Hadron Spectroscopy from Lattice QCD

Robert Edwards
Jefferson Lab

JLab Users Group Meeting 2012

under the auspices of the Hadron Spectrum Collaboration
Hadron Spectroscopy from Lattice QCD

Robert Edwards
Jefferson Lab

JLab Users Group Meeting 2012

under the auspices of the Hadron Spectrum Collaboration

Recent publications:
“Excited and exotic charmonium spectroscopy from lattice QCD”, accepted PRD, 1204.5425
“S and D-wave phase shifts in isospin-2 pi pi scattering from lattice QCD”, accepted PRD, 1201.2349
“Hybrid baryons”, PRD85, 1201.2349
“Helicity operators for mesons in flight”, PRD85, 1107.1930
“Lightest hybrid meson supermultiplet”, PRD84, 1106.5515
“Excited state baryon spectroscopy”, PRD84, 1104.5152
“Isoscalar meson spectroscopy”, PRD83, 1102.4299
“Phase shift of isospin-2 scattering”, PRD83, 1011.6352
“Toward the excited meson spectrum”, PRD82, 1004.4930
“Highly excited and exotic meson spectrum”, PRL103, 0909.0200
What is a gauge theory?

Maxwell’s eqns: field strength tensor and vector potentials

\[ \vec{B} = \nabla \times \vec{A} \quad \vec{E} = \frac{\partial}{\partial x_0} \vec{A} - \nabla A_0 \]

\[
F_{\mu\nu} = \begin{bmatrix}
0 & E_1 & E_2 & E_3 \\
-E_1 & 0 & B_3 & -B_2 \\
-E_2 & -B_3 & 0 & B_1 \\
-E_3 & B_2 & -B_1 & 0 \\
\end{bmatrix} = \frac{\partial}{\partial x_\mu} A_\nu(x) - \frac{\partial}{\partial x_\nu} A_\mu(x)
\]

Action

\[
S = \int d^4x \frac{1}{4} \sum_{\mu,\nu} F_{\mu\nu} F^{\mu\nu} = \int d^4x \frac{1}{2} (\vec{E}^2 + \vec{B}^2)
\]
**QCD**

Dirac operator: $A_\nu$ (vector potential), $m$ (mass), $\gamma_\nu$ (4x4 matrices)

$$D(A, m) = -\sum_\mu \gamma_\mu D_\mu + m; \quad D_\mu = \frac{\partial}{\partial x_\mu} + igA_\mu(x)$$

**Observables**

$$\langle \mathcal{O} \rangle = \int dA(x) \mathcal{O}(A) e^{-S} \det(D(A, m))$$

**QCD:** Vector potentials now 3x3 complex matrices (SU(3))
QCD

Dirac operator: \( A_\nu \) (vector potential), \( m \) (mass), \( \gamma_\nu \) (4x4 matrices)

\[
D(A, m) = -\sum_\mu \gamma_\mu D_\mu + m; \quad D_\mu = \frac{\partial}{\partial x_\mu} + igA_\mu(x)
\]

Observables

\[
\langle \mathcal{O} \rangle = \int dA(x) \mathcal{O}(A) \ e^{-S} \det(D(A, m))
\]

QCD: Vector potentials now 3x3 complex matrices (SU(3))

Lattice QCD: finite difference
QCD

Dirac operator: \( A_\nu \) (vector potential), \( m \) (mass), \( \gamma_\nu \) (4x4 matrices)

\[
D(A, m) = -\sum_\mu \gamma_\mu D_\mu + m; \quad D_\mu = \frac{\partial}{\partial x_\mu} + igA_\mu(x)
\]

Observables

\[
\langle \mathcal{O} \rangle = \int dA(x) \mathcal{O}(A) e^{-S} \det(D(A, m))
\]

QCD: Vector potentials now 3x3 complex matrices (SU(3))

Lattice QCD: finite difference

Lots of "flops/s"
Harness GPU-s
Patterns in hadron spectrum

Observed meson state flavor & $J^{PC}$ systematics suggest $q\bar{q}$

$q\bar{q}[S, L] \rightarrow (J = L \otimes S)^{PC}$, $P = (-1)^{L+1}$, $C = (-1)^{L+S}$

`Exotic' quantum numbers

<table>
<thead>
<tr>
<th>0---</th>
<th>0+-</th>
<th>1-+</th>
<th>2++</th>
</tr>
</thead>
</table>

`constituent quarks'
Patterns in hadron spectrum

Observed meson state flavor & $J^{PC}$ systematics suggest $q\bar{q}$

\[ q\bar{q}[S, L] \rightarrow (J = L \otimes S)^{PC}, \quad P = (-1)^{L+1}, C = (-1)^{L+S} \]

`constituent quarks’

`Exotic’ quantum numbers

\[
\begin{array}{cccc}
0^{-+} & 0^{++} & \\
1^{--} & 1^{+-} & 1^{++} & 1^{+-} & 1^{++} \\
2^{--} & 2^{-+} & 2^{++} & \\
\vdots & \\
0^{--}, 0^{+-}, 1^{+-}, 2^{+-} & \\
\end{array}
\]

Could excited gluonic fields play a role – hybrid mesons $q\bar{q}G$ ?

Possibly exotic $J^{PC}$ and extra `non-exotic’ states
**Patterns in hadron spectrum**

Observed meson state flavor & $J^{PC}$ systematics suggest $q\bar{q}$

$\frac{q\bar{q}[S,L]}{J = L \otimes S}^{PC}$, $P = (-1)^{L+1}$, $C = (-1)^{L+S}$

`constituent quarks`

`Exotic` quantum numbers

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$C^P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0--</td>
<td>0++</td>
</tr>
<tr>
<td>1--</td>
<td>1++</td>
</tr>
<tr>
<td>1+-</td>
<td>1++</td>
</tr>
<tr>
<td>2--</td>
<td>2++</td>
</tr>
<tr>
<td>2++</td>
<td></td>
</tr>
</tbody>
</table>

Could excited gluonic fields play a role – *hybrid* mesons $q\bar{q}G$?

Possibly exotic $J^{PC}$ and extra `non-exotic` states

Constituent quark picture $qqq$ predicts rich baryon spectrum not all experimentally observed

No exotic $J^P$ in *hybrid* baryons $qqqG$

However, might lead to extra states
Hybrid meson models

With minimal quark content, $q\bar{q}G$, gluonic field can in a color singlet or octet.
Hybrid meson models

With minimal quark content, $q\bar{q}G$, gluonic field can in a color singlet or octet

- `constituent' gluon in S-wave

$G \sim 1_{8}^{--}$

$q\bar{q}L=0$

$(0, 1, 2)^{++}, 1^{+-}$

$q\bar{q}L=1$

$0^{--}, (1^{-+})^{3}, 3^{-+} \ldots$
Hybrid meson models

With minimal quark content, $q\bar{q}G$, gluonic field can in a color singlet or octet

- `constituent' gluon in S-wave
  \[ G \sim 1^- \]
  \[
  \begin{align*}
  q\bar{q}_L &= 0 \\
  q\bar{q}_L &= 1 \\
  (0, 1, 2)^{++}, 1^{+-} \\
  0^{--}, (1^{+-})^3, 3^{--} \ldots
  \end{align*}
  \]

- Bag model
  \[ G \sim 1^+ \]
  \[
  \begin{align*}
  q\bar{q}_L &= 0 \\
  q\bar{q}_L &= 1 \\
  (0, 1, 2)^{-+}, 1^{--} \\
  0^{+-}, (2^{-+})^2 \ldots
  \end{align*}
  \]
Hybrid meson models

With minimal quark content, $q\bar{q}G$, gluonic field can in a color singlet or octet

- `constituent' gluon in S-wave

  \[ G \sim 1_8^{--} \quad q\bar{q}L=0 \quad (0, 1, 2)^{++}, 1^{+-} \]
  \[ q\bar{q}L=1 \quad 0^{--}, (1^{+-})^3, 3^{--} \ldots \]

- bag model

  \[ G \sim 1_8^{++} \quad q\bar{q}L=0 \quad (0, 1, 2)^{-+}, 1^{-+} \]
  \[ q\bar{q}L=1 \quad 0^{+-}, (2^{-+})^2 \ldots \]

- `constituent' gluon in P-wave

- flux-tube model

  \[ (0, 1, 2)^{-+}, 1^{--}, (0, 1, 2)^{+-}, 1^{++} \]
Hybrid baryon models

Minimal quark content, \(qqqG\), gluonic field can in color singlet, octet or decuplet

Now must take into account \textit{permutation} symmetry of quarks and gluonic field
Hybrid baryon models

Minimal quark content, \(qqqG\), gluonic field can in color singlet, octet or decuplet.

Now must take into account \textit{permutation} symmetry of quarks and gluonic field.

- Bag model

\[
G \sim 1^+_8, \quad (N\frac{1}{2}^+)^2, \quad (N\frac{3}{2}^+)^2, \quad (\Delta\frac{1}{2}^+), \quad (\Delta\frac{3}{2}^+), \quad (N\frac{5}{2}^+)
\]
Hybrid baryon models

Minimal quark content, $qqqG$, gluonic field can in color singlet, octet or decuplet

Now must take into account *permutation* symmetry of quarks and gluonic field

- **bag model**
  \[ G \sim 1^+_\text{gc} \]
  \[ (N^{1^+}_{\frac{1}{2}})^2, (N^{3^+}_{\frac{3}{2}})^2, (\Delta^{1^+}_{\frac{1}{2}}), (\Delta^{3^+}_{\frac{3}{2}}), (N^{5^+}_{\frac{5}{2}}) \]

- **flux-tube model**
  \[ (N^{1^+}_{\frac{1}{2}})^2, (N^{3^+}_{\frac{3}{2}})^2, (\Delta^{1^+}_{\frac{1}{2}}), (\Delta^{3^+}_{\frac{3}{2}}), (\Delta^{5^+}_{\frac{5}{2}}) \]
Spectrum from variational method

Two-point correlator

\[ C_{ij}(t) = \langle 0 | \Phi_i(t) \Phi_j^\dagger(0) | 0 \rangle \]

\[ C_{ij}(t) = \sum_n e^{-E_n t} \langle 0 | \Phi_i(0) | n \rangle \langle n | \Phi_j^\dagger(0) | 0 \rangle \]

\[ Z_i^n \equiv \langle n | \Phi_i^\dagger | 0 \rangle \]
Spectrum from variational method

Two-point correlator

\[ C_{ij}(t) = \langle 0 | \Phi_i(t) \Phi_j^\dagger(0) | 0 \rangle \]

\[ C_{ij}(t) = \sum_n e^{-E_n t} \langle 0 | \Phi_i(0) | n \rangle \langle n | \Phi_j^\dagger(0) | 0 \rangle \]

Matrix of correlators

\[
C(t) = \begin{pmatrix}
\langle 0 | \Phi_1(t) \Phi_1^\dagger(0) | 0 \rangle & \langle 0 | \Phi_1(t) \Phi_2^\dagger(0) | 0 \rangle & \cdots \\
\langle 0 | \Phi_2(t) \Phi_1^\dagger(0) | 0 \rangle & \langle 0 | \Phi_2(t) \Phi_2^\dagger(0) | 0 \rangle & \cdots \\
\vdots & \vdots & \ddots
\end{pmatrix}
\]

\[ Z_i^n \equiv \langle n | \Phi_i^\dagger | 0 \rangle \]
Spectrum from variational method

Two-point correlator

\[ C_{ij}(t) = \langle 0 | \Phi_i(t) \Phi_j^\dagger(0) | 0 \rangle \]

\[ C_{ij}(t) = \sum_n e^{-E_n t} \langle 0 | \Phi_i(0) | n \rangle \langle n | \Phi_j^\dagger(0) | 0 \rangle \]

Matrix of correlators

\[
C(t) = \begin{pmatrix}
\langle 0 | \Phi_1(t) \Phi_1^\dagger(0) | 0 \rangle & \langle 0 | \Phi_1(t) \Phi_2^\dagger(0) | 0 \rangle & \cdots \\
\langle 0 | \Phi_2(t) \Phi_1^\dagger(0) | 0 \rangle & \langle 0 | \Phi_2(t) \Phi_2^\dagger(0) | 0 \rangle & \cdots \\
\vdots & \vdots & \ddots
\end{pmatrix}
\]

“Rayleigh-Ritz method”

Diagonalize:

- eigenvalues → spectrum
- eigenvectors → spectral “overlaps” \( Z_i^n \)
Spectrum from variational method

Two-point correlator

\[ C_{ij}(t) = \langle 0 | \Phi_i(t) \Phi_j^\dagger(0) | 0 \rangle \]

\[ C_{ij}(t) = \sum_n e^{-E_n t} \langle 0 | \Phi_i(0) | n \rangle \langle n | \Phi_j^\dagger(0) | 0 \rangle \]

\[ Z_i^n \equiv \langle n | \Phi_i^\dagger | 0 \rangle \]

Matrix of correlators

\[
C(t) = \begin{pmatrix}
    \langle 0 | \Phi_1(t) \Phi_1^\dagger(0) | 0 \rangle & \langle 0 | \Phi_1(t) \Phi_2^\dagger(0) | 0 \rangle & \cdots \\
    \langle 0 | \Phi_2(t) \Phi_1^\dagger(0) | 0 \rangle & \langle 0 | \Phi_2(t) \Phi_2^\dagger(0) | 0 \rangle & \cdots \\
    \vdots & \vdots & \ddots
\end{pmatrix}
\]

“Rayleigh-Ritz method”
Diagonalize:
  - eigenvalues → spectrum
  - eigenvectors → spectral “overlaps” \( Z_i^n \)

Each state optimal combination of \( \Phi_i \)

\[ \Omega^{(n)} = \sum_i \nu_i^{(n)} \Phi_i \]
Spectrum from variational method

Two-point correlator

\[ C_{ij}(t) = \langle 0| \Phi_i(t) \Phi_j^\dagger(0)|0 \rangle \]

\[ C_{ij}(t) = \sum_n e^{-E_n t} \langle 0| \Phi_i(0)|n \rangle \langle n| \Phi_j^\dagger(0)|0 \rangle \]

Matrix of correlators

\[
C(t) = 
\begin{pmatrix}
\langle 0| \Phi_1(t) \Phi_1^\dagger(0)|0 \rangle & \langle 0| \Phi_1(t) \Phi_2^\dagger(0)|0 \rangle & \cdots \\
\langle 0| \Phi_2(t) \Phi_1^\dagger(0)|0 \rangle & \langle 0| \Phi_2(t) \Phi_2^\dagger(0)|0 \rangle & \cdots \\
\vdots & \vdots & \ddots
\end{pmatrix}
\]

“Rayleigh-Ritz method”
Diagonalize:
- eigenvalues → spectrum
- eigenvectors → spectral “overlaps” \( Z_i^n \)

Each state optimal combination of \( \Phi_i \)

\[ \Omega^{(n)} = \sum_i \nu_i^{(n)} \Phi_i \]

Benefit: orthogonality for near degenerate states
Operators

Mesons: fermion bi-linears

\[ \psi \Gamma \psi \rightarrow 0, 1 \]
\[ \bar{\psi} \Gamma \vec{D} \psi \rightarrow 0, 1, 2 \]
\[ \bar{\psi} \Gamma \vec{D} \vec{D} \psi \rightarrow 0, 1, 2, 3 \]
\[ \bar{\psi} \Gamma \vec{D} \vec{D} \vec{D} \psi \rightarrow 0, 1, 2, 3, 4 \]

gauge-covariant derivatives

coupling \[ \langle 1m_1; 1m_2|L_{12}m_{12}\rangle \vec{D}_{m_1} \vec{D}_{m_2} \]

2 derivatives can give chromo B field \( 1^+ \)
Operators

Mesons: fermion bi-linears

\[ \psi \Gamma \psi \rightarrow 0, 1 \]
\[ \bar{\psi} \Gamma \hat{D} \psi \rightarrow 0, 1, 2 \]
\[ \bar{\psi} \Gamma \hat{D} \hat{D} \psi \rightarrow 0, 1, 2, 3 \]
\[ \bar{\psi} \Gamma \hat{D} \hat{D} \hat{D} \psi \rightarrow 0, 1, 2, 3, 4 \]

gauge-covariant derivatives

coupling \[ \langle 1m_1; 1m_2 | L_{12} m_{12} \rangle \hat{D}_{m_1} \hat{D}_{m_2} \]

2 derivatives can give chromo B field $1^+$

Baryons: three quarks

\[ \Phi^{JM} \leftarrow (CGC's)_{i,j,k} \left[ \hat{D}_i \right] \left[ \hat{D}_j \right] \left[ \psi \psi \psi \right]_k \]
\[ 1 \otimes 1 \otimes S \rightarrow \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} \]
Isovector meson spectrum

$m_\pi = 396$ MeV
Hadron Spectrum Collab.
Phys.Rev.D82, 034508
Isovector meson spectrum

Identify spin and content of states with “spectral” overlaps

\[ Z^n_i \equiv \langle n \mid \Phi^\dagger_i \mid 0 \rangle \]

negative parity

positive parity

exotics

\[ m_\pi = 396 \text{ MeV} \]

Hadron Spectrum Collab.

Phys.Rev.D82, 034508

Jefferson Lab

Thomas Jefferson National Accelerator Facility
Isovector meson spectrum

Spectral overlap suggests “hybrid” (gluonic) components

$$Z_i^n \equiv \langle n | \Phi_i^\dagger | 0 \rangle$$

$E / \text{GeV}$

- Negative parity
  - $2S$
  - $1D$
  - “1S”
  - $0^{--}$

- Positive parity
  - $1F$
  - $2P$
  - $1P$
  - $0^{++}$

Exotics

$m_\pi = 396 \text{ MeV}$

Hadron Spectrum Collab.

Phys.Rev.D82, 034508
Hybrid mesons

Over-population of states compared to non-rel. qqbar model
Excess forms a “super-multiplet” with exotic states
Hybrid mesons

Over-population of states compared to non-rel. qqbar model
Excess forms a “super-multiplet” with exotic states

Picture of internal structure

\[ G \sim 1^{++}_8 \]
\[ q\bar{q}_{L=0} \]
\[ (0, 1, 2)^{-+}, 1^{--} \]
\[ q\bar{q}_{L=1} \]
\[ 0^{+-}, (2^{+-})^2 \ldots \]

Lowest supermultiplet
1st excited supermultiplet
Chromo-magnetic excitation
Charmonium

\[ M - M_{\eta c} \text{ (MeV)} \]

- \[ \Psi(3S) \]
- \[ \Psi(1D) \]
- \[ \Psi(2S) \]
- \[ \chi_{c1}(2P) \]
- \[ \chi_{c1}(1P) \]
- \[ J/\Psi \]
- \[ D_s\bar{D}_s \]
- \[ D\bar{D} \]

arXiv:1204.5425

Thomas Jefferson National Accelerator Facility

Jefferson Lab
Charmonium

Lightest hybrid supermultiplet

First excited hybrid supermultiplet

$Y(4260)$

$X(3872)$

$\Psi(3S)$

$\Psi(1D)$

$\Psi(2S)$

$\chi_{c1}(2P)$

$\chi_{c1}(1P)$

$D_S\bar{D}_S$

$D\bar{D}$

$J/\Psi$

$M-M_{\eta_c}$ (MeV)

Jefferson Lab

Thomas Jefferson National Accelerator Facility

arXiv:1204.5425
Spin identified Nucleon & Delta spectrum

Full non-relativistic quark model counting
Plus additional levels consistent with color octet $1^+$ in 70-plet
Hybrid hadrons

“subtract off” the quark mass

$m - m_0 / \text{MeV}$

$m_u = m_d = m_s$
$m_\pi = 702 \text{ MeV}$

$m_\pi = 524 \text{ MeV}$

$m_\pi = 396 \text{ MeV}$

$m_0 = \begin{cases} 
  m_\rho & \text{light mesons} \\
  m_N & \text{baryons} \\
  m_\eta_c & \text{charmonium} 
\end{cases}$
Hybrid hadrons

“subtract off” the quark mass

Appears to be a single scale for gluonic excitations \( \sim 1.3 \text{ GeV} \)

Gluonic excitation transforming like a color octet with \( J^{PC} = 1^{+-} \)

\[ m_{\pi} = 524 \text{ MeV} \]
\[ m_{\pi} = 396 \text{ MeV} \]

\[ m_{\rho} = m_d = m_s \]
\[ m_\pi = 702 \text{ MeV} \]

\[ m_0 = \begin{cases} 
  m_\rho & \text{light mesons} \\
  m_N & \text{baryons} \\
  m_{\eta_c} & \text{charmonium}
\end{cases} \]
Scattering in finite volume field theory

The idea: 1 dim quantum mechanics
Scattering in finite volume field theory

The idea: 1 dim quantum mechanics

Two spin-less bosons: \( \psi(x,y) = f(x-y) \to f(z) \)

\[
\left[ -\frac{1}{m} \frac{d^2}{dz^2} + V(z) \right] f(z) = E f(z)
\]

Solutions

\[
f(z) \to \cos \left[ k|z| + \delta(k) \right], \quad E = \frac{k^2}{m}
\]

Quantization condition when \(-L/2 < z < L/2\)

\[
kL + 2\delta(k) = 0 \mod 2\pi
\]
Scattering in finite volume field theory

The idea: 1 dim quantum mechanics

Two spin-less bosons: $\psi(x,y) = f(x-y) \rightarrow f(z)$

Solutions

$$f(z) \rightarrow \cos [k|z| + \delta(k)], \quad E = \frac{k^2}{m}$$

Quantization condition when $-L/2 < z < L/2$

$$kL + 2\delta(k) = 0 \pmod{2\pi}$$

Same physics in 4 dim version (but messier)
Provable in a QFT (and relativistic)
Summary & prospects
Summary & prospects

First picture of highly excited spectrum from lattice QCD
• Broadly consistent with non-relativistic quark model
• Extra bits interpreted as hybrid states with color octet (magnetic) structure
• Electric field structure higher in energy
• Add multi-particle ops → spectrum becomes denser
Summary & prospects

First picture of highly excited spectrum from lattice QCD
- Broadly consistent with non-relativistic quark model
- Extra bits interpreted as hybrid states with color octet (magnetic) structure
- Electric field structure higher in energy
- Add multi-particle ops → spectrum becomes denser

Observe significant overlap of hybrid structure with ground levels
- Could be other observed consequences – like gluon distributions
Summary & prospects

First picture of highly excited spectrum from lattice QCD
• Broadly consistent with non-relativistic quark model
• Extra bits interpreted as hybrid states with color octet (magnetic) structure
• Electric field structure higher in energy
• Add multi-particle ops → spectrum becomes denser

Observe significant overlap of hybrid structure with ground levels
• Could be other observed consequences – like gluon distributions

Short-term plans: resonance determination!
• Lighter pion masses (230MeV available)
• Extract couplings in multi-channel systems (with π, η, K…)
Summary & prospects

First picture of highly excited spectrum from lattice QCD
- Broadly consistent with non-relativistic quark model
- Extra bits interpreted as hybrid states with color octet (magnetic) structure
- Electric field structure higher in energy
- Add multi-particle ops → spectrum becomes denser

Observe significant overlap of hybrid structure with ground levels
- Could be other observed consequences – like gluon distributions

Short-term plans: resonance determination!
- Lighter pion masses (230MeV available)
- Extract couplings in multi-channel systems (with \( \pi, \eta, K \ldots \))

Optimistic: see confluence of methods (an “amplitude analysis”)
- Develop techniques concurrently with decreasing pion mass