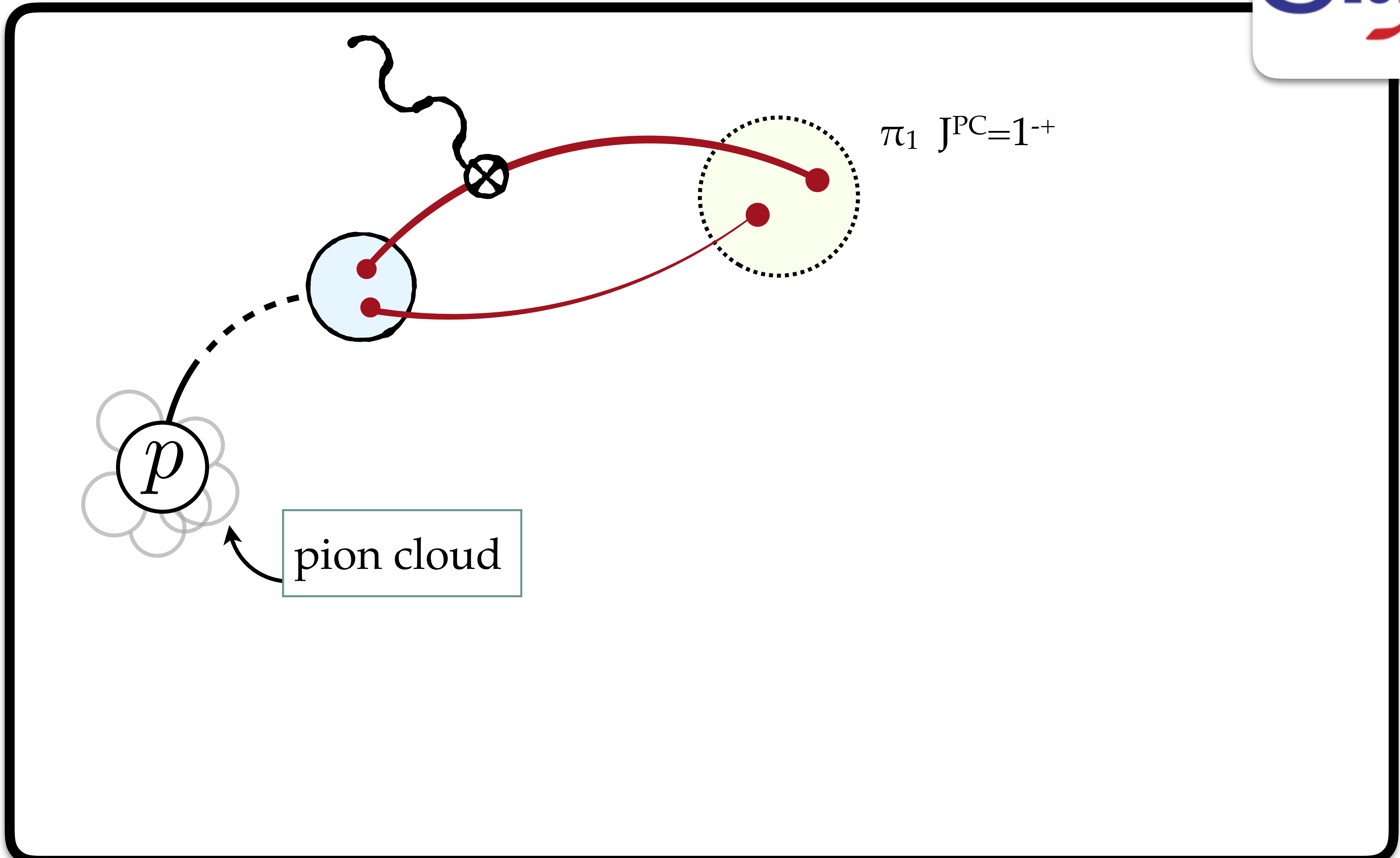
Scattering processes and resonances from QCD

Raúl Briceño

Resonances in experiment



- The vast majority of QCD states are either:
  - unstable under QCD (resonances)
  - *accidentally* stable (bound states)
  - depending on the QCD parameters, a state can *transition*

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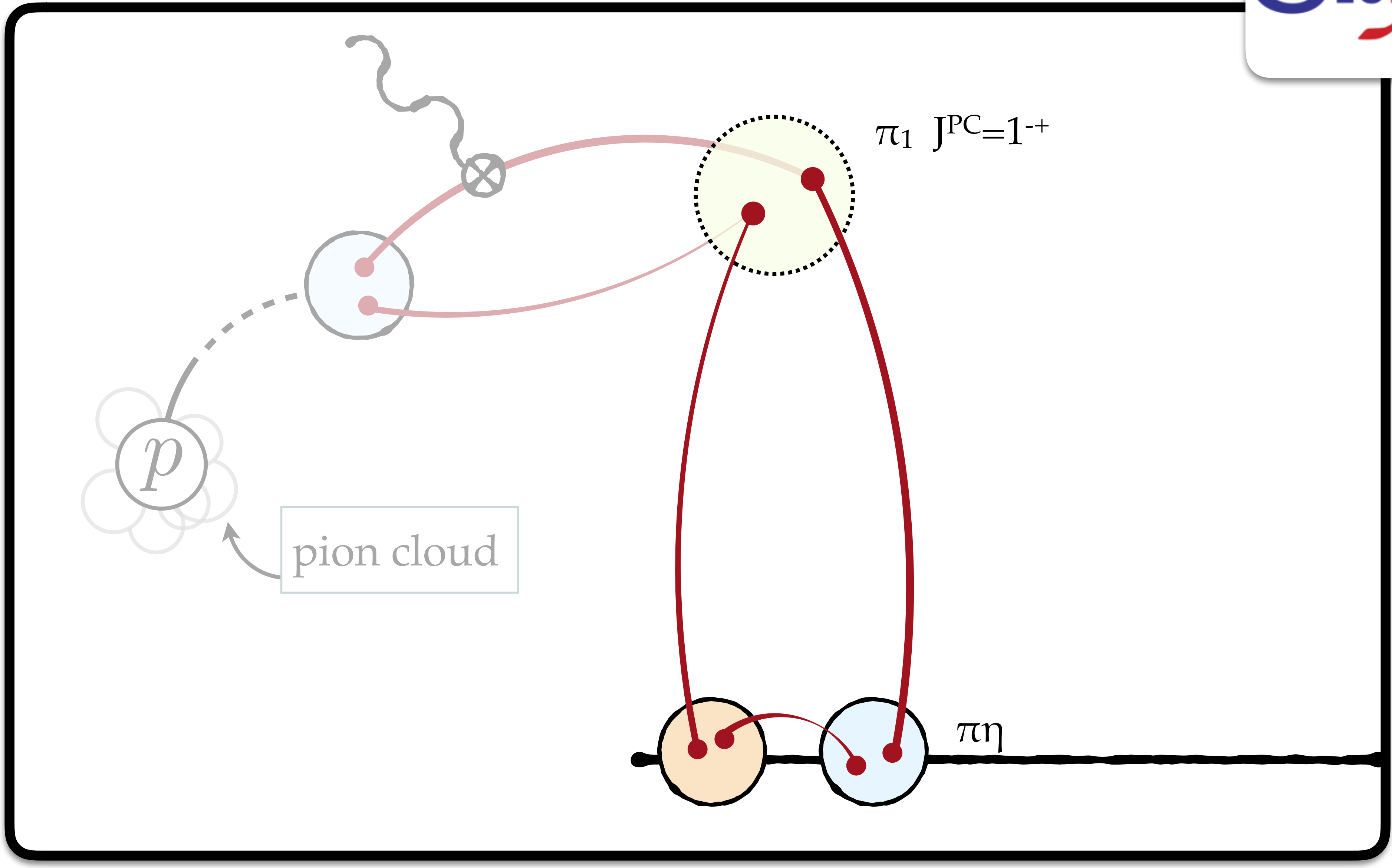

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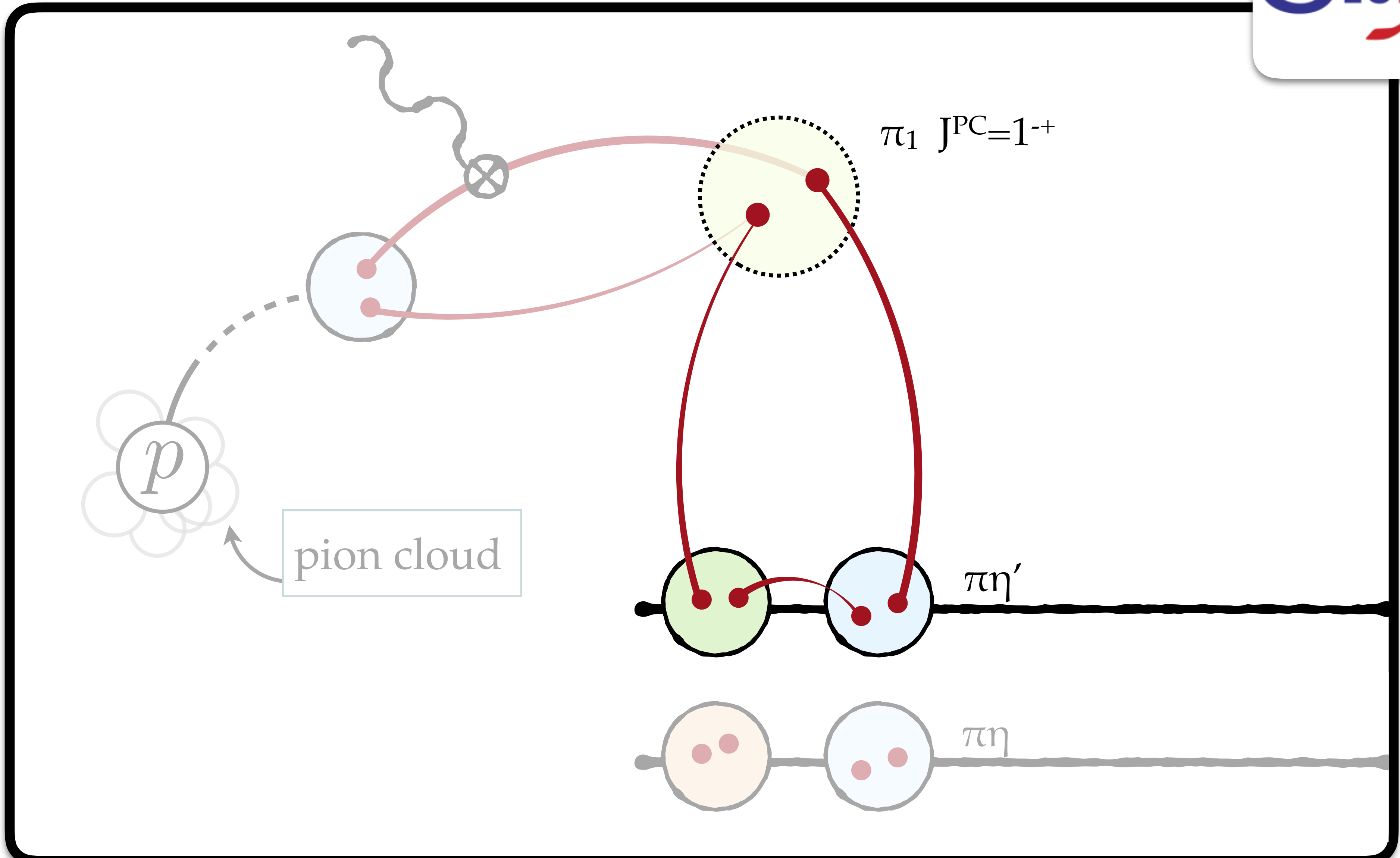

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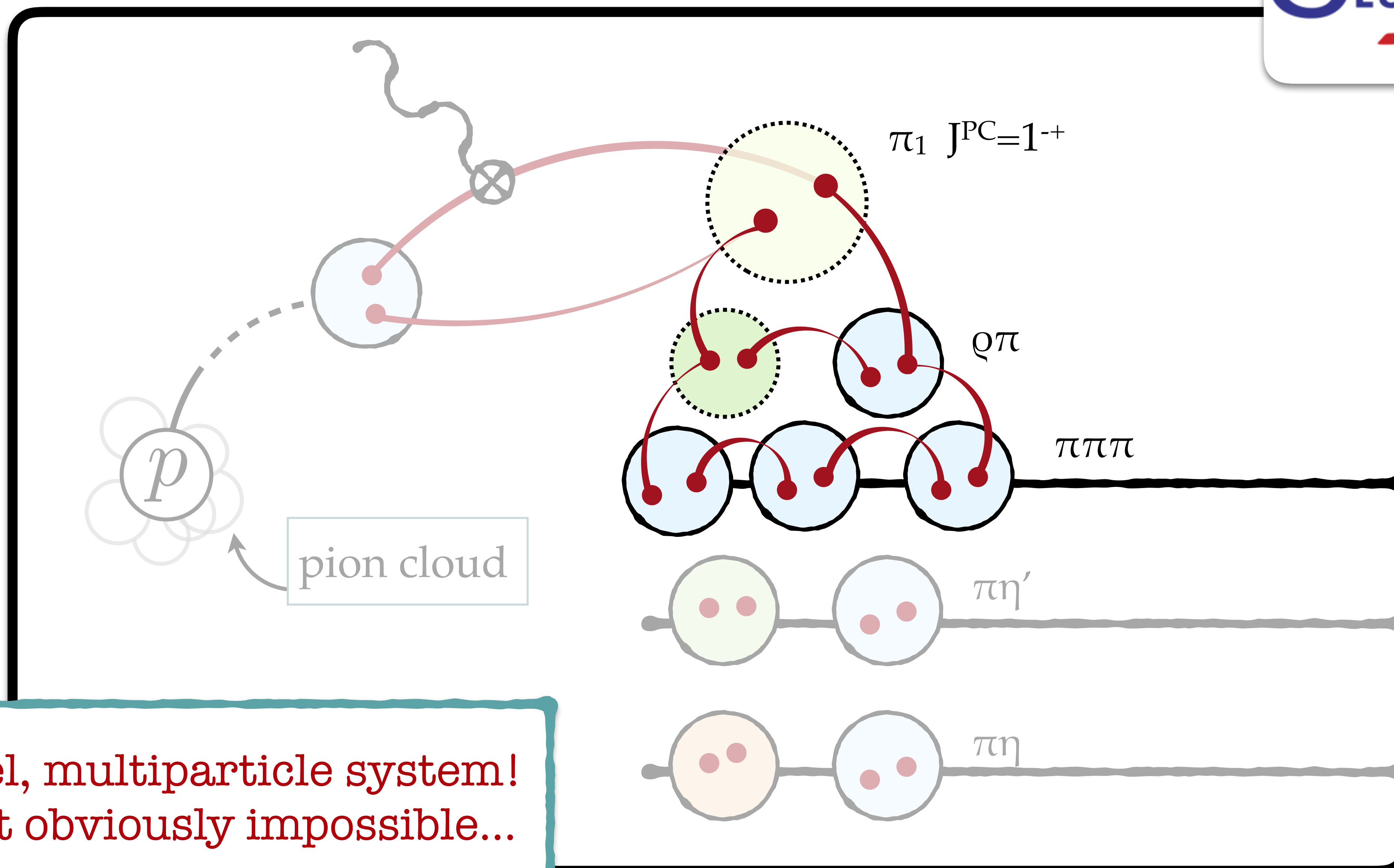


multichannel, multiparticle system!
hard, but not obviously impossible...

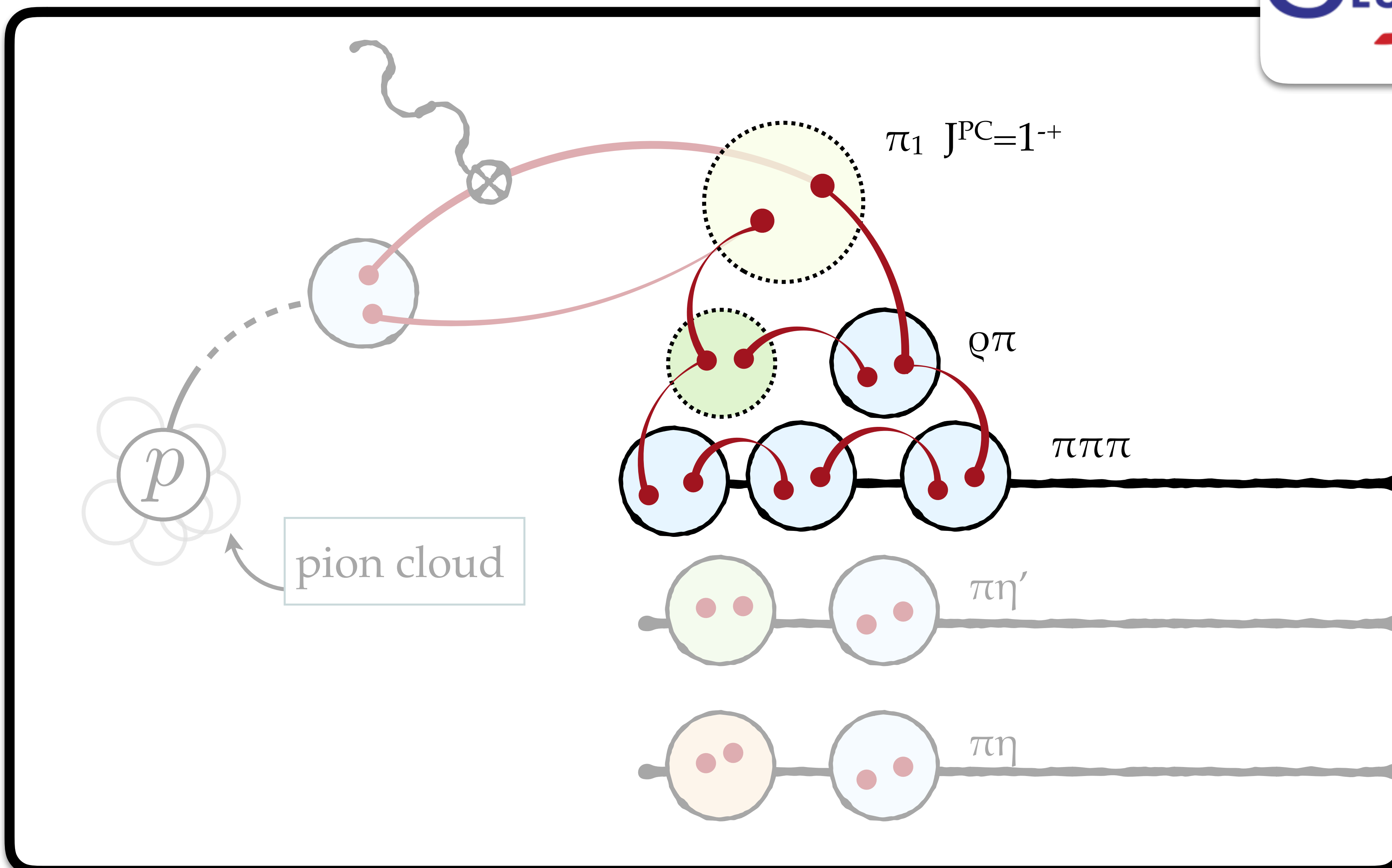
experimental needs

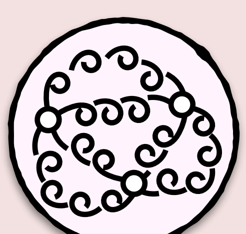
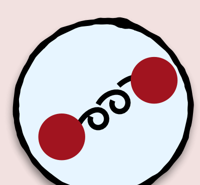
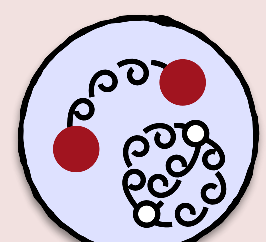
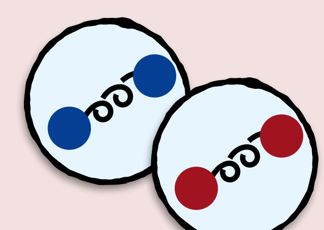
- confirmation
- production mechanism [couplings]
- identification of prominent decay channels
- couplings to decay channels

theoretical needs

- structural understanding

Resonances in experiment

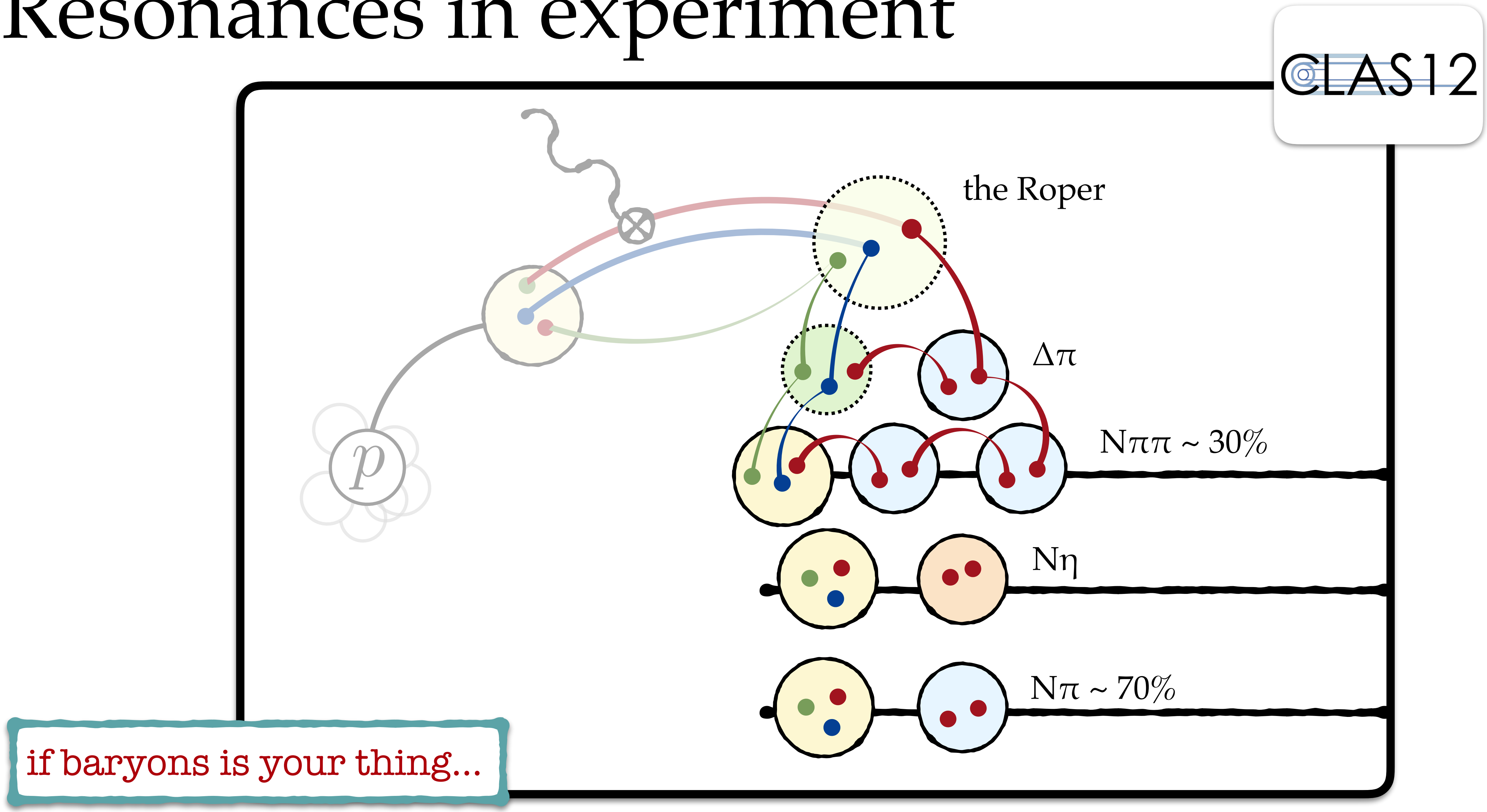


$|n\rangle_{\mathrm{QCD}} = c_0$  $+\ c_1$  $+\ c_2$  $+\ c_3$  $+\cdots$

theoretical needs

structural understanding

Resonances in experiment

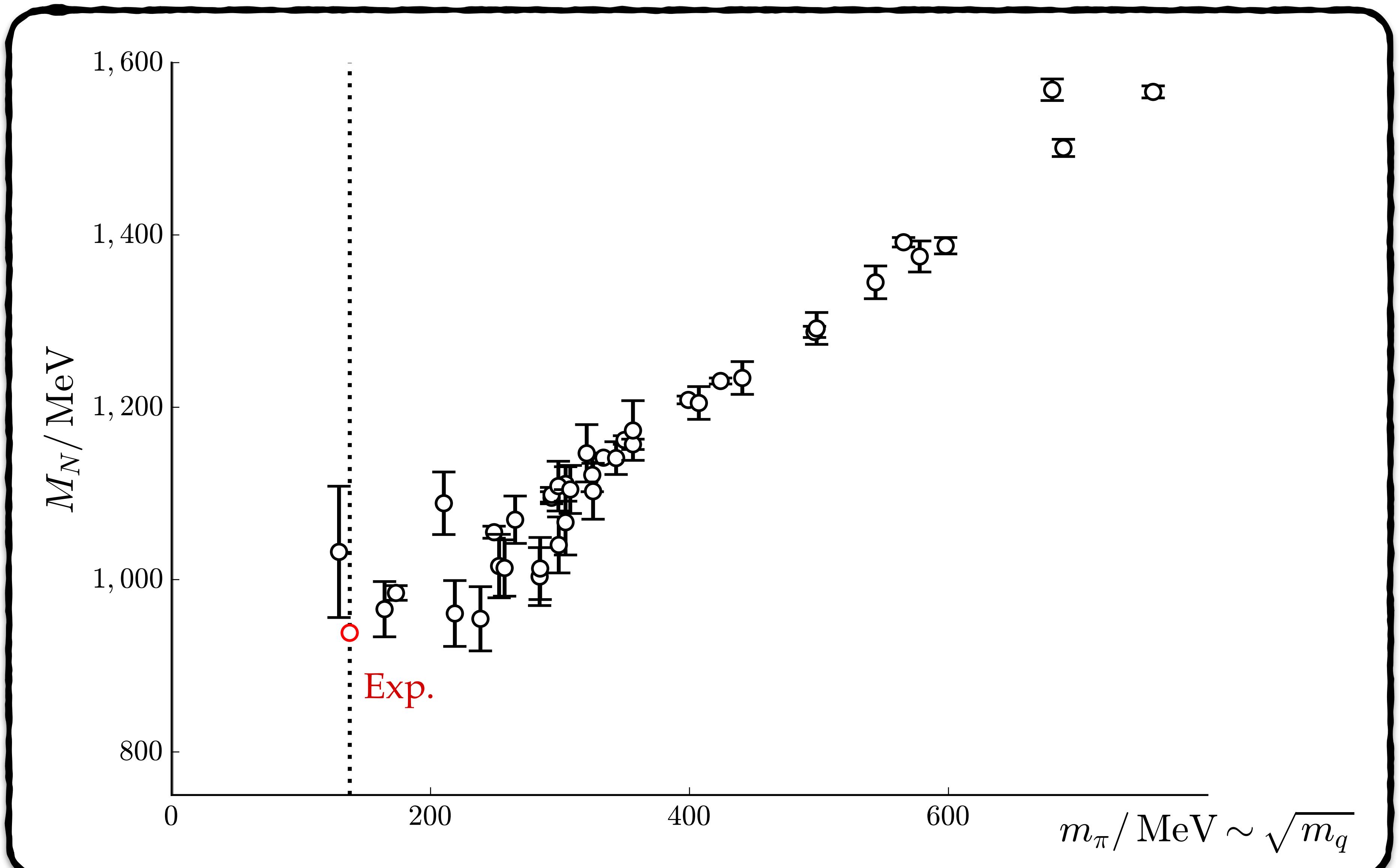


demand for lattice:

- Stable states generated *"exactly"*
- Resonant/non-resonant amplitudes are generated *"exactly"*
- QED/weak can be introduced perturb. or non-perturb.

Status of lattice QCD

- *Simple* properties of QCD stable states [non-composite states]
  - physical or lighter quark masses [down to m_π~120 MeV] ✓
  - non-degenerate light-quark masses: N_f=1+1+1+1 ✓
  - dynamical QED ✓



Status of lattice QCD

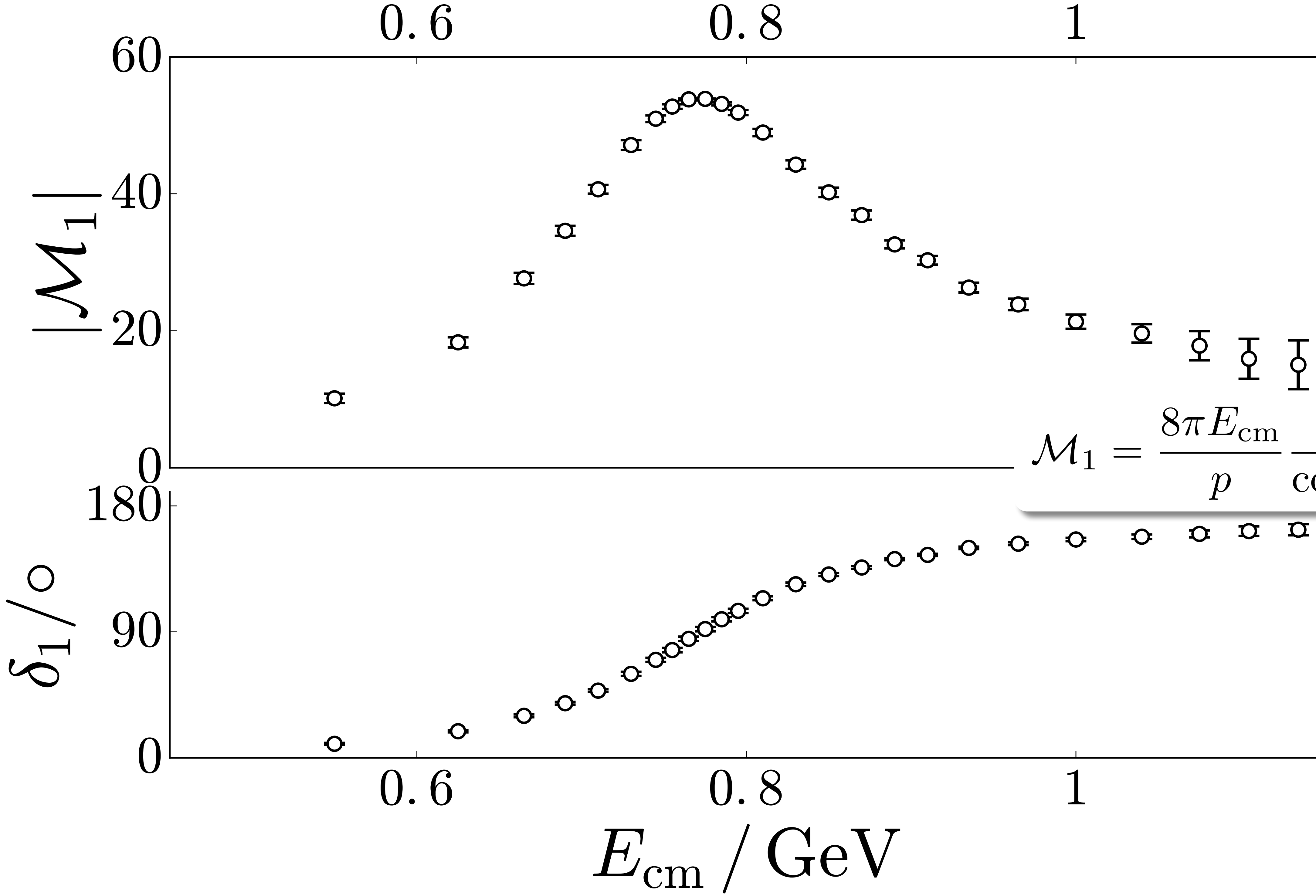
- *Simple* properties of QCD stable states [non-composite states]
  - physical or lighter quark masses [down to m_π~120 MeV]
  - non-degenerate light-quark masses: N_f=1+1+1+1
  - dynamical QED
- One of the frontiers of lattice: multi-particle physics
  - scattering/reactions
  - composite states
    - bound states
    - hadronic resonances

Formal development:
- under way
- more needed

Benchmark calculations:
- exploratory
- proof of principle
- unphysical quark masses [m_π=236, 391 MeV]
- …

A pseudo-quantitative definition

(bump in cross sections/amplitude - e.g., ππ scattering in ϱ-channel)

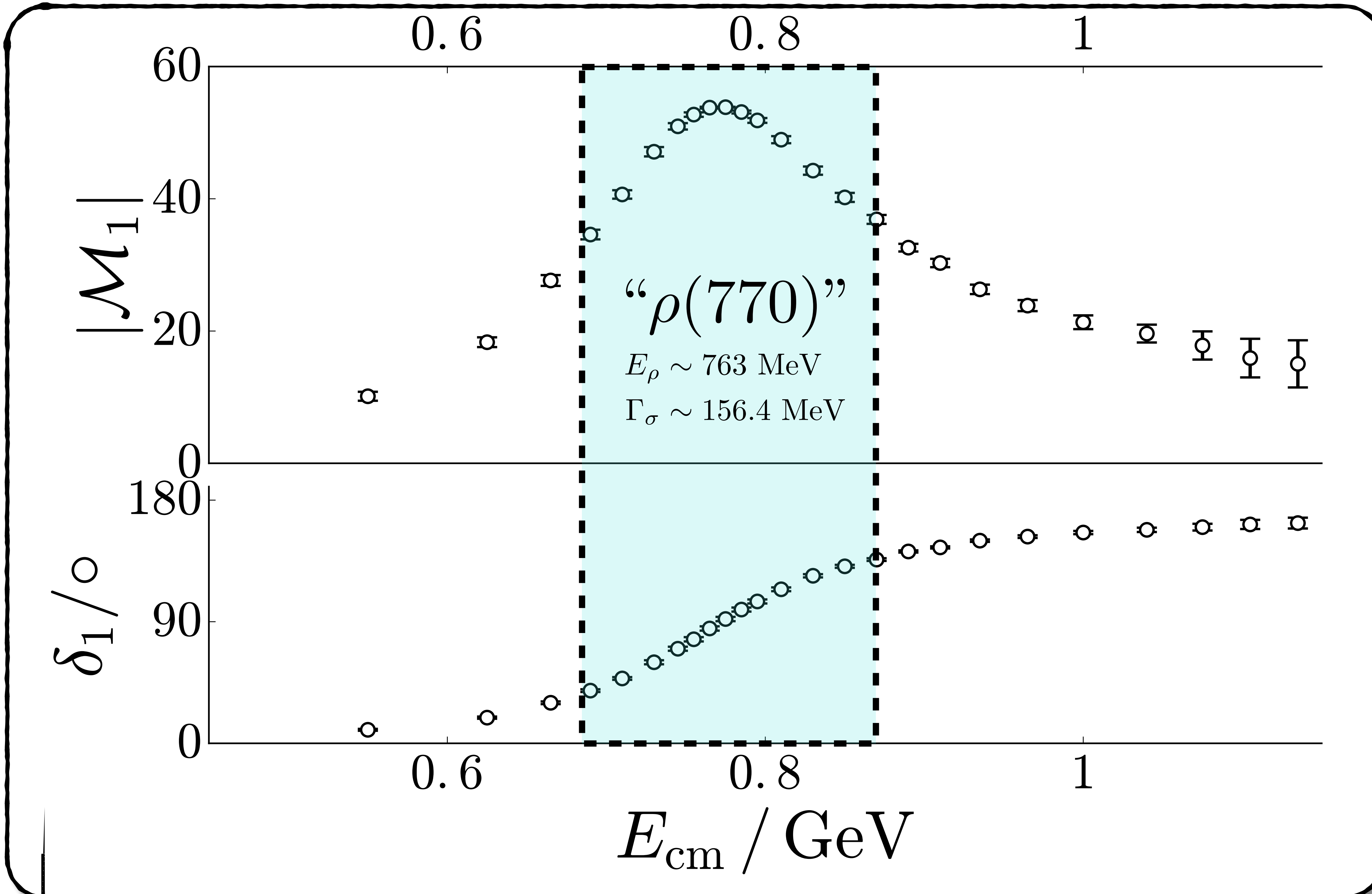


$$\mathcal{M}_1 = \frac{8\pi E_{\mathrm{cm}}}{p}\frac{1}{\cot\delta_1 - i}$$

Protopopescu et al. (1972)

A pseudo-quantitative definition

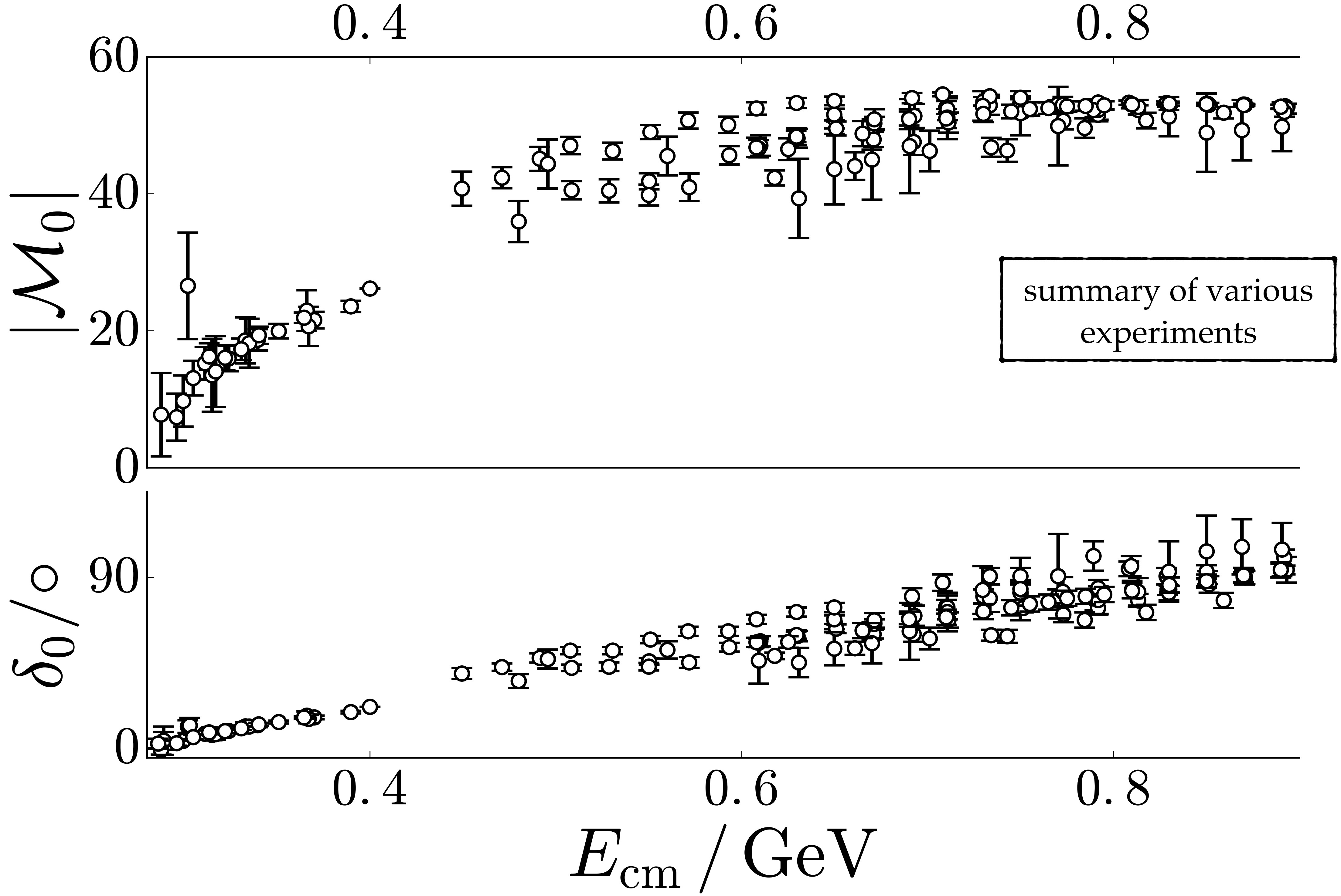
(bump in cross sections/amplitude - e.g., $\pi\pi$ scattering in ϱ-channel)



Protopopescu et al. (1972)

A counter example

(Isoscalar, scalar $\pi\pi$ scattering)



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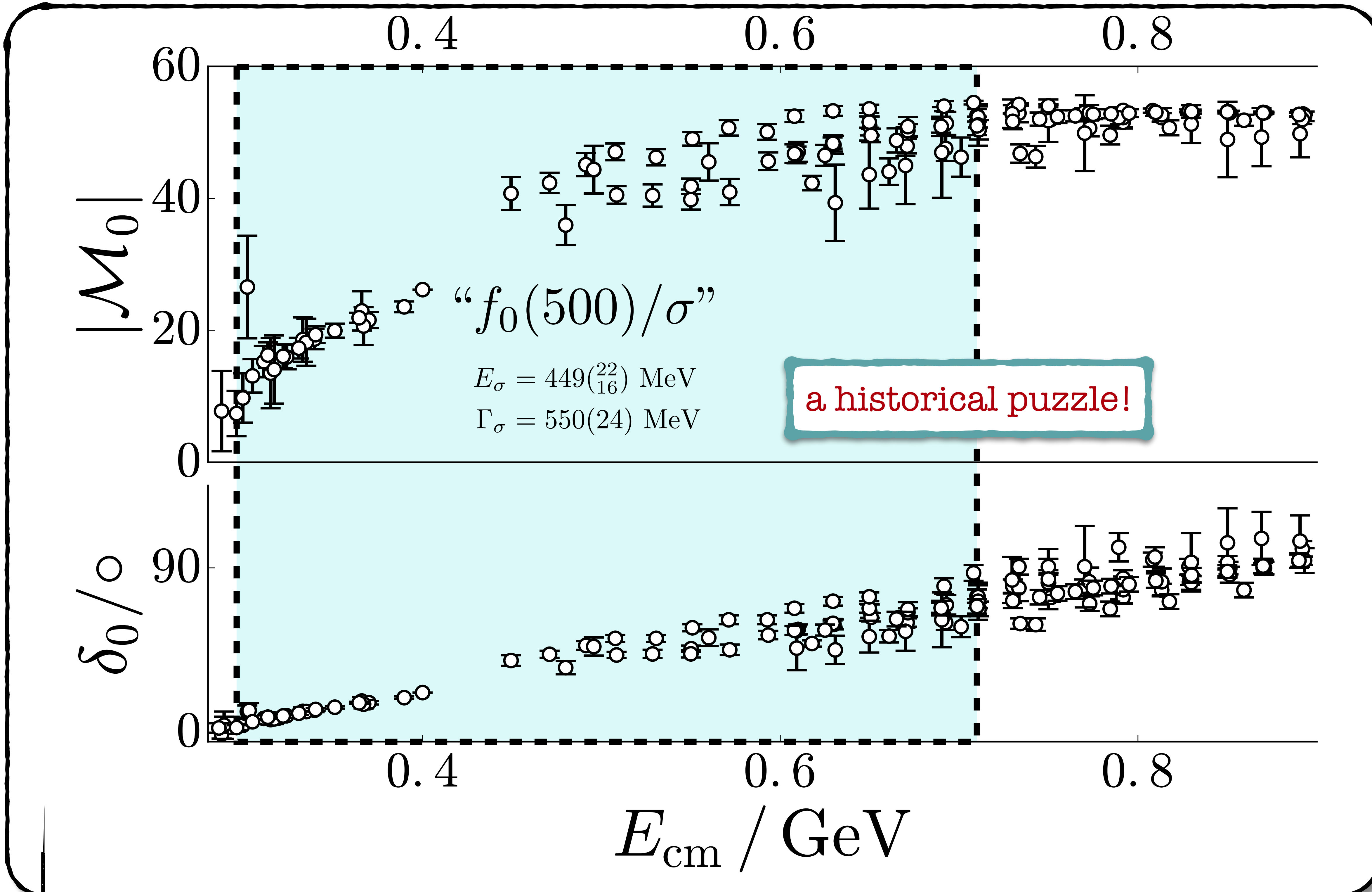
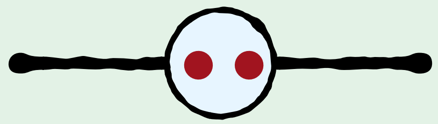


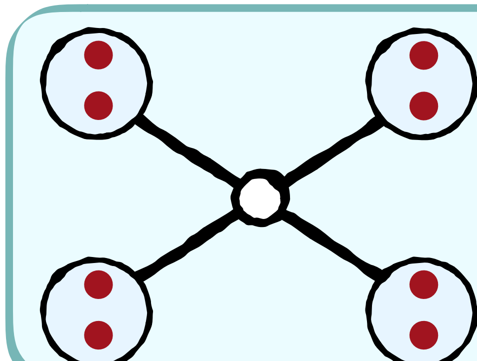
Quantitative definition

single-particle propagator:



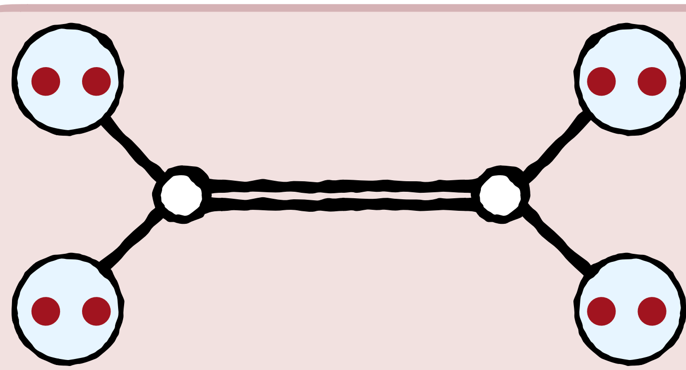
$$\sim \frac{iZ}{p^2 - m^2} = \frac{iZ}{s - m^2}$$

scattering amplitude:



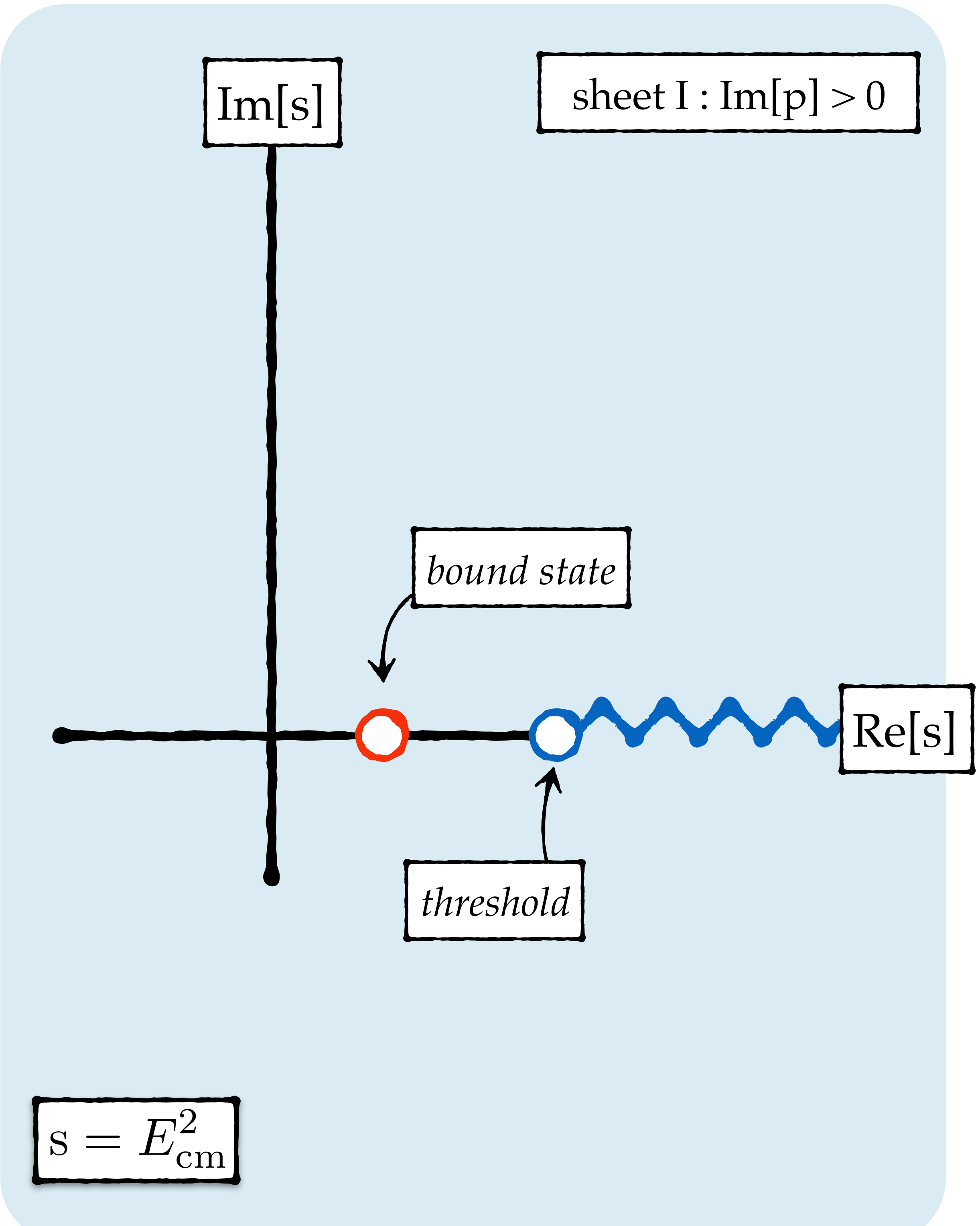
$$= i\mathcal{M}$$

…near bound state or resonance:



$$\sim \frac{i(ig)^2}{s - s_0}, \quad s_0 = (m_R - \tfrac{i}{2}\Gamma_R)^2$$

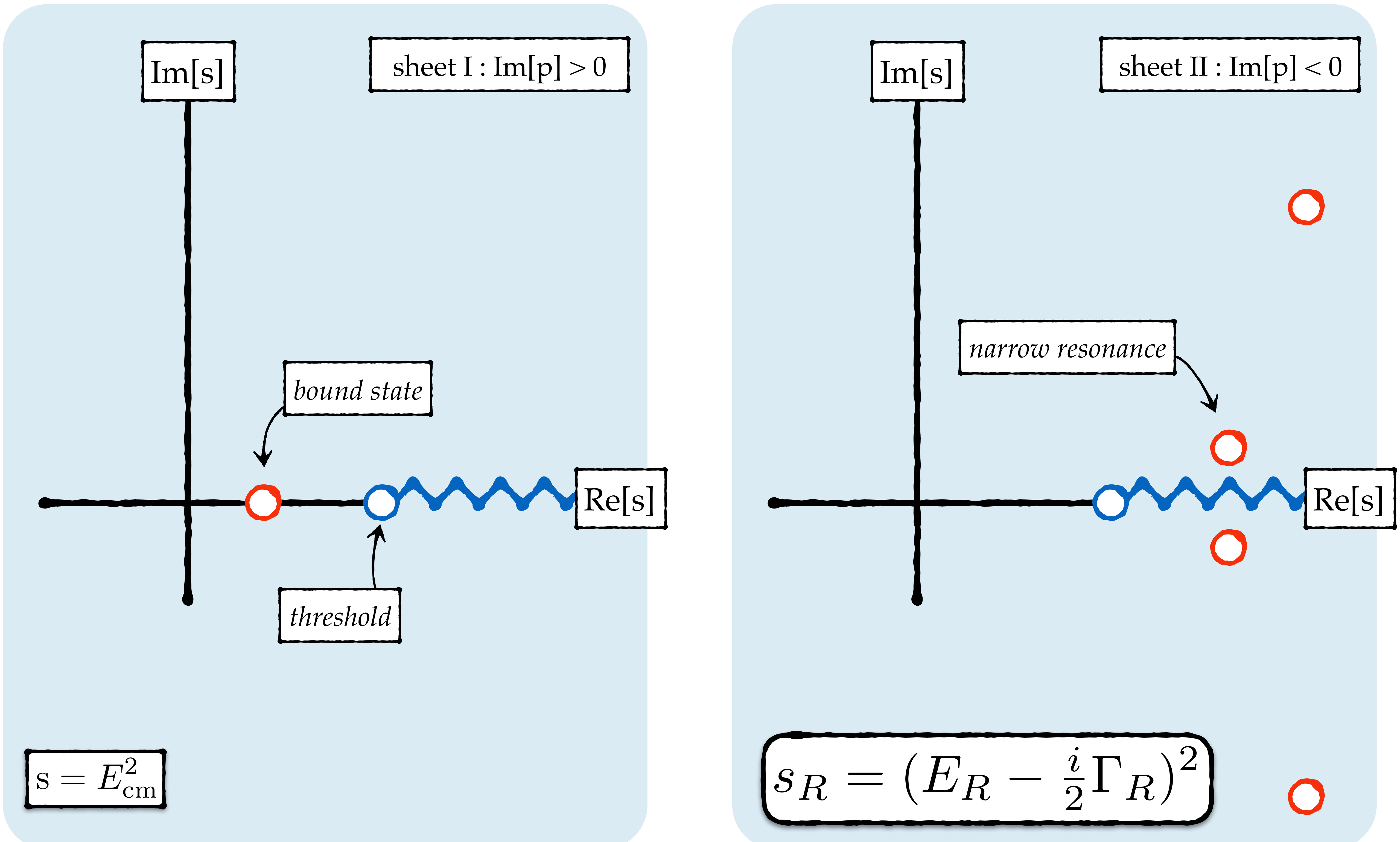
Composite states as poles



unitarity: $\mathcal{M} \sim \dfrac{1}{p\cot\delta - ip}$

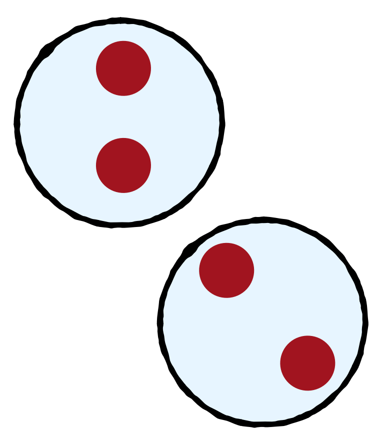
square-root singularity
at threshold: $p = \sqrt{s/4 - m^2}$

Composite states as poles



Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling
- quark masses: $m_q \to m_q^{\text{phys.}}$



Advantage over experiment!

Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
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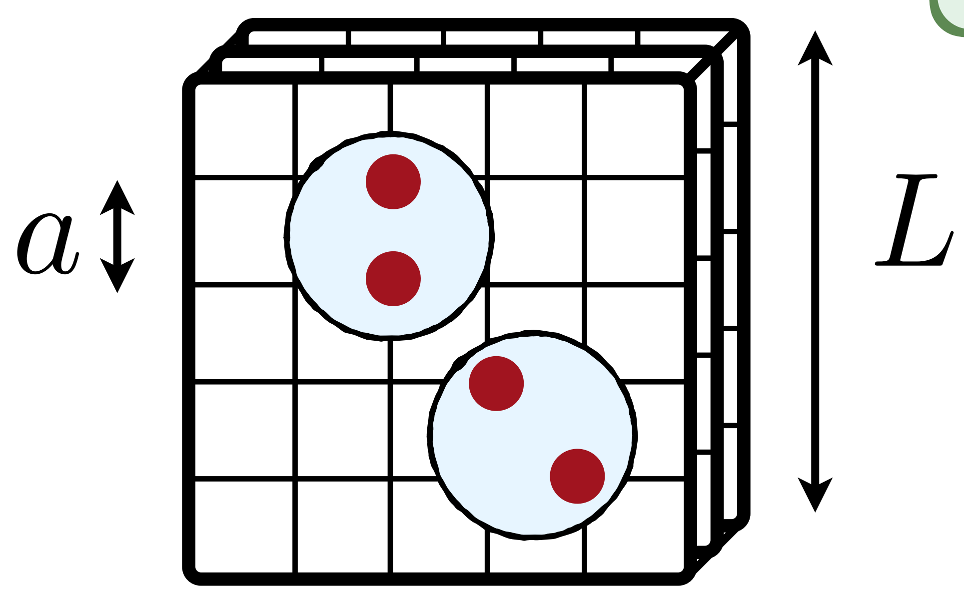
- quark masses: $m_q \to m_q^{\text{phys.}}$
- lattice spacing: $a \sim 0.03 - 0.15 \text{ fm}$
- finite volume

$$D_\mu = \left(\qquad \right) \updownarrow (L/a)^3 \times (T/a)$$



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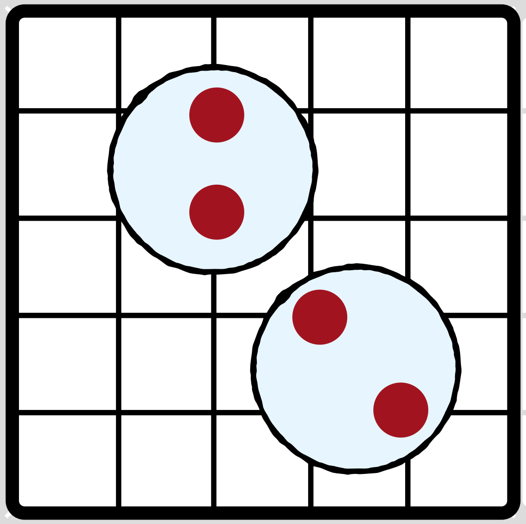

Never free!
No asymptotic states!
No scattering!

Lattice QCD

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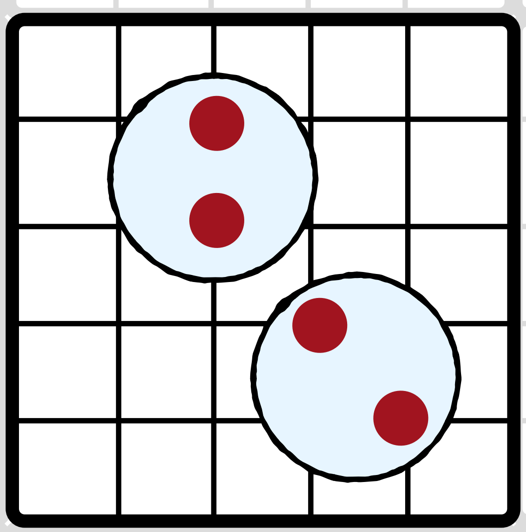

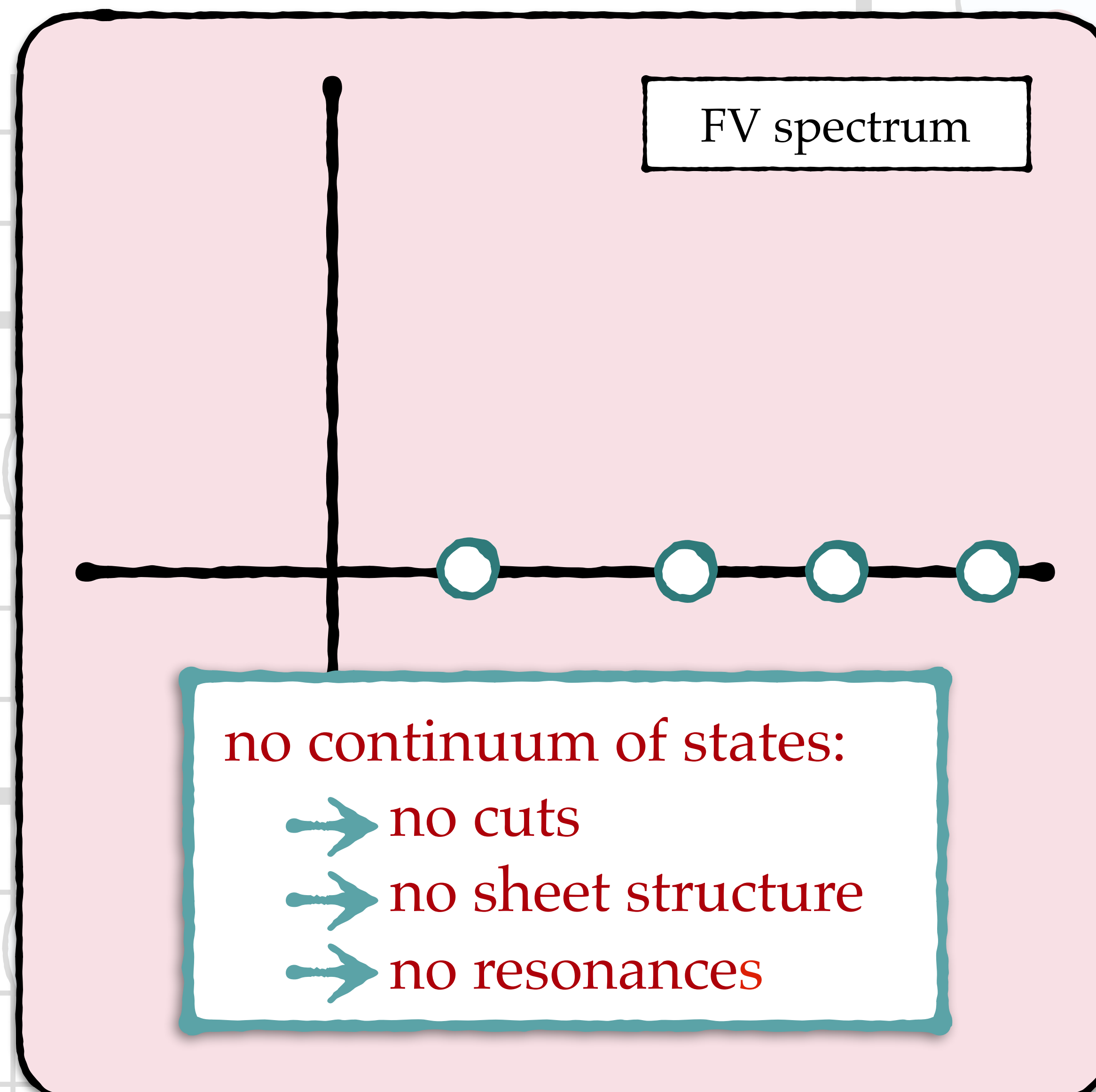

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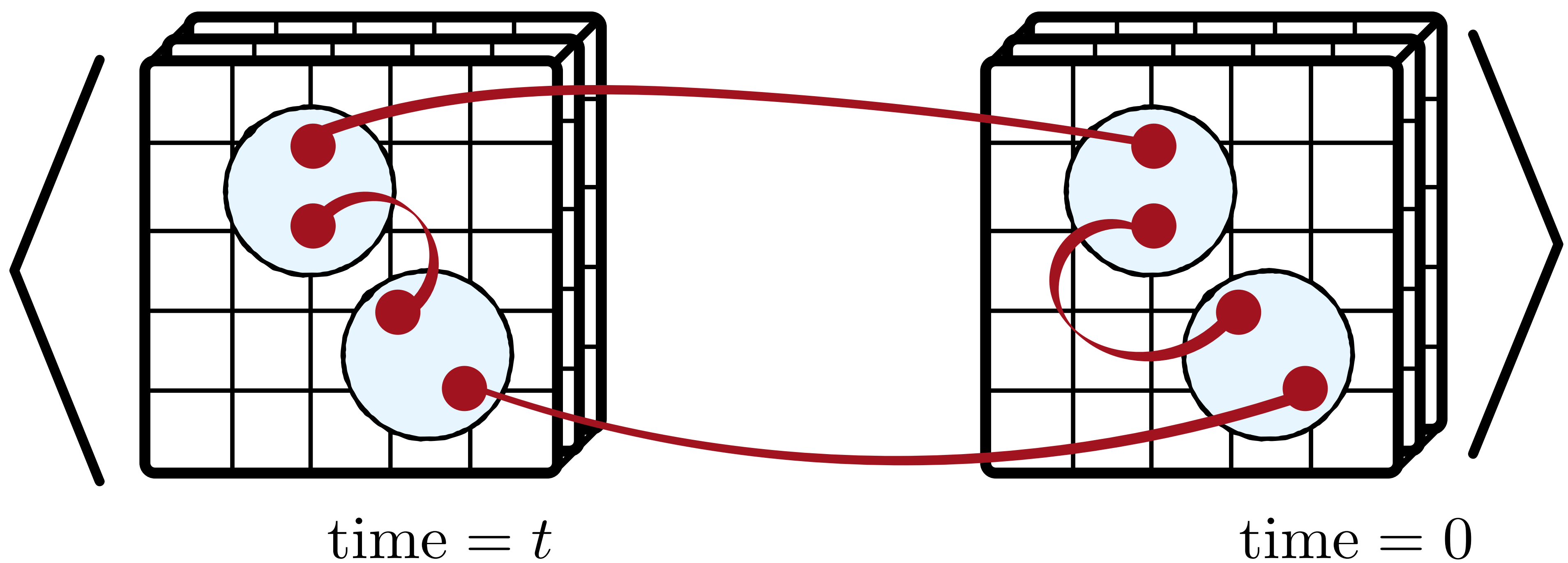
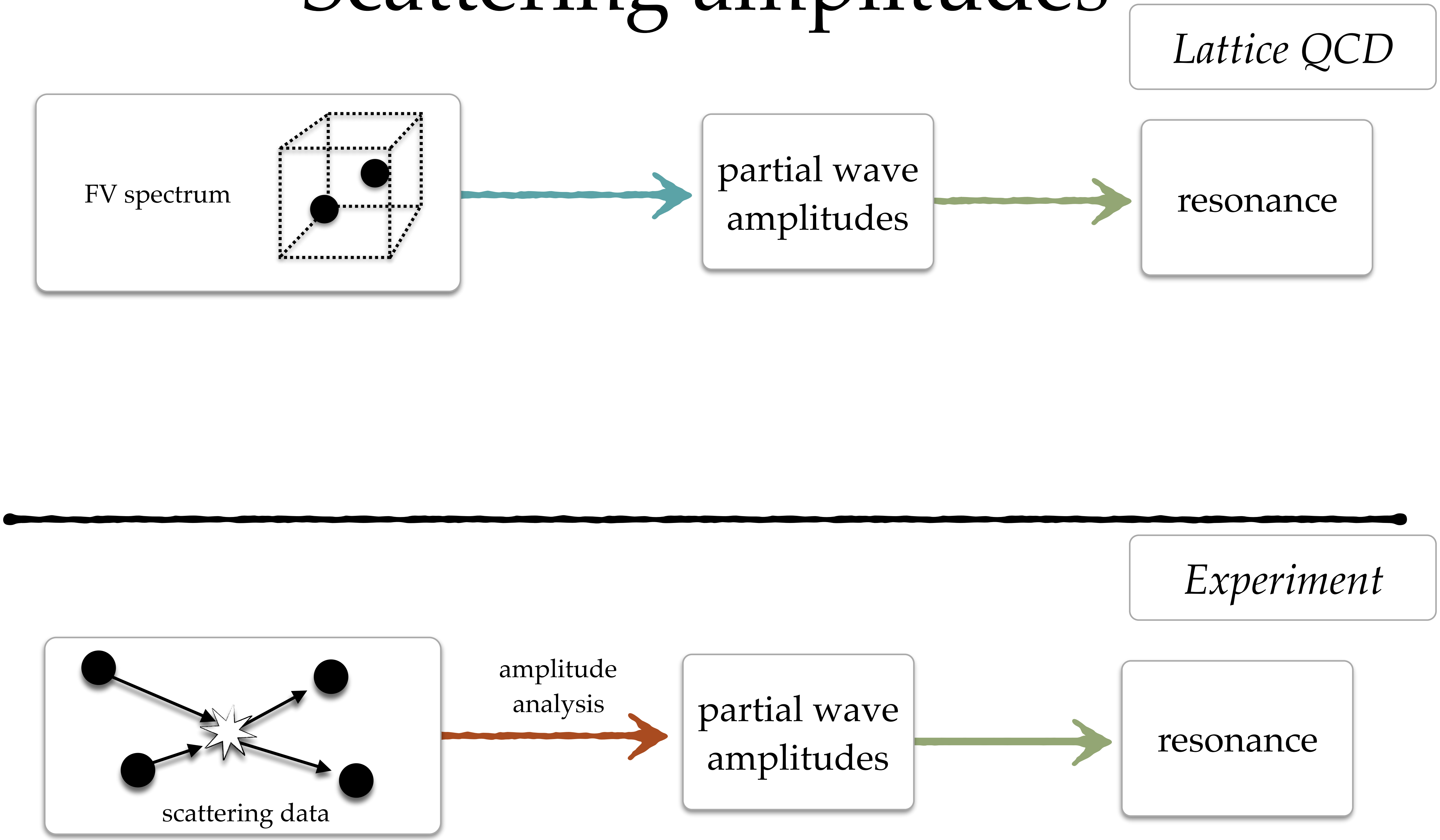
- Correlation functions: spectrum, matrix elements

$C_L(t, \mathbf{P}) =$



Scattering amplitudes



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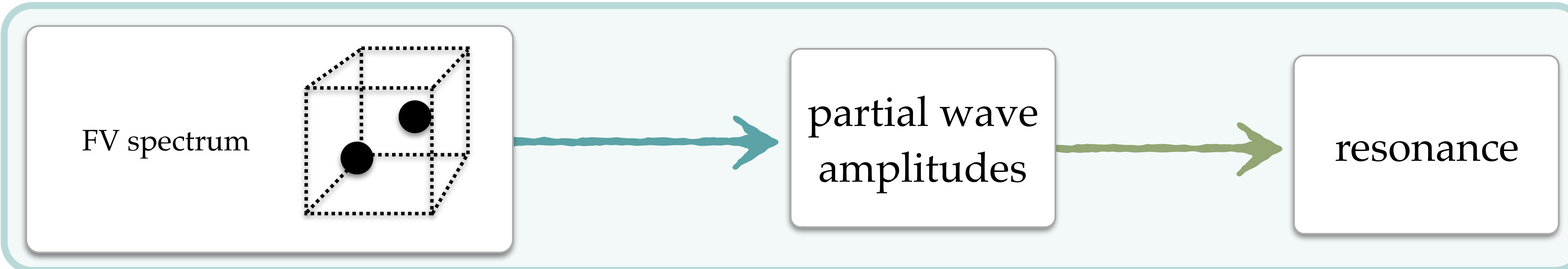


$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$

E_L = finite volume spec.
L = finite volume
F = known function
$\mathcal{M}$ = scattering amp.

not an extrapolation!

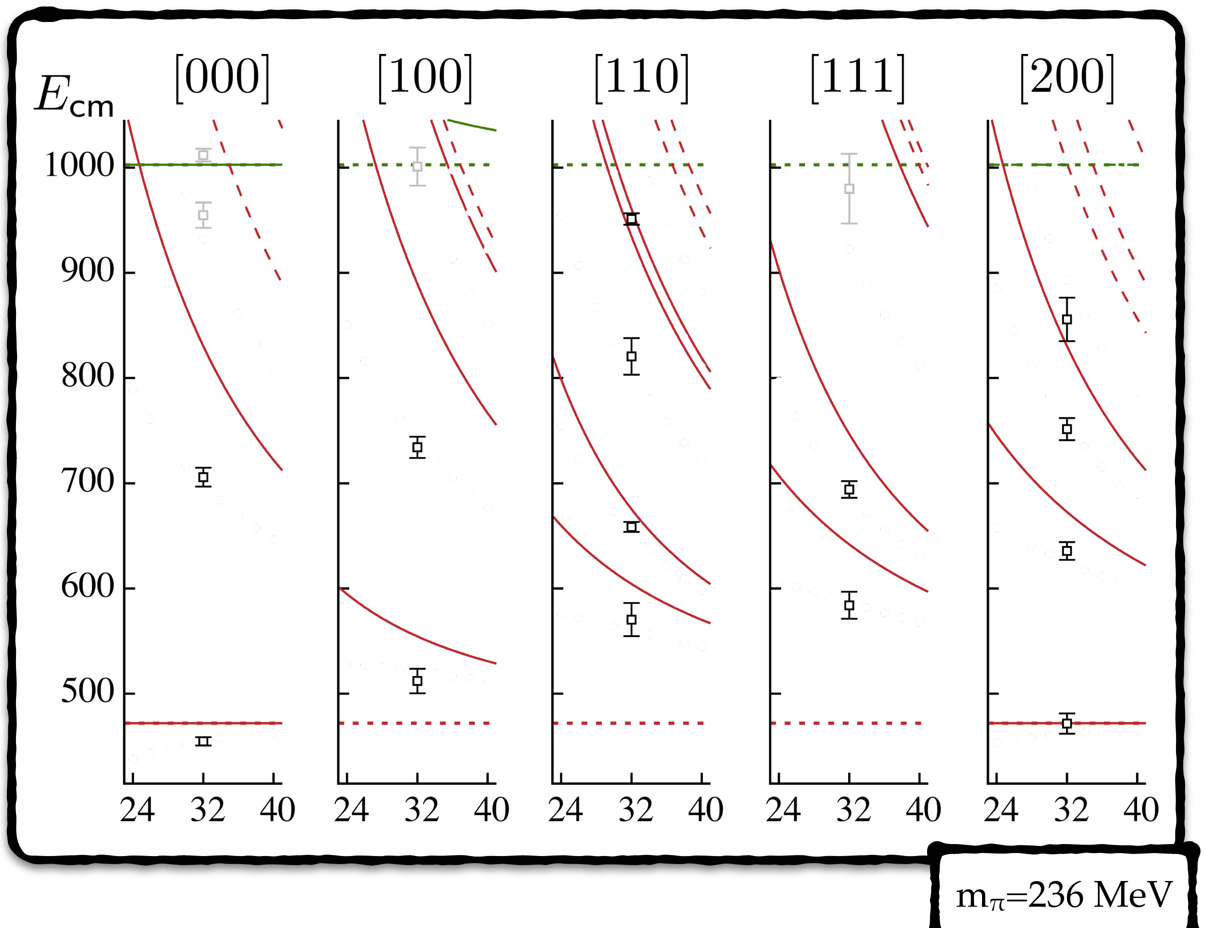
- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
- RB (2014) [general 2-body result]

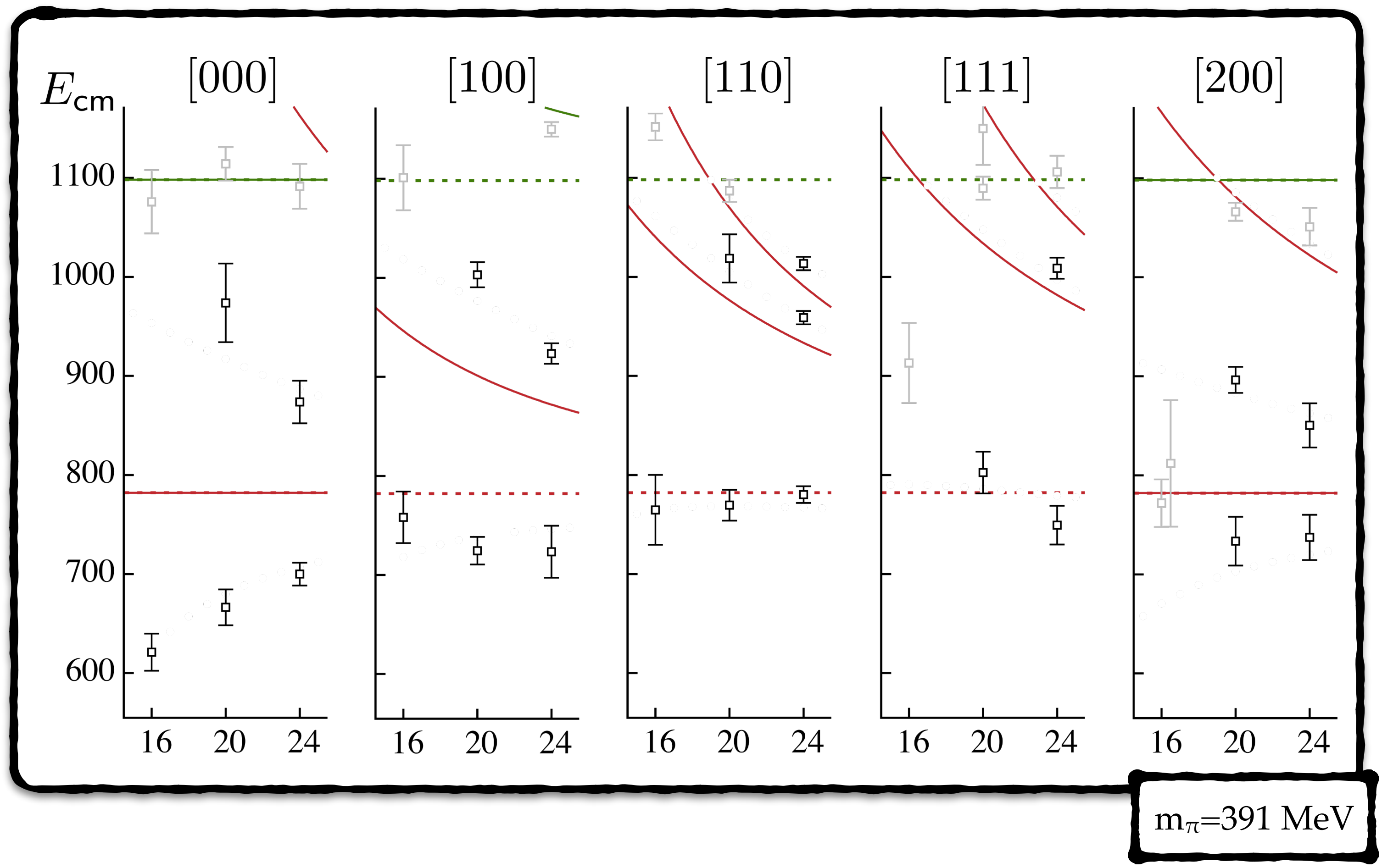
Extracting the spectrum

Two-point correlation functions:

$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n}Z_{a,n}^\dagger e^{-E_n t}$$

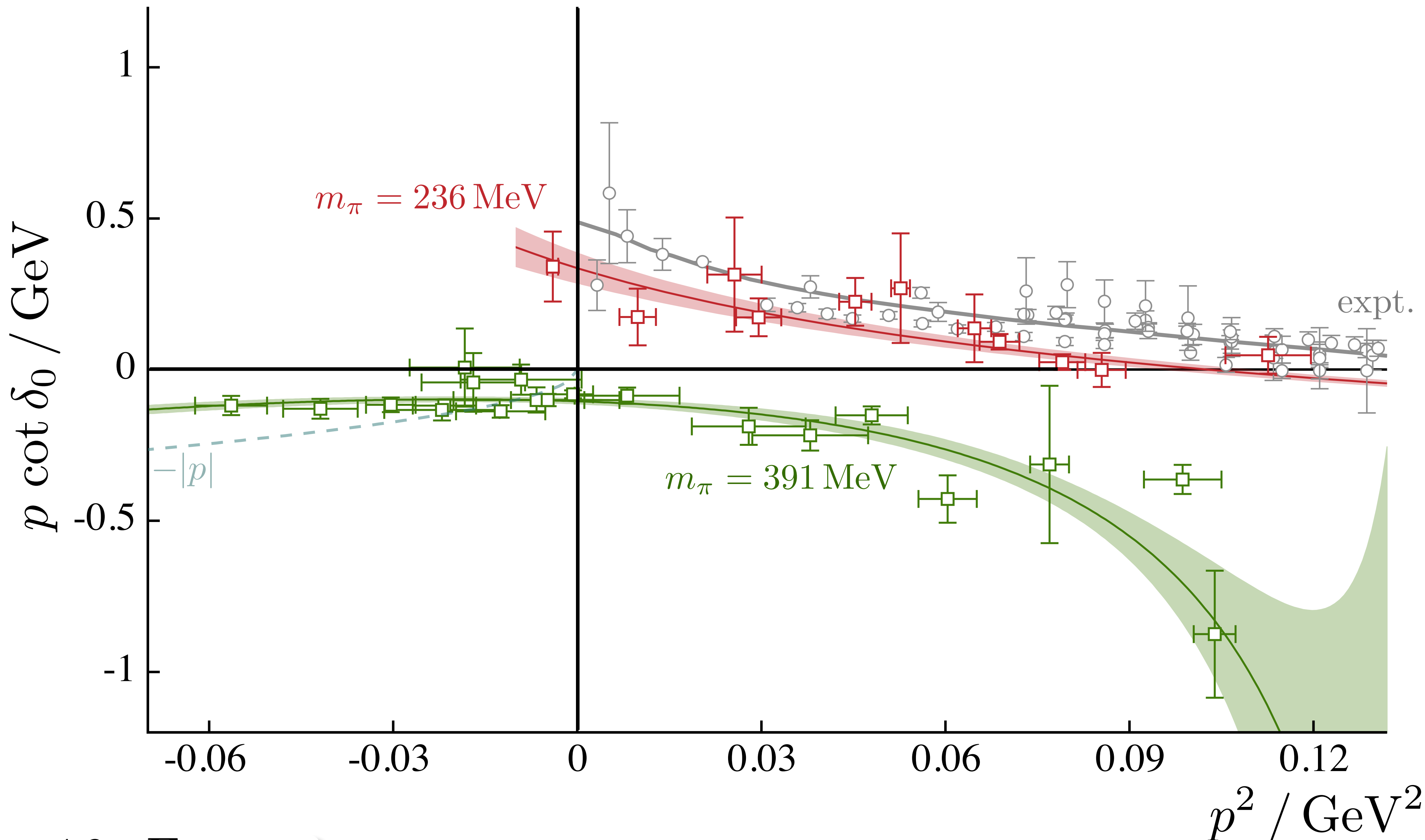
- Evaluate all Wick contraction
- Use a large basis of operators ~ 20-30 ops





RB, Dudek, Edwards, Wilson - PRL (2017)

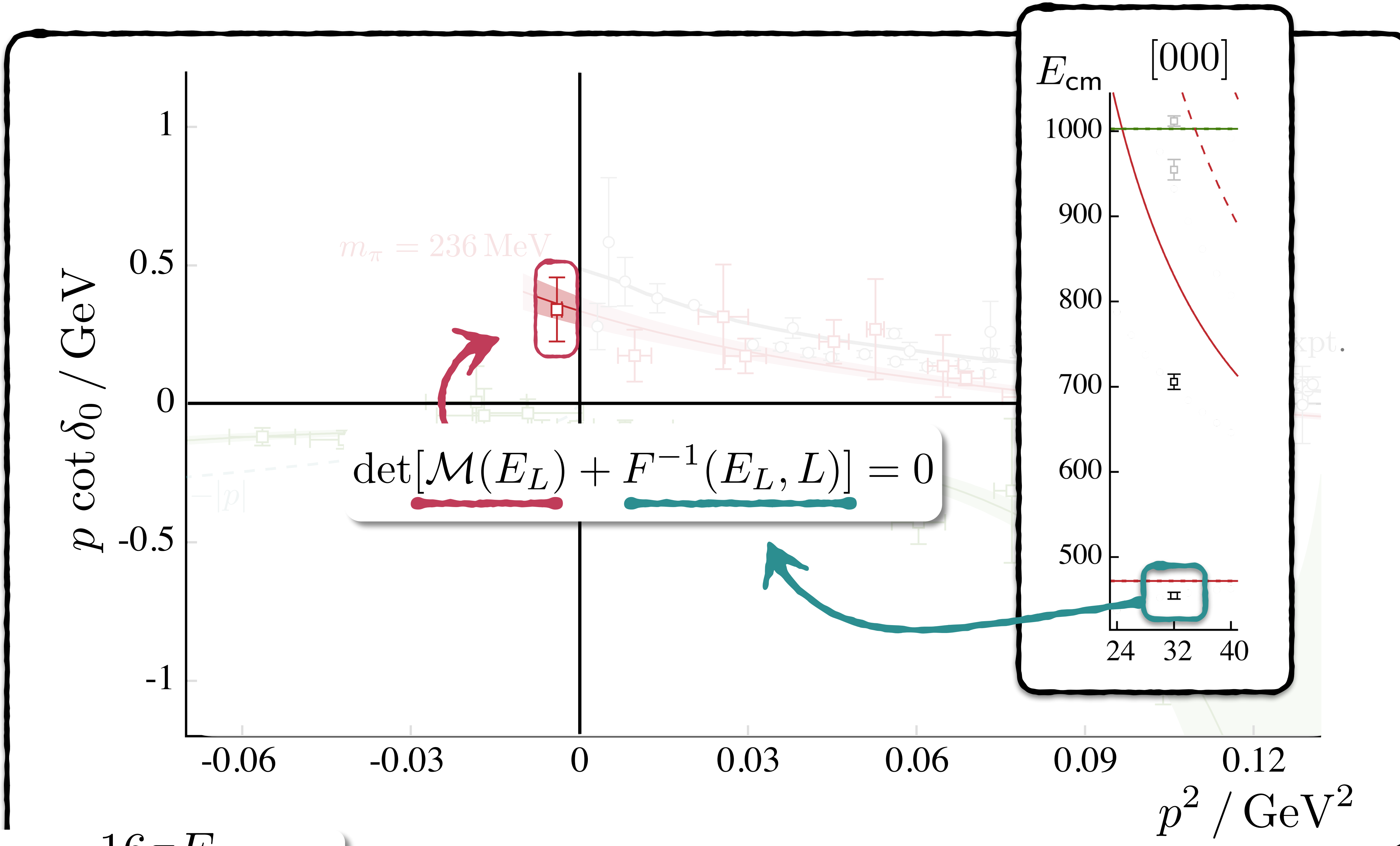
Isoscalar $\pi\pi$ scattering



$$\mathcal{M}_0 = \frac{16\pi E_{\mathrm{cm}}}{p\cot\delta_0 - ip}$$

RB, Dudek, Edwards, Wilson - PRL (2017)

Isoscalar $\pi\pi$ scattering

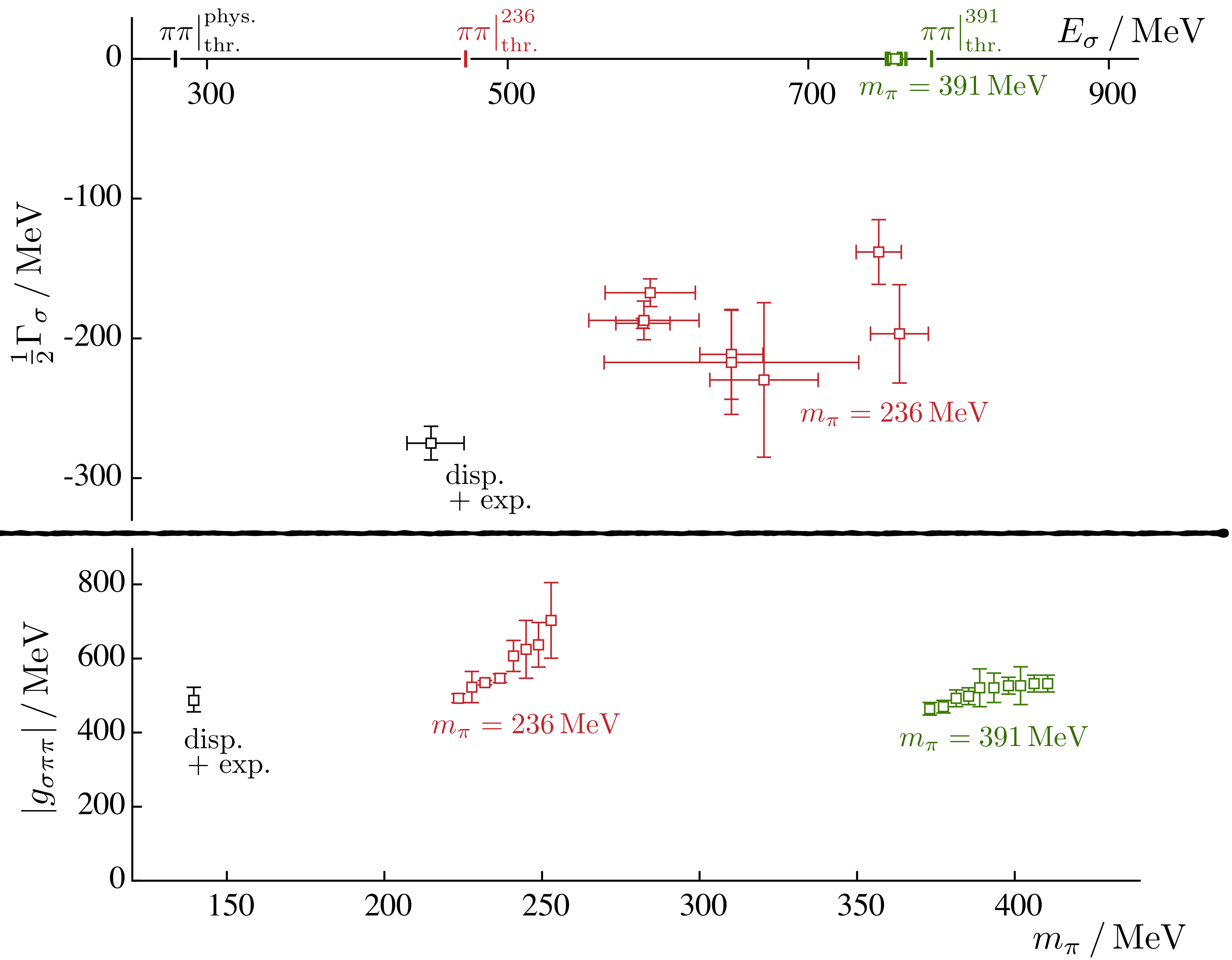


$$\det[\mathcal{M}(E_L) + F^{-1}(E_L, L)] = 0$$

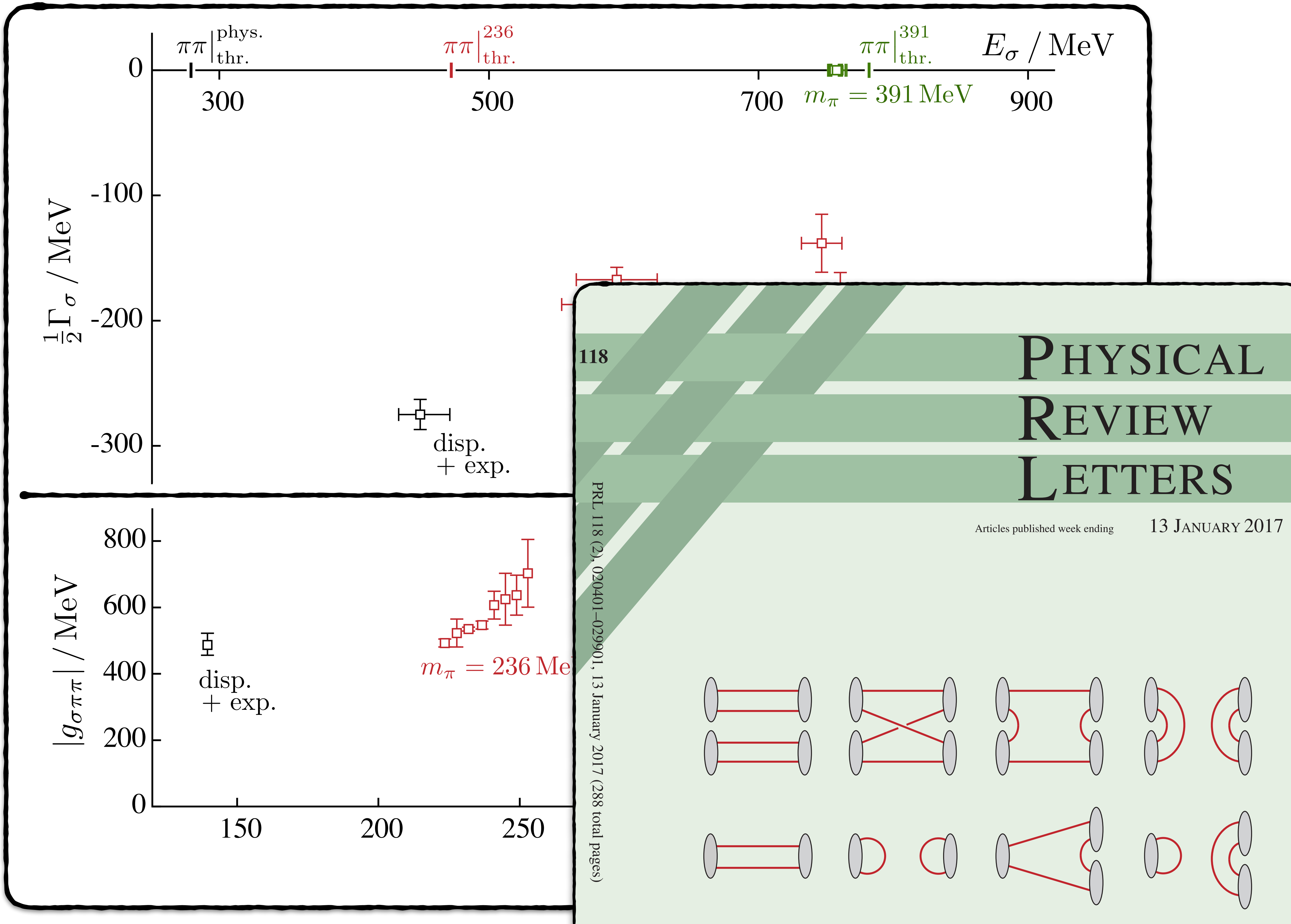
$$\mathcal{M}_0 = \frac{16\pi E_{\mathrm{cm}}}{p \cot \delta_0 - ip}$$

RB, Dudek, Edwards, Wilson - PRL (2017)

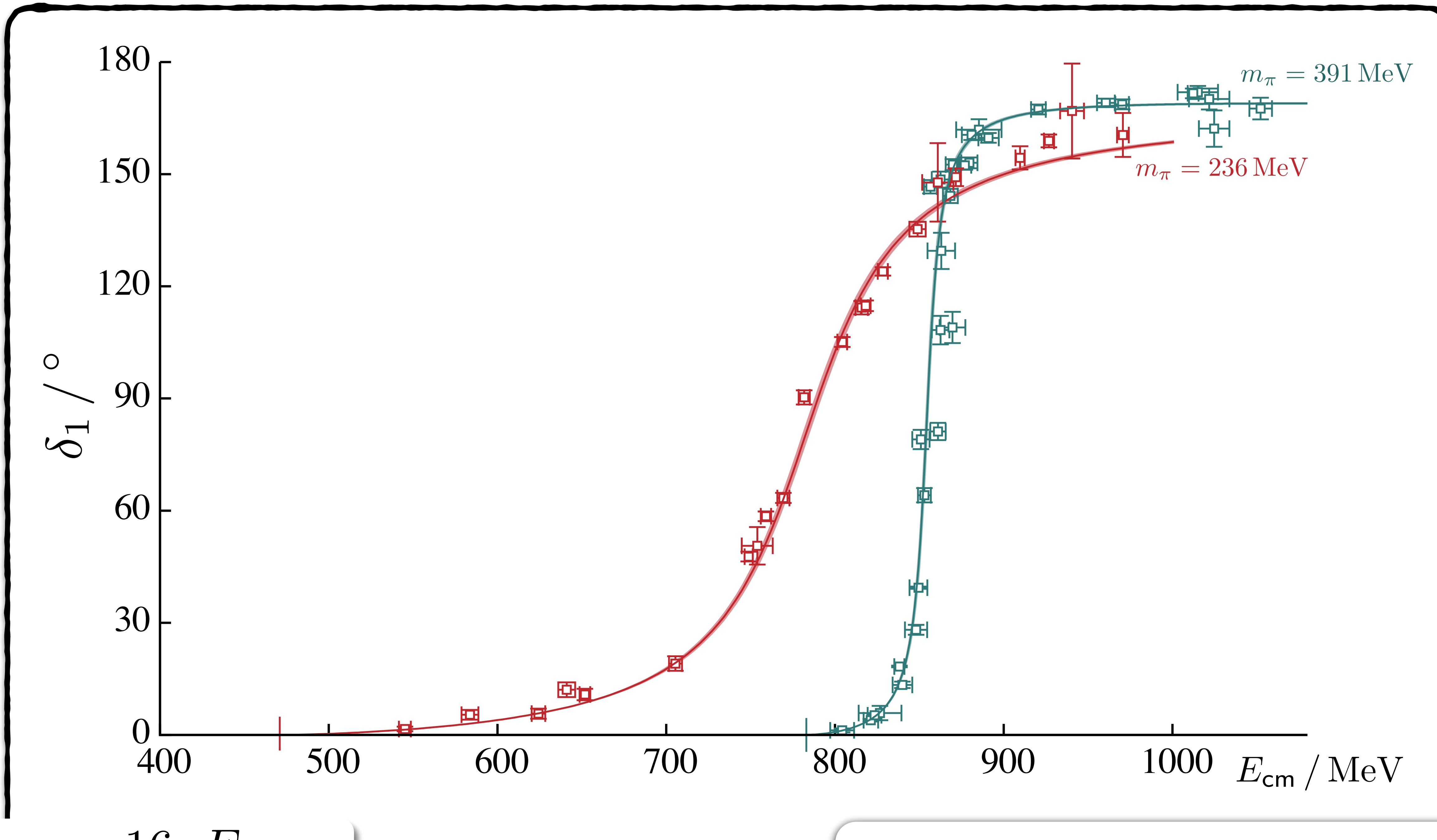
The σ / f$_0$(500) vs m$_\pi$



The σ / $f_0(500)$ vs m_π



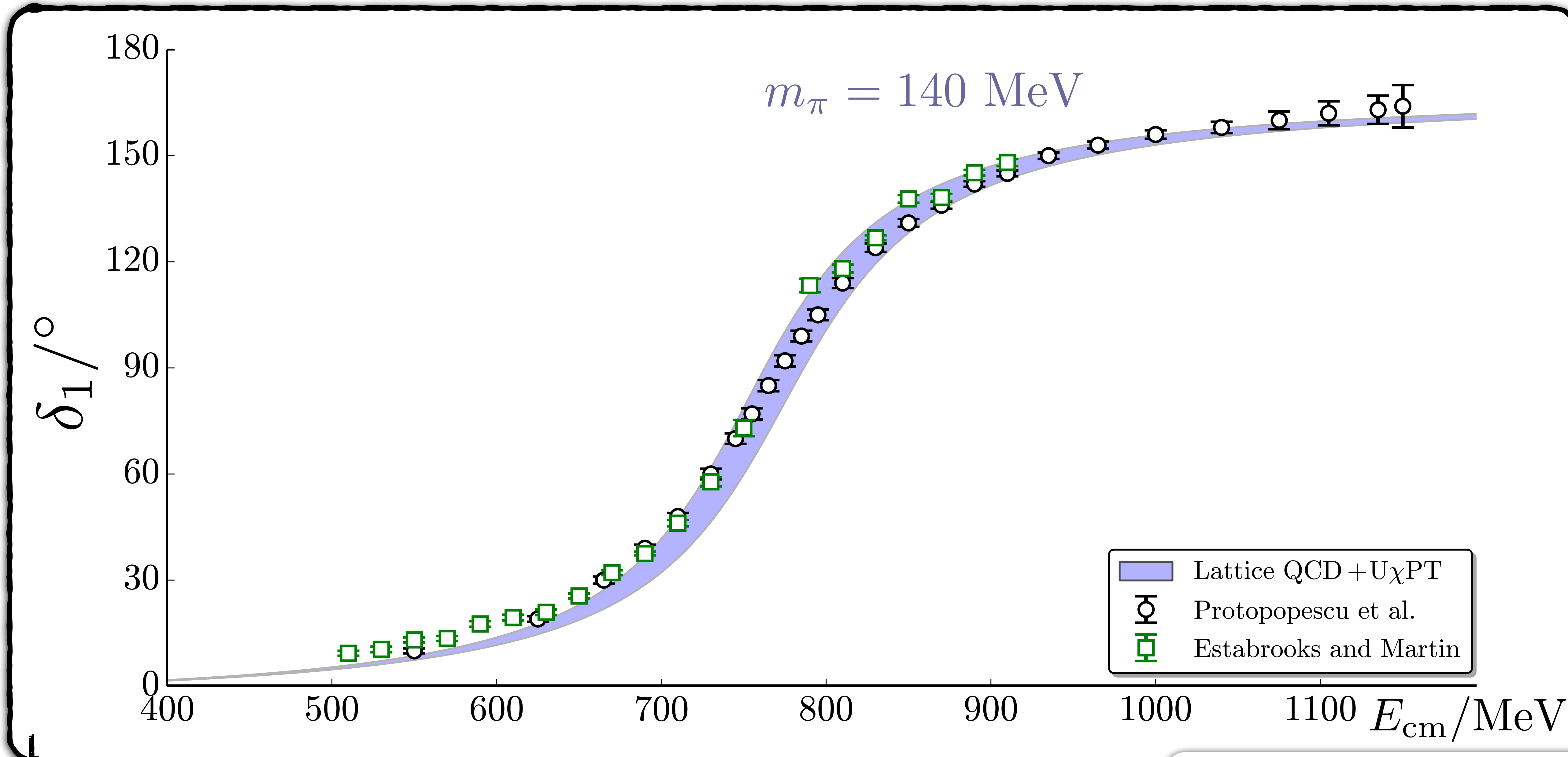
Isovector $\pi\pi$ scattering



$$\mathcal{M}_1 = \frac{16\pi E_{\rm cm}}{p\cot\delta_1 - ip}$$

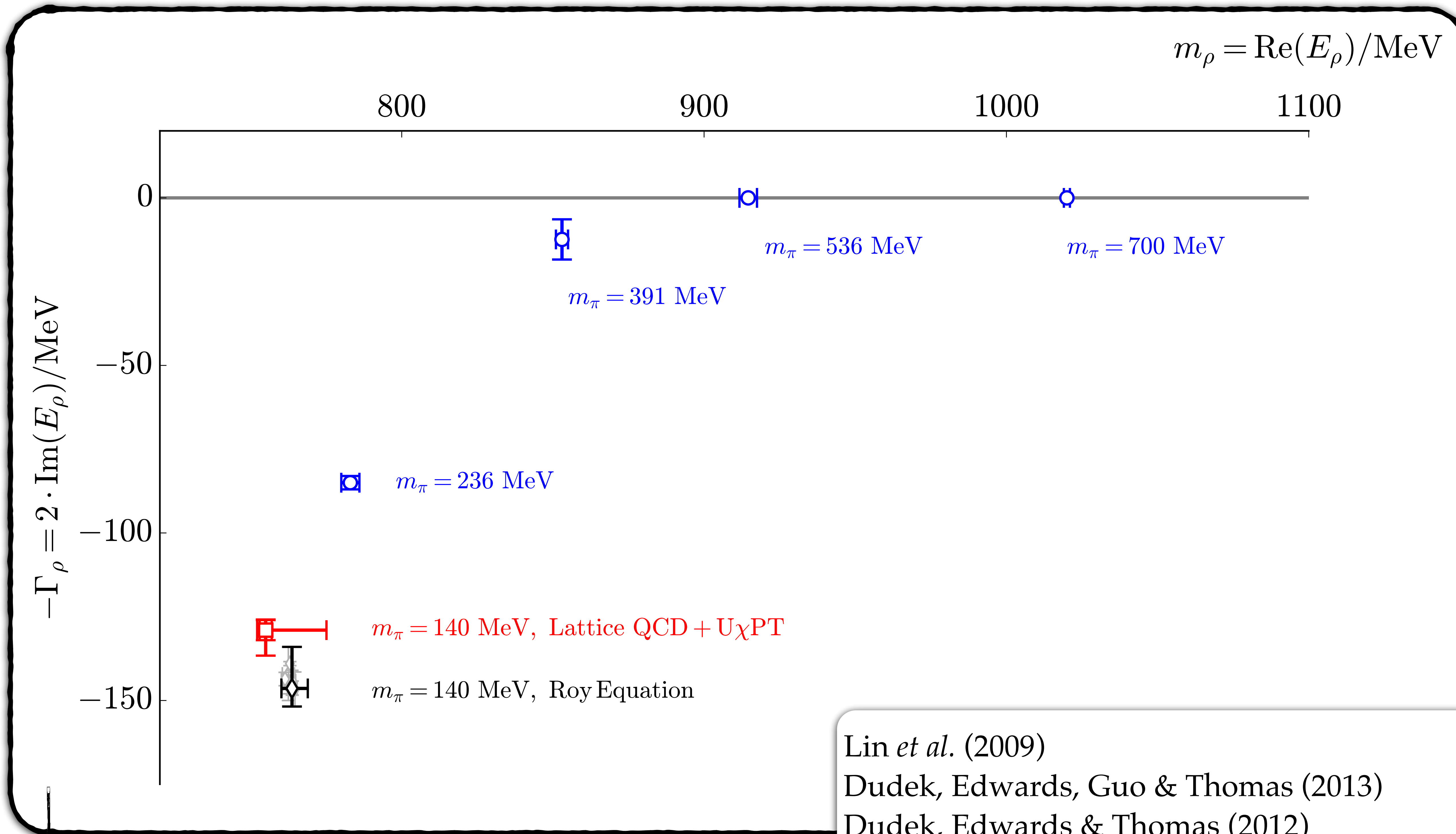
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

Comparison with experiment



Bolton, RB & Wilson (2016)

The ϱ vs m_π



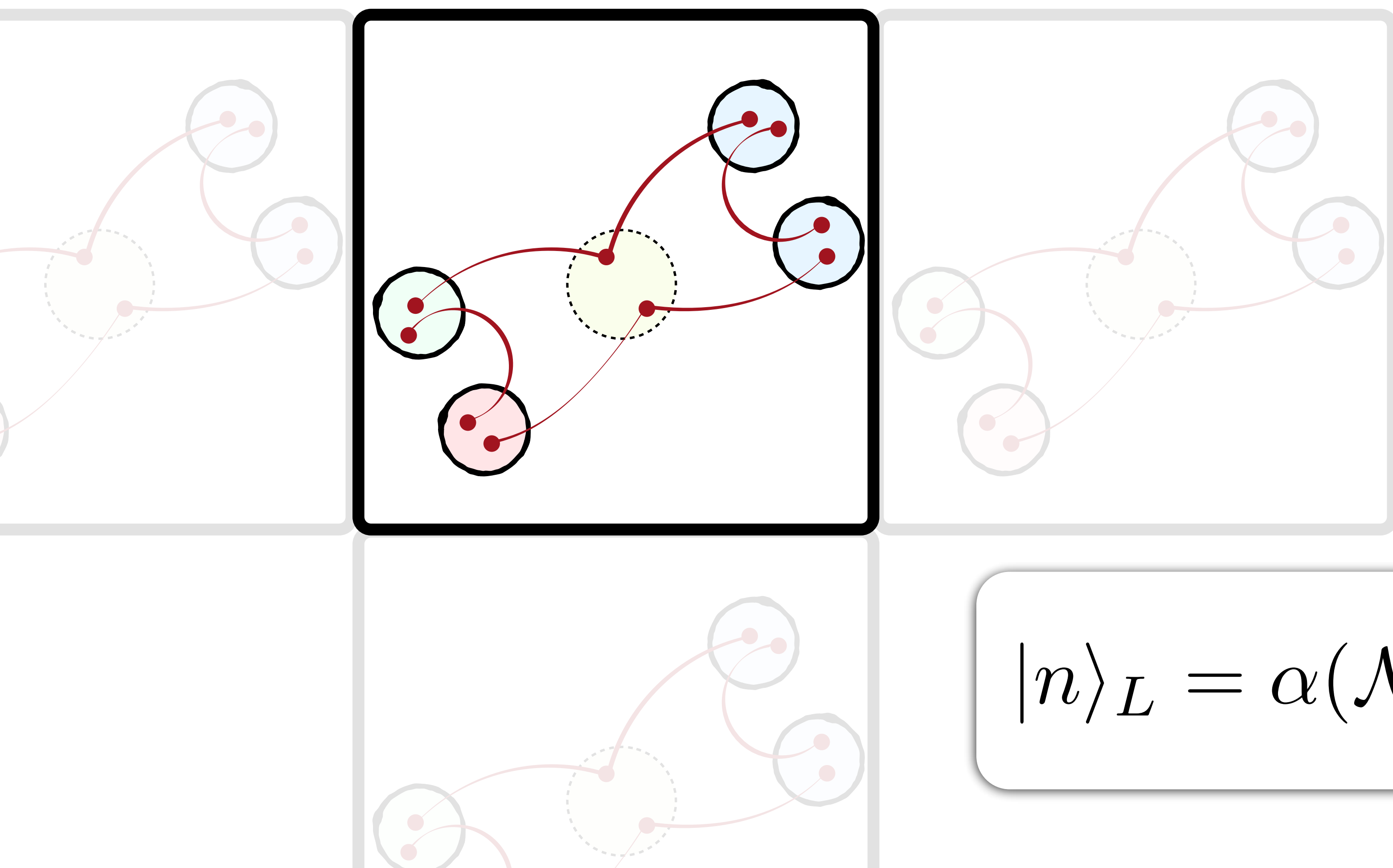
Lin *et al.* (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

More than one channel open

- *Coupled channels:* e.g., $\pi\eta$, $K\overline{K}$

$$\det\begin{bmatrix} F_{\pi\eta}^{-1}+\mathcal{M}_{\pi\eta,\pi\eta} & \mathcal{M}_{\pi\eta,K\overline{K}} \\ \mathcal{M}_{\pi\eta,K\overline{K}} & F_{K\overline{K}}^{-1}+\mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix}=0$$

Hansen & Sharpe / RB & Davoudi (2012)
RB (2014) / RB & Hansen (2015)

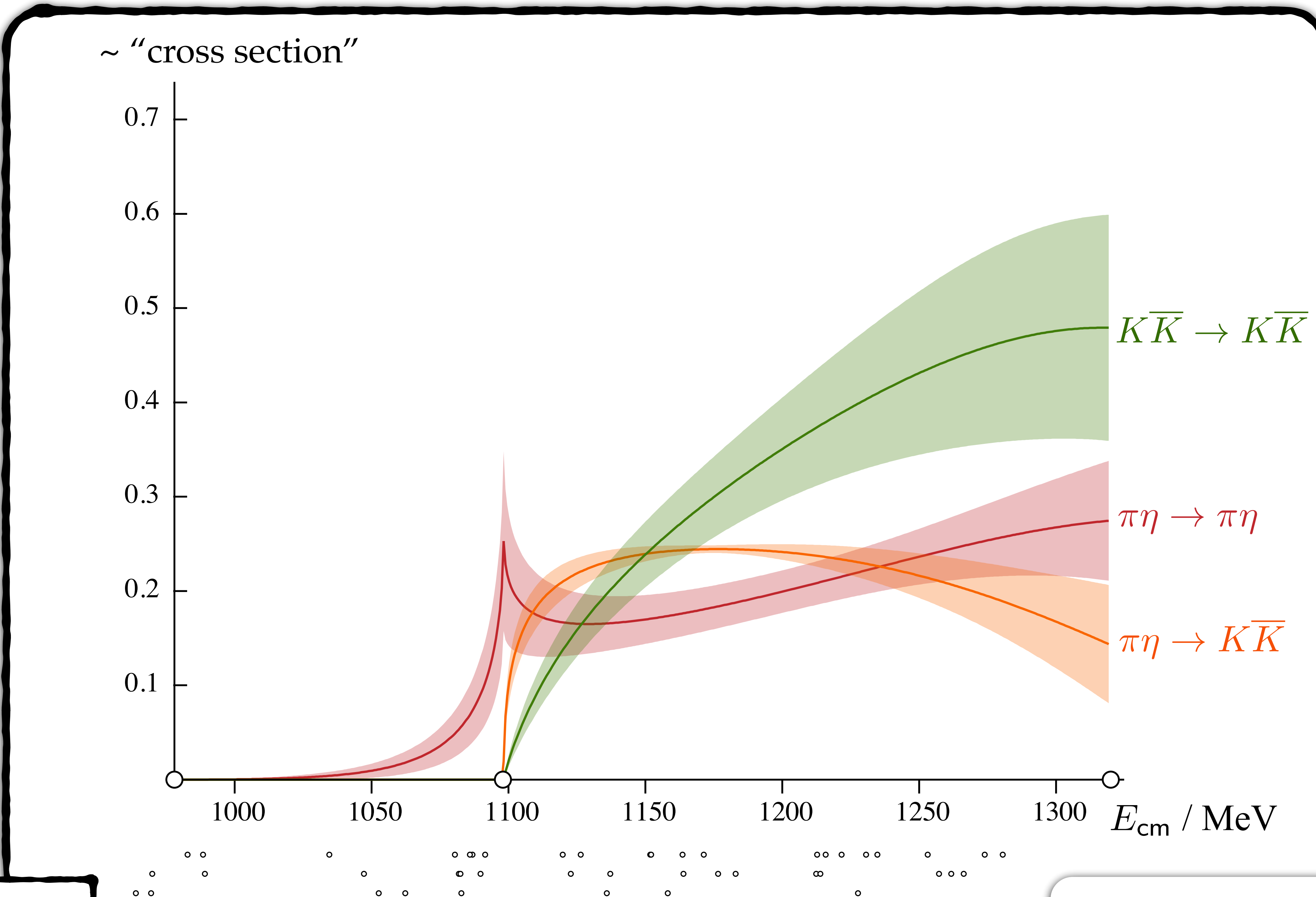


can't pull states apart!

$$|n\rangle_L=\alpha(\mathcal{M},L)|\pi\eta\rangle+\beta(\mathcal{M},L)|K\overline{K}\rangle$$

More than one channel open

- *Coupled channels:* e.g., $\pi\eta$, $K\overline{K}$



m_π=391 MeV

Dudek, Edwards & Wilson (2016)

More than one channel open

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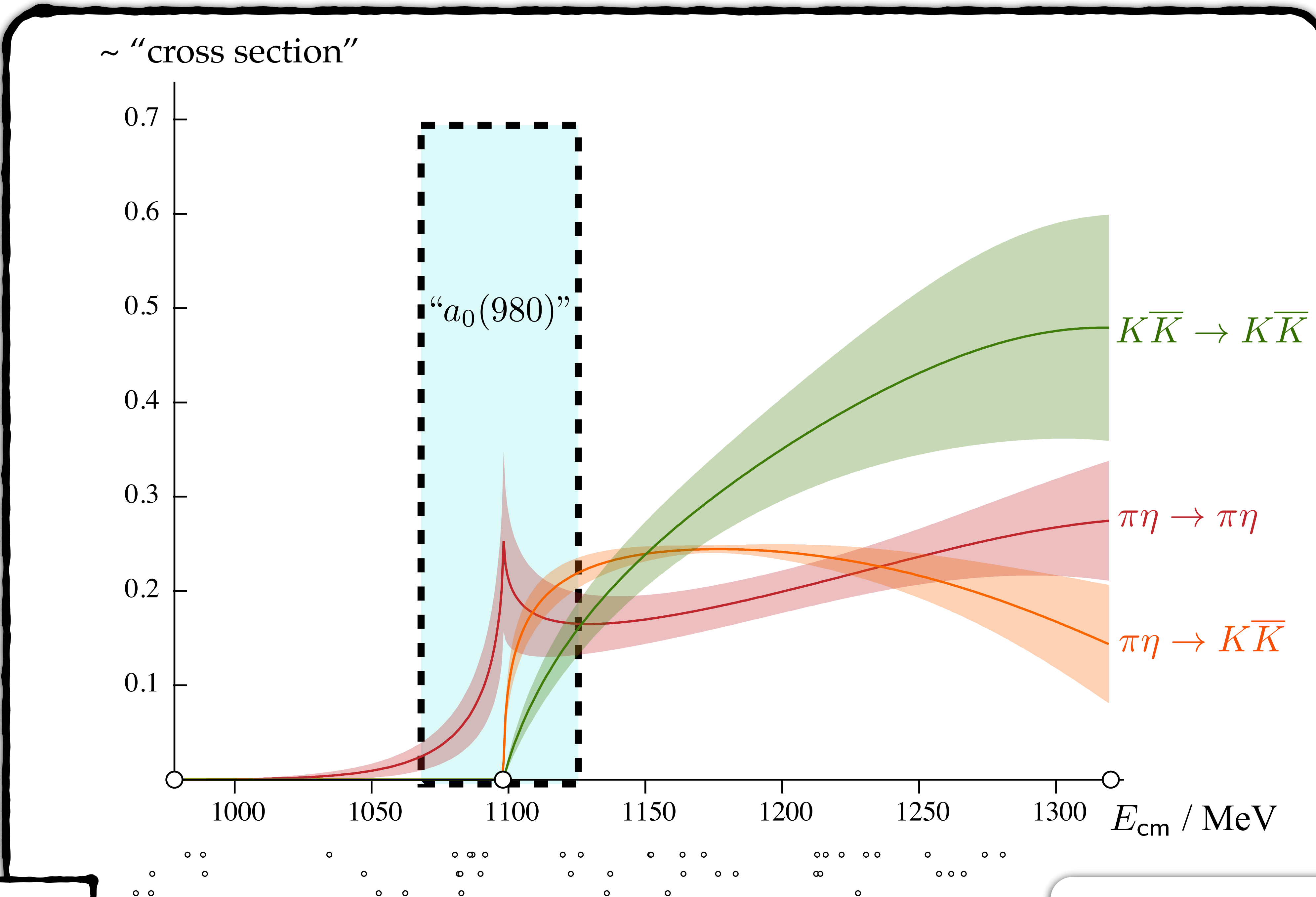
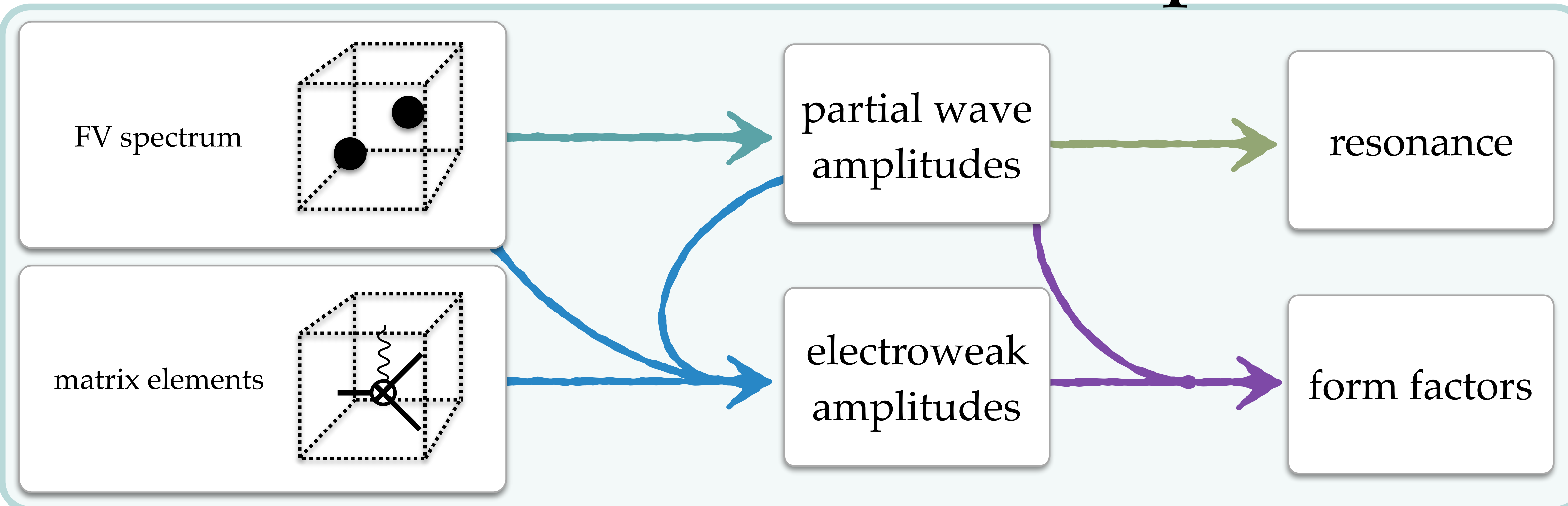


m_π=391 MeV

Dudek, Edwards & Wilson (2016)

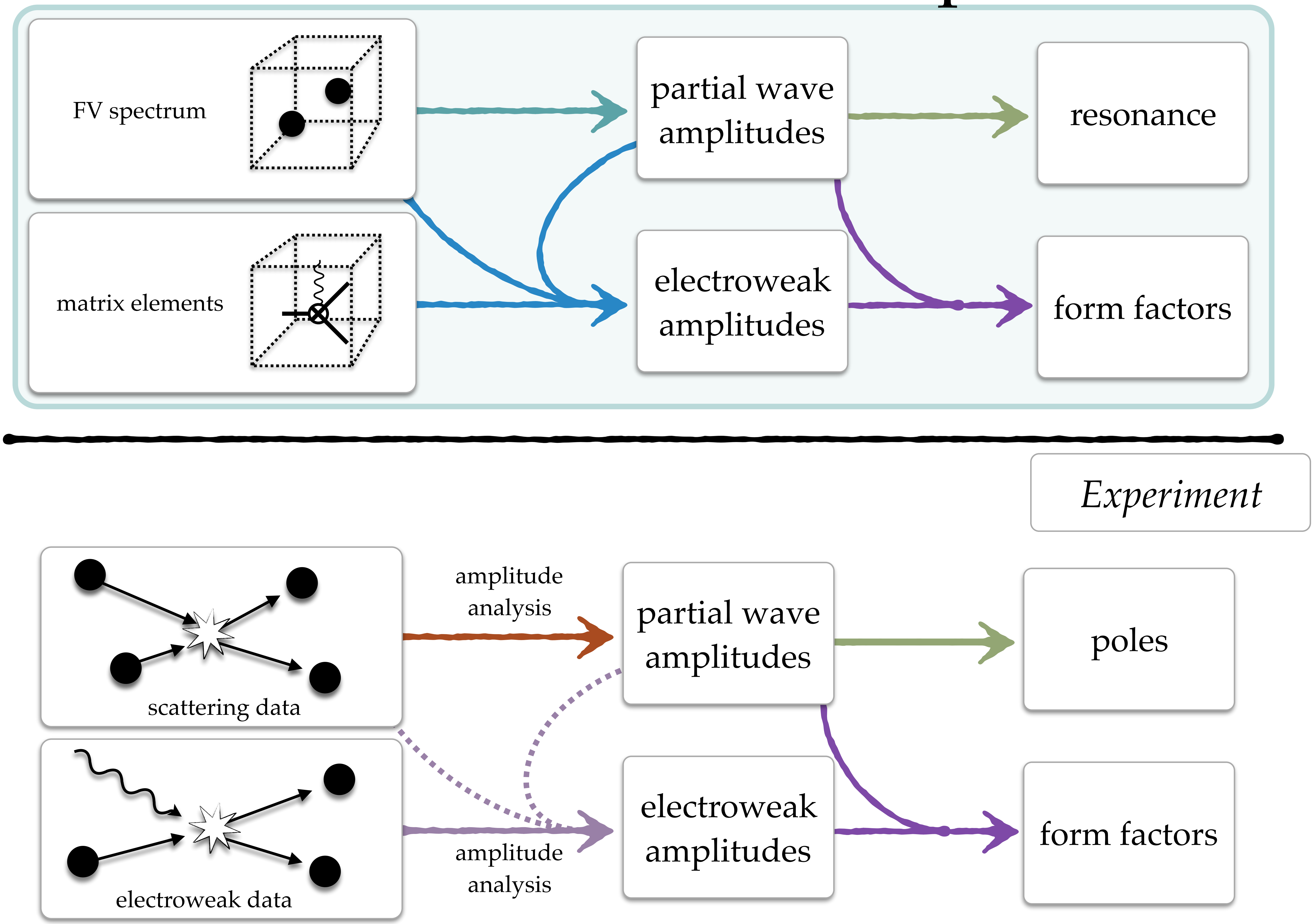
~~RB~~

Resonant electroweak amplitudes

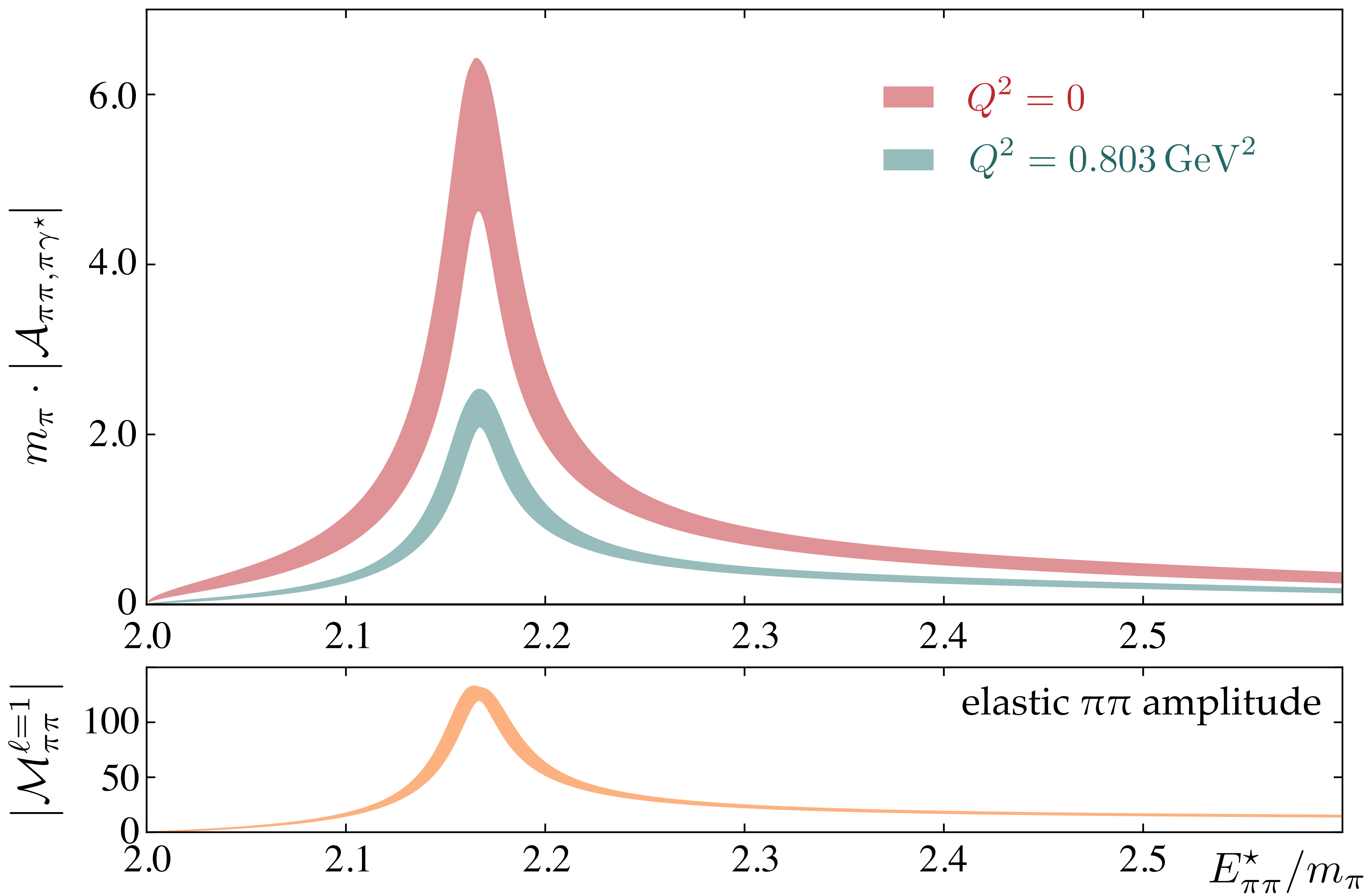


- Lellouch & Lüscher (2000) [K-to-$\pi\pi$ at rest]
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [moving K-to-$\pi\pi$]
- …
- Hansen & Sharpe (2012) [D-to-$\pi\pi$ / KK]
- RB, Hansen Walker-Lou / RB & Hansen (2014-2015) [general 1-to-2 result]

Resonant electroweak amplitudes



$\pi\gamma^*$-to-$\pi\pi$ amplitude

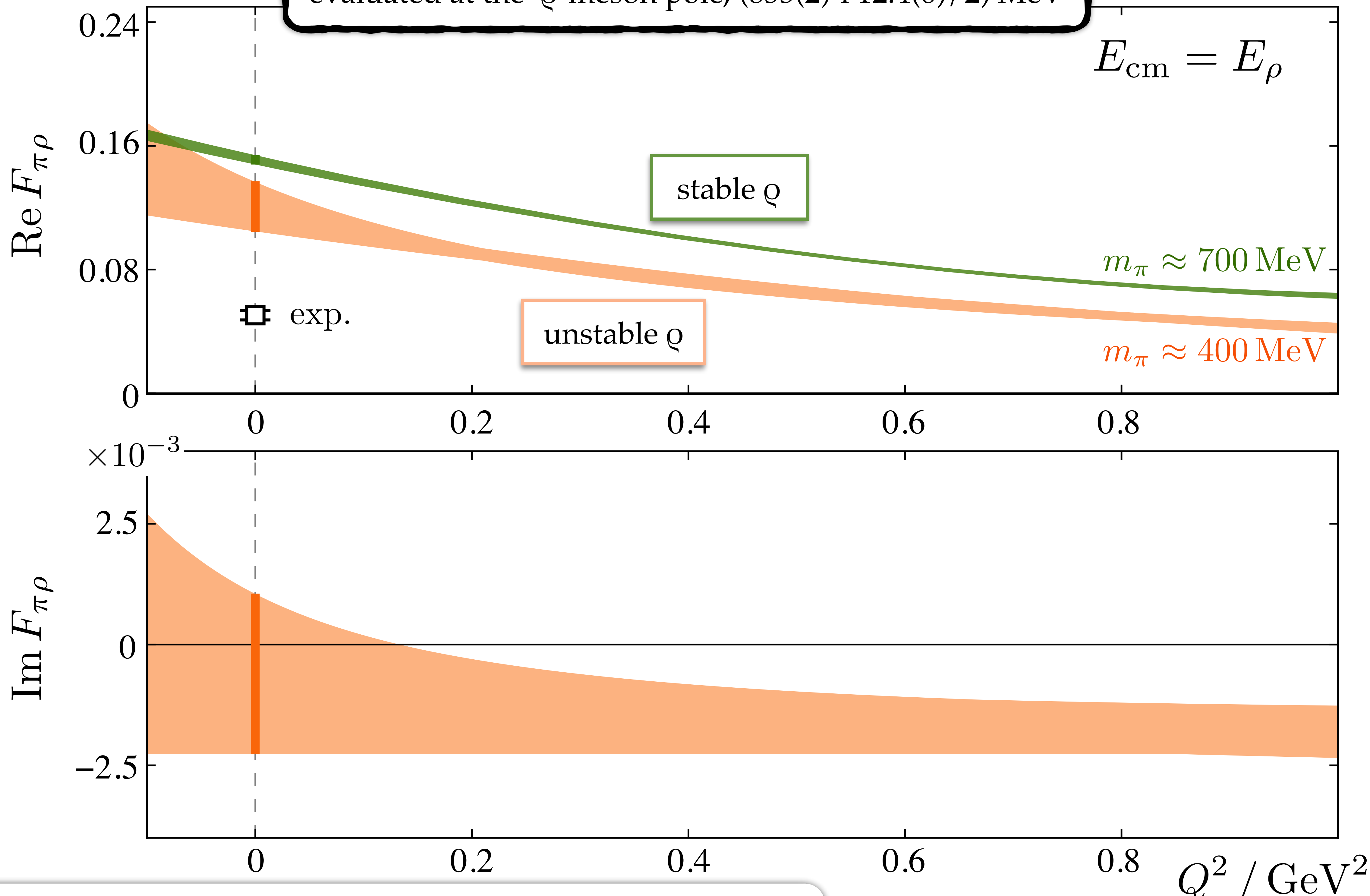


m_π=391 MeV

RB, Dudek, Edwards, Thomas, Shultz, Wilson - PRL (2015)

π-to-ρ form factor

evaluated at the ρ-meson pole, (853(2)-i 12.4(6)/2) MeV

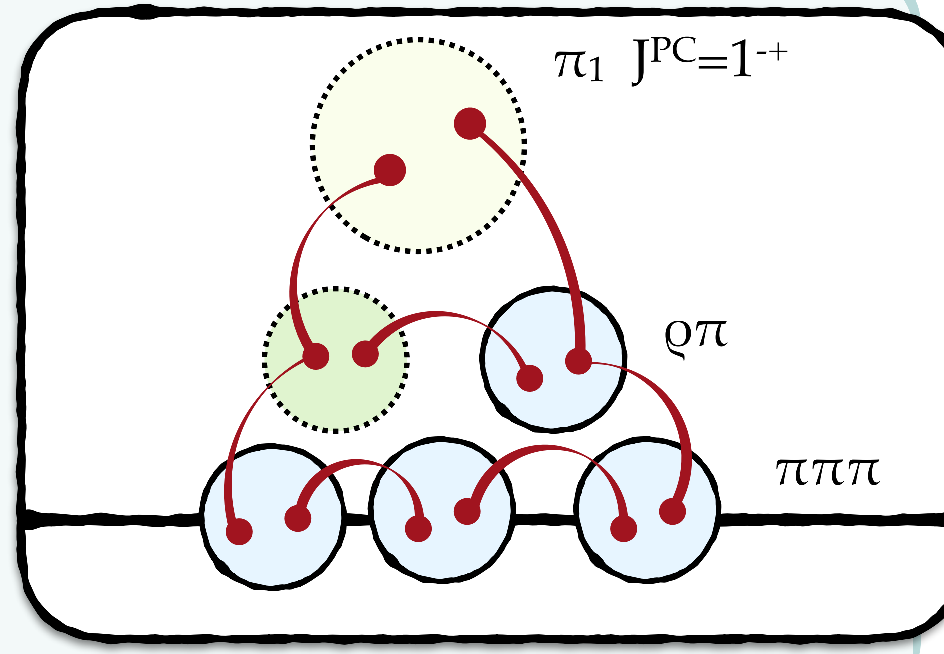


Shultz, Dudek, & Edwards (2014)
RB, Dudek, Edwards, Shultz, Thomas & Wilson - PRL (2015)

The future of few-body physics

- three-body scattering
- needed to study excited states
- will give access to nnn-force!

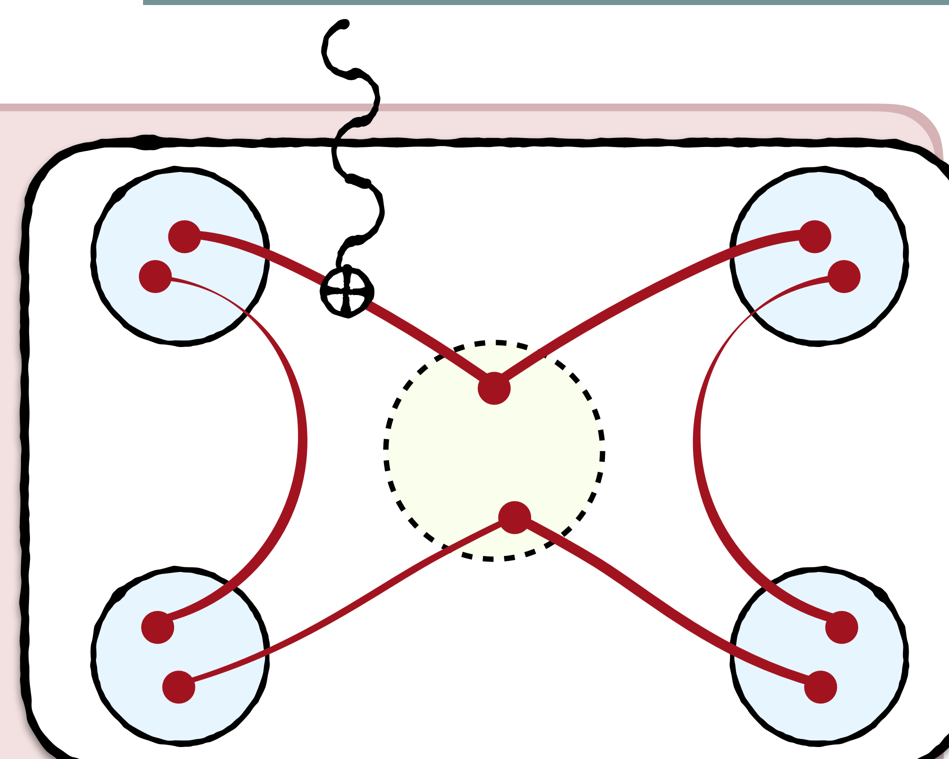
Complimentary to experiment!



RB, Hansen & Sharpe (2017)

- elastic form factors of:
  - resonances
  - bound states
- give information about structure
- formalism largely developed
- untested

Complimentary to experiment!



RB & Hansen (2016)

If you like to learn more

- *A detailed review and introduction to the quickly growing field….*

Scattering processes and resonances from lattice QCD

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[2] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*

[3] *Special Research Center for the Subatomic Structure of Matter (CSSM), Department of Physics, University of Adelaide, Adelaide 5005, Australia*

to appear…this week?

hadspec

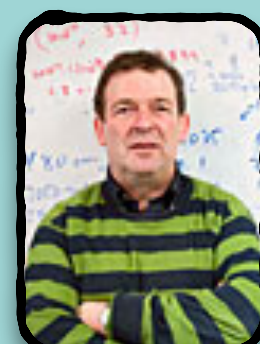

Chakraborty


Dudek


Edwards


Winter


Joó


Richards

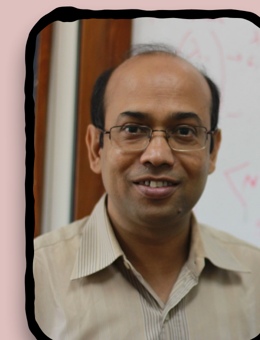

Wilson


Moir


Peardon


Ryan


Thomas


Mathur

Meson Spectrum

JHEP05 021 (2013)
PRD88 094505 (2013)
JHEP07 126 (2011)
PRD83 111502 (2011)
PRD82 034508 (2010)
PRL103 262001 (2009)

Baryon Spectrum

PRD91 094502 (2015)
PRD90 074504 (2014)
PRD87 054506 (2013)
PRD85 054016 (2012)
PRD84 074508 (2011)

Scattering

PRL118 022002 (2017)
JHEP011 1610 (2016)
PRD93 094506 (2016)
PRD92 094502 (2015)
PRD91 054008 (2015)
PRL113 182001 (2014)
PRD87 034505 (2013)
PRD86 034031 (2012)
PRD83 071504 (2011)

Electroweak

PRD93 114508 (2016)
PRL115 242001 (2015)
PRD91 114501 (2015)
PRD90 014511 (2014)

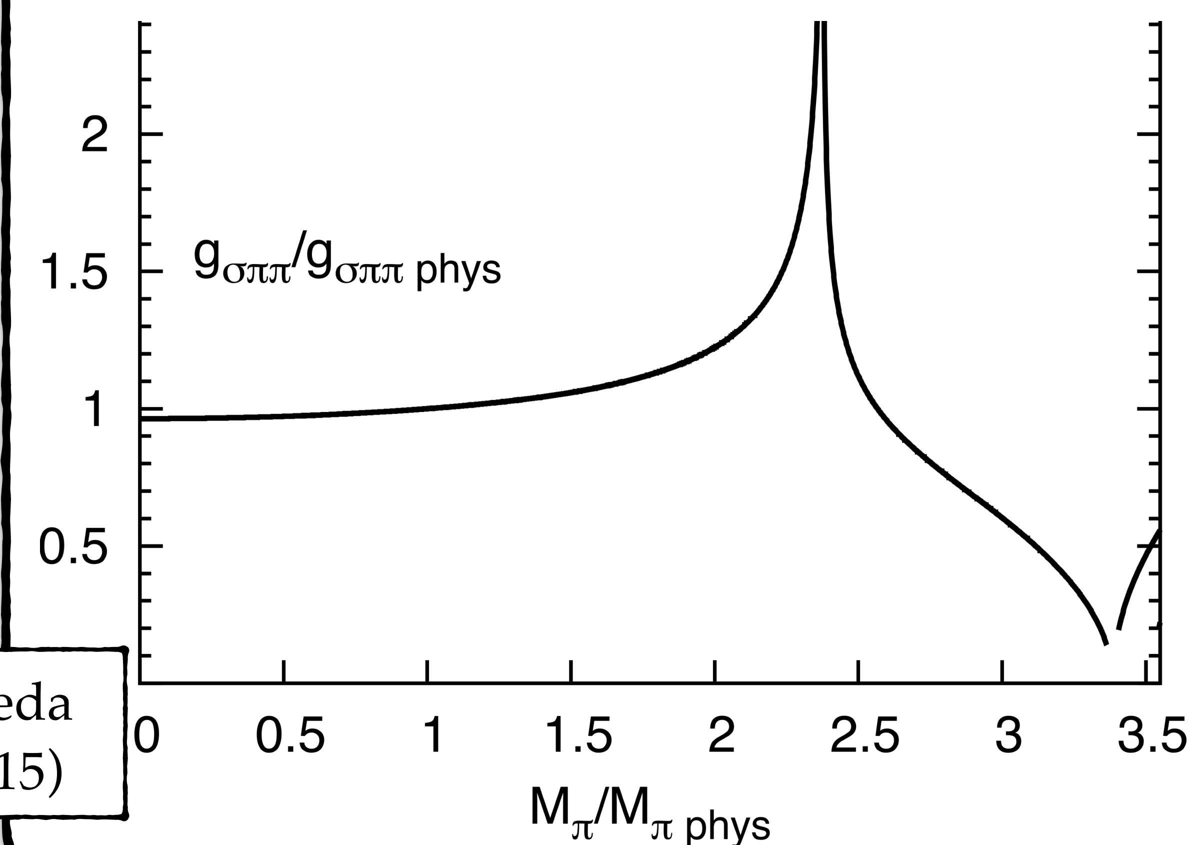
Techniques

PRD85 014507 (2012)
PRD80 054506 (2009)
PRD79 034502 (2009)

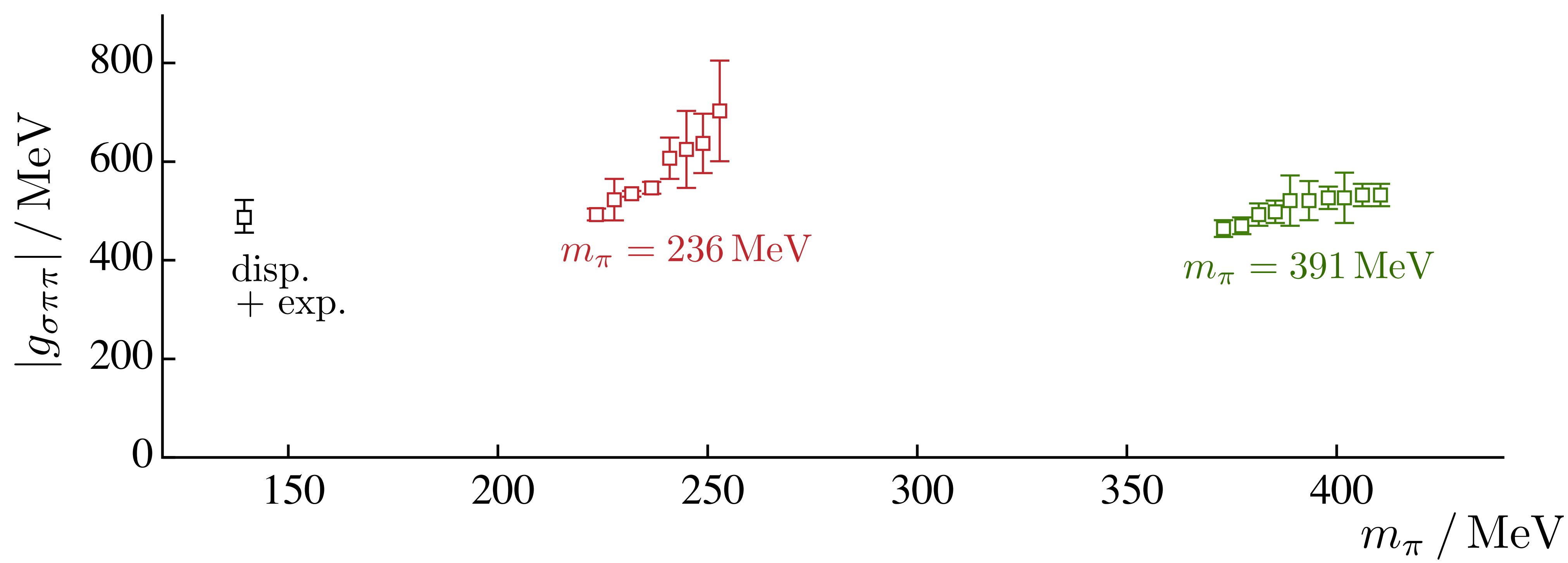
Formalism

PRD95 074510 (2017)
PRD94 013008 (2016)
PRD92 074509 (2015)
PRD91 034501 (2015)
PRD89 074507 (2014)

The σ / f_0(500) vs m_π

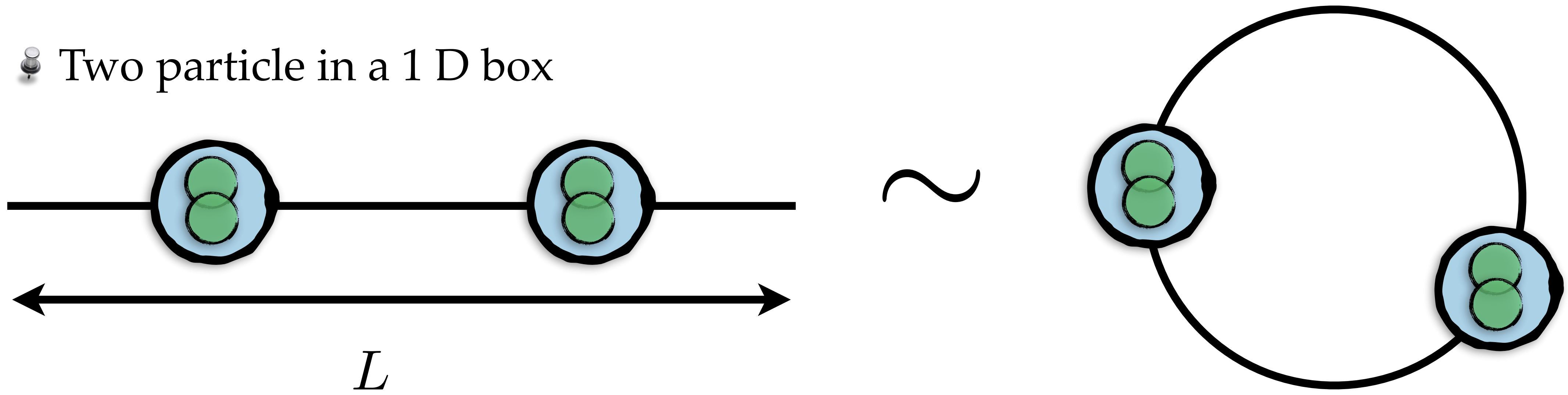


UχPT - Nebreda & Peláez (2015)



Two particles in 1+1 Dimensions

- Two particle in a 1 D box



- Asymptotic wavefunction: $\varphi_p(x) \sim \cos(p|x| + \delta)$

- Imposing periodicity:

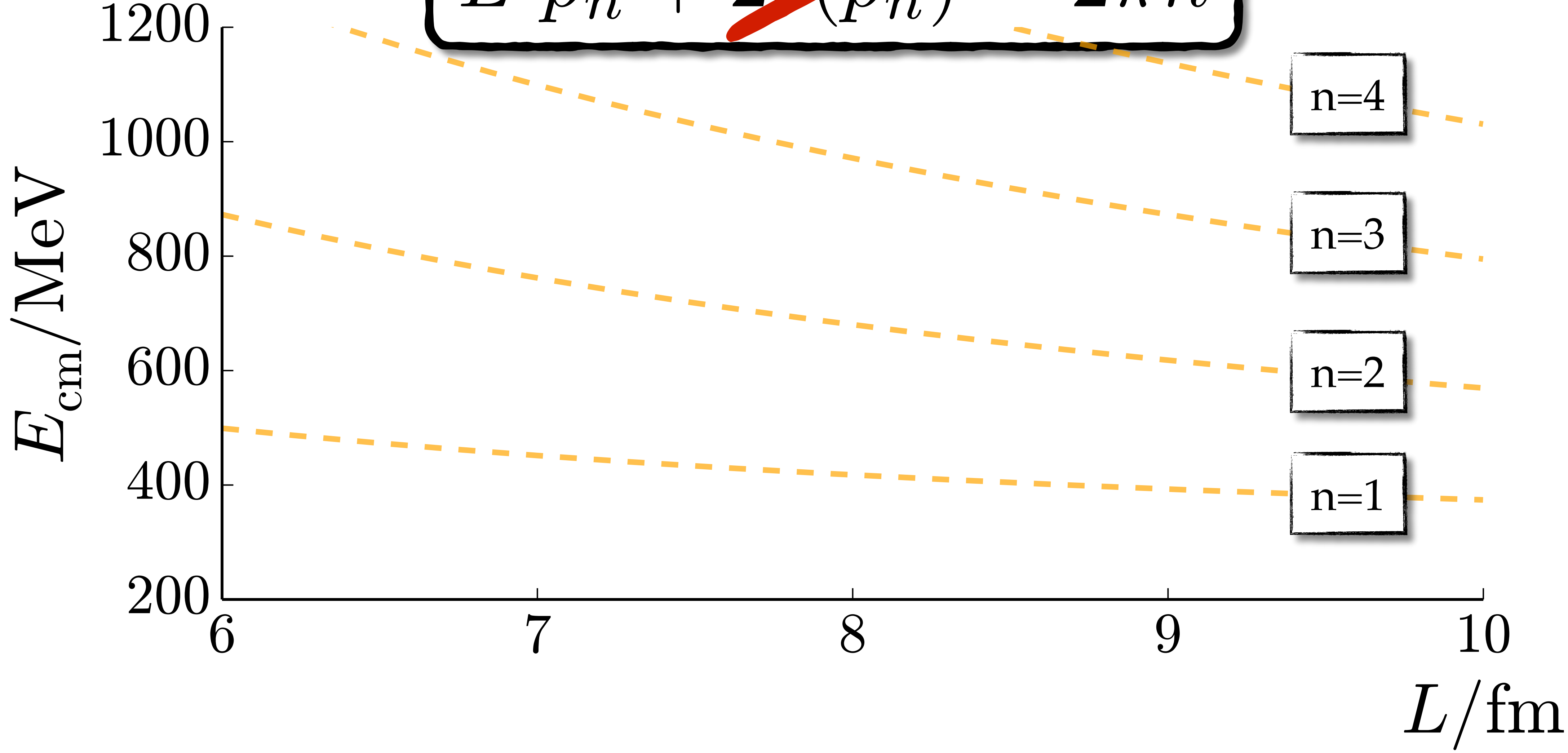
$$\varphi_p(L/2) = \varphi_p(-L/2)$$
$$\varphi'_p(L/2) = \varphi'_p(-L/2)$$

- Quantization condition:

$$\boxed{L\ p_n + 2\delta(p_n) = 2\pi n}$$

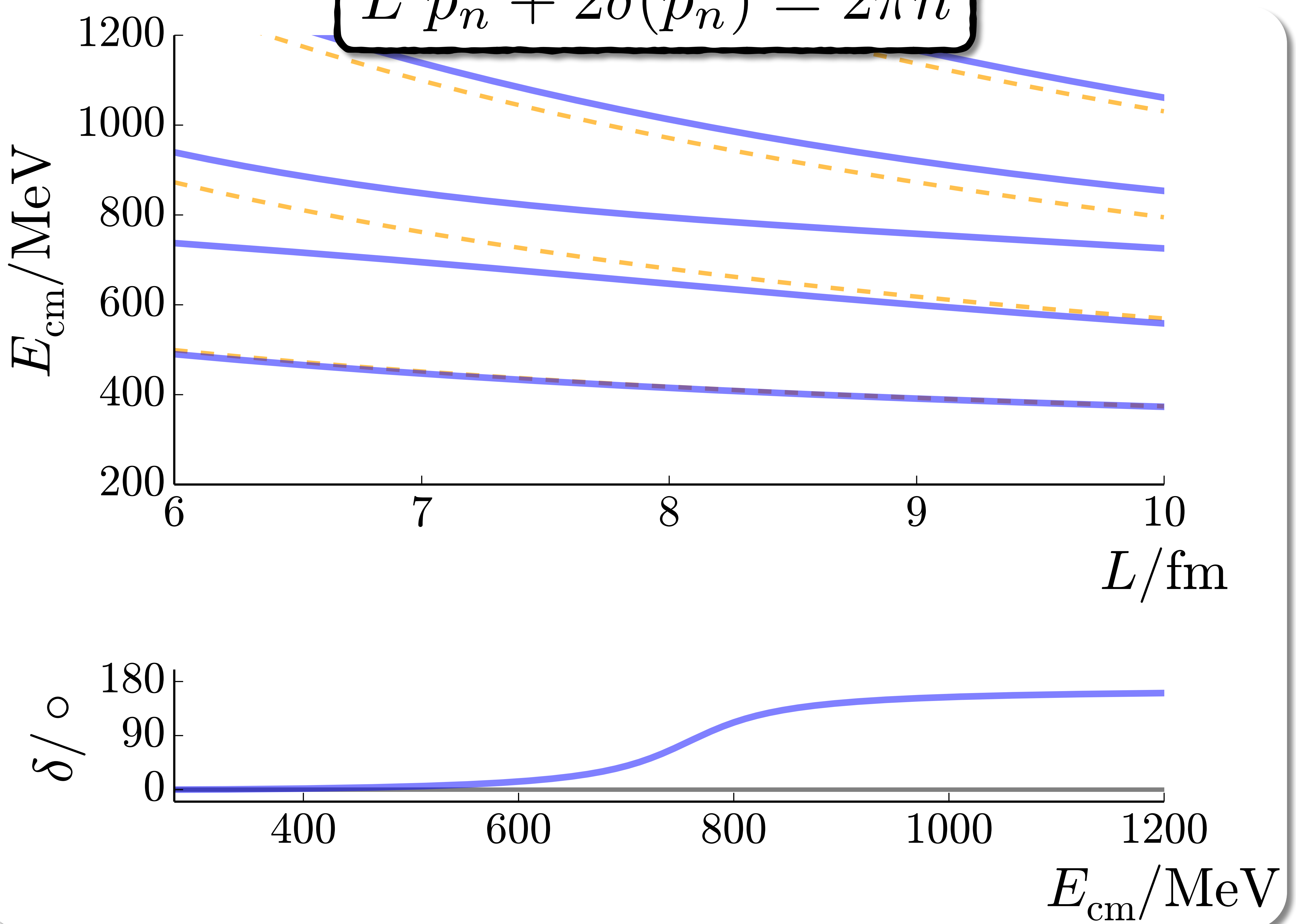
Spectrum in a 1+1D box

$$L\ p_n + \cancel{2\delta(p_n)} = 2\pi n$$



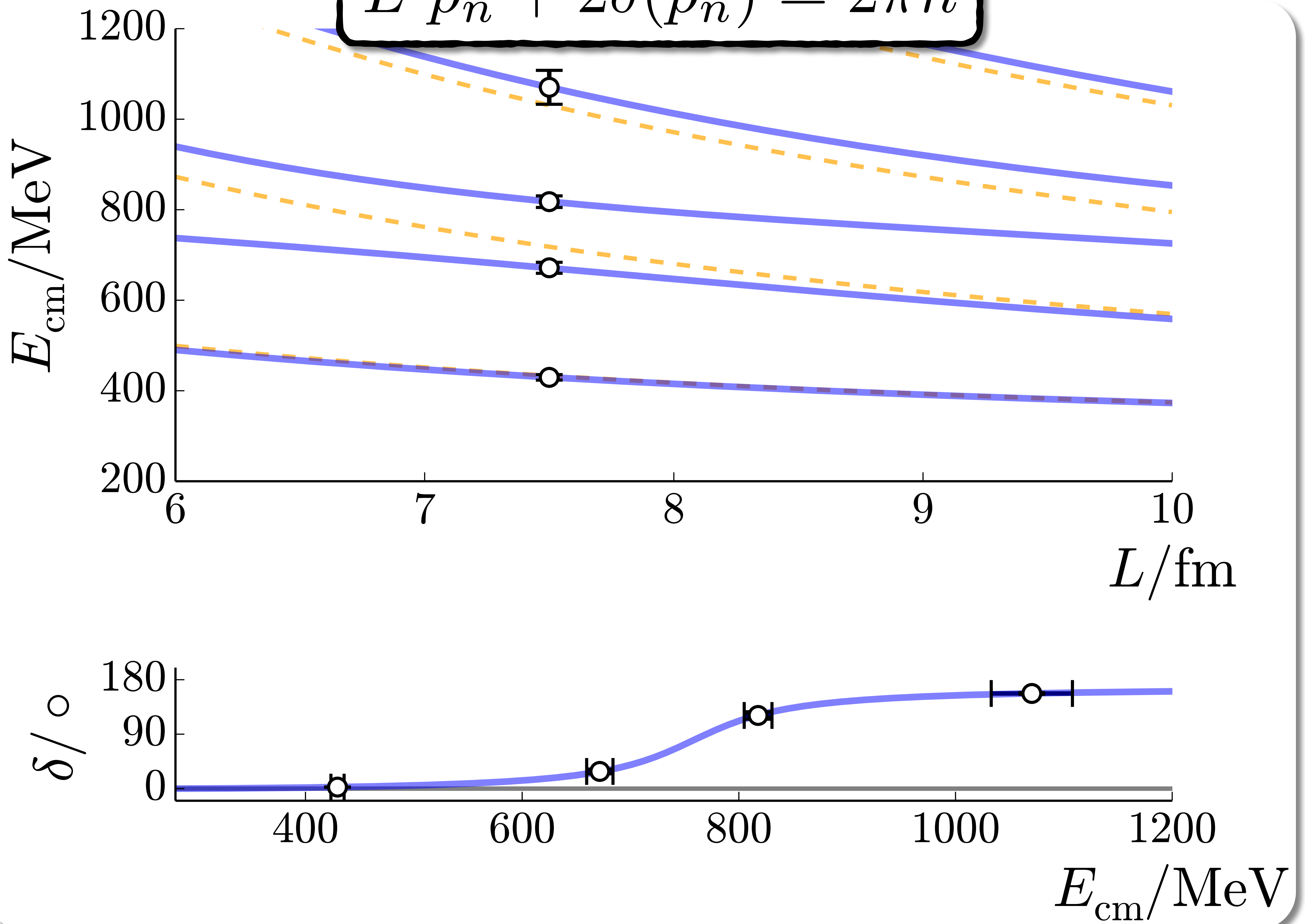
Spectrum in a 1+1D box

$$\boxed{L\ p_n + 2\delta(p_n) = 2\pi n}$$

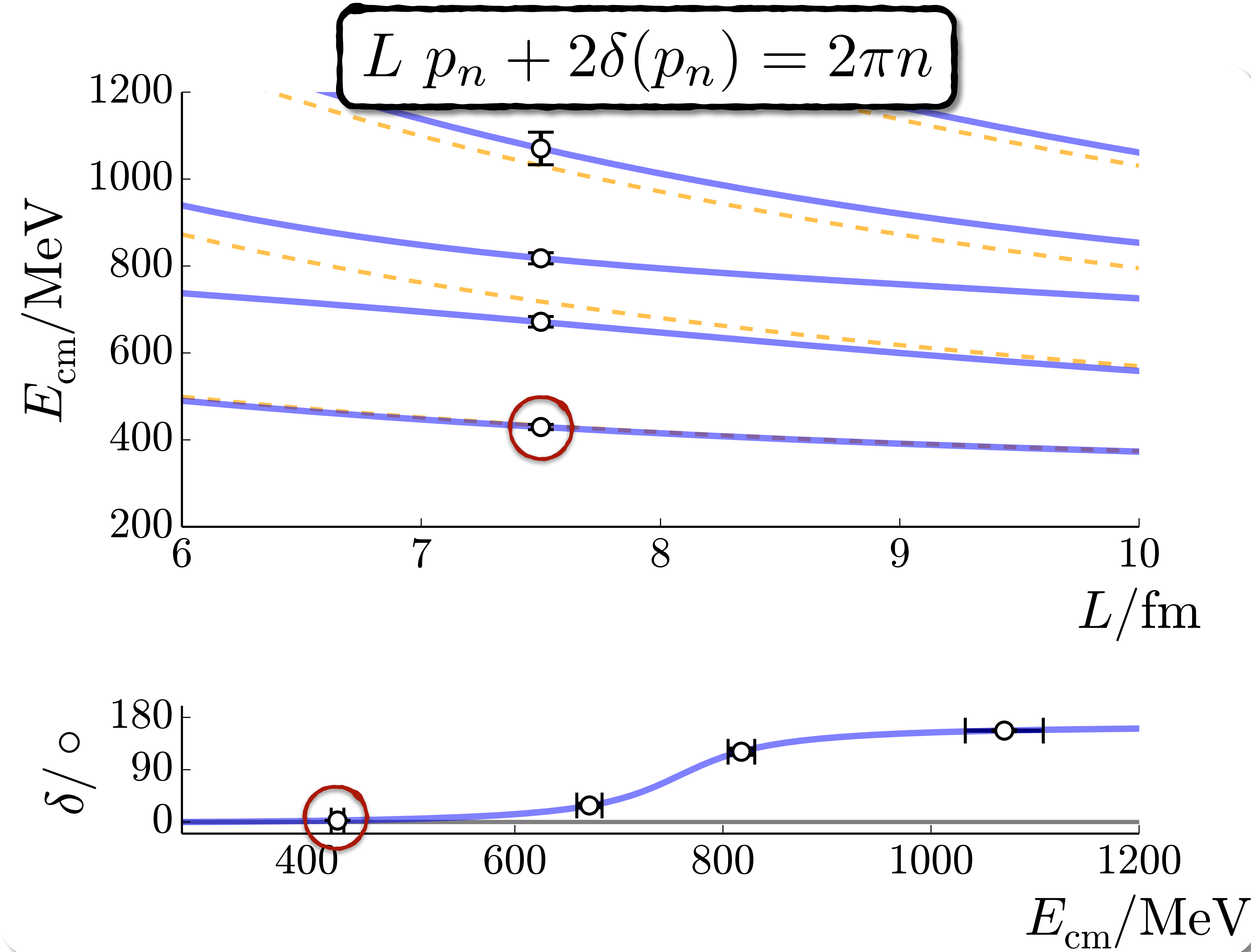


Spectrum in a 1+1D box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$

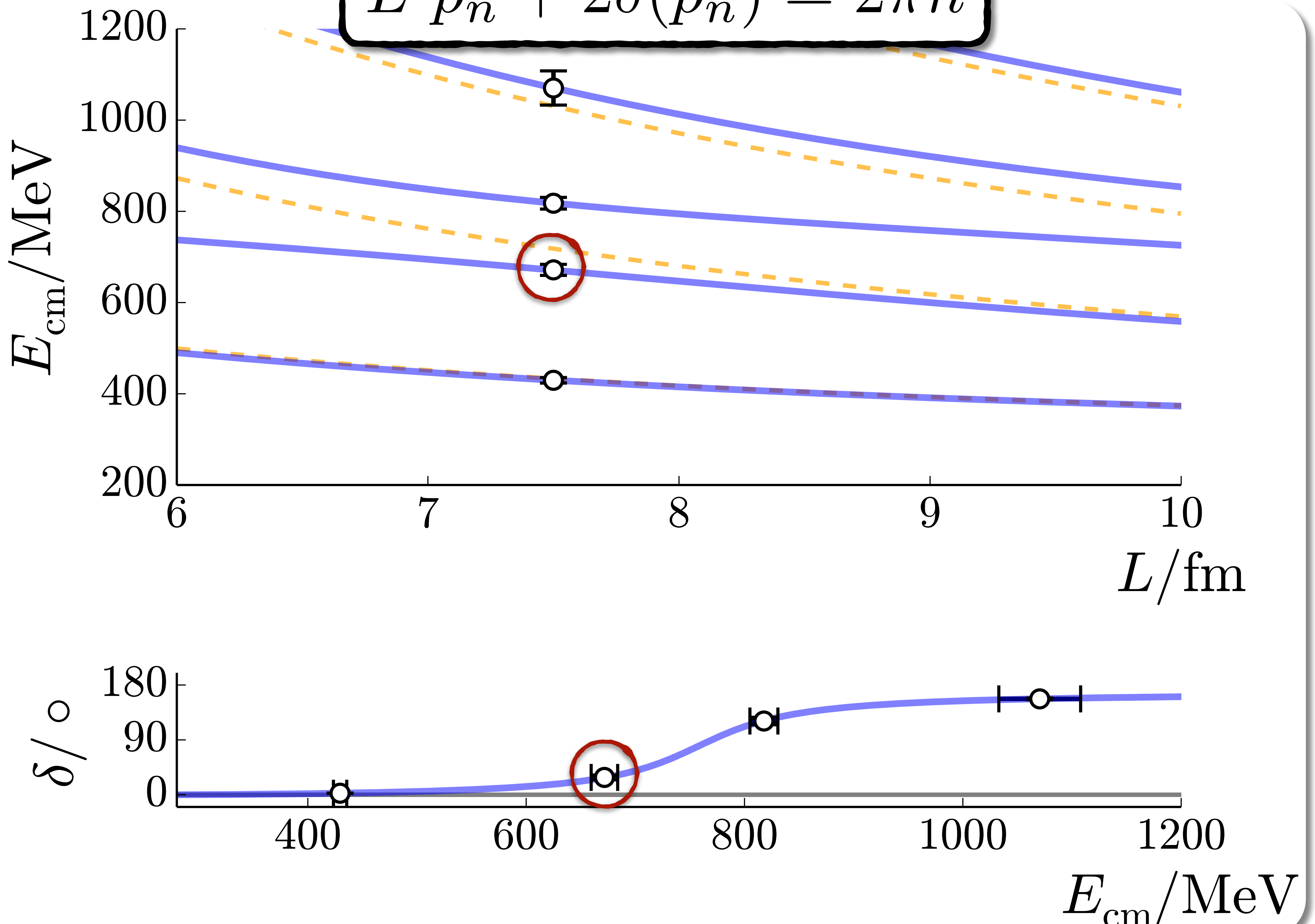


Spectrum in a 1+1D box



Spectrum in a 1+1D box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$



Spectrum in a 1+1D box