Nuclear axial currents and selected applications to few-nucleon systems

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Nuclear weak processes

- $\beta$ decays (single and double) important for
  - Precision tests of the Standard Model
  - $g_A$ quenching (implications for $0\nu\beta\beta$)
  - Nuclear astrophysics (Sun chain reaction)

- $\nu -$ nucleus scattering important for
  - Neutrino oscillations (SNO, ...)
  - Leptonic CP violation
  - Nuclear astrophysics (Supernovae, ..)

Well - known experimentally excellent test for the theory

Less - known experimentally need of theoretical input
• Solar neutrino problem

\( \Phi_{8B}^{\text{Expt.}} \sim \Phi_{8B}^{\text{SSM}} \)

Heavy-water Cherenkov counter built to study neutrinos coming from \(^8\text{B} \beta\)-decay (5-15 MeV)

• CC: \( \nu_e + d \rightarrow e^- + p + p \)

NC: \( \nu_l + d \rightarrow \nu_l + n + p \)

ES: \( \nu_l + e^- \rightarrow \nu_l + e^- \)
Nuclear electroweak interactions?

Atomic nuclei are a complex quantum-many body systems of strongly interacting nucleons.

Hadronic matrix elements difficult because of QCD

Lattice QCD (non perturbative method)

Effective field theories (expansion in kinematic variables)

http://www.tunl.duke.edu/nucldata/HalfLife.shtml
Build the most general Lagrangian with hadronic d.o.f. with the same exact symmetries and approximate symmetries of the underlying theory.

\[ \mathcal{L}_{\chi EFT} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots \]

\[ \mathcal{L}^{(n)} \sim \left( \frac{Q}{\Lambda_{\chi}} \right)^n \] nucleon momenta .. \( \sim 1 \text{ GeV} \)

Approximate chiral symmetry requires the pion to couple to other pions and to nucleons by powers of its momentum.

- S. Weinberg (1968-1979)
Nuclear $\chi$ EFT I

Nuclear bound states cannot be obtained from perturbation theory alone

LECs $\mathcal{L}_{\chi EFT}$

Calculate amplitudes+prescription to obtain potentials + regularization (of high momentum components)

$\pi$, $N$

Predictions

NOT YET!

$\psi_{2N}, \psi_{3N}, \cdots$

$J_{1N}^\mu, J_{2N}^\mu, \cdots$

observables for $\pi \pi, \pi N, \cdots$

Nuclear observables in two and three body systems

$ab\ initio$ methods for $A>2$: HH, QMC, NCS, CC, ….
Nuclear $\chi$EFT II

Nuclear observables in two and three body systems

LECs $\mathcal{L}_{\chi EFT}$

Calculate amplitudes + some prescription to obtain potentials + regularization

LECs $v_{2N}, v_{3N}, \ldots$ $J_{1N}^{\mu}, J_{2N}^{\mu}, \ldots$

Predictions

Observed for $\pi\pi, \pi N, \ldots$

Nuclear bound states cannot be obtained from perturbation theory alone...

$\tau^N$ $\pi^N$

Perturbation theory? Useful to keep in mind...
• Define a weak transition potential \( v_5 = A_0^a \rho_{5,a} - \mathbf{A} \cdot \mathbf{j}_{5,a} \) (similar to EM)

• We require the weak interaction potential to match the on shell scattering amplitude

\[
T_5 = v_5 + v_5 \frac{1}{E_i - H_0 + i\epsilon} T_5
\]

• Perturbative expansion in powers of the nucleon momenta

\[
T_5 = T_5^{\text{LO}} + T_5^{\text{NLO}} + T_5^{\text{N2LO}} + \cdots
\]
\[
v_5 = v_5^{\text{LO}} + v_5^{\text{NLO}} + v_5^{\text{N2LO}} + \cdots
\]

• Matching order by order

\[
v_5^{\text{LO},a} = T_5^{\text{LO}}
\]
\[
v_5^{\text{NLO},a} = T_5^{\text{NLO}} - \left( v_5^{\text{LO},a} \frac{1}{E_i - E_I + i\epsilon} v_5^{\text{LO}} + \text{permutations} \right)
\]
\[
\cdots
\]
\[
\rho_{5,a} = \rho_{5,a}^{\text{LO}} + \rho_{5,a}^{\text{NLO}} + \rho_{5,a}^{\text{N2LO}} + \cdots
\]
\[
\mathbf{j}_{5,a} = \mathbf{j}_{5,a}^{\text{LO}} + \mathbf{j}_{5,a}^{\text{NLO}} + \mathbf{j}_{5,a}^{\text{N2LO}} + \cdots
\]
• A subtle point: operators derived are not unique!

\[ v_{5,a}^{\text{LO}} = T_5^{\text{LO}} \]
\[ v_{5,a}^{\text{NLO}} = T_5^{\text{NLO}} - \left( \frac{1}{v_{5,a}^{\text{LO}}} \frac{1}{E_i - E_I + i\epsilon} v^{\text{LO}} + \text{permutations} \right) \]

• Biblio


N. Kaiser et al. for nuclear potentials Feynman diagrams (1998)

S. Pastore et al. (2008-2011) for em currents, M. Piarulli et al. (2013) for em currents, TOPT

AB et al. (2016) for axial currents, TOPT

• Alternative approach using unitary transformations:

Epelbaum, Krebs, Meissner, et al. (1998-2017), for nuclear potentials, em and axial currents
Summary

- Axial current and charge derived up to N4LO

- Self consistency checks:
  - Current conservation in the chiral limit
    \[ \mathbf{q} \cdot \mathbf{j}_{5,a} = [H, \rho_{5,a}] \rightarrow \text{satisfied order by order} \]
  - Renormalization of the axial charge (delicate cancellation of divergences)
  - Independence of the choice of the parametrization of the pion field

- Technical challenges:
  - "New" class of diagrams appear respect to EM currents, formalism had to be
    adjusted to include them
  - >1000 diagrams in TOPT (no software infrastructure available)

- Difference for some loop topologies with another recent derivation
  - H. Krebs, E. Epelbaum, and Meissner, Unitary transformation
Axial currents

Strong and EM LECs partially known

1+4 “Weak” LECs ??

How do we fix them before?
Actually…

LO chiral 3N force $\cdots \cdots \cdots$ $c_E$ Fixed to 3N binding energies $\cdots \cdots$

Gazit et al. (2009), Marcucci et al. (2011), AB et al. (2017)
R. Schiavilla private communications (2018) (correct relation)

Axial current 2N contact term $\cdots$ Can be fixed with beta decays $\cdots$
• We look at tritium beta decay rate (simplest beta decay), transition rate well known experimentally:

\[
(1 + \delta_R) \, t f_V = \frac{K / G_V^2}{\langle F \rangle^2 + f_A / f_V \, g_A^2 \langle GT \rangle^2} \quad \langle \bar{3}\text{He} | \rho^+ | 3\text{H} \rangle \quad \langle 3\text{He} | j_5^+ | 3\text{H} \rangle
\]

• Wave functions are obtained solving the 3-body Schrödinger equation
  (Pisa group specialty, Hyperspherical Harmonics, \textit{ab initio} method)

\[
\hat{H}_{\chi\text{EFT}}(c_D) | 3\text{H}(c_D) \rangle = E_{3\text{H}} | 3\text{H}(c_D) \rangle \\
\hat{H}_{\chi\text{EFT}}(c_D) | 3\text{He}(c_D) \rangle = E_{3\text{He}} | 3\text{He}(c_D) \rangle
\]

• Since the 3N potential depends on 2 unknown LECs we fix \( c_E \) to three-nucleon binding energies and we get a family of wave functions

Marcucci, Kievsky, Viviani, Rosati (1990-2018)
Fix the LEC in the axial current II

- Fitting of the triton GT matrix element using AV18+UIX and N4LO currents

AB, Schiavilla, Marcucci et al. (2016)
Fix the LEC in the axial current III

- Fitting of the triton GT matrix element

\[ \langle {}^3\text{He}(c_D)|H_{\text{EFT}}(c_D)|^3\text{H}(c_D)\rangle = f(c_D) \]

Wave functions from Entem-Machleidt Chiral potential N3LO currents

<table>
<thead>
<tr>
<th>$\Lambda$</th>
<th>500 MeV</th>
<th>600 MeV</th>
</tr>
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<tbody>
<tr>
<td>$c_D$</td>
<td>0.65-1.24</td>
<td>0.92-1.37</td>
</tr>
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</table>

AB, Schiavilla, Marcucci et al. (2017)
L. Marcucci et al. (2018)
AB, Schiavilla, Marcucci et al. (2018), in preparation

Courtesy of L. Marcucci
## CONTRIBUTIONS

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<td><strong>N2LO</strong></td>
<td>$-0.569 \times 10^{-2}$</td>
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<td><strong>N3LO($^{1}\pi$)</strong></td>
<td>$0.825 \times 10^{-2}$</td>
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<td><strong>N4LO(3Bd)</strong></td>
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<td>$-0.153 \times 10^{-2}$</td>
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- **Major contribution**
- **Relativistic correction to 1-body**
- **2-body tree level, pion range**
- **Loop big effect**
- **3-body currents, suppressed**
Take home message

- LEC in the axial current determined using tritium beta decay
- Loop give important contribution
- Axial current acquires predictive power
- For axial charge as a first step we will assume LECs $\sim 1$
- Second not trivial application is low energy neutrino deuteron scattering
Neutrino deuterium

\[
\frac{d\sigma}{de'd\Omega} \propto G_F^2 L_{\mu\nu} W_{\mu\nu}^{CC} \frac{1}{3} \sum_{M,\{f\}} \delta(E_i - E_f) \langle f | j_{\mu CC}^\nu | d, M \rangle \langle d, M | j_{\mu CC}^\nu | f \rangle^* \]

Can be computed numerically

• Similar for neutral current process

• Nakamura et al. (2002), Phenomelogical interactions
• Shen et al. (2011), Phenomenological interactions
• AB and Schiavilla 2017 (first chiral EFT calculation)
Results I: Differential cross sections

$\theta=90^\circ$  $E_\nu=10$ MeV

\[ \frac{d\sigma}{d\varepsilon'} \text{[10}^{-17} \text{fm}^2/(\text{MeV sr})] \]

\[ \varepsilon' (\text{MeV}) \]

\[ \nu_e-\text{CC} \]

\[ \bar{\nu}_e-\text{CC} \]

This paper  Nakamura et al. (2002)
Results II: Total cross sections

\( \Lambda = 500 \text{ MeV} \)

\( \nu_e\text{-CC} \)

\( \bar{\nu}_e\text{-CC} \)

\( \nu_e\text{-NC} \)

for \( \Lambda = 600 \text{ MeV} \) variation \( \leq 1\% \)

\( \chi \text{EFT} \)

\( \text{Nakamura et al. (2002)} \)
Currents used for beta decays

Electroweak currents, with phenomenological potentials, GFMC 2-body currents play a big role

Pastore et al. (2018)
Outlook

- Currents derived up to N4LO
- LEC in the axial current fixed with experimental GT matrix element
- Prediction for neutrino deuteron→confirm phenomenological approaches
- Hybrid calculations in beta decays denote big effect of two-body currents
- Systematic study of theoretical uncertainty
- LQCD to determine the LEC in the axial current (validation) (Savage et al. 2017)
- Refine calculations for beta decays/include delta in the currents (?) (Goity et al. 2012)
<table>
<thead>
<tr>
<th>Collaborators</th>
<th>Institution</th>
</tr>
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<tbody>
<tr>
<td>L. Girlanda</td>
<td>INFN Lecce, Italy</td>
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<td>L. E. Marcucci</td>
<td>Univ. Pisa, Italy</td>
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<td>INFN Pisa, Italy</td>
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<td>S. Pastore</td>
<td>LANL, Washington University, USA</td>
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<td>R. Schiavilla</td>
<td>JLab/ODU, USA</td>
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<td>M. Viviani</td>
<td>INFN Pisa, Italy</td>
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**Advisor**

Thank you!
Collaborators
Axial current

For $n$ vertices $n!$ diagrams (TOPT)

- Loop diagrams have been regularized (dim reg)

No new contacts
Convergence pattern

\[ E_{\nu} \text{(MeV)} \]

\[ N(1|2)_{\text{LO}} / N(1|2)_{\text{LO}} \]

\[ N(2|3)_{\text{LO}} / N(1|2)_{\text{LO}} \]

\[ N(3|4)_{\text{LO}} / N(2|3)_{\text{LO}} \]

\[ \nu_e - \text{CC} \]

\[ \bar{\nu}_e - \text{CC} \]

\[ \nu_e - \text{NC} \]

\[ \bar{\nu}_e - \text{NC} \]
Axial charge

Loop diagrams have been regularized (dim reg) divergences are reabsorbed by contact terms and higher order $\pi N$ couplings

Many thousands of diagrams for $n$ vertices $n!$ diagrams (TOPT)
Comparison with others?
\beta decays Saori
Results: Neutral currents

\[ \Lambda = 500 \text{ MeV} \]

\( \nu_e \)-NC

\( \bar{\nu}_e \)-NC

for \( \Lambda = 600 \text{ MeV} \) variation \( \leq 1\% \)
Triton calculation

N3LO/N2LO chiral potentials

Hyperpherical harmonics \textit{ab initio} method (developed by Pisa group over last two decades)

family of wave functions

Standard Monte Carlo techniques

Matrix element

Pisa group citations
## Triton Results

- Loop give not negligible contribution
- Preliminary three-body currents seem negligible

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- N3LO/N2LO full chiral
- AV18/UIX hybrid
Matching holds for on shell scattering amplitude
• Matching is not unique → Nuclear operators are not unique
  • iterations of LS depend on the off-the-energy-shell extension of lower order currents and potentials
  • Not unique operators **should** be related by a unitary transformation (**no general proof at the moment**)
Outlook
Backup slides
Effective field theory

- Pion and nucleons degrees of freedom
- Exact
- Lagrangian is an expansion in powers of $Q/\Lambda_{\chi}$
  \[ \mathcal{L}_{\chi\text{EFT}} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots \]
- Low energy constants (encode our ignorance)

Experiments (past and present)  Lattice QCD (near future)
Theory approaches

Theory \rightleftharpoons \text{Experiment}

Lattice QCD

Effective theories
Effective field theories

Low energy approximations of an underlying theory

- Exploit separation of scales
- Build the most general Lagrangian consistent with the symmetries of the underlying theory

- Weinberg 1979
Strategy

Theory ➔ ? ➔ Experiment
Pipeline

$\mathcal{L}_{QCD}$

$\mathcal{L}_{\chi EFT}$

Derivation → Interactions and currents → ab initio methods → Fix LECs

ab initio methods → Predictions → Validation → Expt.

Input for expts.