

## Proton charge radius

Michael O. Distler for the A1 collaboration @  
MAMI

Institut für Kernphysik  
Johannes Gutenberg-Universität Mainz



JOHANNES GUTENBERG  
UNIVERSITÄT MAINZ

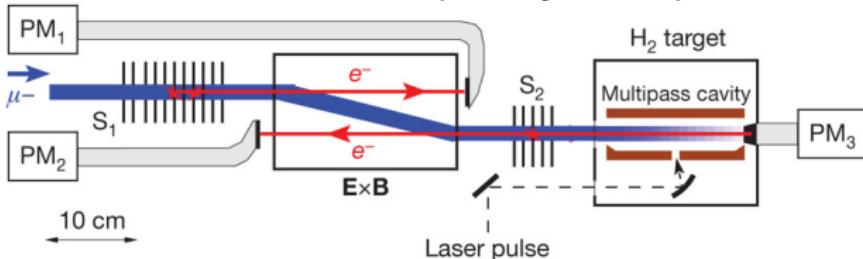
# Outline

- ① Introduction I: The size of the proton from the Lamb shift in muonic hydrogen and electron scattering
- ② Introduction II: Electric and magnetic form factors of the Proton
- ③ The Mainz high-precision  $p(e,e')p$  measurement
  - Design considerations
  - Covered kinematical region
- ④ Results
  - Analysis technique
  - Cross section results
  - Checks: Rosenbluth and model dependence
- ⑤ Conclusion and Outlook
- ⑥ Discussion of the Lamb shift / electron scattering discrepancy

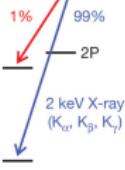
# Introduction I: The size of the proton



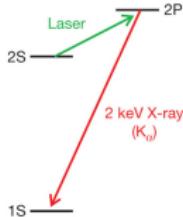
Nature 466, 213-216 (8 July 2010)



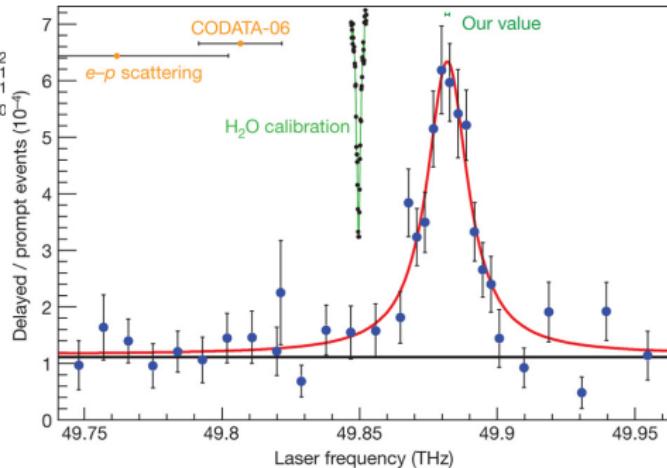
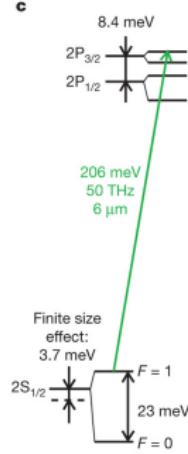
a  $n = 14$



b



c



# Cross section and form factors for elastic e-p scattering

The cross section:

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \frac{1}{\varepsilon(1+\tau)} \left[ \varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2) \right]$$

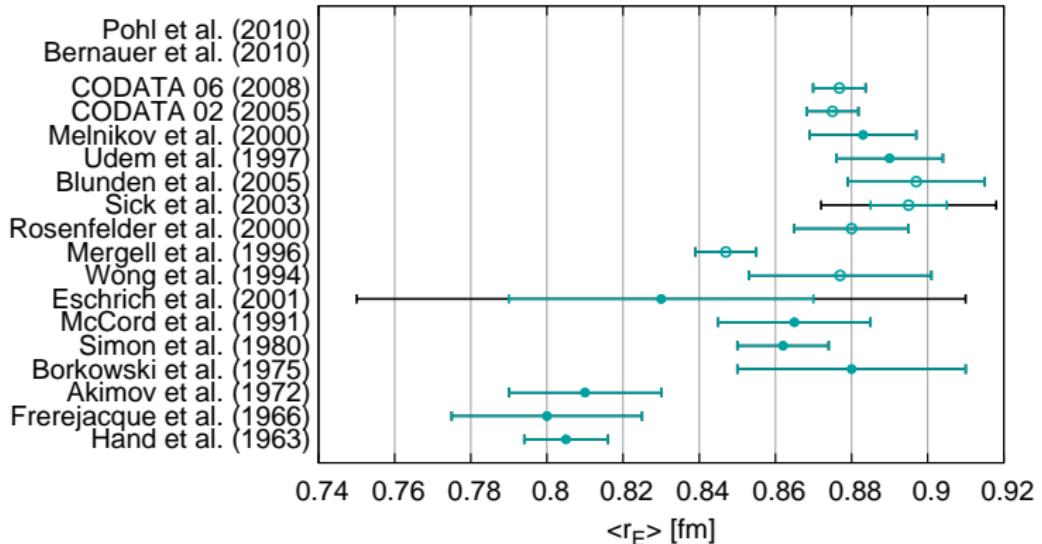
with:

$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left( 1 + 2(1+\tau) \tan^2 \frac{\theta_e}{2} \right)^{-1}$$

Fourier-transform of  $G_E$ ,  $G_M \rightarrow$  spatial distribution (Breit frame)

$$\langle r_E^2 \rangle = -6\hbar^2 \left. \frac{dG_E}{dQ^2} \right|_{Q^2=0} \quad \langle r_M^2 \rangle = -6\hbar^2 \left. \frac{d(G_M/\mu_p)}{dQ^2} \right|_{Q^2=0}$$

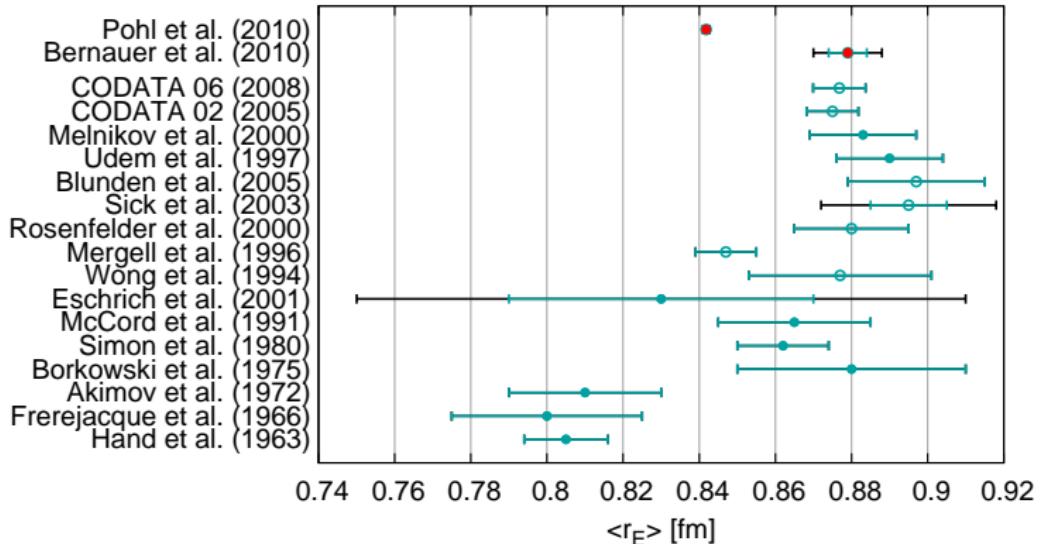
# Overview of different proton charge-radius results



Filled dots: Results from new measurements.

Hollow dots: Reanalysis of existing data.

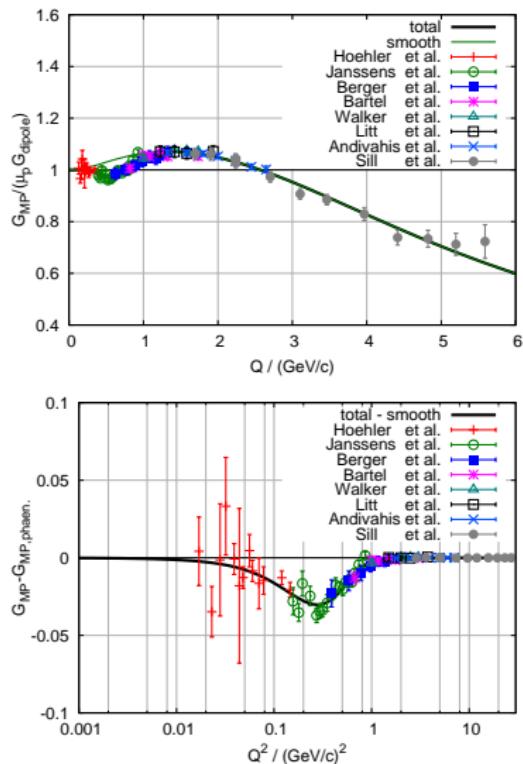
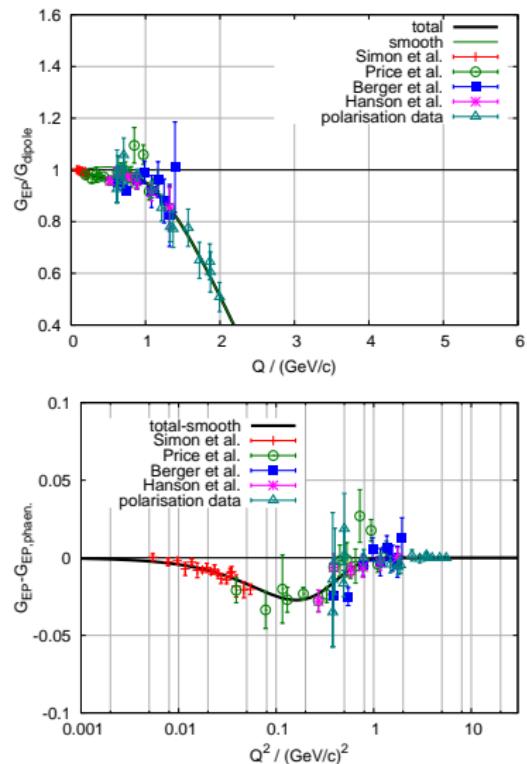
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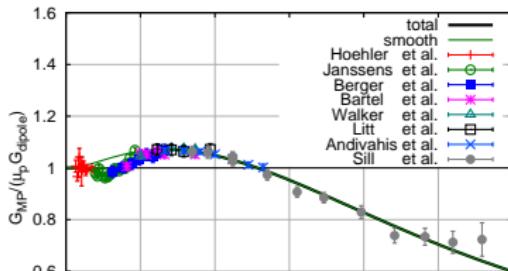
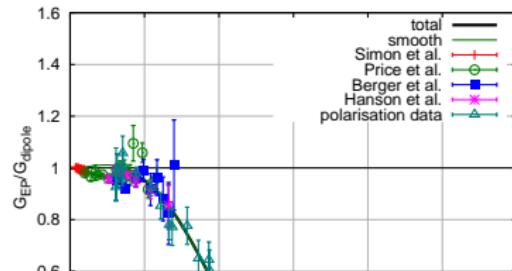
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# Introduction II: Original Motivation



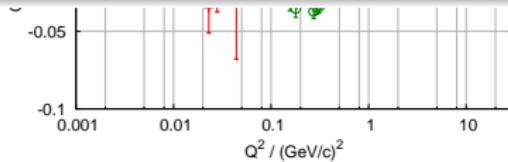
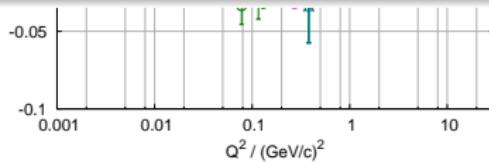
(see J. Friedrich and Th. Walcher, Eur. Phys. J. A 17 (2003) 607)

# Introduction II: Original Motivation



Discrepancy of existing values for proton electric radius:

- 0.809(11) fm: standard dipole at HEPL (Hand et al. 1963)
- 0.862(12) fm: low  $Q^2$  at Mainz (Simon et al. 1979)
- 0.847(09) fm: dispersion relation (Mergell et al. 1996)
- 0.890(14) fm: Hydrogen Lamb shift (Udem et al. 1997)



(see J. Friedrich and Th. Walcher, Eur. Phys. J. A 17 (2003) 607)

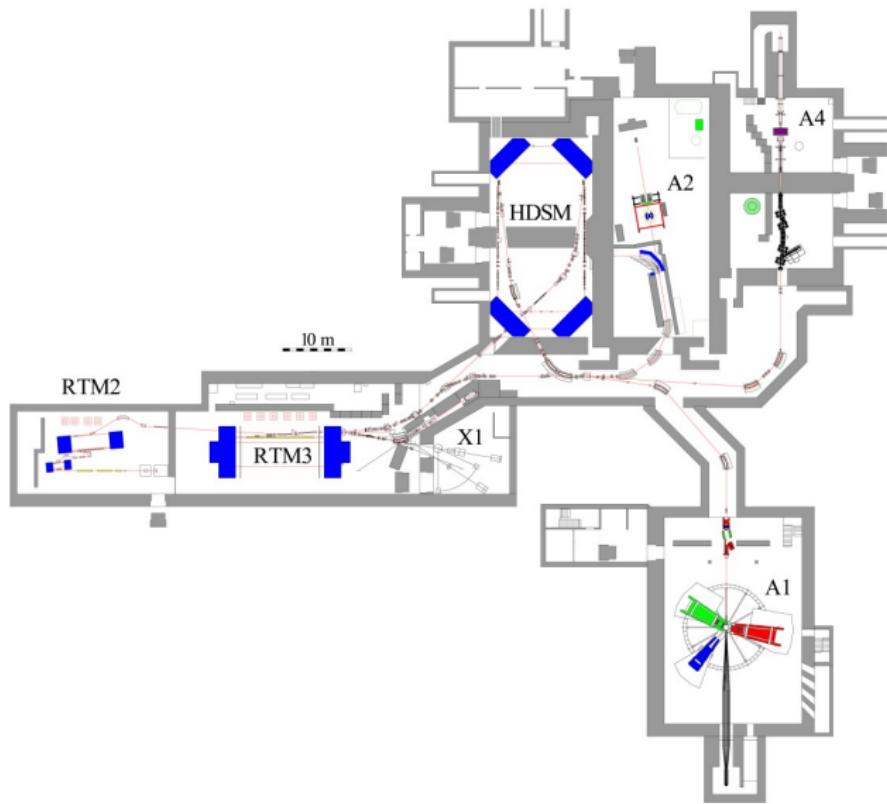
# Location of Mainz, Germany



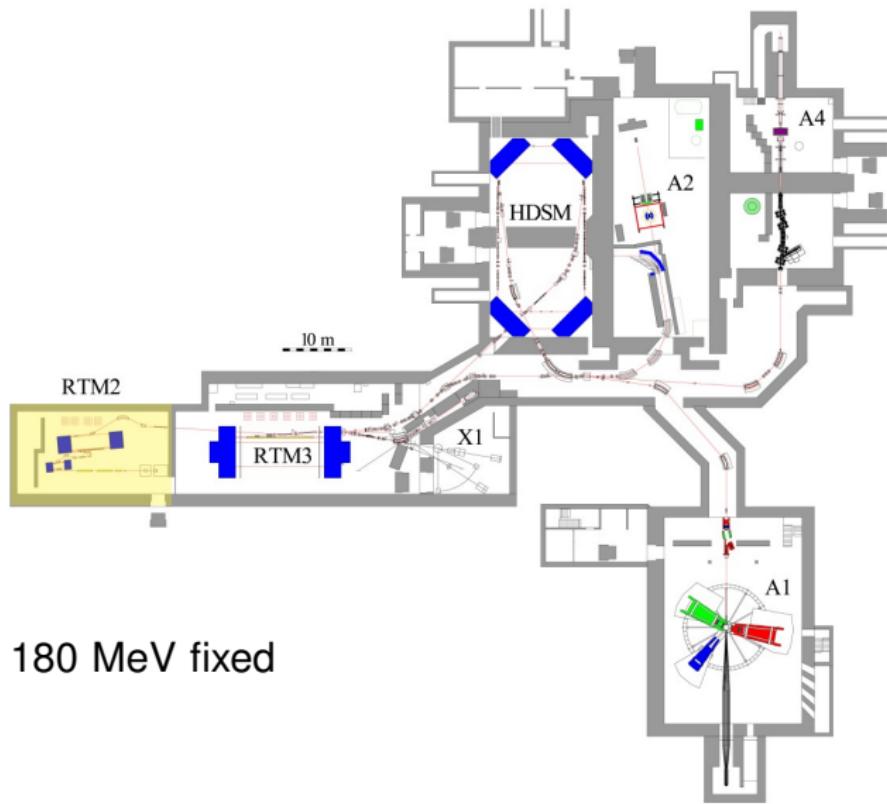
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# The Mainz Microtron MAMI

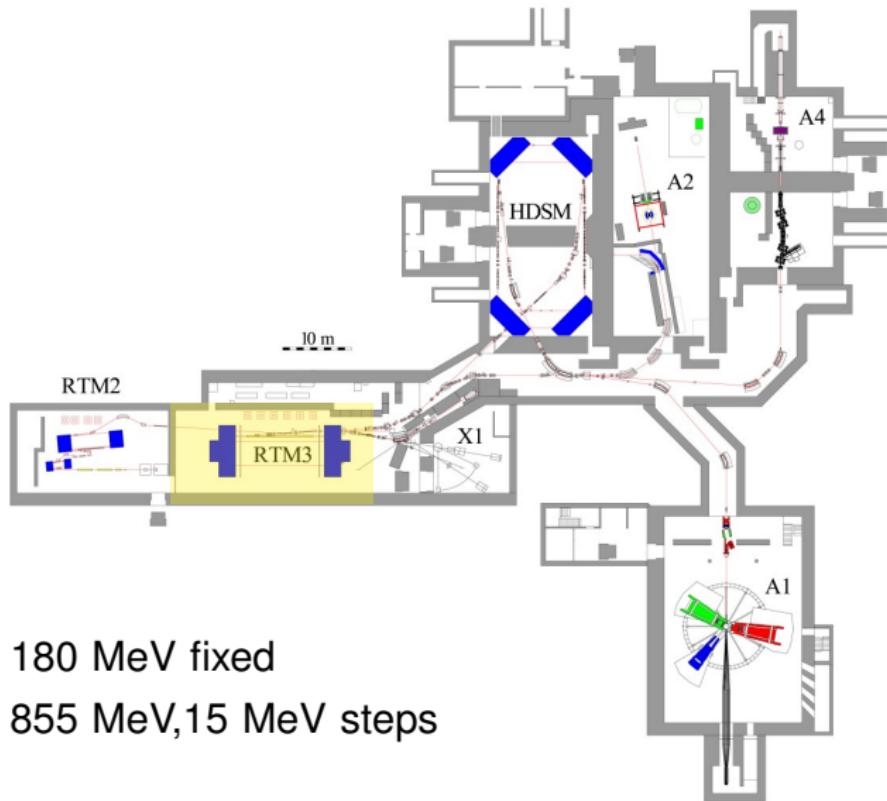


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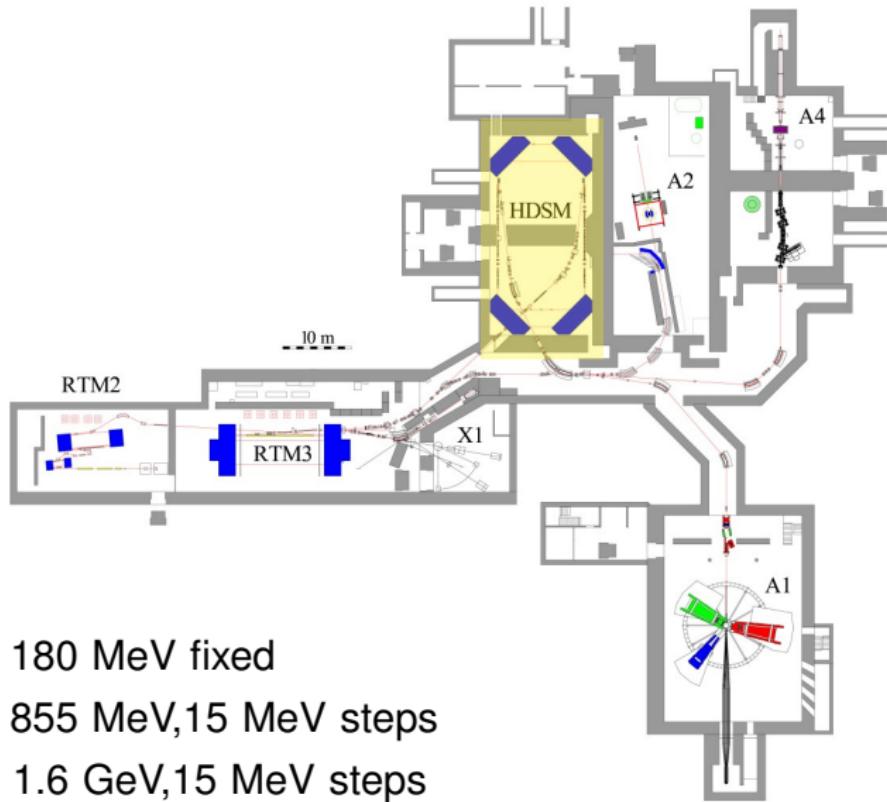
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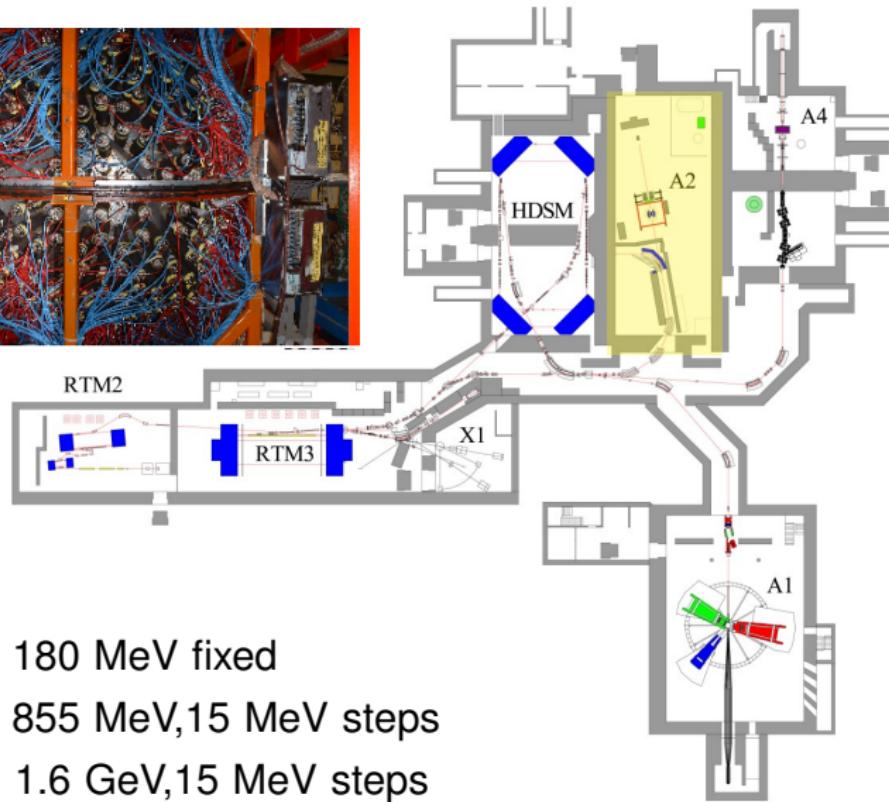
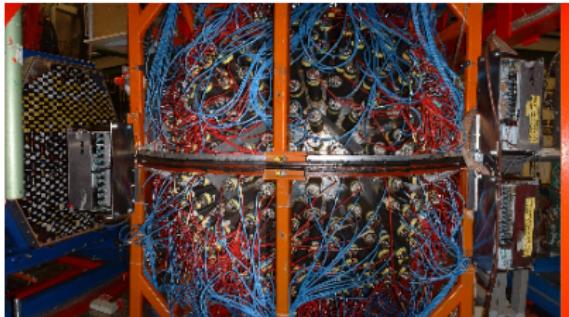


- MAMI-A: 180 MeV fixed
- MAMI-B: 855 MeV, 15 MeV steps

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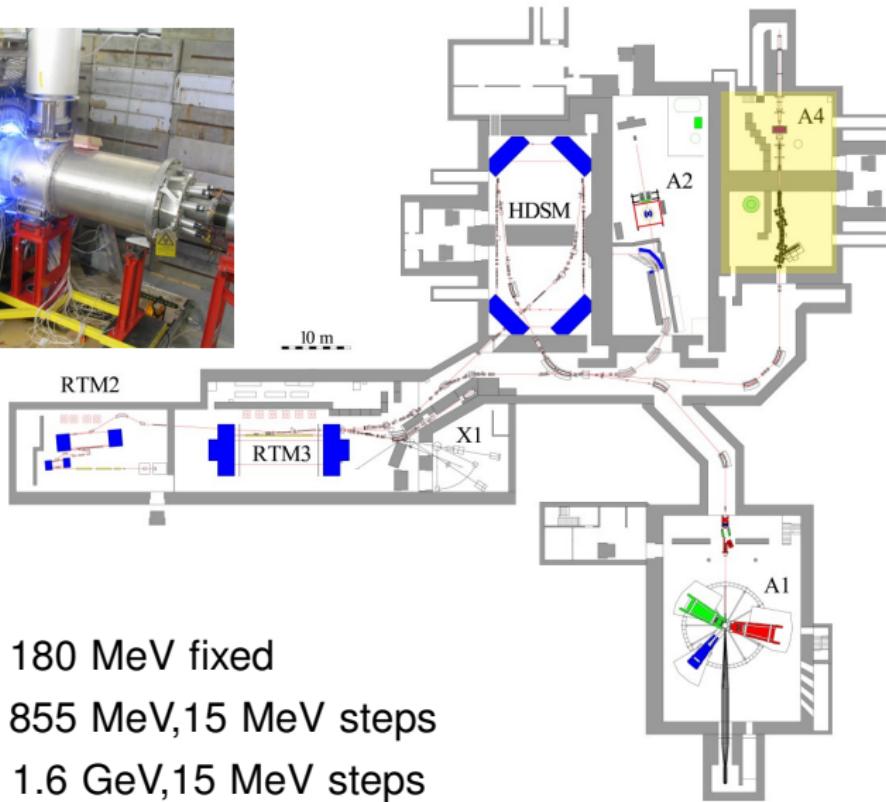


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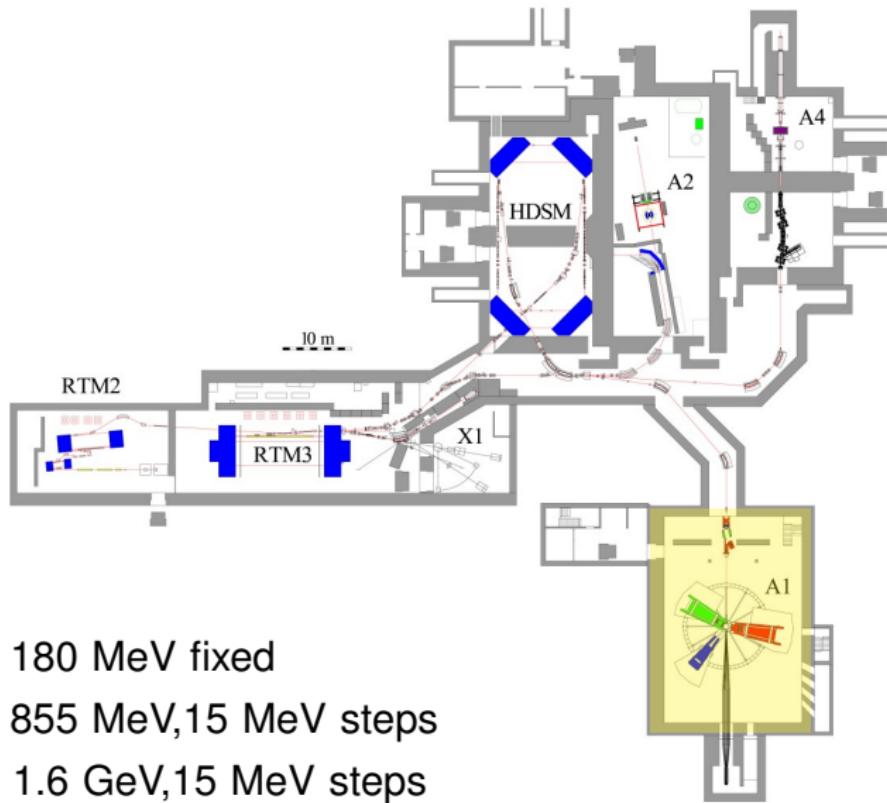
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# The Mainz Microtron MAMI



# The Mainz high-precision $p(e,e')p$ measurement: Three spectrometer facility of the A1 collaboration



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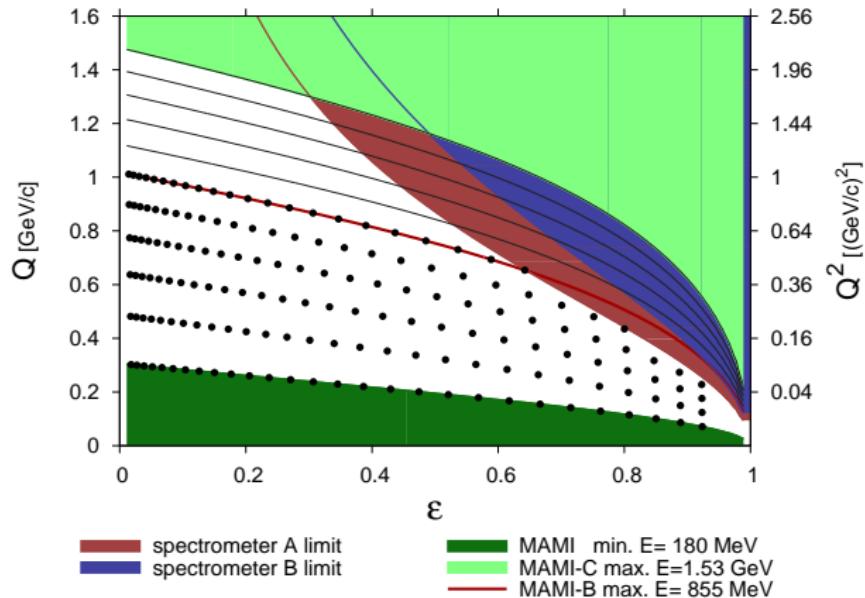
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current  $\times$  density  $\times$  target length  
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  - Overlapping acceptance
  - Where possible: Measure at the same scattering angle with two spectrometers

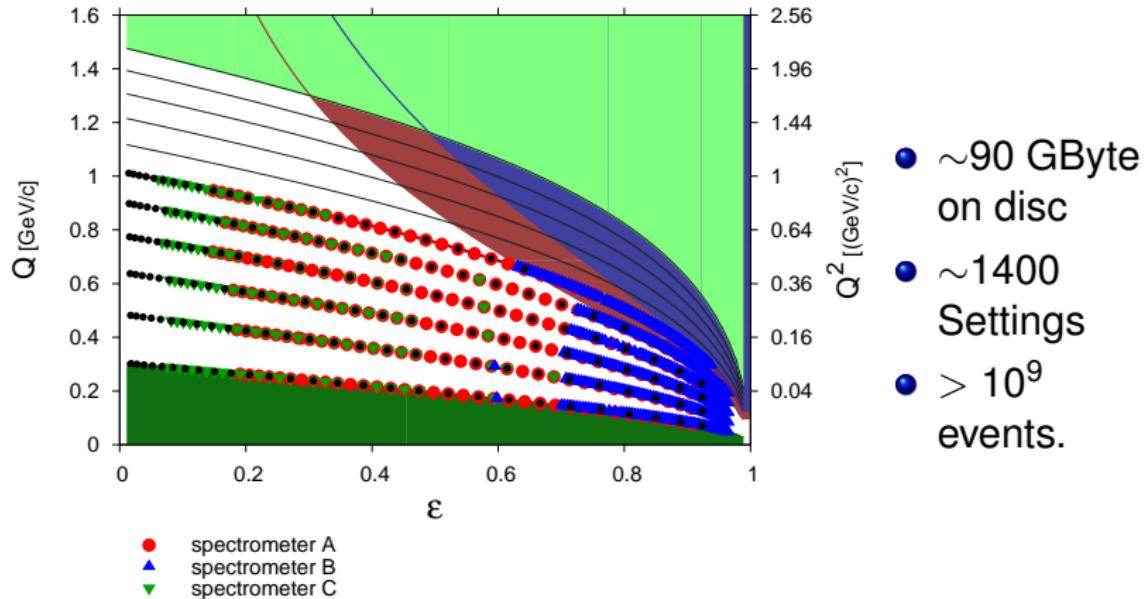
# Measured settings and future (high $Q^2$ ) expansion

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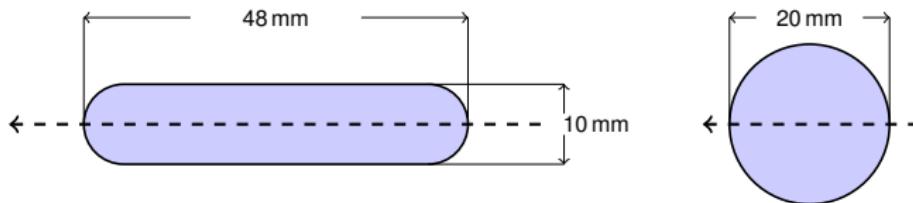


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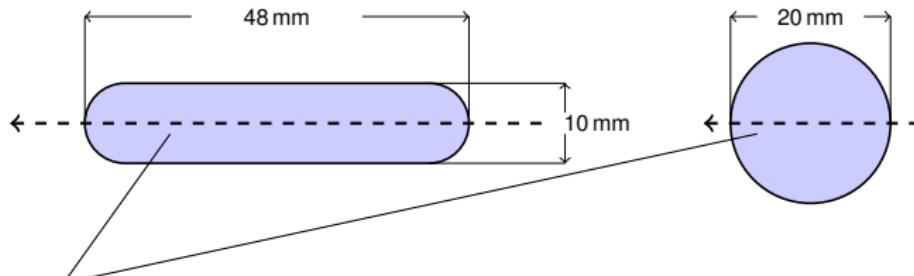
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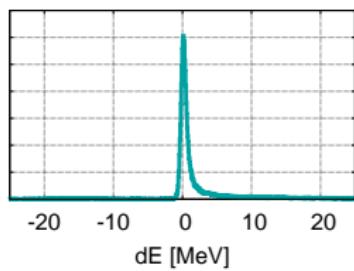
# Background



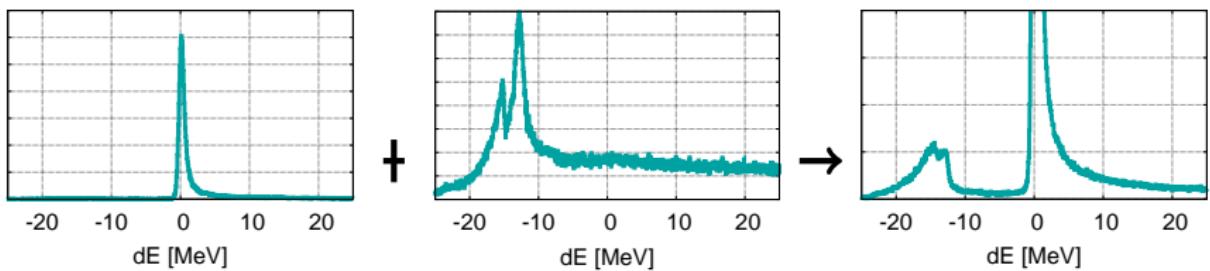
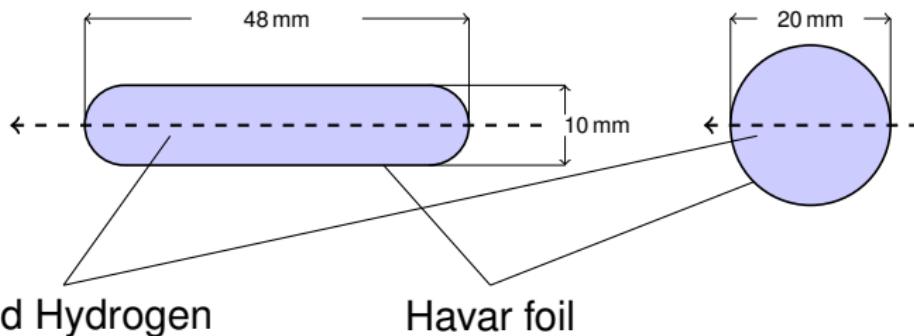
# Background



Liquid Hydrogen



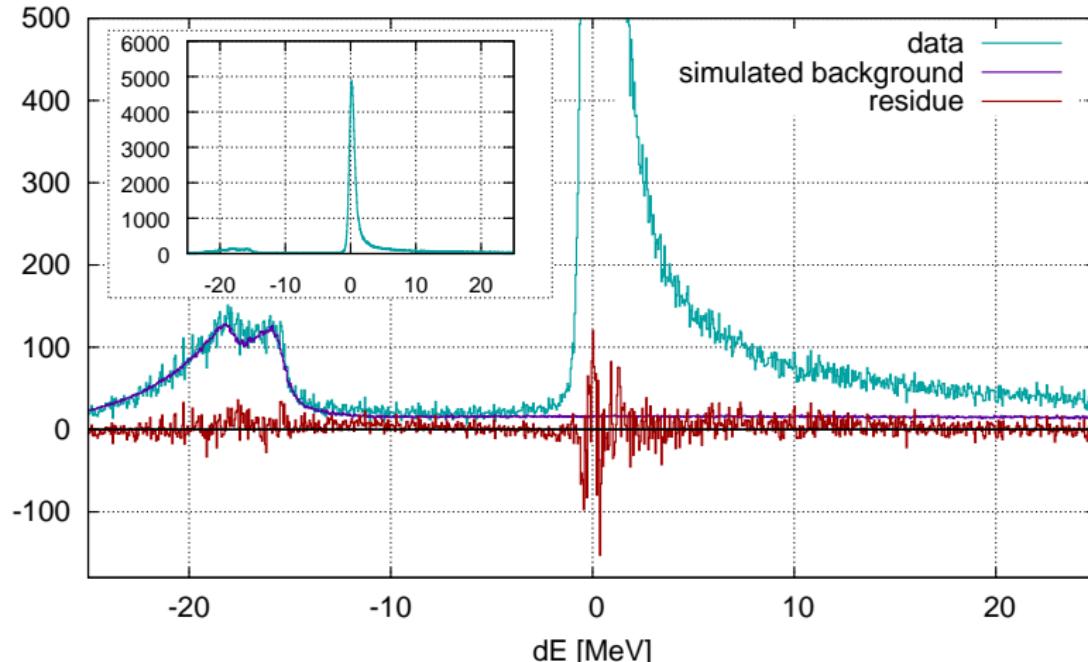
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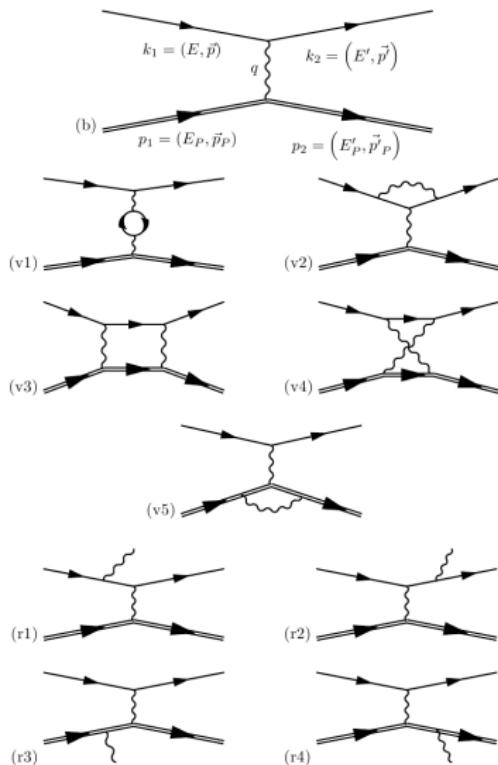
# Data $\iff$ Simulation matching

## Simulation:

- Model for energy loss and small angle scattering
- Input: momentum-, angular-, vertex resolution



# Feynman graphs of leading and next to leading order for elastic scattering



All graphs are taken into account:

- **vacuum polarization (v1):**  
e, ( $\mu$ ,  $\tau$ )  
*Maximon/Tjon (2000) and  
Vanderhaeghen et al. (2000)*
- **electron vertex correction**
- **Coulomb distortion  
(two photon exchange)**
- **real photon emission**

# Comments on Coulomb distortion and TPE

- Coulomb distortion:

Exchange of one hard and multiple soft photons

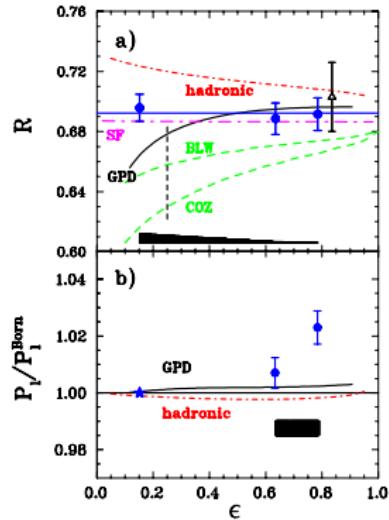
Feshbach (1948), Mo and Tsai (1969).

- Two-photon exchange (TPE) with and w/o excited intermediate states:

Exchange of two hard photons

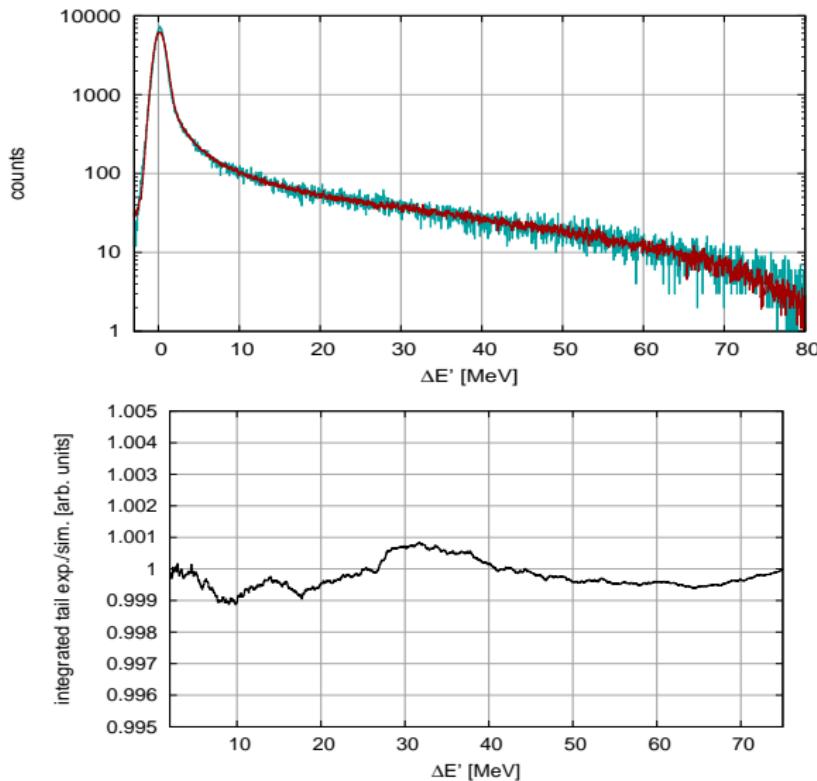
Still not reliable and highly debated

Figure shows a recent experimental result from JLab.

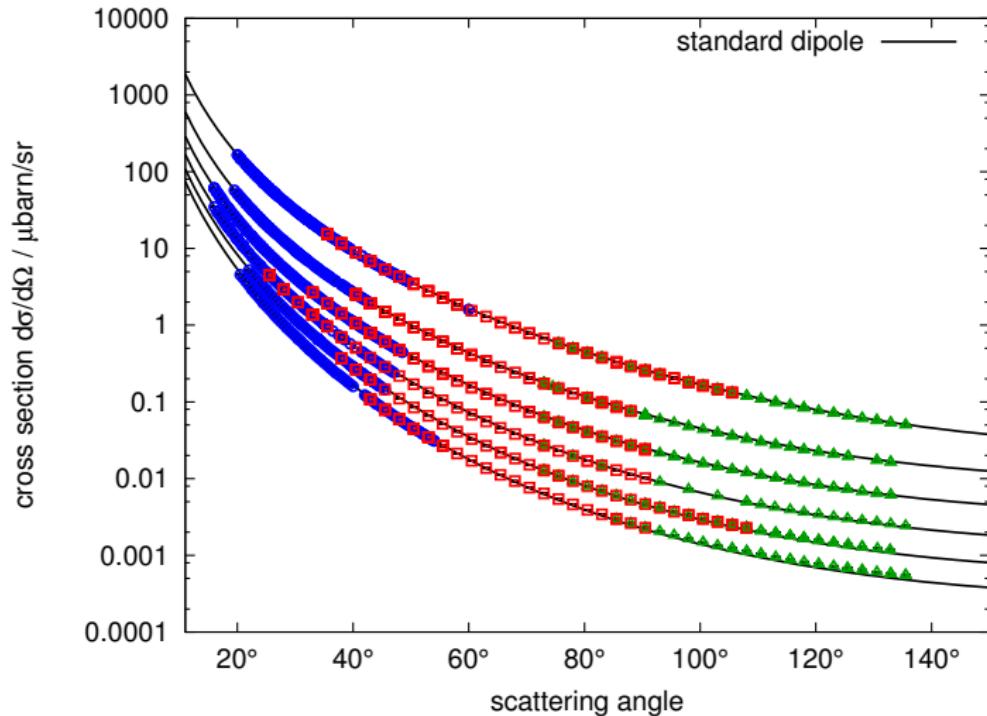


Meziane, M. et al.: *Search for effects beyond the Born approximation in polarization transfer observables in  $\bar{e}p$  elastic scattering*,  
PRL 106, 132501 (2011), arXiv:1012.0339

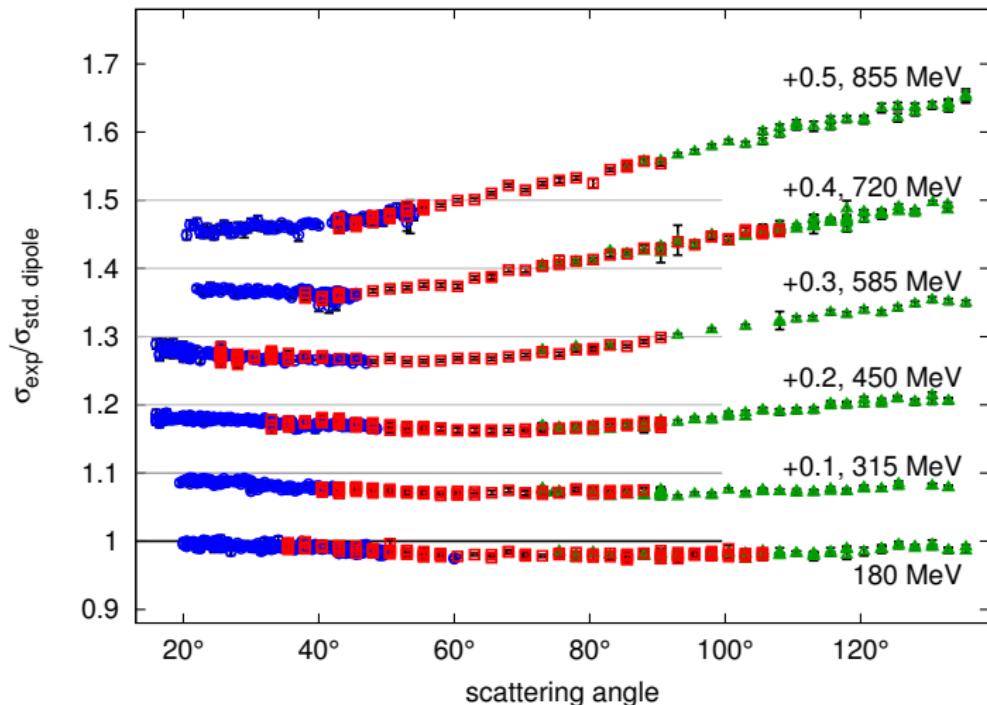
# Description of the radiative tail



# Cross sections



# Cross sections / standard dipole



# How to extract the form factors?

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  - Feasible due to fast computers.
  - All data at all  $Q^2$  and  $\varepsilon$  values contribute to the fit, i.e. full kinematical region used, no projection (to specific  $Q^2$ ) needed.
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For radii extraction: Needs a fit anyway!

Classical Rosenbluth: Extracted  $G_E$  and  $G_M$  highly correlated!  
⇒ Error propagation very involved.

## Models: Dipols

Dipole (different b for  $G_E$  and  $G_M$ ):

$$G_D(Q^2, b) = \frac{1}{\left(1 + \frac{Q^2}{b}\right)^2}$$

Double Dipole (as in Friedrich/Walcher phenomenological fit [Eur. Phys. J. A **17** (2003) 607]):

$$G_{DD}(Q^2, a, b_1, b_2) = a G_D(Q^2, b_1) + (1 - a) G_D(Q^2, b_2)$$

# Models: Polynomial

## Polynomial

$$G_P(Q^2, a_1, \dots, a_n) = 1 + \sum_{i=1}^n a_i Q^{2 \cdot i}$$

## Polynomial + standard Dipole

$$G_{PAD}(Q^2, a_1, \dots, a_n) = G_D(Q^2, 0.71) + \sum_{i=1}^n a_i Q^{2 \cdot i}$$

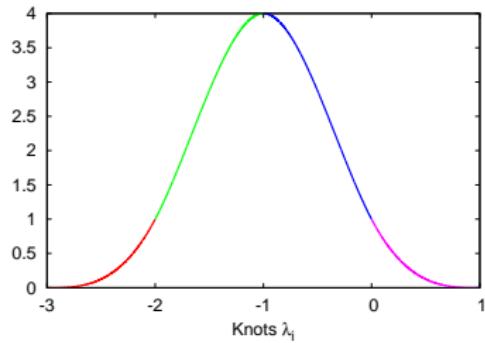
## Polynomial $\times$ standard Dipole

$$G_{PMD}(Q^2, a_1, \dots, a_n) = G_D(Q^2, 0.71) \cdot \left(1 + \sum_{i=1}^n a_i Q^{2 \cdot i}\right)$$

# Models: Splines

Uniform cubic splines

$$spline(Q^2, a_1, \dots, a_n)$$



Spline:

$$G_{Spline}(Q^2, a_1, \dots, a_n) = 1 + Q^2 \cdot spline(Q^2)$$

Spline  $\times$  standard Dipole

$$G_{SMD}(Q^2, a_1, \dots, a_n) = G_D(Q^2, 0.71) \cdot (1 + Q^2 \cdot spline(Q^2))$$

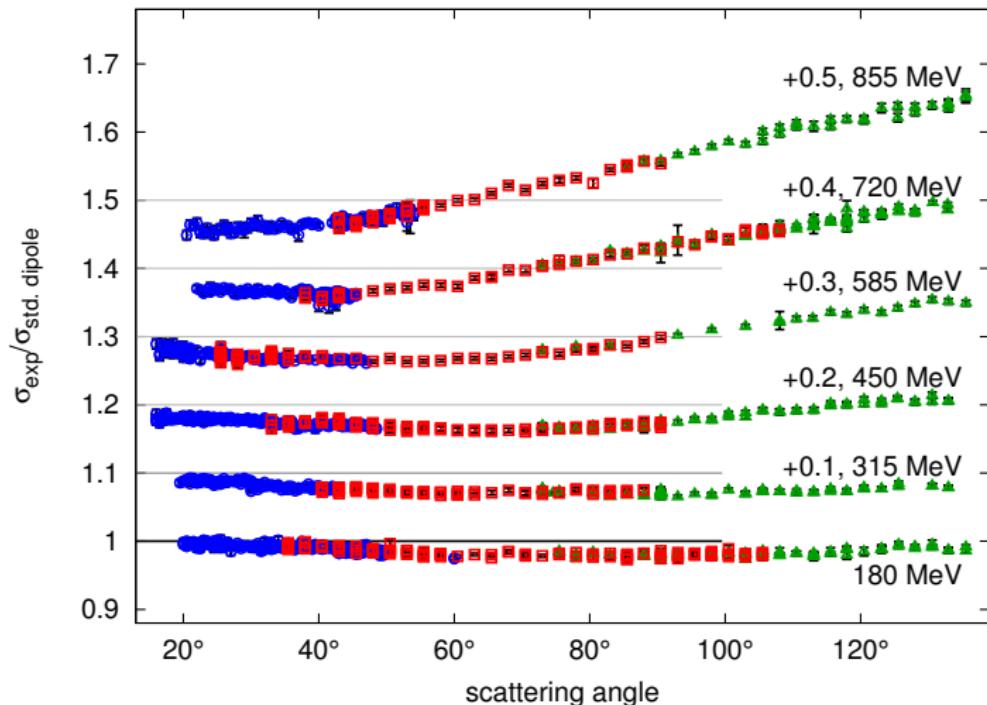
## Models: Misc

Also:

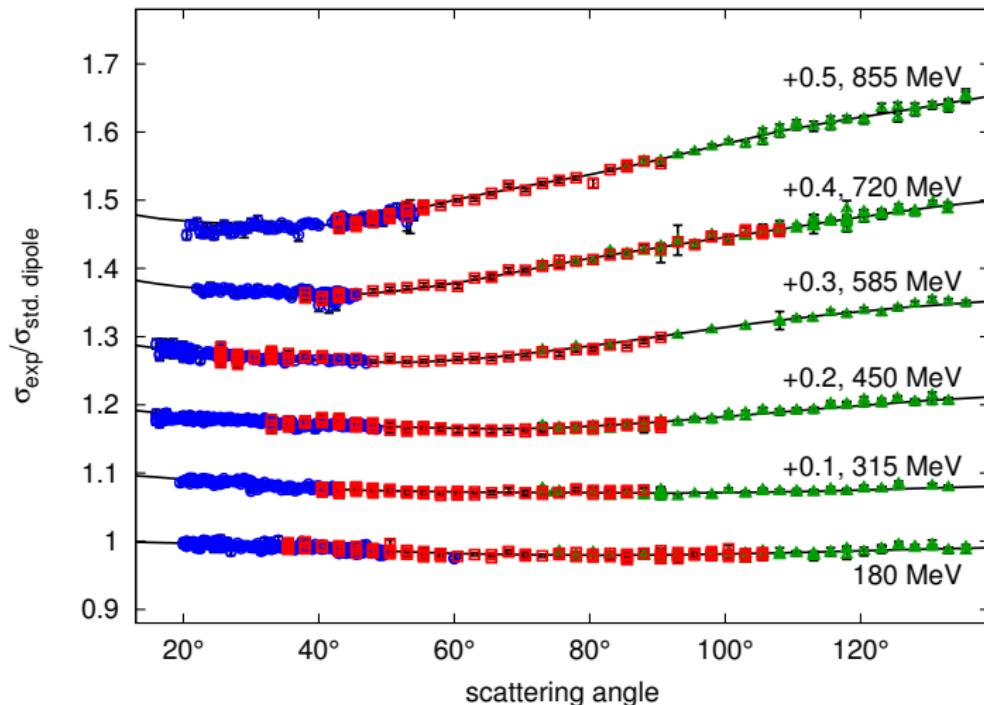
- Friedrich / Walcher phenomenological ansatz
- extended Gari-Krümpelmann (VMD), Lomon et al.
- Arrington type:

$$\frac{P^N}{P^{N+2}}$$

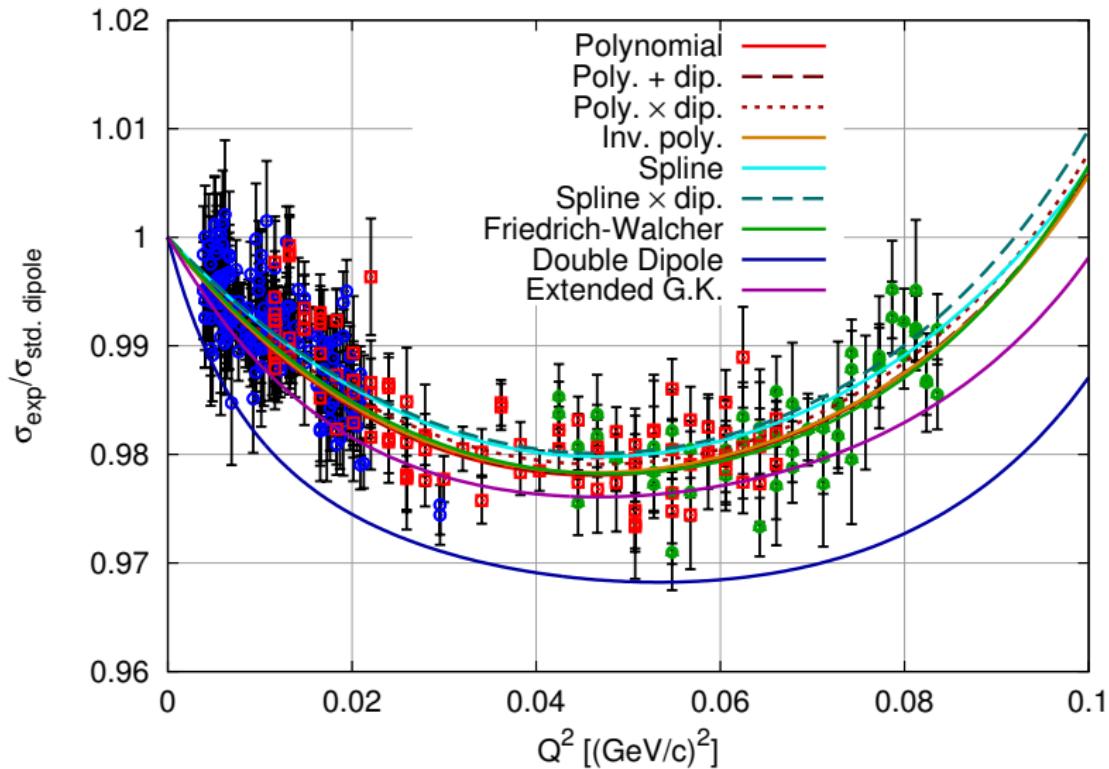
# Cross sections / standard dipole



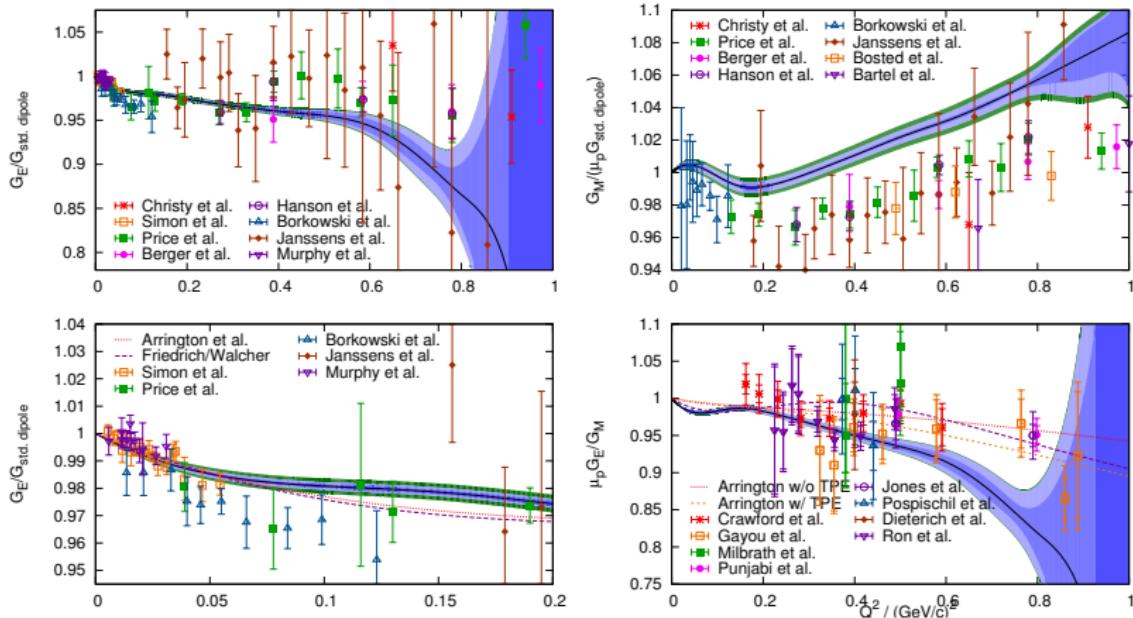
# Cross sections + spline fit



# Cross sections: 180 MeV

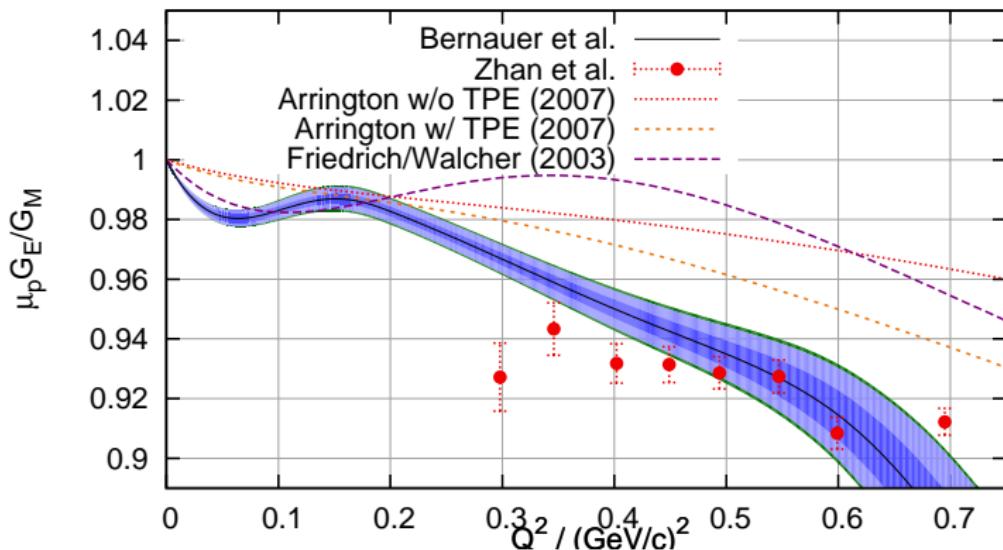


# Form factor results



Jan C. Bernauer *et al.*, “High-precision determination of the electric and magnetic form factors of the proton”,  
PRL 105, 242001 (2010), arXiv:1007.5076

# Form factor results: $G_E/G_M$ ratio

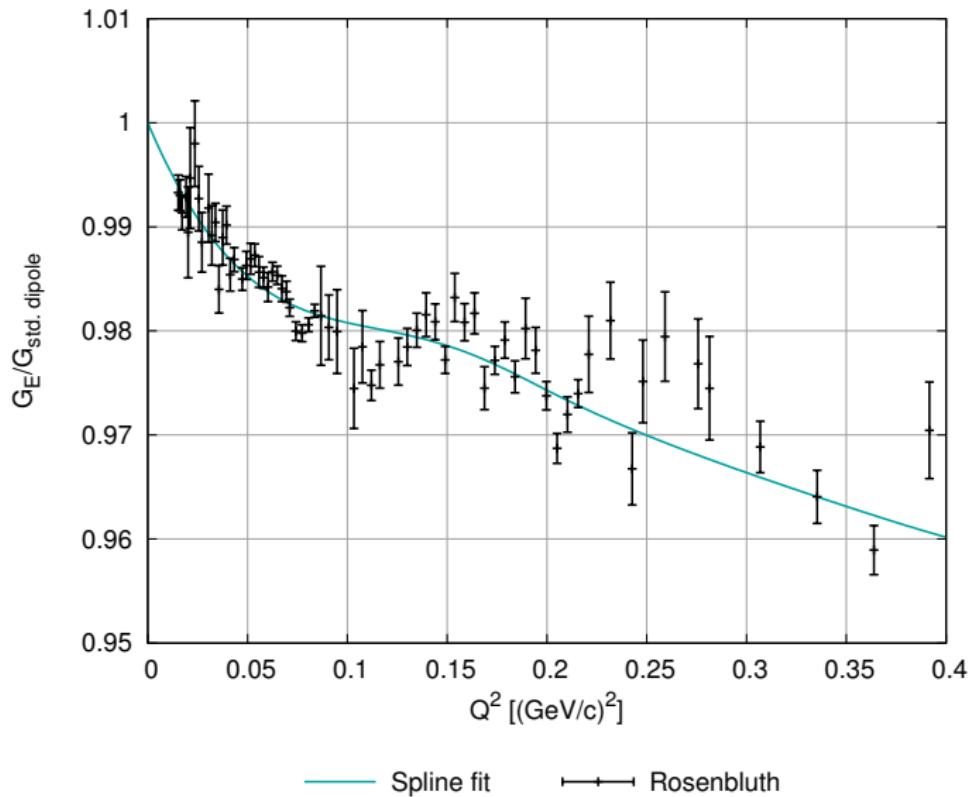


Jan C. Bernauer *et al.*, PRL 105, 242001 (2010), arXiv:1007.5076

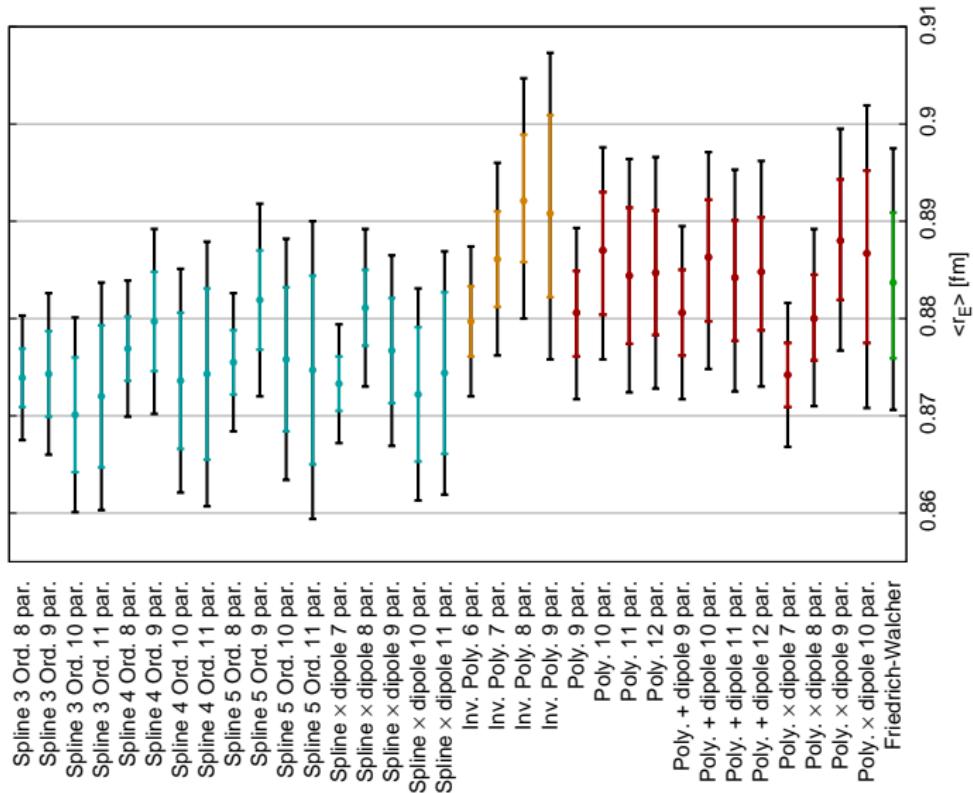
X. Zhan *et al.*, arXiv:1102.0318

J. Arrington *et al.*, Phys. Rev. C76 (2007) 035205, arXiv:0707.1861

# Comparison: Rosenbluth vs. Spline fit



# The electric rms radius - extracted by different models



# Conclusion – Part I

- High precision e-p scattering data from MAMI.  
PRL 105, 242001 (2010), arXiv:1007.5076.
- $Q^2$  range from 0.003 to 1 (GeV/c) $^2$ .
- Consistent data set.
- “Super-Rosenbluth” fit to determine form factors and radii.
- The charge and magnetic rms radii are determined as

$$\begin{aligned}\langle r_e \rangle &= 0.879 \pm 0.005_{\text{stat.}} \pm 0.004_{\text{syst.}} \pm 0.002_{\text{model}} \pm 0.004_{\text{group}} \text{ fm}, \\ \langle r_m \rangle &= 0.777 \pm 0.013_{\text{stat.}} \pm 0.009_{\text{syst.}} \pm 0.005_{\text{model}} \pm 0.002_{\text{group}} \text{ fm}.\end{aligned}$$

# Discussion of the Lamb shift / electron scattering discrepancy

- **Muonic hydrogen (Lamb Shift)**

$$r_p = 0.84184(67) \text{ fm}$$

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**Discrepancy is between  
muonic and electronic measurements**

## Possible explanations of the discrepancy

- **Exotic particles**

e.g. V. Barger *et al.*, arXiv:1011.3519 and references.

- **Contributions to the Lamb shift in  $\mu p$**

C.E. Carlson und M. Vanderhaeghen, arXiv:1011.3519

U.D. Jentschura, Annals Phys. **326**, 500-515 (2011)

E. Borie, arXiv:1103.1772

- **Higher moments of the charge distribution and Zemach radii**

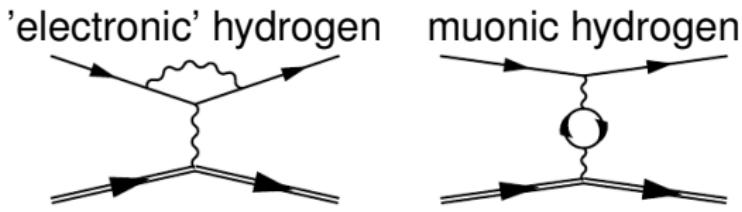
M.O.D., J.C. Bernauer, and Th. Walcher,  
Phys. Lett. **B696**, 343-347 (2011)

## Speculation about the discrepancy

- Reminder: The muon g-2 experiment has a  $2 - 3\sigma$  discrepancy. Hadronic corrections may provide an explanation.
- The main contribution to the **Lamb shift** in ...

## Speculation about the discrepancy

- Reminder: The muon g-2 experiment has a  $2 - 3\sigma$  discrepancy. Hadronic corrections may provide an explanation.
- The main contribution to the **Lamb shift** in ...



vertex and self-energy  
vacuum polarization  
anom. magn. moment  
+ higher order

1011.41 MHz  
-27.13 MHz  
67.82 MHz

-205.028 meV

theoretical value  
experimental value

1057.864(14) MHz  
1057.862(20) MHz

-206.057 meV  
 $\Delta : 0.341$  meV

## Conclusion – Part II

- High precision form factors from MAMI provide constraints for the charge distribution of the proton.
- Standard dipole approximation is not sufficient for correction of the muonic hydrogen Lamb shift.
- The proton size discrepancy is between the Lamb shift of muonic hydrogen and every "electronic" determination.
- Explanation for the discrepancy?
- Outlook: Low- and high  $Q^2$  measurements @ MAMI
- Lamb shift measurements on D,  $^{3,4}\text{He}$  @ PSI  
Form factor and polarizability of D,  $^{3,4}\text{He}$  @ MAMI

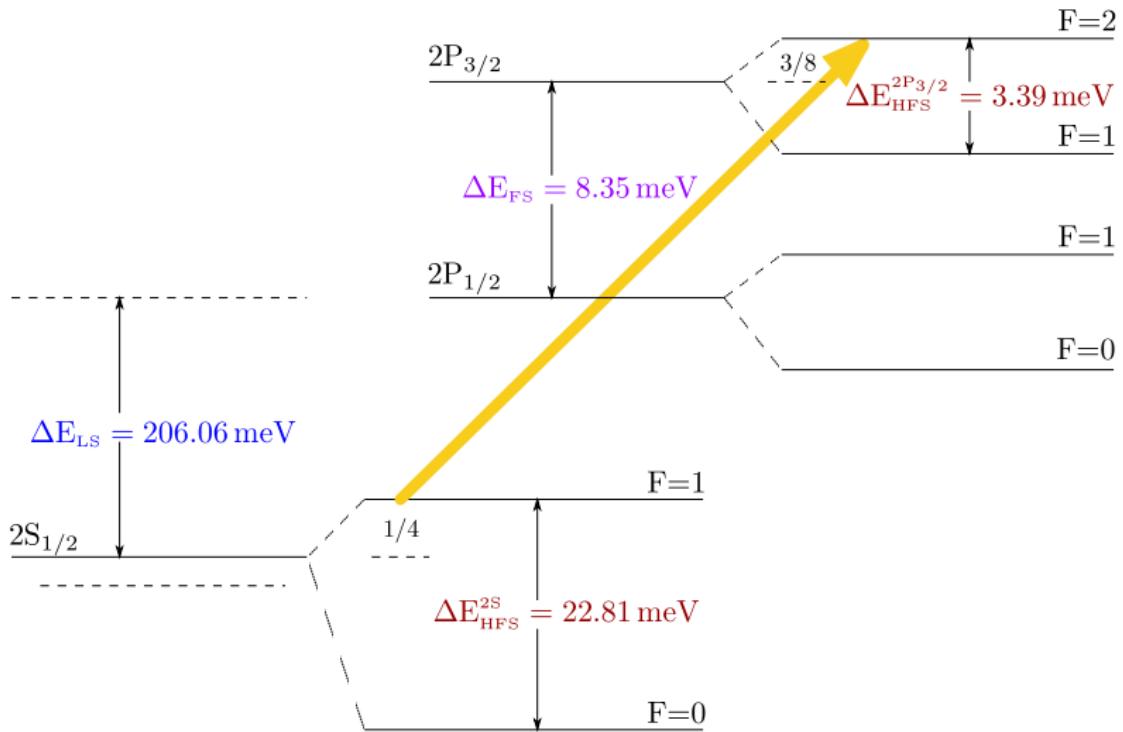
# Backup

# Discussion of the Lamb shift / electron scattering discrepancy

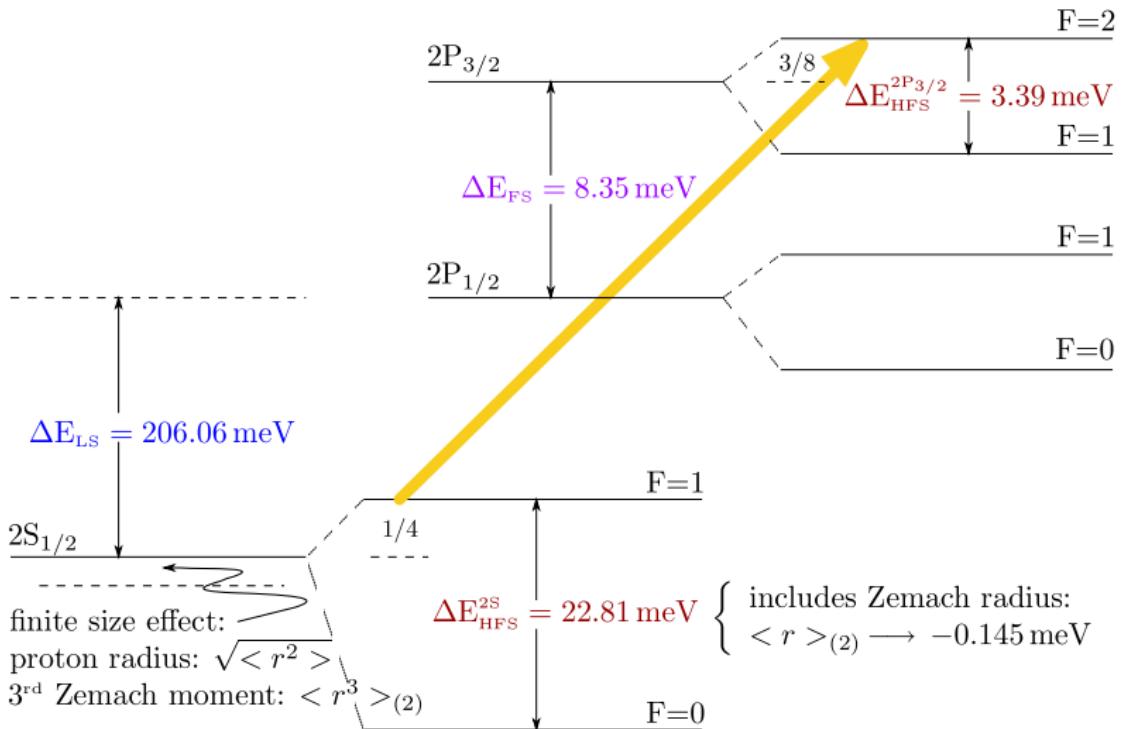
The following tables are taken from the 'QED supplement' published in *Nature* **466**, 213-216 (8 July 2010).

'All known radius-independent contributions' and 'all relevant radius-dependent contributions' to the Lamb shift in  $\mu_p$  from different authors are listed.

# 2S – 2P splitting in muonic hydrogen



# 2S – 2P splitting in muonic hydrogen



# Discussion of the Lamb shift / electron scattering discrepancy

#	Contribution	Ref.	Our selection Value	Unc.	Pachucki <sup>1-3</sup> Value	Unc.	Borie <sup>4</sup> Value	Unc.
1	NR One loop electron VP	1,2			205.0074			
2	Relativistic correction (corrected)	1-3,5			0.0169			
3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	-0.00103				-0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	-0.00500	0.0010	-0.006	0.001	-0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17-19	-0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21-23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	22,23	-0.000015					
17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	-0.00410				-0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2,7	-0.66770		-0.6677		-0.66788	
21	Muon Lamb shift 4th order	5	-0.00169				-0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	2,5-7	-0.04497		-0.045		-0.04497	
23	Recoil of order $\alpha^6$	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	1,2,7	-0.00960		-0.0099		-0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	-0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

# Discussion of the Lamb shift / electron scattering discrepancy

$$\Delta E = 209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3$$

Values are in meV and radii in fm.

Contribution	Ref.	our selection	Pachucki <sup>2</sup>	Borie <sup>5</sup>
Leading nuclear size contribution	<sup>26</sup>	-5.19745 $\langle r_p^2 \rangle$	-5.1974	-5.1971
Radiative corrections to nuclear finite size effect	<sup>2,26</sup>	-0.0275 $\langle r_p^2 \rangle$	-0.0282	-0.0273
Nuclear size correction of order $(Z\alpha)^6 \langle r_p^2 \rangle$	<sup>1,27-29</sup>	-0.001243 $\langle r_p^2 \rangle$		
Total $\langle r_p^2 \rangle$ contribution		-5.22619 $\langle r_p^2 \rangle$	-5.2256	-5.2244
Nuclear size correction of order $(Z\alpha)^5$	<sup>1,2</sup>	0.0347 $\langle r_p^3 \rangle$	0.0363	0.0347

# Discussion of the Lamb shift / electron scattering discrepancy

Zemach-Moments:

- A. C. Zemach, *Proton Structure and the Hyperfine Shift in Hydrogen*, Phys. Rev. **104**, 1771 (1956).

$$\langle r^3 \rangle_{(2)} = \int_0^\infty \frac{dq}{q^4} \left( G_E^2(q^2) - 1 + q^2 \langle r^2 \rangle_p / 3 \right)$$

$$\langle r^3 \rangle_{(2)} = 2.27 \text{ fm}^3 \quad \longrightarrow \quad r_p = 0.84184(67) \text{ fm}$$

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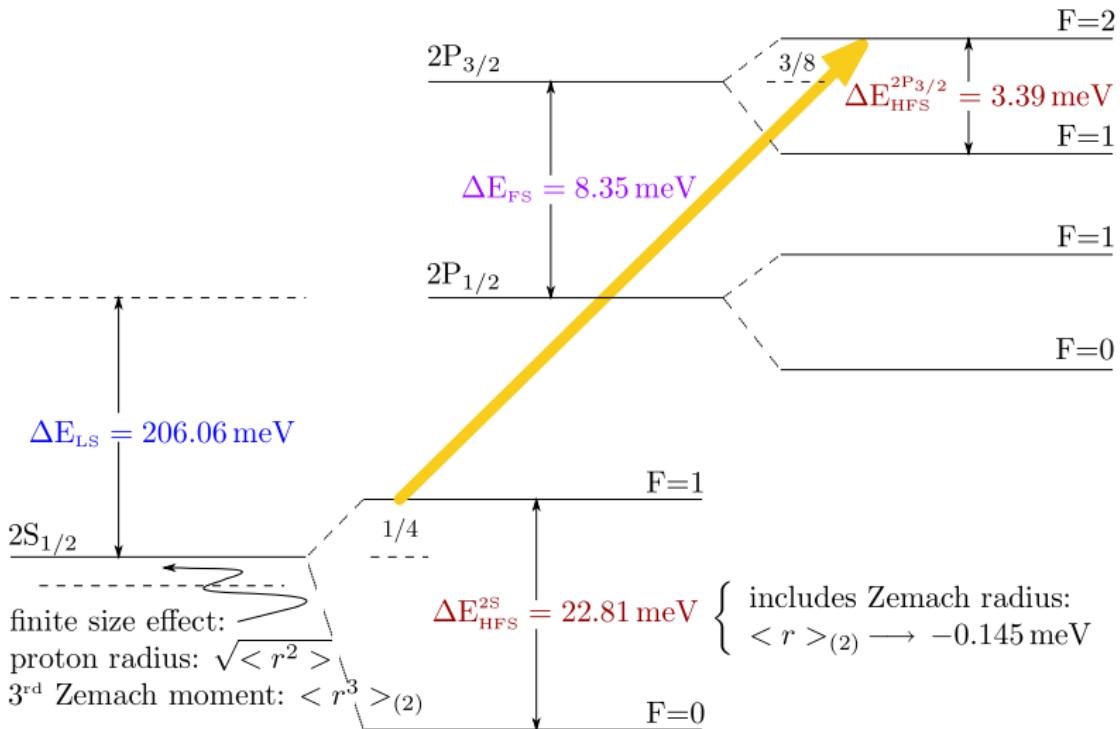
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$$\langle r^3 \rangle_{(2)} = 2.85(8) \text{ fm}^3 \quad \longrightarrow \quad r_p = 0.84245(67) \text{ fm}$$

- M.O.D., J.C. Bernauer, and Th. Walcher, *The RMS Charge Radius of the Proton and Zemach Moments*, in Press  
doi:10.1016/j.physletb.2010.12.067, arXiv:1011.1861.

# 2S – 2P splitting in muonic hydrogen



## De Rújula's toy model

- A. De Rújula, “QED is not endangered by the proton’s size”, Phys. Lett. **B693**, 555 (2010).
- Sum of “single pole” and “dipole”

$$\begin{aligned}\rho_{\text{Proton}}(r) &= \frac{1}{D} \left[ \frac{M^4 e^{-Mr} \cos^2(\theta)}{4\pi r} + \frac{m^5 e^{-mr} \sin^2(\theta)}{8\pi} \right] \\ D &\equiv M^2 \cos^2(\theta) + m^2 \sin^2(\theta)\end{aligned}$$

using  $M = 0.750 \text{ GeV}/c^2$ ,  $m = 0.020 \text{ GeV}/c^2$ , and  $\sin^2(\theta) = 0.3$  and

$$\rho_{(2)}(r) = \int d^3 r_2 \rho_{\text{charge}}(|\vec{r} - \vec{r}_2|) \rho_{\text{charge}}(r_2)$$

we get the **third Zemach moment**:

$$\langle r^3 \rangle_{(2)} = \int d^3 r r^3 \rho_{(2)}(r) = 36.2 \text{ fm}^3$$

## De Rújula's toy model – . . .

We put  $\langle r^3 \rangle_{(2)} = 36.2 \text{ fm}^3$  in the Lamb shift formula:

$$L^{5th}[\langle r^2 \rangle, \langle r^3 \rangle_{(2)}] = \\ \left( 209.9779(49) - 5.2262 \frac{\langle r^2 \rangle}{\text{fm}^2} + 0.00913 \frac{\langle r^3 \rangle_{(2)}}{\text{fm}^3} \right) \text{ meV}$$

and get  $r_p = 0.878 \text{ fm}$

## De Rújula's toy model – . . .

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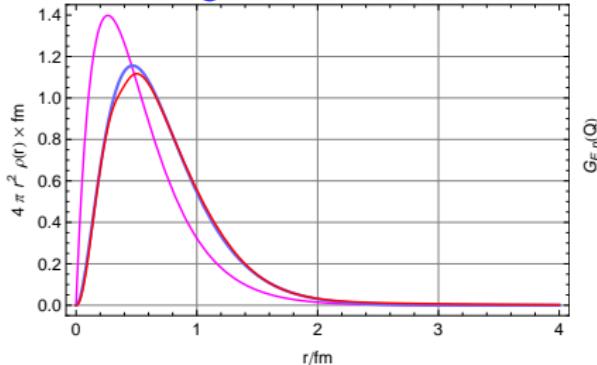
$$L^{5th}[\langle r^2 \rangle, \langle r^3 \rangle_{(2)}] = \\ \left( 209.9779(49) - 5.2262 \frac{\langle r^2 \rangle}{\text{fm}^2} + 0.00913 \frac{\langle r^3 \rangle_{(2)}}{\text{fm}^3} \right) \text{ meV}$$

and get  $r_p = 0.878 \text{ fm}$

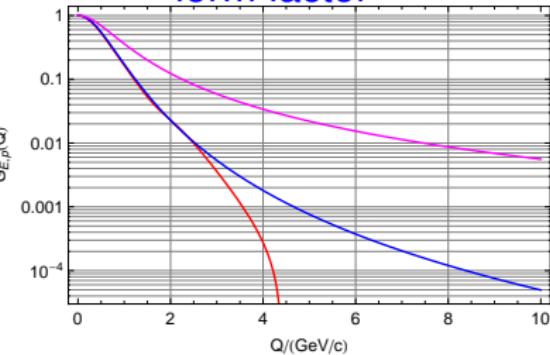
**problem solved**

# De Rújula's toy model – is excluded by experiment

charge distribution

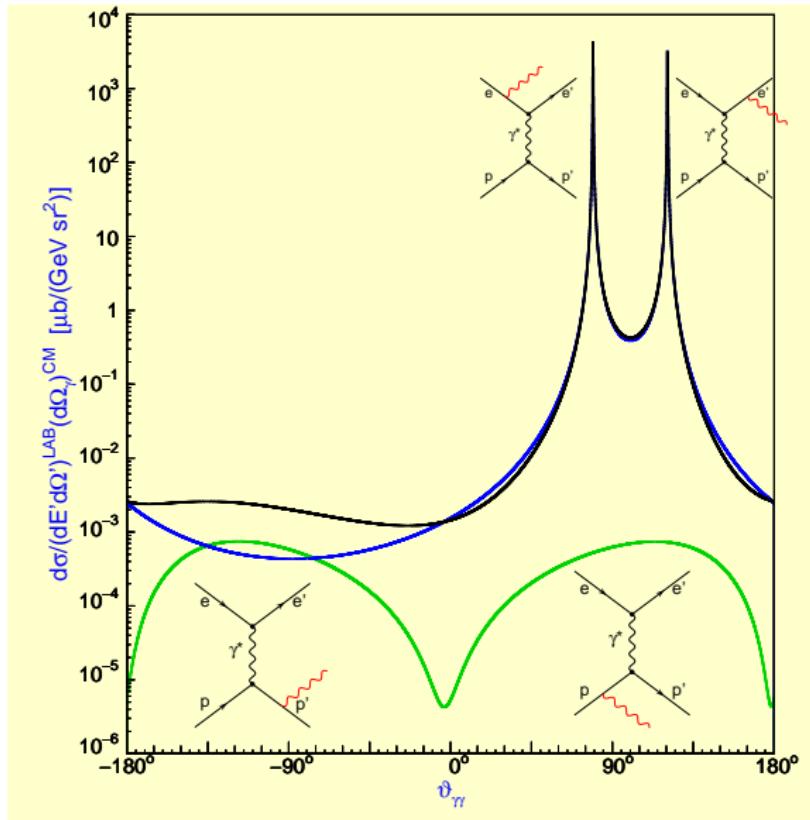


form factor



- De Rújula's toy model
- standard dipole
- Bernauer-Arrington fit assembly

# Outlook: Initial state radiation



# Outlook: Initial state radiation

