The Story of the Proton Spin
Status & Prospects

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June 1st, 2015
the cookies we crumble: fundamental questions driving spin physics

how do quarks and gluons carry the proton spin

\[ \Delta f(x) \equiv f_{\rightarrow}(x) - f_{\leftarrow}(x) \]

- what is the role of gluons and sea quarks
- what is the role of orbital angular momentum
- how does it compare to models/lattice QCD

reveal mechanisms behind transverse spin phenomena

- further develop TMD framework / evolution
- role of multi-parton correlations & their matching to TMD’s
- explore connections to unintegrated PDFs

what is the distribution of partons in the transverse plane

- physics of exclusive processes & generalized parton distributions
- possible access to quark and gluon angular momentum
- high-level connections of TMDs & GPDs to Wigner functions
the cookies we crumble: fundamental questions driving spin physics

how do quarks and gluons carry the proton spin

\[ \Delta f(x) = f_+(x) - f_-^+(x) \]

- what is the role of gluons and sea quarks
- what is the role of orbital angular momentum
- how does it compare to models/lattice QCD

reveal mechanisms behind transverse spin phenomena

- further development of TMD framework / evolution
- role of transverse spin in QCD and its matching to TMD’s
- integrated PDFs
- possible access to quark and gluon angular momentum
- high-level connections of TMDs & GPDs to Wigner functions

what is the distribution of partons in the transverse plane

see other talks in this meeting
topics to be covered

1. some helicity PDF and global analysis basics
2. global fits through the years, current status & issues
3. theoretical developments
4. future avenues
some basics
formally defined as matrix elements of bi-local operators on the light-cone, e.g.:

$$\Delta q(x) = \frac{1}{4\pi} \int dy^- e^{iy^- xP^+} \langle P, S | \bar{\Psi}_q(0, y^-, \bar{0}) \gamma^+ \gamma_5 \mathcal{F} \psi_q(0) | P, S \rangle$$

- non-perturbative but universal objects (through factorization theorem)
- not measurable quantities — need to be extracted from cross section data
- scale dependence is key prediction of pQCD (factorization again)

$$\frac{d}{d \ln Q^2} \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix} = \Delta \mathcal{P}[\alpha_s] \otimes \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix}$$

- PDF at \((x, Q_2)\) “knows about” PDF values in range \([x, 1]\) at smaller scales \(Q_1 < Q_2\) through evolution
why do we care?

- proton helicity sum rule

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + \Delta G + L_g
\]

- all quantities depend on scale \( Q \) & factorization scheme
- ignoring all the complications about choice of gauge, etc ...

- indirect handle on net OAM (\( Q^2 \)) once we know \( \Delta \Sigma \) and \( \Delta g \)

- \( \Delta f(x, Q^2) \) are fundamental building blocks of nucleon structure

- compare to attempts to model/compute nucleon structure theoretically
  - valence region and \( x \to 1 \) behavior
  - flavor structure, e.g., \( \Delta \bar{u} - \Delta \bar{d} \) large-\( N_C \), chiral quark model, meson cloud, ...
  - lattice QCD calculations
    - traditional: some lower moments
    - quasi parton densities: promising & very active field
      
      Ji; Ma, Qiu; Lin et al.; Alexandrou et al.; Liu et al., ...
guiding principle for theoretical description: **factorization**

e.g. **DIS**

\[
d\Delta\sigma = \sum_{f=q,\bar{q},g} \int dx \ \Delta f(x, Q^2) \ d\Delta\hat{\sigma} \gamma^* f(xP, \alpha_s(Q^2))
\]

universal PDF

**calculable in pQCD**

**essential: QCD corrections**

\[
d\Delta\hat{\sigma} = d\Delta\hat{\sigma}^{LO} + \alpha_s \ d\Delta\hat{\sigma}^{NLO} + \ldots
\]

to reduce theoretical uncertainties
tremendous exp. efforts in the past 30++ years

SLAC
... E142, E143, E154, E155, ...

JLab
Hall A, B, C
upcoming 12 GeV program

CERN
EMC, SMC, COMPASS

DESY
HERMES

BNL
PHENIX, STAR
polarized pp collisions up to 510 GeV

How to make the most of it?
### Global QCD Analysis: Synergy of All Probes

<table>
<thead>
<tr>
<th>Probe</th>
<th>Dominant LO Process</th>
<th>PDF Probed</th>
<th>x Range</th>
<th>Scale $^{2}$ [GeV$^{2}$]</th>
<th>Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIS</td>
<td>$\gamma^* q \rightarrow q$</td>
<td>$\Delta q + \Delta \bar{q}$</td>
<td>0.003 - 0.8</td>
<td>1 - 70</td>
<td>Only sum probed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta g$</td>
<td>0.003 - 0.8</td>
<td>1 - 70</td>
<td>Very clean</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>dg$_{\perp}$/dlnQ$^{2}$ needs large Q$^{2}$ range at fixed x</td>
</tr>
<tr>
<td>SIDIS</td>
<td>$\gamma^* q \rightarrow q$</td>
<td>$\Delta u, \Delta d, \Delta \bar{u}, \Delta \bar{d}$</td>
<td>0.005 - 0.5</td>
<td>1 - 60</td>
<td>Pion FF</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta s$</td>
<td>0.005 - 0.5</td>
<td>1 - 60</td>
<td>Kaon FF</td>
</tr>
<tr>
<td></td>
<td>$\gamma^* g \rightarrow q\bar{q}, c\bar{c}$</td>
<td>$\Delta g$</td>
<td>0.05 - 0.2</td>
<td>5 - 20</td>
<td>Statistics</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Theoretical Unc.</td>
</tr>
<tr>
<td></td>
<td>gg $\rightarrow$ gg</td>
<td>$\Delta g$</td>
<td>0.05 - 0.3</td>
<td>40 - 900</td>
<td>Very clean</td>
</tr>
<tr>
<td></td>
<td>qg $\rightarrow$ qg</td>
<td>$\Delta g$</td>
<td>0.02 - 0.2</td>
<td>4 - 100</td>
<td>Pion FF</td>
</tr>
<tr>
<td></td>
<td>$u_L \bar{d}_R \rightarrow W^+$</td>
<td>$\Delta u, \Delta \bar{d}$</td>
<td>0.05 - 0.4</td>
<td>$M_Z^2$</td>
<td>Very clean</td>
</tr>
<tr>
<td></td>
<td>$d_L \bar{u}_R \rightarrow W^-$</td>
<td>$\Delta \bar{u}, \Delta d$</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
anatomy of a global QCD analysis

**cross sections at NLO**

- Jager, MS, Vogelsang; de Florian;
- Bojak, MS; Riedl, Schafer, MS;
- Gordon, Vogelsang; Signer et al.;
- Hendlemez, Schafer, MS;
- Contogouris et al.; ....

**evolution kernels at NLO**

- NLO: Mertig, van Neerven; Vogelsang; ...
- NNLO: Moch, Vermaseren, Vogt 1409.5131

**obtain helicity PDFs through global $\chi^2$ optimization**

- model ansatz for pdfs with initial set of parameters
- evolve pdfs to relevant scale with DGLAP
- calculate observable and $\chi^2$
- $\chi^2$ minimum? yes no

**set of optimum parameters for assumed functional form**

**plus a prescription to estimate & propagate uncertainties**

**non-pert. inputs**

- e.g. frag. fcts.
- de Florian, Sassot, MS; ...

**novel techniques**

- e.g. in complex Mellin space
- Vogt et al.;
- MS, Vogelsang; ...

**resolution scale** $\mu$

- NLO: Mertig, van Neerven; Vogelsang; ...
- NNLO: Moch, Vermaseren, Vogt 1409.5131
# Overview of Recent Helicity PDF Fits @ NLO

<table>
<thead>
<tr>
<th>Latest Paper</th>
<th>Uncertainties</th>
<th>Features &amp; Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NNPDF</strong> 1406.5539</td>
<td>jets W's</td>
<td>pp data w/ reweighing method</td>
</tr>
<tr>
<td>Ball, Forte, Guffanti, Nocera, Rodolfi, Rojo</td>
<td>100 MC replicas</td>
<td></td>
</tr>
<tr>
<td><strong>DSSV</strong> 0904.3821 1404.4293</td>
<td>jets pions</td>
<td>Lagrange mult. (Hessian)</td>
</tr>
<tr>
<td>de Florian, Sassot, MS, Vogelsang</td>
<td></td>
<td>pp data fitted</td>
</tr>
<tr>
<td><strong>JAM</strong> 1403.3355</td>
<td></td>
<td>fast Mellin method</td>
</tr>
<tr>
<td>Jimenez-Delgado, Accardi, Avakian, Melnitchouk, Sato, ...</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>LSS</strong> 1010.0574</td>
<td></td>
<td>Hessian</td>
</tr>
<tr>
<td>Leader, Sidorov, Stamenov</td>
<td></td>
<td>large x / JLab region</td>
</tr>
<tr>
<td><strong>BBS</strong> 1502.02517 1408.7057</td>
<td></td>
<td>Hessian</td>
</tr>
<tr>
<td>Bourrely, Buccella, Soffer</td>
<td></td>
<td>higher twist, Δs</td>
</tr>
<tr>
<td><strong>BB</strong> 1010.3113</td>
<td></td>
<td>Hessian</td>
</tr>
<tr>
<td>Blumlein, Bottcher</td>
<td></td>
<td>statistical approach</td>
</tr>
<tr>
<td></td>
<td></td>
<td>unpol/pol simult. fit</td>
</tr>
<tr>
<td><strong>GRSV</strong> 9508347 0011215</td>
<td></td>
<td>Hessian</td>
</tr>
<tr>
<td>Gluck, Reya, MS, Vogelsang</td>
<td></td>
<td>α_s, higher twist</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1st NLO analysis 1995</td>
</tr>
</tbody>
</table>
extra challenges compared to unpolarized PDF fits

- rather limited $x$-$Q^2$ coverage
  - difficult to get $\Delta g$ from scaling violations
    - need to rely on “direct probes” for $\Delta g$
  - need to use data down to $Q^2 = 1$ GeV$^2$
    - applicability of pQCD? higher twist?
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- no neutrino DIS data
  - no quark/anti-quark separation from DIS
    - largely rely on SIDIS for flavor separation
    - need fragmentation functions to analyze data

extra source of uncertainties

new data on FFs: ALICE, BABAR, BELLE, COMPASS, HERMES
extra challenges compared to unpolarized PDF fits

- **rather limited x-Q^2 coverage**
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- **sum rules on shaky (?) grounds**
  - 1st moments of non-singlet combinations <-> hyperon decays
    - constraint on unmeasured small-x behavior of Δs and ΔΣ
    - doubts, however, on applicability of SU(3) relation
      Savage, Walden; ...; Bali et al. 1112.3354 (lattice)

- extra source of uncertainties
  - new data on FFs: ALICE, BABAR, BELLE, COMPASS, HERMES

\[ F + D = 1.2701 \pm 0.0025 \]
\[ 3F - D = 0.585 \pm 0.025 \]
\[ \Delta s(x) ?? \]
global fits through the years

current status & remaining issues
emerging picture from 1st global analysis back in ’08

de Florian, Sassot, MS, Vogelsang; PRL 101 (2008) 072001; PR D80 (2009) 034030

well constrained total quark densities

$x \rightarrow 1$ behavior to be determined

gluon small (node?) in $x$-region constrained by data

large errors though!

indications for non-trivial sea quark polarizations

$\Delta \bar{u} > 0$, $\Delta \bar{d} < 0$

surprising strangeness polarization sizable SU(3) breaking?
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\(\Delta \bar{u} > 0\)
\(\Delta \bar{d} < 0\)

surprising strangeness polarization sizable SU(3) breaking?

gluon small (node?) in x-region constrained by data large errors though!
some hints at non-trivial sea polarizations

\[ \Delta \bar{u} > 0 \quad \Delta \bar{d} < 0 \]

- similar size but opposite sign than in unp. fits
- uncertainties still large
- driven by SIDIS data on \( \pi^+ \) and \( \pi^- \)
- connection to model calculations

Thomas, Signal, Cao; Holtmann, Speth, Fassler; Diakonov, Polyakov, Weiss; Schäfer, Fries; Kumano; Wakamatsu; Gluck, Reya; Bourrely, Soffer, ...
**some hints at non-trivial sea polarizations**

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  Diakonov, Polyakov, Weiss; Schafer, Fries;
  Kumano, Wakamatsu; Gluck, Reya; Bourrely, Soffer, ...

latest COMPASS data prefer smaller net sea polarizations though

- changes well within uncertainties
- smaller $\Delta \bar{u}(x) - \Delta \bar{d}(x)$

---

**WARNING**

BEWARE OF $W^{+/−}$ DATA

$Q^2 = 10 \text{ GeV}^2$
• $\Delta s$ always *assumed* to be negative in fits to DIS data …

sign change & small uncertainties entirely driven by

$$3F - D = 0.585 \pm 0.025$$

leads to

$$\int_0^1 dx [\Delta s + \Delta \bar{s}](x) \approx -0.1$$

• new SIDIS data at smaller $x$ compatible

**remarks/caveats:**

• knowledge of parton-to-kaon FFs

• do we really now unpolarized strangeness?

• can we trust 3F-D constraint?

  Lipkin; Zhu, Puglia, Ramsey-Musolf; Savage, Walden; …

• lattice also finds smallish strangeness

$$\int_0^1 dx [\Delta s + \Delta \bar{s}](x) \approx -0.020(10)(1)$$

<–> large breaking of 3F-D relation

• recall old GRSV “valence scenario” with $\Delta s = 0$!
large $x$ behavior of helicity PDFs

several different theoretical expectations for $x \rightarrow 1$

Close; Isgur; Close, Melnitchouk; Cloet, Bentz, Thomas; Roberts, Holt, Schmidt; …

\[
\frac{\Delta u^+ + \Delta \bar{u}}{u + \bar{u}}
\]

\[
\frac{\Delta d^+ + \Delta \bar{d}}{d + \bar{d}}
\]

compilation by E. Nocera

arXiv:1410.7290
large $x$ behavior - cont'd

to settle this, measure $A_1^p$ & $A_1^n$ at large $x$ —> JLab-12 program

<table>
<thead>
<tr>
<th>Model</th>
<th>$A_1^p$</th>
<th>$A_1^n$</th>
<th>Model</th>
<th>$A_1^p$</th>
<th>$A_1^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(6)</td>
<td>0</td>
<td>5/9</td>
<td>NJL</td>
<td>0.35</td>
<td>0.77</td>
</tr>
<tr>
<td>RCQM</td>
<td>1</td>
<td>1</td>
<td>DSE (realistic)</td>
<td>0.17</td>
<td>0.59</td>
</tr>
<tr>
<td>QHD ($\sigma_{1/2}$)</td>
<td>1</td>
<td>1</td>
<td>DSE (contact)</td>
<td>0.34</td>
<td>0.88</td>
</tr>
<tr>
<td>QHD ($\psi_{p}$)</td>
<td>1</td>
<td>1</td>
<td>pQCD</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>NNPDF ($x = 0.7$)</td>
<td>0.41 ± 0.31</td>
<td>0.75 ± 0.07</td>
<td>NNPDF ($x = 0.9$)</td>
<td>0.36 ± 0.61</td>
<td>0.74 ± 0.34</td>
</tr>
</tbody>
</table>

compilation by E. Nocera
arXiv:1410.7290
RHIC sees first evidence for non zero $\Delta g$

major contributor to proton spin - - - perhaps little room left for OAM
Δg: DSSV '14 adds latest RHIC jet and π⁰ data

de Florian, Sassot, MS, Vogelsang; PRL 113 (2014) 012001

- jet data pull on Δg
- relevant x-range:

\[ \langle x \rangle \]

\[ \begin{array}{c}
0.1 \\
10 \\
30 \\
\end{array} \]

\[ \begin{array}{c}
\pi^0 \text{'s well described by DSSV '08} \\
\text{probes lower } x \text{ values:} \\
\end{array} \]

good combined fit of STAR & PHENIX data complementarity!
resulting DSSV ‘14 gluon

**functional form in DSSV:**

\[ x \Delta g(x, \mu_0^2) = N_g x^{\alpha_g} (1 - x)^{\beta_g} (1 + \eta_g x^{\kappa_g}) \]

- "famous" node in DSSV’08
- \( \eta_g \) essential for good fit
- DSSV ‘14 \( \Delta g \) much larger and positive
- \( \eta_g \) only to produce small \( x \) variations

uncertainty 90% C.L.
towards the first moment of $\Delta g$
towards the first moment of $\Delta g$
towards the first moment of $\Delta g$

before Run 9

after Run 9

$\Delta g^1, |x| > 1 < 1$

$\Delta g^1, |x| > 1 > 0$

gluon contribution down to $x \sim 10^{-3}$ tends to be positive and smaller than 1
Comparison DSSV vs NNPDF
The graph compares the NNPDF 1.0 and 1.1, DSSV, and DSSV update PDFs with respect to the positivity bound. The x-axis represents the scale $Q^2$ in GeV$^2$, while the y-axis shows the variable $x\Delta g$. The shaded regions indicate the uncertainty bands for different PDF sets.
no $\pi^0$ data in NNPDF fit

comparison DSSV vs NNPDF
best fits within each others bands

no π^0 data in NNPDF fit

comparison DSSV vs NNPDF
best fits within each others bands

no $\pi^0$ data in NNPDF fit

comparable uncertainties in x-region dominated by jets

comparison DSSV vs NNPDF

\begin{itemize}
\item \textcolor{red}{NNPDF 1.1}
\item \textcolor{green}{NNPDF 1.0}
\item \textcolor{blue}{DSSV update}
\item \textcolor{black}{DSSV}
\end{itemize}
some recent theoretical developments
towards NNLO accuracy

- major achievement: 

\[
\Delta P_{ij} = a_s \Delta P_{ij}^{(0)} + a_s^2 \Delta P_{ij}^{(1)} + a_s^3 \Delta P_{ij}^{(2)}
\]

1977
Ahmed, Ross; Altarelli, Parisi, ...

1995
Mertig, van Neerven; Vogelsang

- obtained from calculating several fixed N moments in graviton-exchange DIS, e.g.:

\[
\Delta P_{gq}^{(2)}(N = 25) = -C_A^3 \frac{1890473255283802937678830745102921869938637}{23^{4}19^{4}17^{4}13^{5}11^{4}7^{4}5^{10}3^{5}2^{12}} + \ldots
\]

- full N dependence reconstructed from fixed moments through some sophisticated math

- x space expressions from algebraic procedure:

  harmonic sums in N space  \(\leftrightarrow\) harmonic polylog's in x space

- requires extra care with \(\gamma_5\) \(\rightarrow\) finite renormalization to avoid problems with non-singlet axial currents and Bjorken sum rule

\[ \Delta P_{gg}^{(2)}(x) = 16 C_A^2 \left( 4 \Delta P_{gg}(x) - 11/8 \delta_{zz}^2 + H_{-3,0} - 4 H_{-2,1} - 2 H_{-2,1,0} + 3 H_{-2,2,0} + 9/2 H_{-2,1,0,0} + 3 H_{-1,1,0,0} - 2 H_{-1,1,0} + 4 H_{-1,1,1,0} + 5/4 H_{0,0,0} + 2 H_{0,0,0} - H_{0,0,0} - 1 H_{0,0,0} - H_{0,0,0} - 1/2 H_{0,0,0} + 2 H_{0,0,0} + 11/24 H_{0,0,0} + 67/36 \right) \]

\[ + 4 \Delta P_{gg}(x) (245/96 - 3/40 \delta_{zz}^2 - H_{-3,0} + 3/2 H_{-2,1} + H_{-2,1,0} - H_{-2,1,0} - H_{-2,2,0} - 7/4 H_{-2,1} + 2 H_{0,0,0} + 2 H_{0,0,0} - 3/2 H_{1,1,0} - 3/2 H_{1,1,0} + 2 H_{1,1,0}) \]

\[ + 2 H_{1,1,0} + 2 H_{2,1,0} + 2 H_{1,1,0} - H_{2,1,0} + 5/2 H_{2,1,0} + 2 H_{2,1,0} + 5/2 H_{2,1,0} + 2 H_{3,1} + 2 H_{4,1} + 11/2 H_{4,0,0} + 11/24 H_{0,0,0} + 11/24 H_{0,0,0} \]

\[ - 67/36 (2 \delta_{zz} - H_{0,0,0} - 2 H_{2,1,0} + 2 H_{2,1,0} + 1 H_{2,1,0} + 1/3 (72 - 185 x - 22 x^2) H_{0,0,0} + 1/3 (32 - 161 x - 11 x^2) H_{0,0,0} - 4 (1 - 6 x) H_{3,0} - 1/6 (312 - 393 x - 55 x^2) H_{0,0,0} + 1/6 (x - 5) (5579/18 + 4 H_{-2,1,0} + 8 H_{-2,1,0} + 12 H_{-2,1,0} + 24 H_{-2,1,0} + 37 H_{0,0,0} + 1/18 H_{1,1,0} + 1/5 (43 + 33 x) H_{0,0,0} - 8 (1 + x) H_{0,0,0} - 2 (11 + 13 x) H_{0,0,0} + 1 (1 + x) (21 H_{1,1,0} - 25/2 H_{1,1,0} + 65 H_{1,1,0} + 23 H_{1,1,0} - 4 H_{2,1,0} + 10 H_{2,1,0} + 16 H_{2,1,0} + 26 H_{2,1,0} - 215/3 H_{1,1,0}) - 1/9 (74 - 97 x) H_{2,1,0} + 1/3 (77 - 115 x) H_{2,0,0} + 1/3 (40 - 185 x - 11 x^2) H_{0,0,0} + 1/9 (571 + 97 x) H_{0,0,0} + 1/3 (158 - 87 x + 11 x^2) H_{0,0,0} + 1/12 (1019 - 1489 x) H_{0,0,0} + 1/216 (24625 + 40069 x) H_{0,0,0} - 11/6 (x^{-1} + x^2) H_{1,1,0} + 28 H_{0,0,0} - 11/2 (x^{-1} + x^2) H_{1,1,0} + 2 (3/3 H_{-1,0,0} - 4/3 H_{-2,1,0}) + 6 (1 - x) (79/32 + 5 x^2 + 67/6 H_{0,0,0} + 1/6 H_{0,0,0} - 2 H_{0,0,0} + 11/24 H_{0,0,0}) \]
phenomenological relevance

- impact on evolution of helicity PDFs:
  
  \[ \frac{d \Delta q_S}{d \ln \mu^2} \]
  
  \[ \frac{d \Delta g}{d \ln \mu^2} \]

- crucial to match expected precision of future EIC DIS data

Moch, Vermaseren, Vogt arXiv:1409.5131
• at LO find:

\[ P_{ij}^{(0)} - \Delta P_{ij}^{(0)} \sim (1 - x)^2 + \ldots \]

in accordance with large \( x \) arguments by Brodsky, Burkardt, Schmidt

• at NLO: all fine and well, except for

\[ P_{gq}^{(1)} - \Delta P_{gq}^{(1)} \sim \log(1 - x) + \text{const} + \ldots \]

- can be fixed by simple scheme transformation to yield

\[ P_{gq}^{(1)} - \Delta P_{gq}^{(1)} \sim (1 - x)^2 + \ldots \]

- phenomenological relevance for analysis of large \( x \) JLab-12 data?

- can be extended to NNLO
• going back to $\Delta q/q \rightarrow 1$:

$\Delta d(x)/d(x) \sim (1 - x)^\beta \log^2(1 - x)$

fit containing

inspired by OAM arguments a la Avakian, Brodsky, Deur, Yuan

• however: pQCD develops same type of logarithms in each order

  - can be systematically resummed to all orders
  - related to soft gluons $\rightarrow$ log’s the same in unpol. and pol. case
**main idea:** inhibited radiation near exclusive boundary; IR cancellation leaves large logs

- example DIS: \( C_q \sim e_q^2 \left[ \delta(1-x) + a_s C_F \left\{ (1 + x^2) \left( \frac{\ln(1-x)}{1-x} \right)_+ - \frac{3}{2} \frac{1}{(1-x)_+} + \ldots \right\} \right] \)

\[
\frac{1}{2} \log^2(1-x) \frac{q(x)}{x} \text{ in convolution with PDFs}
\]

- well defined class of higher order corrections: \( a_s^k L^{2k}, a_s L^{2k-1}, \ldots \) at \( k \)th order

**terminology:** logs can be resummed to all orders (\( \rightarrow \) exponentiated)

<table>
<thead>
<tr>
<th>Resummation</th>
<th>Fixed Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>1</td>
</tr>
<tr>
<td>NLO</td>
<td>( \alpha_s L^2 )</td>
</tr>
<tr>
<td>NNLO</td>
<td>( \alpha_s^2 L^4 )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( N^k )LO</td>
<td>( \alpha_s^k L^{2k} )</td>
</tr>
</tbody>
</table>

\[ ... \]

\[ LL \quad NLL \quad NNLL \]
• often relevant for pheno. studies (mostly at lower energies but also at colliders)

• recent example: resummation for (SI) DIS spin asymmetries

Anderle, Ringer, Vogelsang 1304.1373

resummations should be included in analyses of JLab-12 data
fragmentation functions provide "analyzing power" for different quark flavors in SIDIS and hadron production in pp

- determine parton -> hadron FFs & uncertainties from global fit to $\pi^+$, $\pi^-$, $K^+$, $K^-$ yields in $e^+e^-$, DIS multiplicities, and pp

$\rightarrow$ DSSV uses DSS '07 FFs for SIDIS and RHIC π⁰ data


- good global description of all data sets
- only set of FFs which describes DIS multiplicities
- only set which constrains gluon FF through pp data

lots of new data after DSS’07 analysis:
e$^+e^-$: BaBar, Belle  DIS: COMPASS, HERMES; pp: ALICE, STAR, PHENIX

$\rightarrow$ DSS’14 update for pion FFs
kaon FF update in the making

arXiv:1410.6024
future avenues
near and long term
exciting new result: $W$ boson asymmetry

another key measurement at RHIC

neat idea: measure parity-violating single-spin asymmetry

\[ A_L^e \sim \frac{\Delta \bar{u}(x_1) d(x_2)(1 - \cos \theta)^2 - \Delta d(x_1) \bar{u}(x_2)(1 + \cos \theta)^2}{\bar{u}(x_1) d(x_2)(1 - \cos \theta)^2 + d(x_1) \bar{u}(x_2)(1 + \cos \theta)^2} \]

backward lepton rapidity  
forward lepton rapidity

- Repeat for $W^+$ to get $\Delta u/u$, $\Delta \bar{d}/\bar{d}$
- Leads to flavor separation at $Q = M_W$ free of fragmentation uncertainties
- Accessible $x$-range at RHIC limited to $\langle x_{1,2} \rangle \sim \frac{M_w}{\sqrt{S}} e^{\pm \frac{m}{2}} \sim 0.04 \div 0.4$ @ $Q=M_W$ evolution!
- No access to strangeness (would require $W+$charm)
current situation

uses new analytic NLO calculation for incl. lepton production  Ringer, Vogelsang arXiv:1503.07052
current situation

- Uses new analytic NLO calculation for incl. lepton production.

- Potential tension w/ DSSV (based on SIDIS).

- Old GRSV fit with large 3F-D violation and $\Delta s = 0$.

- New fit: $W^+$ and $W^-$.

potential impact on sea polarizations

STAR arXiv:1404.6880

published data already have a significant impact

\[ \Delta \bar{u} > 0 \]

\[ \Delta \bar{u} \]

\[ \Delta d \]

\[ \Delta \bar{d} \]

\[ \Delta u \]

\[ \Delta \bar{d} < 0 \]

\[ x \Delta \bar{d} \quad Q^2 = 10 \text{ GeV}^2 \]

\[ x \Delta \bar{u} \quad Q^2 = 10 \text{ GeV}^2 \]
potential impact on sea polarizations

- points towards rather sizable positive $\Delta \bar{u}(x) - \Delta \bar{d}(x)$
- same trend seen by NNPDF, see 1406.5539
- starts to test of what we know from SIDIS
- 2013 STAR data soon: further reduce uncertainties by 2
Bj sum rule

- rare example of a well understood quantity in pQCD
- \[ \Gamma_1^{p-n} = \frac{1}{6} C_{Bj}[\alpha_s(Q^2)] \Delta q_3 \]
- QCD corrections up to O(\(\alpha_s^4\))
- Kodaira; Gorishny, Larin; Larin, Vermaseren; Baikov, Chetyrkin, Kuhn

- probes conserved non-singlet \(\Delta q_3\)
- \[ \Delta q_3 \equiv \int_0^1 dx [\Delta u + \Delta \bar{u} - \Delta d - \Delta \bar{d}] (x, Q^2) \]
- all fits usually assume
- \[ \Delta q_3 = F + D \]
- \[ \Delta q_3 = 1.2701 \pm 0.0025 \]
- significant uncertainties if fitted
- \[ \Delta q_3 = 1.19 \pm 0.22 \]
- significantly worse than sometimes claimed!
- needs an EIC to reduce uncertainty

NNPDF, arXiv:1303.7236
status & expectations for $\Delta g$

status for “running integral” of $\Delta g$ ($\rightarrow$ spin sum rule):

- Best fit prefers $\Delta g$ of about 0.36
- 70 - 75% of 1/2

but LARGE uncertainties

Based on RHIC run9 jets and $\pi^0$
status for “running integral” of $\Delta g$ ($\to$ spin sum rule):

- based on RHIC run9 jets and $\pi^0$
- upcoming 200/510 GeV RHIC spin results
- projections: RHIC spin

DSSV 2014
with 90% C.L. band

best fit prefers
$\Delta g$ of about 0.36
70 - 75% of 1/2!
but LARGE uncertainties

510 GeV forward rapidity data will have sensitivity down to few $\times 10^{-3}$
status & expectations for $\Delta g$

status for “running integral” of $\Delta g$ (\textit{\rightarrow} spin sum rule):

- based on RHIC run9 jets and $\pi^0$
- upcoming 200/510 GeV RHIC spin results
- jets and $\pi^0$ @ central & forward $\eta$

ultimate goal: EIC ep collider
projection assumes modest luminosities

EIC can achieve 10% accuracy on $\Delta g$

best fit prefers $\Delta g$ of about 0.36
70 - 75% of 1/2!
but LARGE uncertainties

sensitivity of 15x250 GeV
510 GeV forward rapidity data will have sensitivity down to few $\times 10^{-3}$
what about $\Delta \Sigma$ and OAM?

- running integral for $\Delta \Sigma$

![Graph showing running integral for $\Delta \Sigma$]

- $Q^2 = 10$ GeV$^2$

- we can’t really say we “know” $\Delta \Sigma$
- likewise for the Bj sum rule!

- main culprit: small x behavior of $\Delta s$
- need to scrutinize 3F-D constraint
what about $\Delta \Sigma$ and OAM?

running integral for $\Delta \Sigma$

$Q^2 = 10 \text{ GeV}^2$

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  likewise for the Bj sum rule!

- main culprit: small $x$ behavior of $\Delta s$
  need to scrutinize 3F-D constraint

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![Graph showing running integral for $\Delta \Sigma$ with $Q^2 = 10 \text{ GeV}^2$.](image)

- DSSV 2014 with 90% C.L. band
- Projection: $eRHIC \times (100, 250) \text{ GeV}$ only incl. DIS data used
- PRELIMINARY
- need to add SIDIS projections
what about $\Delta \Sigma$ and OAM?

running integral for $\Delta \Sigma$

\[ Q^2 = 10 \text{ GeV}^2 \]

a little subtraction game

\[ \frac{1}{2} - \int_{x_{\text{min}}}^{1} \Delta \frac{\Delta \Sigma + \Delta g}{x Q^2} \]

- we can’t really say we “know” $\Delta \Sigma$
  likewise for the Bj sum rule!

- main culprit: small $x$ behavior of $\Delta s$
  need to scrutinize 3F-D constraint

best fit has no OAM at 10 GeV$^2$
what about $\Delta \Sigma$ and OAM?

running integral for $\Delta \Sigma$

- we can't really say we "know" $\Delta \Sigma$
  likewise for the Bj sum rule!
- main culprit: small $x$ behavior of $\Delta s$
  need to scrutinize 3F-D constraint

a little subtraction game

- best fit has no OAM at 10 GeV$^2$
- EIC can make definitive statement
  presumably most precise (indirect) handle on OAM

$Q^2 = 10$ GeV$^2$

$1/2 - \int_{x_{min}}^{1} dx \frac{[2 \Delta \Sigma + \Delta g]}{Q^2}$

DSSV 2014 projection:
- eRHIC projection: 15 $\times$ (100, 250) GeV
  only incl. DIS data used

need to add SIDIS projections
spin experiments continue to produce high impact results

theory efforts & global QCD fits try to keep up

$\Delta g$ has gained quite some weight in proton spin sum

RHIC spin and JLab-12 have many more results to come

to close the chapter on the proton spin

an EIC is the only option