# Hadron Spectroscopy from Lattice QCD

Robert Edwards
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*JLab Users Group Meeting 2012*

under the auspices of the Hadron Spectrum Collaboration

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Recent publications:
"Excited and exotic charmonium spectroscopy from lattice QCD", accepted PRD, 1204.5425
"S and D-wave phase shifts in isospin-2 pi pi scattering from lattice QCD", accepted PRD, 1201.2349
"Hybrid baryons", PRD85, 1201.2349
"Helicity operators for mesons in flight", PRD85, 1107.1930
"Lightest hybrid meson supermultiplet", PRD84, 1106.5515
"Excited state baryon spectroscopy", PRD84, 1104.5152
"Isoscalar meson spectroscopy", PRD83, 1102.4299
"Phase shift of isospin-2 scattering", PRD83, 1011.6352
"Toward the excited meson spectrum", PRD82, 1004.4930
"Highly excited and exotic meson spectrum", PRL103, 0909.0200

# What is a gauge theory?

Maxwell's eqns: field strength tensor and vector potentials

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{E} = \frac{\partial}{\partial x_0}\vec{A} - \vec{\nabla} A_0$$

$$F_{\mu\nu} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{bmatrix} = \frac{\partial}{\partial x_\mu} A_\nu(x) - \frac{\partial}{\partial x_\nu} A_\mu(x)$$

Action

$$S = \int d^4x \, \frac{1}{4} \sum_{\mu,\nu} F_{\mu\nu} F^{\mu\nu} = \int d^4x \, \frac{1}{2}(\vec{E}^2 + \vec{B}^2)$$

# QCD

Dirac operator: $A_\nu$ (vector potential), m (mass), $\gamma_\nu$ (4x4 matrices)

$$D(A,m) = -\sum_\mu \gamma_\mu D_\mu \; + \; m; \qquad D_\mu = \frac{\partial}{\partial x_\mu} + ig A_\mu(x)$$

Observables

$$\langle \mathcal{O} \rangle = \int dA(x)\, \mathcal{O}(A)\; e^{-S} \det(D(A,m))$$

QCD: Vector potentials now 3x3 complex matrices (SU(3))

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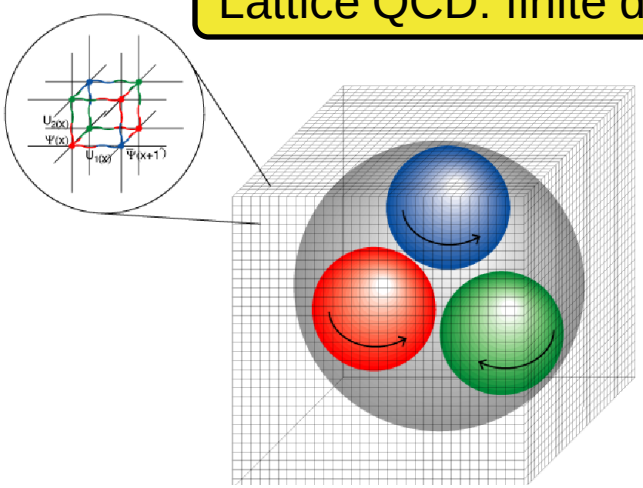
Lattice QCD: finite difference



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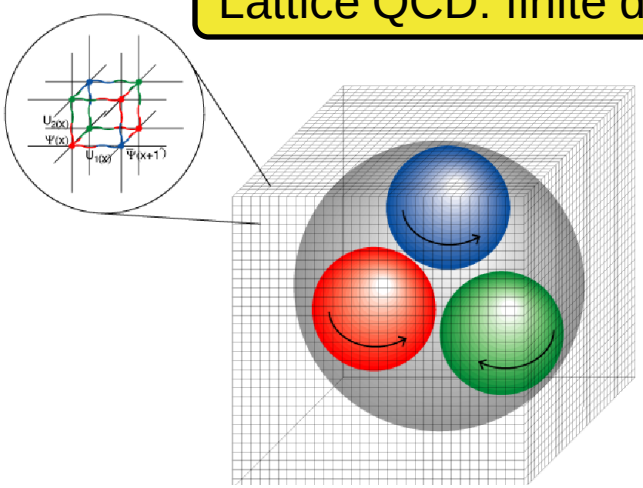

Lots of "flops/s"
Harness GPU-s

# Patterns in hadron spectrum

Observed meson state flavor & $J^{PC}$ systematics suggest $q\bar{q}$

`constituent quarks'

$$q\bar{q}[S,L] \rightarrow (J = L \otimes S)^{PC}, \quad P = (-1)^{L+1}, C = (-1)^{L+S}$$

| | | | |
|---|---|---|---|
| $\cdot$ | $0^{-+}$ | $0^{++}$ | $\cdot$ |
| $1^{--}$ | $\cdot$ | $1^{++}$ | $1^{+-}$ |
| $2^{--}$ | $2^{-+}$ | $2^{++}$ | $\cdot$ |
| $\vdots$ | | | |

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Could excited gluonic fields play a role – *hybrid* mesons $q\bar{q}G$ ?

Possibly exotic $J^{PC}$ and extra `non-exotic' states

Constituent quark picture $qqq$ predicts rich baryon spectrum not all experimentally observed

No exotic $J^{P}$ in *hybrid* baryons $qqqG$

However, might lead to extra states

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- flux-tube model

$(0,1,2)^{-+}, 1^{--}, (0,1,2)^{+-}, 1^{++}$

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# Spectrum from variational method

Two-point correlator

$$C_{ij}(t) = \langle 0|\Phi_i(t)\Phi_j^\dagger(0)|0\rangle$$

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"Rayleigh-Ritz method"
Diagonalize:
eigenvalues $\rightarrow$ spectrum
eigenvectors $\rightarrow$ spectral "overlaps" $Z_i^n$

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Each state optimal combination of $\Phi_i$

$$\Omega^{(\mathfrak{n})} = \sum_i v_i^{(\mathfrak{n})}\Phi_i$$

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Benefit: orthogonality for near degenerate states

# Operators

Mesons: fermion bi-linears

| | | |
|---|---|---|
| $\bar{\psi}\Gamma\psi$ | $\rightarrow$ 0, 1 | |
| $\bar{\psi}\Gamma\overleftrightarrow{D}\psi$ | $\rightarrow$ 0, 1, 2 | gauge-covariant derivatives |
| $\bar{\psi}\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\psi$ | $\rightarrow$ 0, 1, 2, 3 | coupling $\langle 1m_1; 1m_2 \vert L_{12}m_{12}\rangle \overleftrightarrow{D}_{m_1}\overleftrightarrow{D}_{m_2}$ |
| $\bar{\psi}\Gamma\overleftrightarrow{D}\overleftrightarrow{D}\overleftrightarrow{D}\psi$ | $\rightarrow$ 0, 1, 2, 3, 4 | 2 derivatives can give chromo B field $1^{+-}$ |

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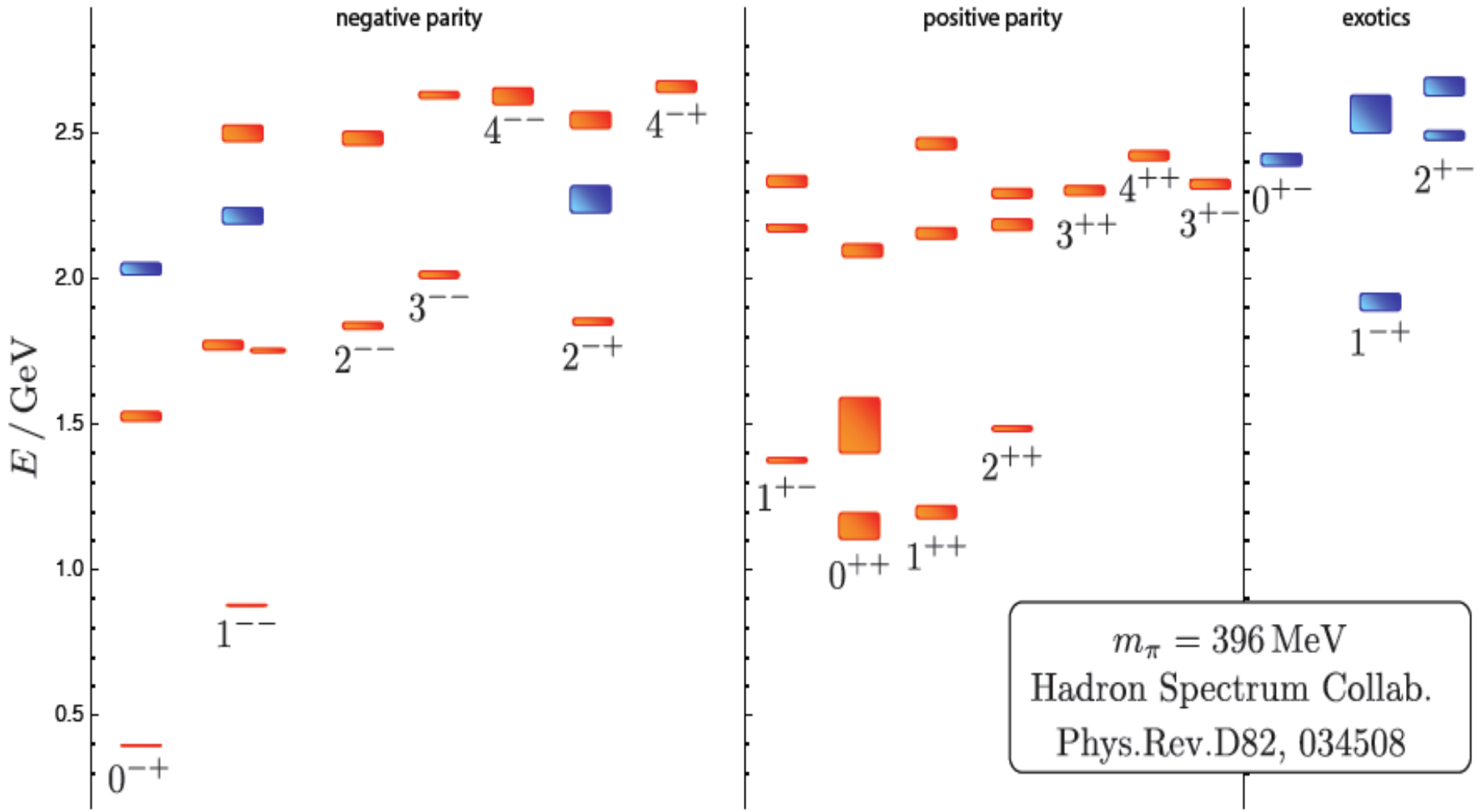
Baryons: three quarks

$$\Phi^{JM} \leftarrow \left(CGC's\right)_{i,j,k}\left[\vec{D}\right]_i\left[\vec{D}\right]_j\left[\psi\psi\psi\right]_k$$

$$\mathbf{1}\otimes\mathbf{1}\otimes\mathcal{S}\rightarrow\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2}$$

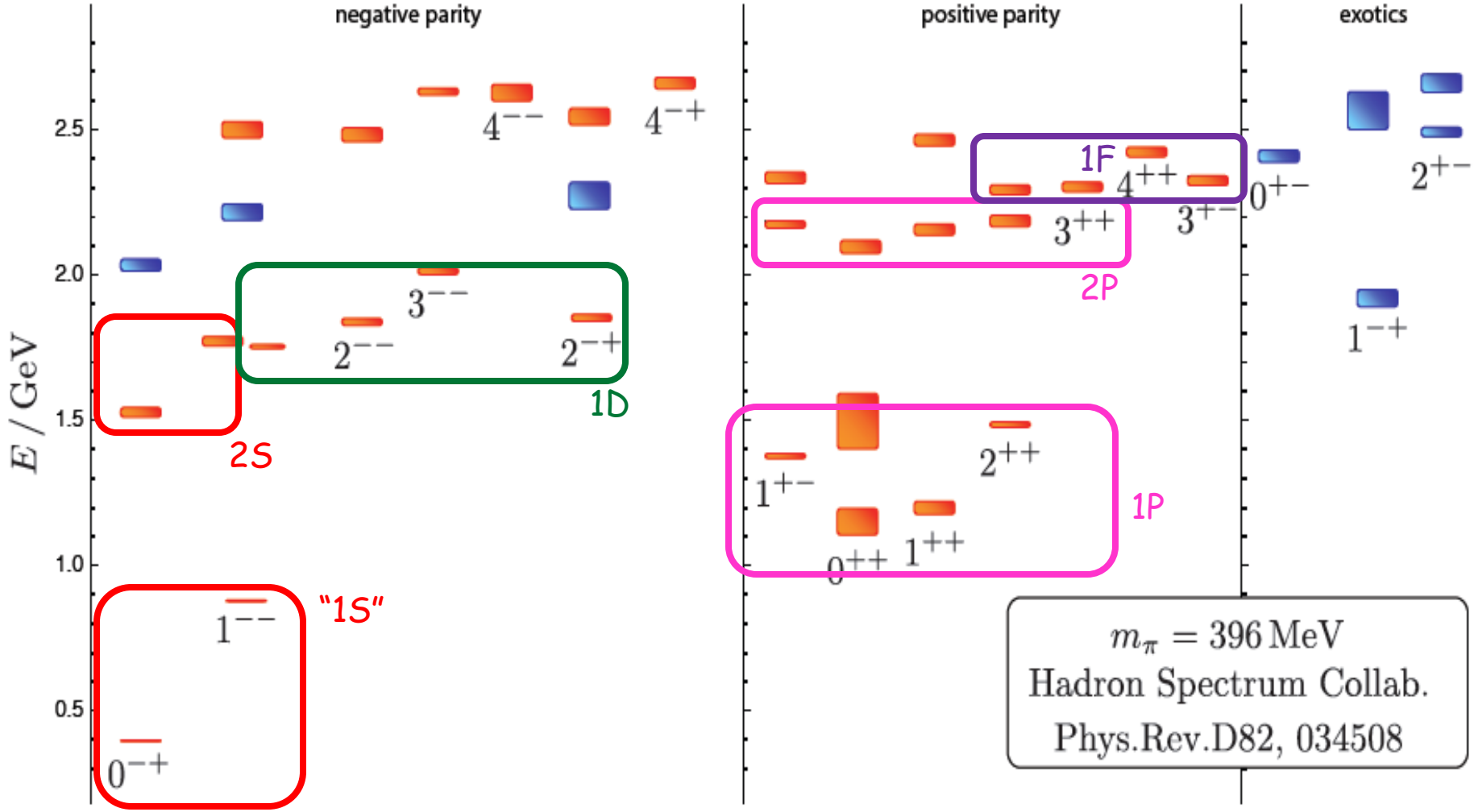
# Isovector meson spectrum



# Isovector meson spectrum

Identify spin and content of states with "spectral" overlaps

$$Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n} | \Phi_i^\dagger | 0 \rangle$$



# Isovector meson spectrum

Spectral overlap suggests "hybrid" (gluonic) components

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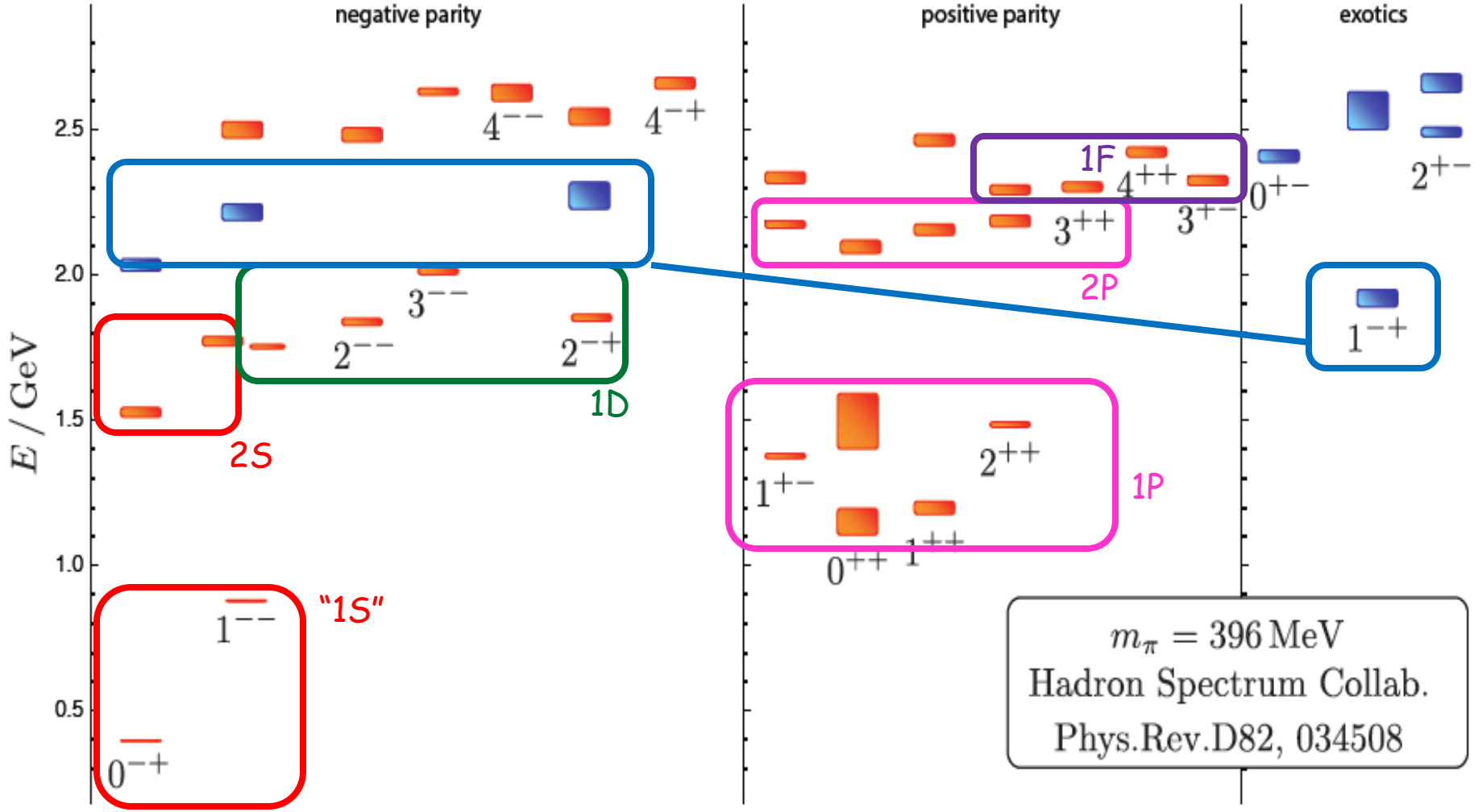
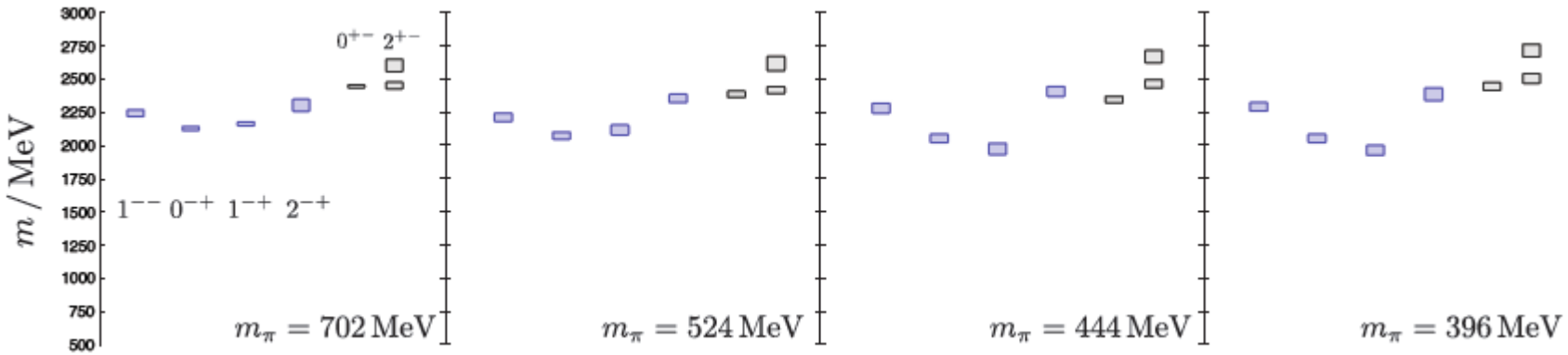


# Hybrid mesons

Over-population of states compared to non-rel. qqbar model
Excess forms a “super-multiplet” with exotic states



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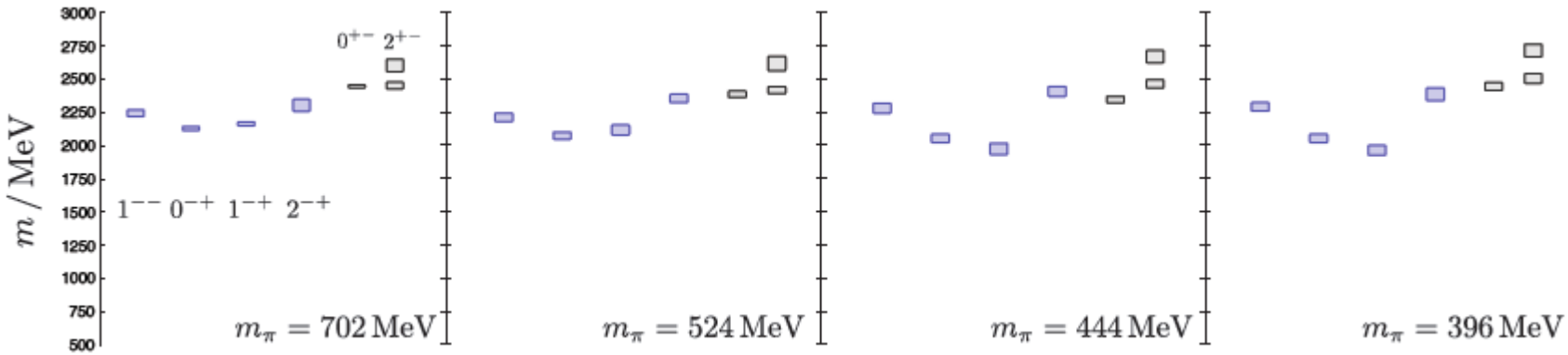



Picture of internal structure

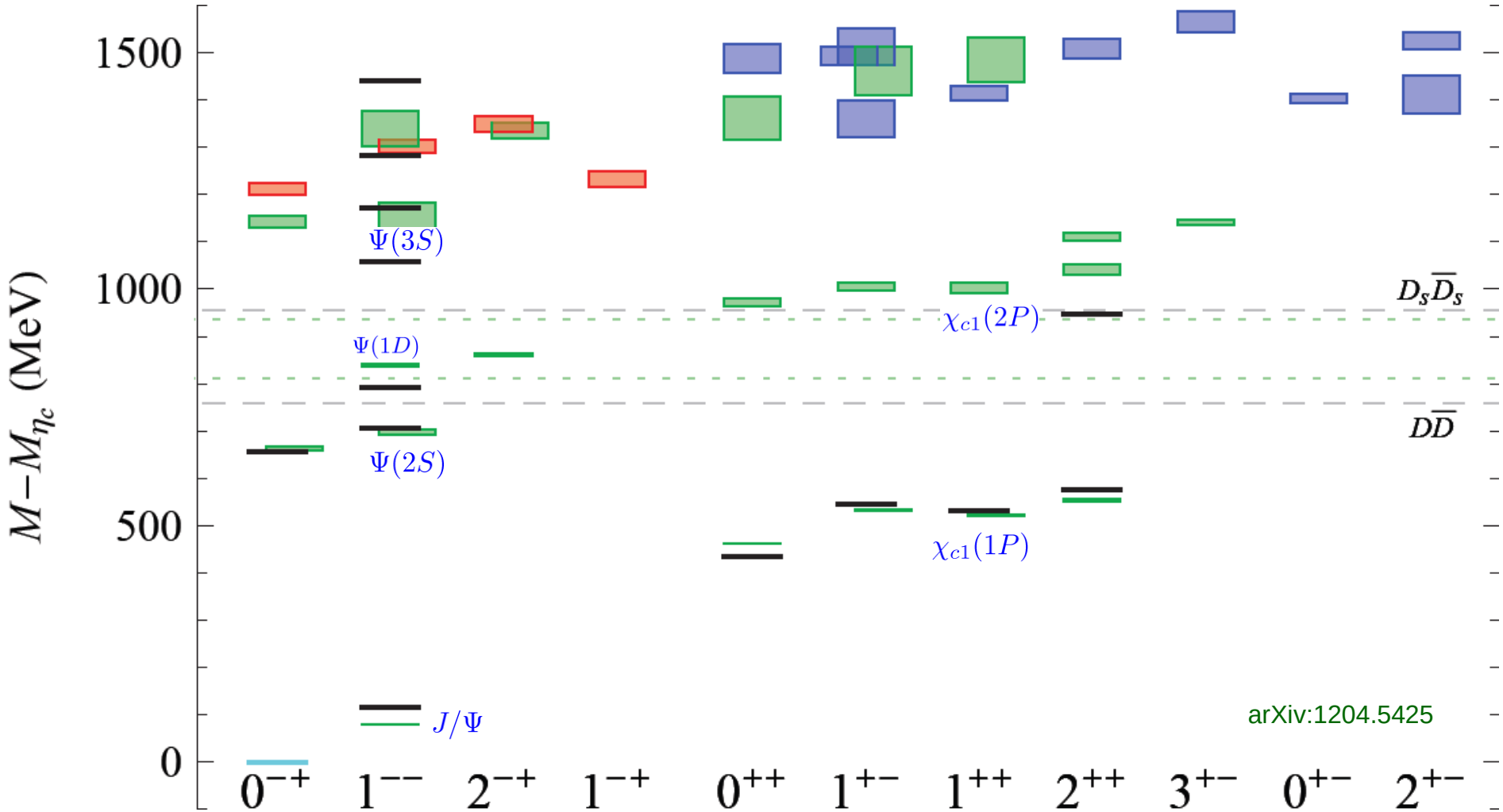
$G \sim 1_8^{+-}$

$q\bar{q}_{L=0}$ $(0,1,2)^{-+}, 1^{--}$ Lowest supermultiplet

$q\bar{q}_{L=1}$ $0^{+-}, (2^{+-})^2 \ldots$ 1st excited supermultiplet

Chromo-magnetic excitation

# Charmonium


$M - M_{\eta_c}$ (MeV)
1500
1000
500
0
$\Psi(3S)$
$\Psi(1D)$
$\Psi(2S)$
$J/\Psi$
$\chi_{c1}(2P)$
$\chi_{c1}(1P)$
$D_s\overline{D}_s$
$D\overline{D}$
arXiv:1204.5425
$0^{-+}$
$1^{--}$
$2^{-+}$
$1^{-+}$
$0^{++}$
$1^{+-}$
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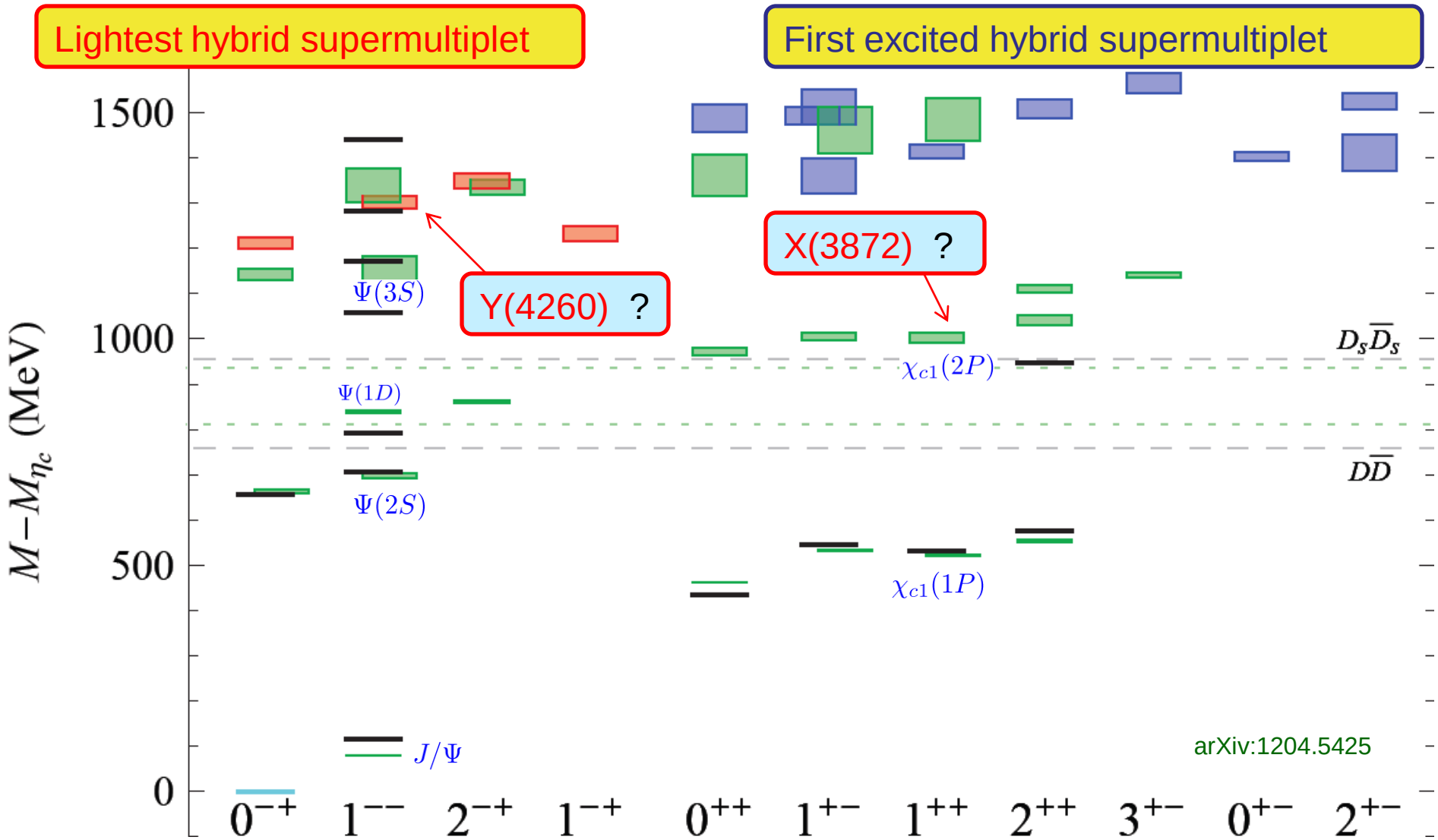
Lightest hybrid supermultiplet
First excited hybrid supermultiplet
1500
1000
500
0
$M - M_{\eta_c}$ (MeV)
$\Psi(3S)$
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Y(4260) ?
X(3872) ?
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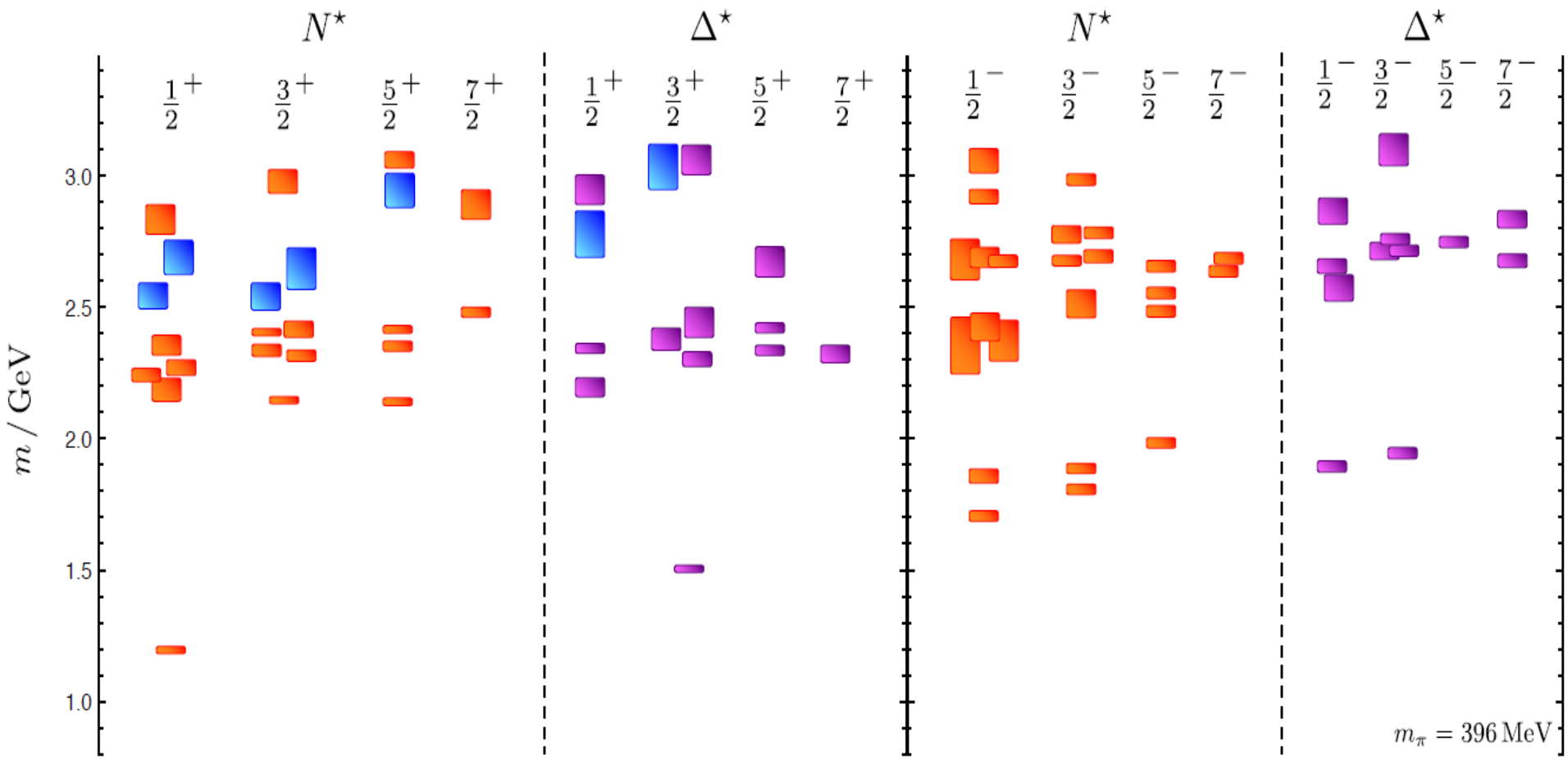
# Spin identified Nucleon & Delta spectrum

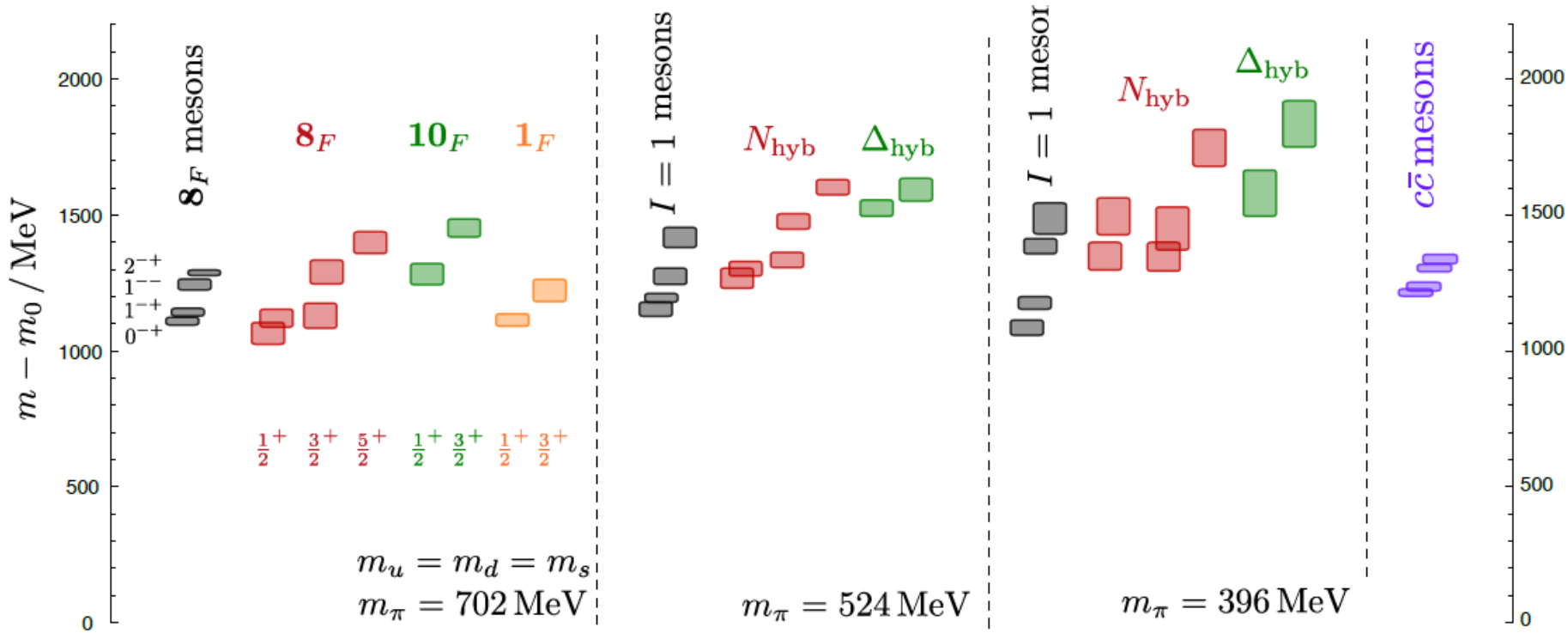
Full non-relativistic quark model counting
Plus additional levels consistent with color octet $1^+$ in 70-plet

arXiv:1104.5152, 1201.2349



# Hybrid hadrons

"subtract off" the quark mass




$$m_0 = \begin{cases} m_\rho & \text{light mesons} \\ m_N & \text{baryons} \\ m_{\eta_c} & \text{charmonium} \end{cases}$$

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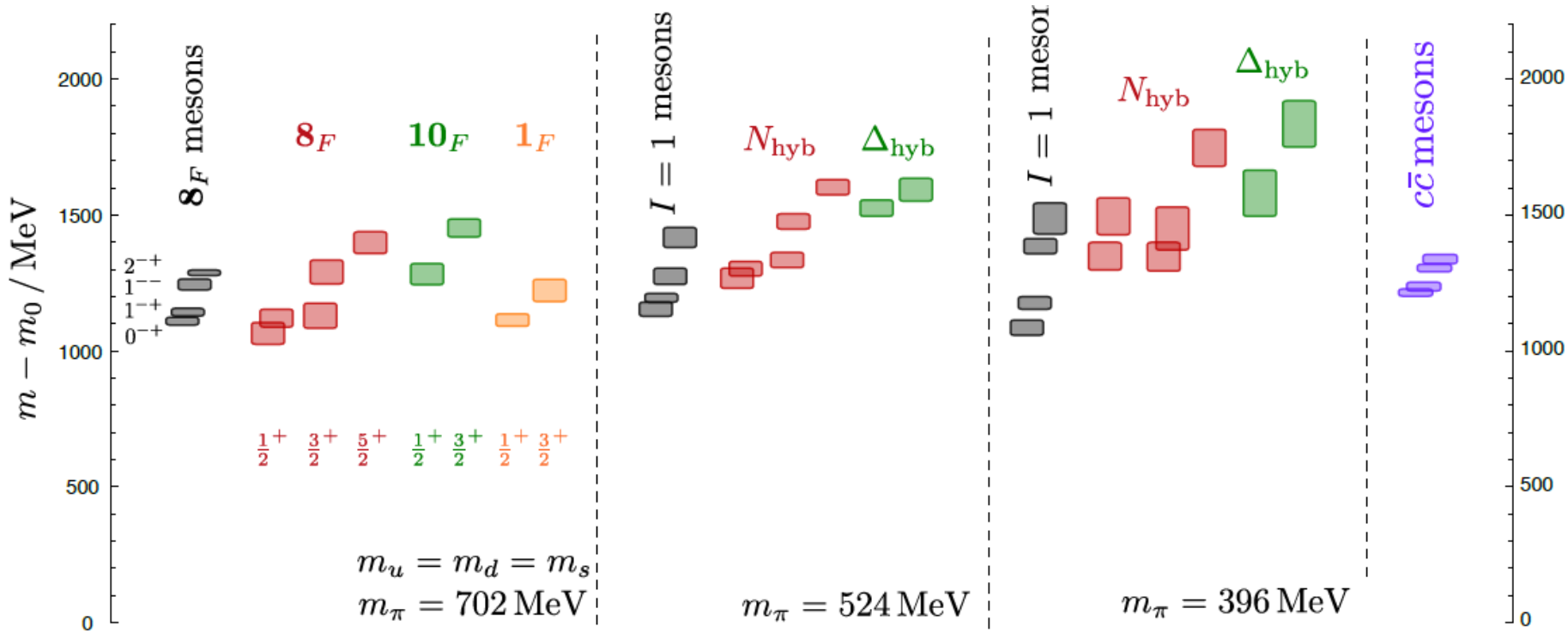


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Appears to be a single scale for gluonic excitations **~ 1.3 GeV**

Gluonic excitation transforming like a color **octet** with **$J^{PC} = 1^{+-}$**

# Scattering in finite volume field theory

The idea: 1 dim quantum mechanics

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Two spin-less bosons: $\psi$(x,y) = f(x-y) $\rightarrow$ f(z)

$$\left[-\frac{1}{m}\frac{d^2}{dz^2} + V(z)\right] f(z) = E\, f(z)$$

Solutions

$$f(z) \rightarrow \cos\left[k|z| + \delta(k)\right], \qquad E = k^2/m$$

Quantization condition when -L/2 < z < L/2

$$kL + 2\delta(k) = 0 \mod 2\pi$$

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Same physics in 4 dim version (but messier)
Provable in a QFT (and relativistic)

# Summary & prospects

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First picture of highly excited spectrum from lattice QCD

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- Extra bits interpreted as hybrid states with color octet (magnetic) structure
- Electric field structure higher in energy
- Add multi-particle ops $\rightarrow$ spectrum becomes denser

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Optimistic: see confluence of methods (an "amplitude analysis")
- Develop techniques concurrently with decreasing pion mass