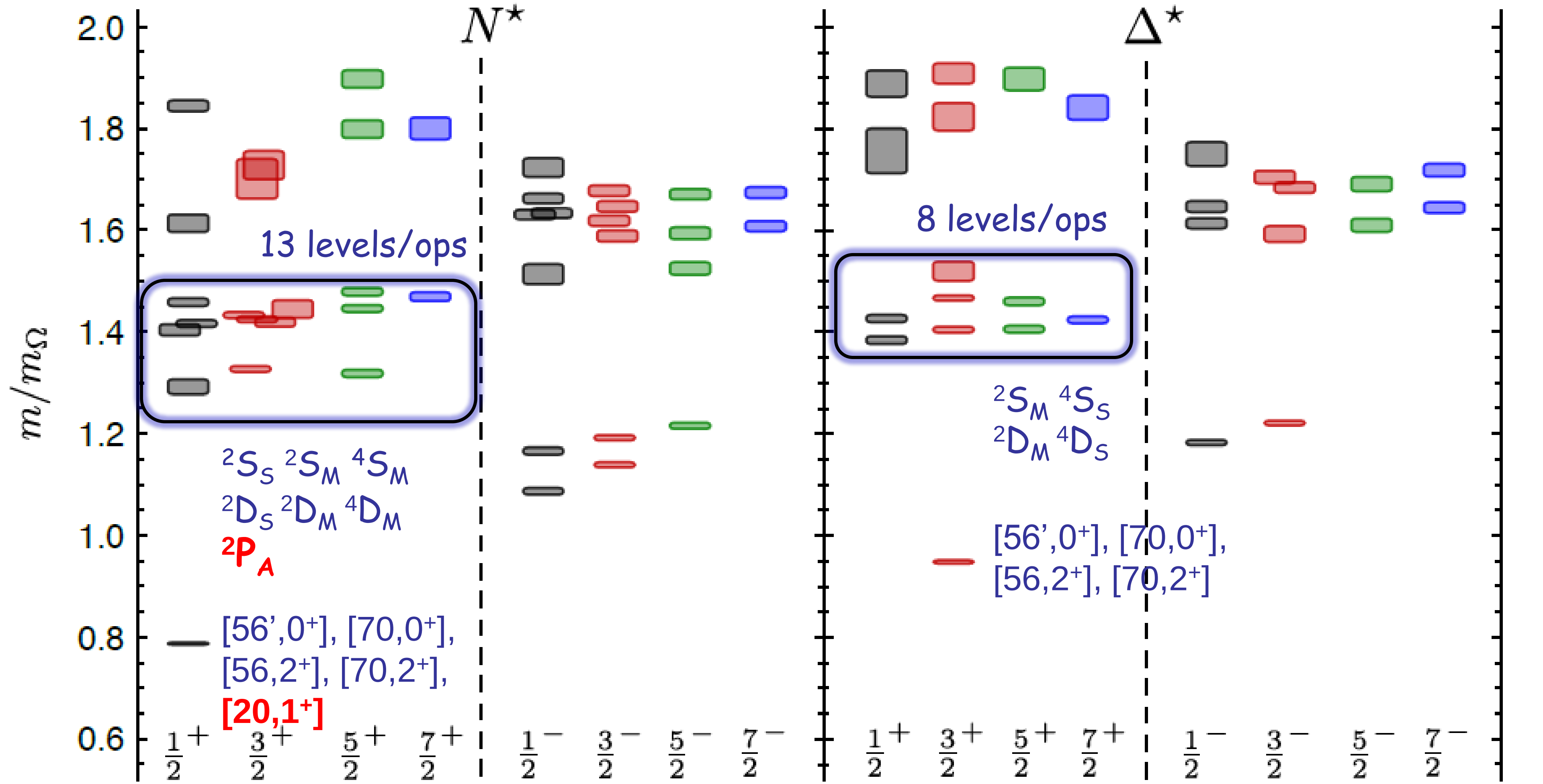
$m_\pi \sim 520$MeV



N=2 J^+ Nucleon & Delta spectrum

Discern structure: spectral overlaps

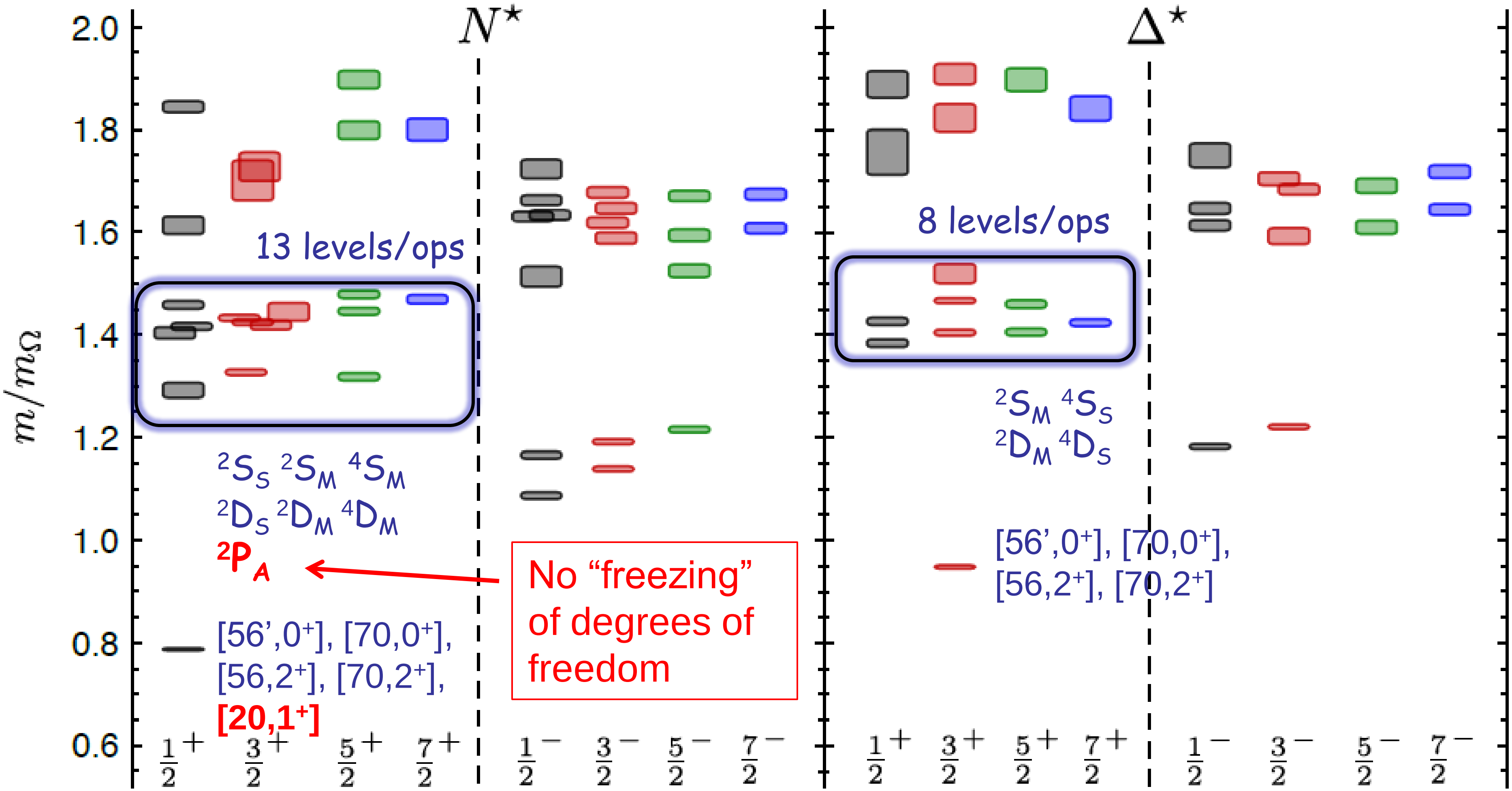
Significant mixing in J^+



N=2 J⁺ Nucleon & Delta spectrum

Discern structure: spectral overlaps

Significant mixing in J⁺



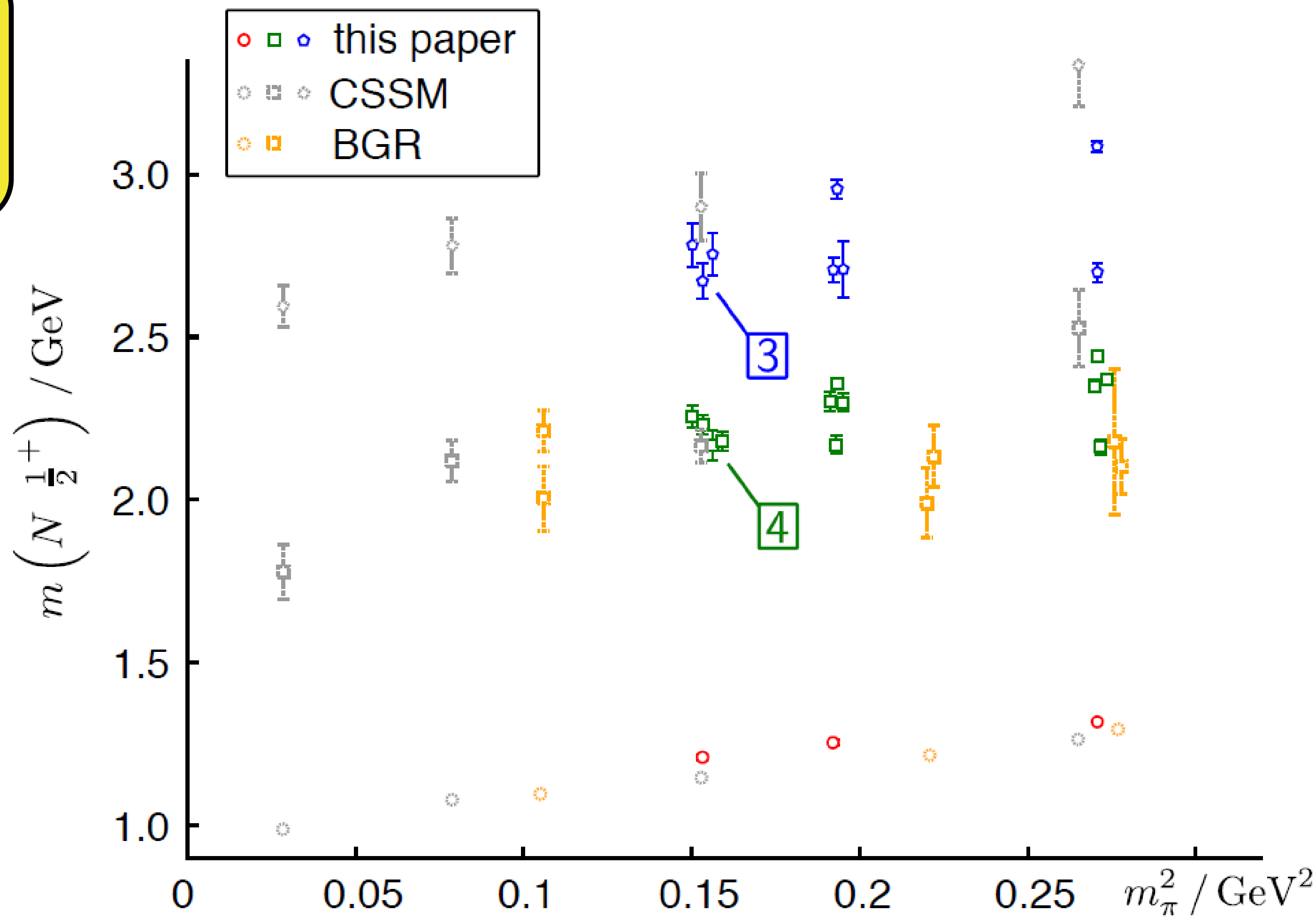
Roper??

Near degeneracy in $\frac{1}{2}^{+}$ consistent with SU(6) O(3) counting, but heavily mixed

Discrepancies??

Operator basis – spatial structure

What else?
Multi-particle operators



Spectrum of finite volume field

The idea: 1 dim quantum mechanics

Two spin-less bosons: ψ(x,y) = f(x-y) $\to$ f(z)

$$\left[-\frac{1}{m}\frac{d^2}{dz^2} + V(z)\right] f(z) = E\, f(z)$$

Solutions

$$f(z) \to \cos\left[k|z| + \delta(k)\right], \qquad E = k^2/m$$

Quantization condition when -L/2 < z < L/2

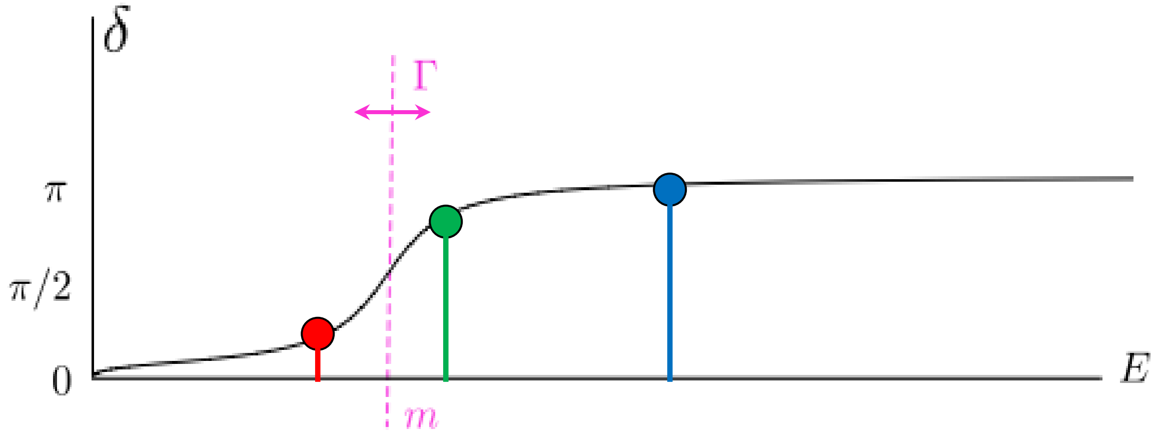
$$kL + 2\delta(k) = 0 \quad \mathrm{mod}\ 2\pi$$

Same physics in 4 dim version (but messier)
Provable in a QFT

Finite volume scattering

Scattering in a periodic cubic box (length **L**)
• Discrete energy levels in finite volume

E.g. just a single elastic resonance



e.g.

$$\pi\pi \to \rho \to \pi\pi$$

$$\pi N \to \Delta \to \pi N$$

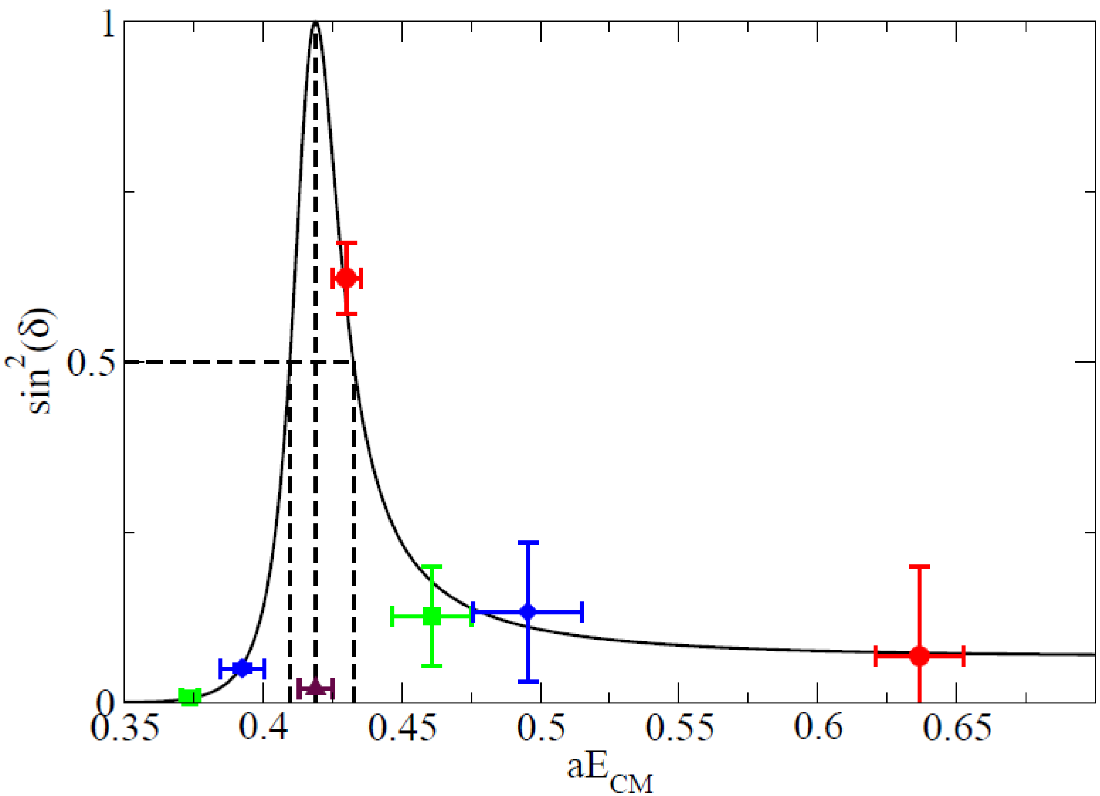
At some **L ,** have discrete excited energies

$$E \to k; \quad kL + 2\delta(k) = 0 \mod 2\pi$$

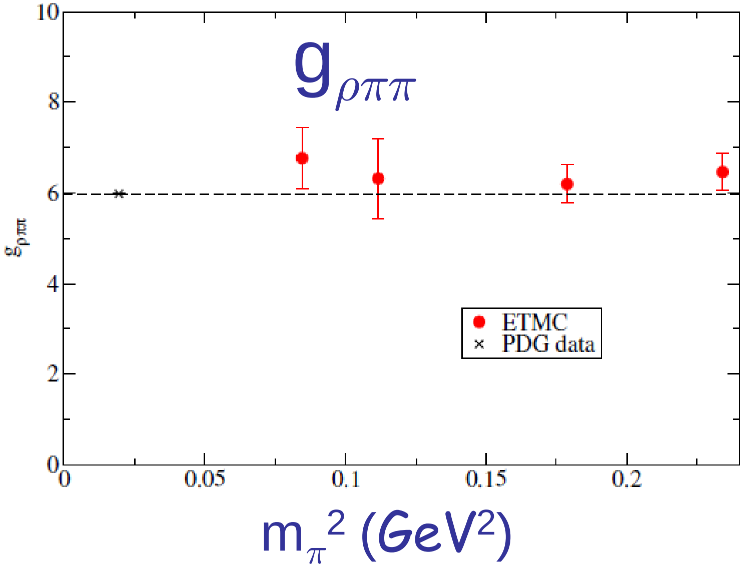
•T-matrix amplitudes $\to$ partial waves
• Finite volume energy levels **E(L)** $\leftrightarrow$ **δ(E)**

I=1 $\pi\pi$: the "ρ"

Extract $\delta_1(E)$ at discrete E



Extracted coupling: stable in pion mass



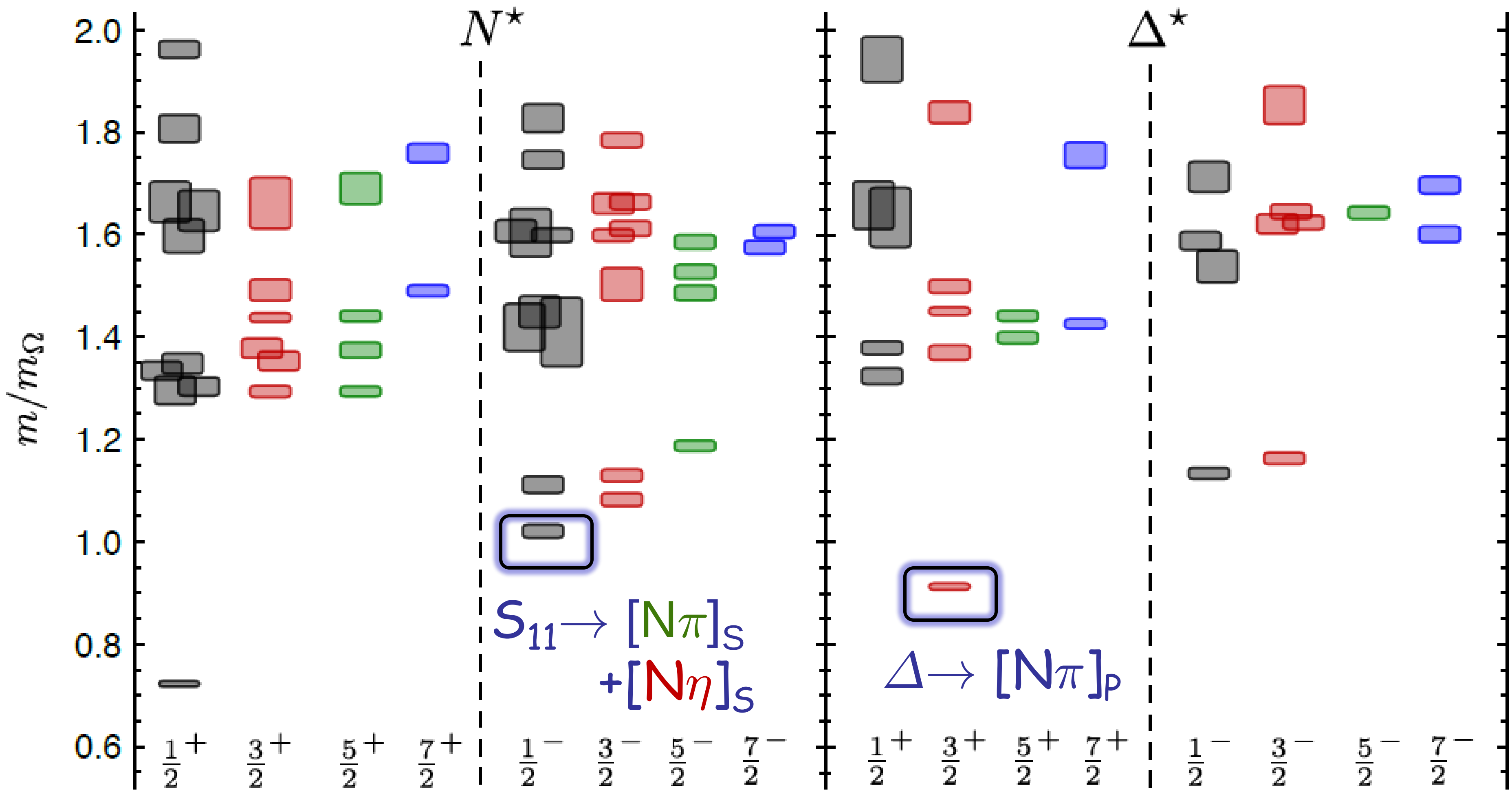
Stability a generic feature of couplings??

Feng, Jansen, Renner, 1011.5288

Hadronic Decays

Some candidates: determine phase shift
Somewhat elastic

$m_\pi \sim 400$ MeV



Summary & prospects

Results for baryon excited state spectrum:
- No "freezing" of degrees of freedom nor parity doubling
- Broadly consistent with non-relativistic quark model
- Add multi-particles $\rightarrow$ baryon spectrum becomes denser

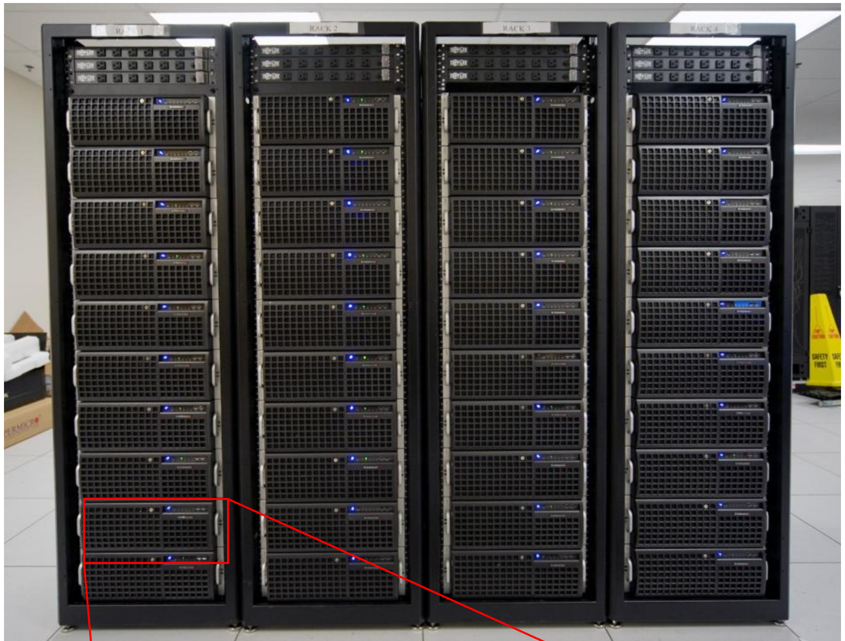
Short-term plans: resonance determination!
- Lighter pion masses (230MeV available)
- Extract couplings in multi-channel systems (with π, η, K…)

T-matrix "poles" from Euclidean space?

Yes! [with caveat]
- Also complicated
- But all Minkowski information is there

Optimistic: see confluence of methods (an "amplitude analysis")
- Develop techniques concurrently with decreasing pion mass

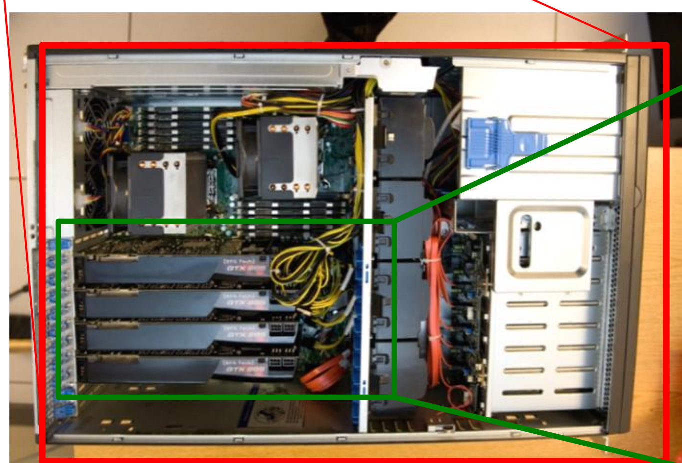
Game changer: Graphic cards (GPU-s)



JLab ARRA GPU cluster: ~ 530 GPUs

~100 Teraflop/s sustained performance

~ All available national resources for LQCD



DOE FY10 Nuclear Theory Accomplishments

Department of Energy

FY 2012 Congressional Budget Request

Nuclear Theory

Selected FY 2010 Accomplishments

Spectrum
- LQCD calculations continue to bring exciting physics results and computational advances. The Hadron Spectrum Collaboration provided new physics insight into how quarks are bound in mesons and hadrons through the calculation of the masses of states with exotic quantum numbers from dynamic LQCD. The quantum numbers of exotic meson states cannot be constructed from the conventional excitations of a bound quark-anti-quark pair, and the existence of these states may signal the explicit influence of the gluons that bind quarks together.

GPU-s
- In a recent computational advance, attained in the framework of the national USQCD collaboration and the SciDAC funded software infrastructure project for Lattice Field Theory, parallelization onto multiple graphical processing units enabled the analysis of large space-time lattices at a performance of over 3 teraflops, an order of magnitude increase in speed in critically important calculations. This was made possible by a project funded under the Recovery Act.

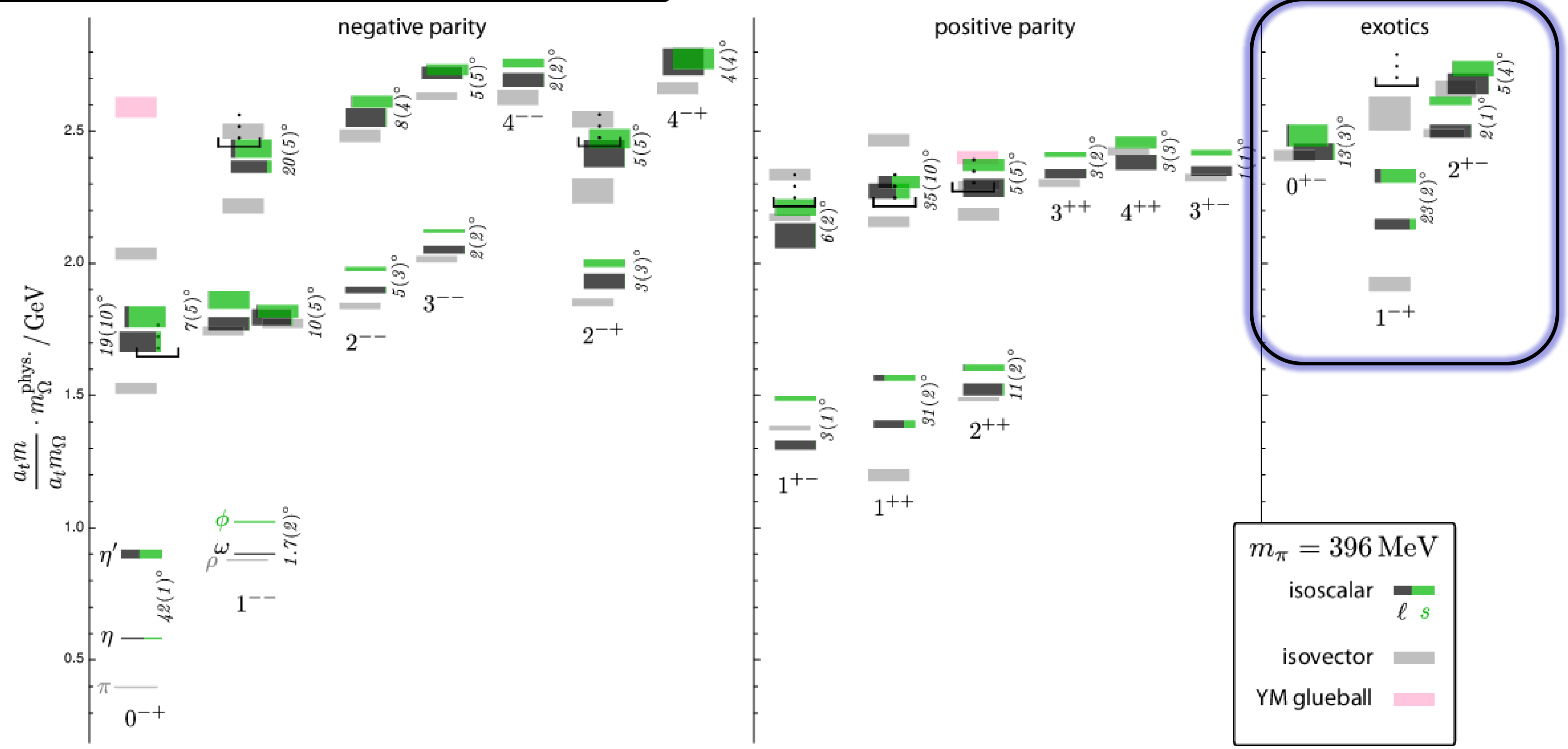
Backup slides

- The end

Isoscalar & isovector meson spectrum

Isoscalars: flavor mixing determined

Exotics



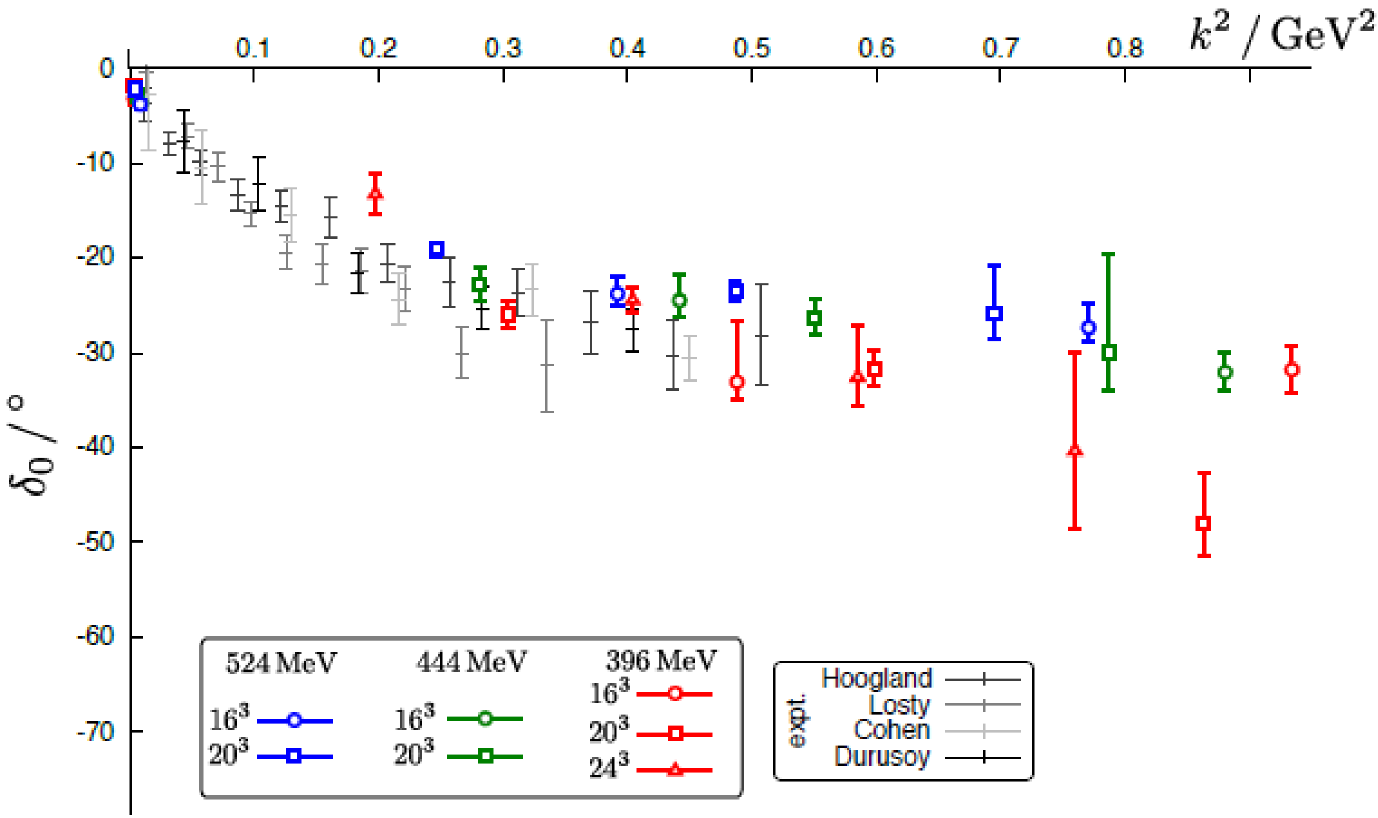
Will need to build PWA within mesons

1102.4299

Phase Shifts demonstration: I=2 $\pi\pi$

$\pi\pi$ isospin=2

Extract $\delta_0(E)$ at discrete E



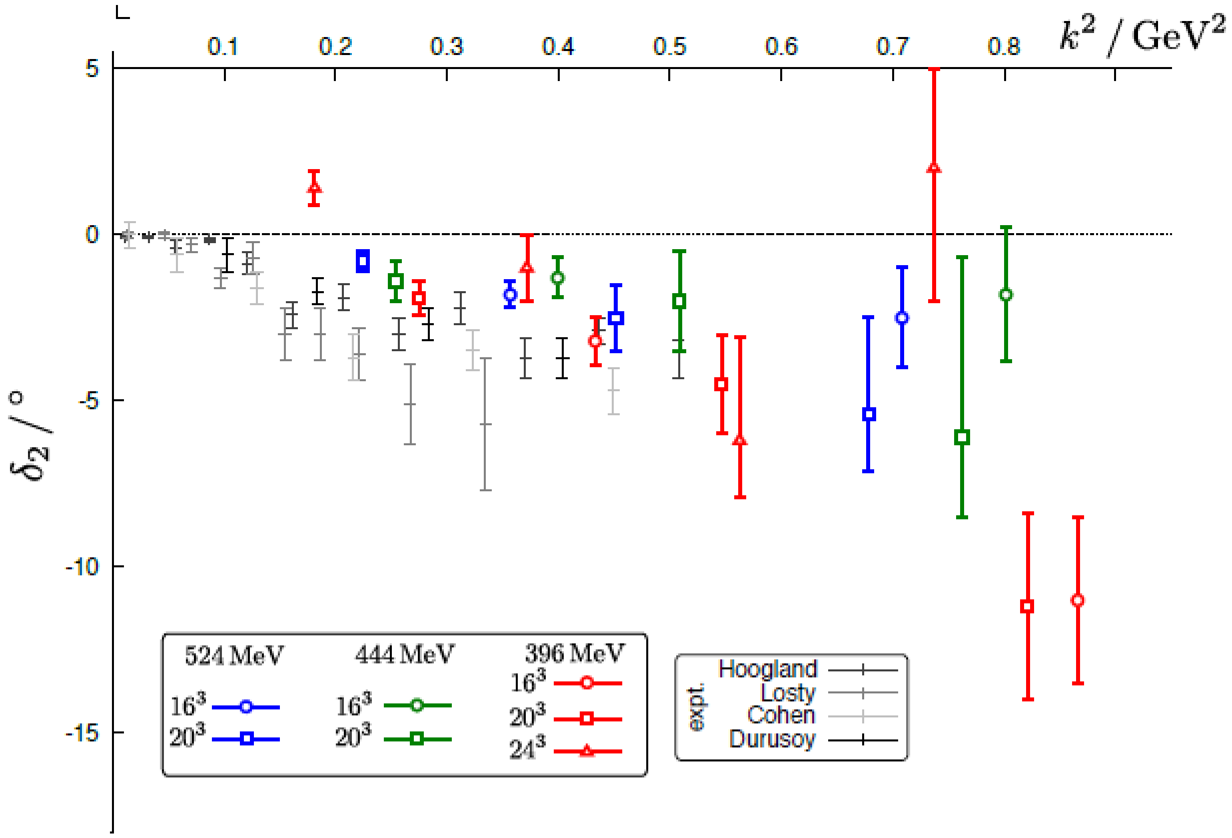
No discernible pion mass dependence

1011.6352 (PRD)

Phase Shifts: demonstration

$\pi\pi$ isospin=2

$\delta_2(E)$



Operators are not states

Two-point correlator

$$C(t) = \langle 0 | \Phi'(t)\, \Phi(0) | 0 \rangle$$

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \Phi(0) | 0 \rangle$$

Full basis of operators: many operators can create same state

Spectral "overlaps"

$$\langle \mathfrak{n};\, J^P \mid \Phi_i \mid 0 \rangle \;=\; Z_i^{\mathfrak{n}}$$

States may have subset of allowed symmetries

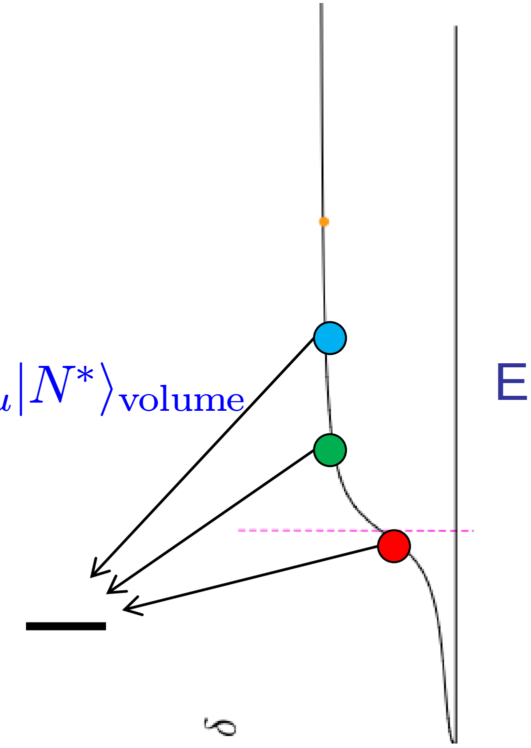
Form Factors

What is a form-factor off of a resonance?

What is a resonance? Spectrum first!

Extension of scattering techniques:
- Finite volume matrix element modified

$$\langle N|J_\mu|N^*\rangle_\infty(Q^2,E) \leftarrow \underbrace{[\delta'(E)}_{\text{Phase shift}} + \underbrace{\Phi'(E)]}_{\text{Kinematic factor}} \langle N|J_\mu|N^*\rangle_{\text{volume}}$$



Requires excited level transition FF's: some experience
- Charmonium E&M transition FF's (1004.4930)
- Nucleon 1st attempt: "Roper"->N (0803.3020)

Range: few GeV^2
Limitation: spatial lattice spacing

Nucleon J^-

Overlaps

$$Z_i^{\mathfrak{n}} = \langle J^- \mid \Phi_i \mid 0 \rangle$$

Little mixing in each J^-

Nearly “pure” [S= 1/2 & 3/2] 1^-