STUDY OF THE AXIAL ANOMALY IN THE
 $\gamma p \rightarrow \pi^+ \pi^0 n$ REACTION AT LOW t
USING THE CLAS AND THE PHOTON TAGGER

A Dissertation Presented

by

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CHAPTER 1

INTRODUCTION

1.1 Physics Motivation

One can study low energy properties of the QCD Lagrangian by considering the effective Lagrangian[1], which takes into account only states with $m \ll \Lambda \sim m_\rho \sim 1$ GeV. At low interaction energies and in the limit of massless quarks, the QCD Lagrangian possesses chiral symmetry. In this limit, the pion is one of the Goldstone bosons with $m_\pi = 0$, and the left-handed and right-handed components of the quark spinor fields decouple. Also, scattering cross sections can be calculated in a power series expansion of the energy. Such a procedure is known as chiral perturbation theory.

Chiral or axial anomaly was first introduced in an effort to understand the decay of a neutral pion into two photons[2]. In the Partially Conserved Axial Current(PCAC) theory, the pion decay constant, f_π , is related to the matrix element of the pion and the vacuum (f_π has been experimentally determined to be 93.1 ± 0.1 MeV). By adding to the PCAC Lagrangian an anomalous axial term, which is now understood to arise from quantum mechanical symmetry breaking, *i.e.* when a symmetry present in the classical Lagrangian is broken due to quantization, one obtains a relation between the π^0 decay amplitude and the number of color quarks, N_c ,

$$F_{th}^\pi = \frac{\alpha N_c}{3\pi f_\pi} \text{ GeV}^{-1} \quad (1.1)$$

where α is the fine structure constant. For $N_c = 3$, $F^\pi = 0.025 \text{ GeV}^{-1}$ [3, 4]. This theoretically determined value is in good agreement with results from experiments, $F_{exp}^\pi = 0.0240 \pm 0.0003 \text{ GeV}^{-1}$.

In a similar fashion, the structure function $F^{3\pi}$ for the $\gamma \rightarrow 3\pi$ vertex was also calculated. $F^{3\pi}$ is related to F^π by[5, 6, 7]

$$F^{3\pi} = \frac{F^\pi}{ef_\pi^2} = \frac{eN_c}{12\pi^2 f_\pi^3} \text{ GeV}^{-3} \quad (1.2)$$

which equals 9.5 GeV^{-3} for $N_c = 3$. Here we use $e = \sqrt{4\pi\alpha}$. The above result was derived in the chiral limit, *i.e.* $q^2 = s = t' = u = 0$.

Figure 1.1 t-channel Feynman diagram for $\gamma\pi^+ \rightarrow \pi^+\pi^0$.

The variables q , s , t' , and u are defined as follows: q is the four-momentum of the incident real photon, $s \equiv (k_1 + k_2)^2$ is the invariant mass squared of the two pions, $t' \equiv (q - k_1)^2$ is the four-momentum transfer squared between the photon and the outgoing π^+ , and $u \equiv (q - k_2)^2$ is the four-momentum transfer squared between the photon and the π^0 (see Figure 1.1). The four-momentum squared transfer to nucleon, $t \equiv (q - k_1 - k_2)^2 = k_3^2$, is related to s , t' , and u by $t \equiv s + t' + u - 2m_\pi^2$.

However the agreement between theoretical calculation and experimental measurements for $F^{3\pi}$ are not as good as the F^π case. Two experiments have attempted to measure $F^{3\pi}$. One was done by Antipov *et al.*[8] at Serpukhov. Antipov's experiment used 40 GeV/c pion beams for the radiative production reaction $\pi^- A \rightarrow \pi^- \pi^0 A$ and obtained the result $F_{exp}^{3\pi} = 12.9 \pm 0.9 \pm 0.5 \text{ GeV}^{-3}$. This result was consistent with $N_c = 4$ which contradicts the basic assumption of the Standard Model in which $N_c = 3$. The other by Amendolia *et al.*[9] studied neutral pion production in $\pi^- e \rightarrow \pi^- \pi^0 e$ inelastic scattering with 300 GeV/c pion beams at the CERN SPS. Both these experiments have an experimental uncertainty of about 20%.

1.2 Goal

This experiment E94-015[10] was among the other five Hall B approved experiments known as G1 group. The G1 group was the first Hall B photoproduction experiments using the photon tagging system and the CEBAF Large Acceptance Spectrometer (CLAS). The G1 group was divided into three running periods which cover the approved 60 beam days. The first period of this experiment was carried out in May and July of 1998 at CEBAF Hall B of the Thomas Jefferson National Accelerator Facility in Newport News, Virginia.

This experiment measured the cross section for the $\gamma p \rightarrow \pi^+ \pi^0 n$ reaction at low s and t . Tagged photons with energies between 1 and 2 GeV, and a liquid

hydrogen target were used to study the reaction. The detected final state particles were positive and neutral pions, the latter through its decay to two photons. The positive pion was detected using the CLAS drift chambers and the time-of-flight system, and the photons were detected by the electromagnetic calorimeters. Finally the neutron was identified using missing mass technique.

$F^{3\pi}(s, t', u)$ will be extracted from this cross section in the region where $4 < s < 50m_\pi^2$ and $0 < -t < 50m_\pi^2$. The Chew-Low extrapolation will be used to extrapolate the $F^{3\pi}(s, t', u)$ in the physical region to the pion pole. The results will be compared to various theoretical calculations.

In summary, the $\gamma \rightarrow 3\pi$ amplitude is not well determined. Further, measurements of $F^{3\pi}$ test fundamental assumptions of chiral quantum field theory. It is in this context that this experiment has been attempted to measure the $F^{3\pi}$ within 10% uncertainty.

The physics of the chiral anomaly and some theoretical calculations will be presented in Chapter 2. Chapter 3 describes the experimental apparatus used in this experiment. Chapter 4 covers the event selection and event reconstruction. The details of event simulations and data analysis will be discussed in subsequent Chapter 5 and Chapter 6. Chapter 7 discusses the results and concludes the work of this dissertation.

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ABSTRACT

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While the agreement between theory and experiment is excellent for the $\pi^0 \rightarrow 2\gamma$ reaction, other reactions that proceed through the axial anomaly have been poorly tested. For example the existing measurement of the $\gamma \rightarrow 3\pi$ amplitude, $F^{3\pi}$, is in poor agreement with theory. In the limit of low s and t , the $\gamma p \rightarrow \pi^+ \pi^0 n$ reaction is sensitive to $F^{3\pi}$. In this thesis preliminary cross sections for the $\gamma p \rightarrow \pi^+ \pi^0 n$ reaction are presented using the CLAS with tagged photon energies between 1 and 2 GeV and over a range in s and t up to 1 GeV². The π^+ was detected using the time-of-flight and tracking systems. The π^0 was detected via reconstruction of the invariant mass of its two decay photons, which were detected by an electromagnetic calorimeter. The presence of the neutron was inferred via missing mass. The sensitivity of these cross sections to $F^{3\pi}$ in the low t region is studied for $s = 18m_\pi^2$ and $s = 22m_\pi^2$ at $E_\gamma = 2$ GeV. The results show a momentum dependence of the $F^{3\pi}$ and are consistent with a calculation that includes the effects of $\pi\pi$ final state interactions on the chiral perturbation prediction for $F^{3\pi}$.

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CHAPTER 2

THEORETICAL OVERVIEW

In this section, various physics topics involved in the chiral anomaly are introduced in order to develop an understanding on this subject. There are excellent reviews on this subject which can be found in the literatures [11, 12, 13, 14, 15, 16, 17].

2.1 Quantum Chromodynamics: QCD

Quantum Chromodynamics or QCD is the Standard Model based theory which describes the color charge interaction between quarks and gluons. Because gluons carry color charges, they can couple among themselves in addition to quarks. This nonlinearity makes QCD difficult to solve exactly. The quark-gluon coupling strength exhibits asymptotic freedom at short distances or at large momentum transfer. That means the couplings are weak and one can treat the interaction perturbatively at high energies. The contrary situation is at low energies, where quarks and gluons are bound together and form hadrons. Therefore nucleons and pions are the appropriate degree of freedoms for the strong interaction at low energies.

2.2 Symmetry

Symmetry plays an important role in developing an understanding of the way nature works. As we shall see later on, the strong interaction at low energies has a symmetry called chiral symmetry. When we apply a transformation to a system

and the properties of the system are not changed, we say that it has a symmetry under such a transformation. A symmetry of a system also implies a conserved quantity. For example, let us consider a Lagrangian which is symmetric under a transformation of the fields

$$\phi \rightarrow \phi + \delta\phi \quad (2.1)$$

and its Lagrangian

$$\mathcal{L}(\phi + \delta\phi) - \mathcal{L}(\phi) = \frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta(\partial_\mu\phi) \quad (2.2)$$

$$0 = \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi \right) \quad (2.3)$$

$$= \partial_\mu J^\mu \quad (2.4)$$

where we define a current J^μ as

$$J^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi \quad (2.5)$$

This implies a conserved current with $\partial_\mu J^\mu = 0$. The conserved current also leads to charge conservation

$$Q = \int J^0 d^3x. \quad (2.6)$$

2.3 Chiral Symmetry

The QCD Lagrangian can be written as

$$\mathcal{L}_{QCD} = \bar{q}(iD_\mu\gamma^\mu - m)q - \frac{1}{2}\text{tr} G_{\mu\nu}G^{\mu\nu} \quad (2.7)$$

where q and $\bar{q}$ are the quark and anti-quark fields, m is the mass of the corresponding quark, D_μ and $G_{\mu\nu}$ are the covariant derivative and the color-field tensor, respectively and are defined as

$$iD_\mu = i\partial_\mu - gA_\mu^a \frac{\lambda^a}{2} \quad (2.8)$$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - g[A_\mu, A_\nu] \quad (2.9)$$

where A_μ^a is the gluon fields, λ^a are the $SU(3)$ Gell-Mann matrices with $a = 1, \dots, 8$ and g is the strong coupling constant.

One can define a vector current V^μ and an axial vector current A^μ as

$$V^\mu = \bar{q}\gamma^\mu q \quad (2.10)$$

$$A^\mu = \bar{q}\gamma^\mu\gamma^5 q \quad (2.11)$$

These two currents correspond to the global phase transformation and the chiral transformation respectively by

$$q \rightarrow e^{i\alpha} q \quad (2.12)$$

$$q \rightarrow e^{i\beta\gamma^5} q \quad (2.13)$$

where α and β are rotation angles. If q satisfies the equation $(iD_\mu\gamma^\mu - m)q = 0$, the divergences of these currents are

$$\partial_\mu V^\mu = 0 \quad (2.14)$$

$$\partial_\mu A^\mu = 2im\bar{q}\gamma^5 q \quad (2.15)$$

Clearly the axial current is conserved provided $m = 0$.

Introducing the chirality operators $(\Gamma_{L,R})$ which project the left- and right-handed components of the quark fields by $q_L = \Gamma_L q$ and $q_R = \Gamma_R q$ and are defined as

$$\Gamma_{L,R} = \frac{1}{2}(1 \pm \gamma_5) \quad (2.16)$$

One can rewrite $\bar{q}(iD_\mu\gamma^\mu - m)q$ in the QCD Lagrangian as

$$\bar{q}(iD_\mu\gamma^\mu - m)q = i\bar{q}_L D_\mu\gamma^\mu q_L + i\bar{q}_R D_\mu\gamma^\mu q_R - \bar{q}_L m q_R - \bar{q}_R m q_L \quad (2.17)$$

where $q = q_L + q_R$. In the limit of massless quarks, the QCD Lagrangian possesses a symmetry called chiral symmetry. This means that the Lagrangian is invariant under global and chiral transformations and there is no term in the Lagrangian that

couples the left-handed quark field to the right-handed quark field. The conserved quantities are both the vector current and axial vector current.

In reality, chiral symmetry is broken not only explicitly by the non-massless quarks but also spontaneously due to the axial current. By spontaneously broken symmetry, one means that while the Hamiltonian is invariant, the ground state is not. This results in $N_f^2 - 1$ massless particles, Goldstone bosons where N_f is the number of flavor quarks. For $N_f = 3(u, d, s)$, the Goldstone bosons are $\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0$ and η with quantum number $J^P = 0^-$. These are not massless particles in nature. They acquire mass via quark masses.

2.4 Chiral Perturbation Theory

Chiral perturbation theory(χ PT) is an effective field theory of QCD at low energies. What this means is that it does not contain full information about the strong interaction. However it is constructed with the observed symmetries of the strong interaction, *i.e.* chiral symmetry at low energies. Despite the name *perturbation*, χ PT itself is a *non-perturbative* theory. The Lagrangian is not expanded in orders of the strong coupling constant. The dynamics are controlled by the pions, the lightest particles, and the chiral symmetry, the symmetry of the QCD. Calculations are carried out in orders of pion momentum divided by pion mass (p_π/m_π). The energy scale is set by $m, p \ll \Lambda \sim m_\rho \sim 1$ GeV. As pointed out by Weinberg [1]

... if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S -matrix consistent with analyticity, perturbative unitary, cluster decomposition and the assumed symmetry principles.

... one might imagine weakening the force of QCD by introducing some sort of infrared cut-off λ , and preserving the physical content of the theory by introducing the bound states of the theory as fictitious elementary particles. These bound states are just the ordinary hadrons, and they must be described by a chiral-invariant phenomenological Lagrangian. The parameters of the phenomenological Lagrangian would have to be functions of λ , defined by differential equations which guarantee the λ -independence of the S -matrix, with initial condition set by the requirement that the theory goes over to pure QCD in the limit $\lambda \rightarrow 0$, where there is no infrared cut-off.

Thus the QCD Lagrangian is replaced by an *effective* Lagrangian as functions of the field $U(x)$ and its derivatives

$$\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{eff}(U, \partial U, \partial^2 U, \dots) \quad (2.18)$$

By Lorentz invariance requirement, only terms with an even number of derivatives are permitted.

$$\mathcal{L}_{eff} = \mathcal{L}_{eff}^2 + \mathcal{L}_{eff}^4 + \mathcal{L}_{eff}^6 + \dots \quad (2.19)$$

2.5 Chiral Anomaly

As mentioned in previous chapter, the chiral anomaly was first introduced in order to resolve the decay rate of the π^0 [2]. Classical current conservation implies that $\partial_\mu A^\mu = 0$ provided the quarks are massless. However the quantum calculation of the one loop diagram for π^0 decay(see Figure 2.1) indicates that it is not the case. In fact the divergence of the axial current picks up an extra term(the so called anomaly term) which is not conserved even though the quarks are massless.

$$\partial_\mu A_{(3)}^\mu = 2i(m_u \bar{u} \gamma^5 u + m_d \bar{d} \gamma^5 d) + \frac{\alpha N_c}{12\pi} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \quad (2.20)$$

where $F_{\alpha\beta}$ is the electromagnetic tensor.

Figure 2.1 The loop diagram of $\pi^0 \rightarrow 2\gamma$.

Later Witten related the F^π amplitude to the number of colors, N_c by using the Wess-Zumino effective Lagrangian[3, 4].

$$\begin{aligned} F^\pi &= \frac{\alpha N_c}{3\pi f_\pi} \\ &= 0.025 \text{ GeV}^{-1}, \text{ for } N_c = 3 \end{aligned} \tag{2.21}$$

which is in a good agreement with the experimental measurement as mentioned in previous chapter.

2.6 Chiral Anomaly and $\gamma \rightarrow 3\pi$

In 1970s, the partially conserved axial-vector current(PCAC) was used to relate the matrix elements for $\gamma\gamma \rightarrow \pi^0$ to $\gamma \rightarrow \pi^0\pi^+\pi^-$ [5, 6, 7]. Later by using the Wess-Zumino-Witten effective Lagrangian the relation between F^π and $F^{3\pi}$ was calculated

$$F^{3\pi} = \frac{F^\pi}{ef_\pi^2} \tag{2.22}$$

$$\begin{aligned} &= \frac{eN_c}{12\pi^2 f_\pi^2} \\ &= 9.5 \text{ GeV}^{-3}, \text{ for } N_c = 3 \end{aligned} \tag{2.23}$$

Note that the F^π and $F^{3\pi}$ mentioned here were calculated in chiral limit where $s = t' = u = 0$. Figure 2.2 shows the loop diagram of $\gamma \rightarrow 3\pi$.

Figure 2.2 The loop diagram of $\gamma \rightarrow 3\pi$.

2.7 Momentum Dependence of $F^{3\pi}$

There are several models which predict the form of $F^{3\pi}(s, t', u)$ in the physical region ($s \geq 4m_\pi^2$). The models of Terent'ev[18] and Rudaz[19] are vector meson dominance models(*i.e.* the photon behaves like a hadronic state and couples to the vector meson *e.g.* ρ or ω). Bijmens'[20] and Holstein's[21] models are based on chiral perturbation theory. Alkofer and Roberts[22], and Ivanov and Mizutani[23] use quark models.

The main features of these models are:

- Terent'ev[18] used the vector meson dominance(VMD) model

$\gamma \rightarrow \rho\pi \rightarrow 3\pi$ and $\gamma \rightarrow \omega \rightarrow 3\pi$ to extrapolate $F^{3\pi}(0)$ to the physical region.

Since we use real photons in this experiment, the ω -pole term that appears in this model does not contribute.

$$F^{3\pi}(s, t', u) = F^{3\pi}(0) \left[1 + C_\rho e^{i\delta} \left(\frac{s}{m_\rho^2 - s} + \frac{t'}{m_\rho^2 - t'} + \frac{u}{m_\rho^2 - u} \right) \right] \quad (2.24)$$

where s, t', u are the Mandelstam variables defined in Chapter 1, δ is a phenomenological phase factor, and

$$C_\rho = \frac{2g_{\rho\pi\pi}g_{\pi\rho\gamma}}{m_\rho^3 F^{3\pi}(0)} = 0.478 \quad (2.25)$$

- Rudaz[19] showed that the $\pi^0 \rightarrow 2\gamma$ and $\gamma \rightarrow 3\pi$ amplitudes can be calculated on the basis of pure vector meson dominance or by using anomalous PCAC.

$$F^{3\pi}(s, t', u) = -\frac{1}{2}F^{3\pi}(0) \left[1 - \left(\frac{m_\rho^2}{m_\rho^2 - s} + \frac{m_\rho^2}{m_\rho^2 - t'} + \frac{m_\rho^2}{m_\rho^2 - u} \right) \right] \quad (2.26)$$

- Bijmans *et al.*[20] introduced an $O(p^6)$ term to the effective Lagrangian to cancel the divergence from one loop diagrams $\eta \rightarrow \pi^+ \pi^- \gamma$ and $\gamma \pi^- \rightarrow \pi^- \pi^0$. These loops arise in an $O(p^4)$ chiral perturbation theory.

$$F^{3\pi}(s, t', u) = F^{3\pi}(0) \left[1 + \frac{3m_\pi^2}{2m_\rho^2} + \frac{m_\pi^2}{24\pi^2 f_\pi^2} \left(\frac{3}{4} \ln \frac{m_\rho^2}{m_\pi^2} + F(s) + F(t') + F(u) \right) \right] \quad (2.27)$$

where

$$F(s) = \begin{cases} \left(1 - \frac{s}{4m_\pi^2}\right) \sqrt{1 - \frac{4m_\pi^2}{s}} \ln \frac{1 + \sqrt{1 - \frac{4m_\pi^2}{s}}}{-1 + \sqrt{1 - \frac{4m_\pi^2}{s}}} - 2, & s > 4m_\pi^2, \\ 2 \left(1 - \frac{s}{4m_\pi^2}\right) \sqrt{\frac{4m_\pi^2}{s} - 1} \tan^{-1} \sqrt{\frac{s}{4m_\pi^2 - s}} - 2, & s \leq 4m_\pi^2. \end{cases} \quad (2.28)$$

- Holstein[21] showed that the amplitudes calculated by Rudaz[19] and Bijmans *et al.*[20] could be consistent in the low s, t', u regions if one includes the final state interactions.

$$F^{3\pi}(s, t', u) = -\frac{1}{2} F^{3\pi}(0) \left[1 - \left(\frac{m_\rho^2}{m_\rho^2 - s} + \frac{m_\rho^2}{m_\rho^2 - t'} + \frac{m_\rho^2}{m_\rho^2 - u} \right) \right] \times \left(\frac{1 - s/m_\rho^2}{D_1(s)} \right) \left(\frac{1 - t'/m_\rho^2}{D_1(t')} \right) \left(\frac{1 - u/m_\rho^2}{D_1(u)} \right) \quad (2.29)$$

with

$$D_1(s) = 1 - \frac{s}{m_\rho^2} - \frac{s}{96\pi^2 f_\pi^2} \ln \frac{m_\rho^2}{m_\pi^2} - \frac{m_\pi^2}{24\pi^2 f_\pi^2} F(s) \quad (2.30)$$

- Alkofer and Roberts[22] used a phenomenological approach to the QCD Dyson-Schwinger equations to calculate $F^{3\pi}(0)$. $F^{3\pi}(s, t', u)$ was calculated by parameterizing the Schwinger functions to fit the low energy pion data.

$$F^{3\pi}(s, t', u) = 1.044 + 0.096x + 0.006x^2 \quad (2.31)$$

with $x = s/4m_\pi^2 - 1$

- Ivanov and Mizutani[23] used a confined quark model to calculate $F^{3\pi}(s, t', u)$ without fixing $F^{3\pi}(0)$ to 9.5 GeV^{-3} . The value of $F^{3\pi}(0)$ predicted by this

model is 5% greater than what is expected in the chiral limit.

$$F^{3\pi}(s, t', u) = F^{3\pi}(0) [0.22 + C_\rho(s)D_\rho(s) + C_\rho(t')D_\rho(t') + C_\rho(u)D_\rho(u)] \quad (2.32)$$

where

$$C_\rho(s) = \frac{2g_{\rho\pi\pi}(s)g_{\rho\pi\gamma}(s)}{m_\pi^3 F^{3\pi}(0)} \quad (2.33)$$

$$D_\rho(s) = \frac{m_\rho^2}{m_\rho^2 - s} \quad (2.34)$$

$$g_{\rho\pi\pi}(s) = \frac{6R_{VPP}(s/\Lambda^2)}{R_{VPP}(m_\rho^2/\Lambda^2)} \quad (2.35)$$

$$g_{\rho\pi\gamma} = \frac{0.16R_{PVV}(s/\Lambda^2)}{R_{PVV}(m_\rho^2/\Lambda^2)} \quad (2.36)$$

$$R_{PVV}(x) = \frac{1}{4} \int_0^1 a\left(-\frac{xu}{4}\right) \ln\left(\frac{1 + \sqrt{1-u}}{1 - \sqrt{1-u}}\right) du \quad (2.37)$$

$$R_{VPP}(x) = 2.26 + \frac{x}{4} \int_0^1 b\left(-\frac{xu}{4}\right) \sqrt{1-u} du \quad (2.38)$$

$$a(u) = 2 \exp(-u^2 - u) \quad (2.39)$$

$$b(u) = 2 \exp(-u^2 + 0.4u) \quad (2.40)$$

$$\Lambda = 460 \text{ MeV} \quad (2.41)$$

Figure 2.3 shows the s dependence of $F^{3\pi}$ as calculated from these models. One can see that for $s < 10m_\pi^2$ the predictions from various model calculations are not so different as shown in Figure 2.4 for s up to $15m_\pi^2$. However the disagreements among these model calculations are obviously seen for $s > 10m_\pi^2$. Note that the calculations of Terent'ev, Rudaz, and Ivanov and Mizutani have a pole at the ρ .

Figure 2.3 The $\gamma \rightarrow 3\pi$ structure function $F^{3\pi}(s, t', u)$ as a function of s (for $t = m_\pi^2$) as predicted by various models. $F^{3\pi}(0) = 9.5 \text{ GeV}^{-3}$.

$$t = m_\pi^2, \quad t' = u$$

Figure 2.4 The $\gamma \rightarrow 3\pi$ structure function $F^{3\pi}(s, t', u)$ as a function of s for $s < 15m_\pi^2$ as predicted by various models. $F^{3\pi}(0) = 9.5 \text{ GeV}^{-3}$.

2.8 $\gamma \rightarrow 3\pi$ Cross Section

This experiment used the reaction $\gamma p \rightarrow \pi^+ \pi^0 n$ to study the $\gamma \rightarrow 3\pi$ amplitude in the physical region. The cross section for $\gamma\pi \rightarrow \pi\pi$ was given by [24, 25]

$$\frac{d\sigma_{\gamma\pi \rightarrow \pi\pi}}{d\tau} = \frac{(F^{3\pi}(s, t', u))^2}{128\pi} \cdot \frac{1}{4} (s - 4m_\pi^2) \sin^2 \theta_\pi \quad (2.42)$$

where $\tau = (k_3 - k_1)^2$ (see Figure 1.1), and θ_π is the π^+ scattering angle relative to the incident photon direction in the center of mass frame. The variable τ is related to θ_π through the kinematic relation

$$\tau = \frac{1}{2} \left[2t + m_\pi^2 - s - (s - m_\pi^2) \sqrt{1 - \frac{4m_\pi^2}{s}} \cos \theta_\pi \right] \quad (2.43)$$

where $t = k_3^2$, the four-momentum of the exchange pion. The cross sections are:

$$\frac{d\sigma_{\gamma\pi \rightarrow \pi\pi}}{d\Omega} = \frac{(F^{3\pi}(s, t', u))^2}{128\pi} \cdot \frac{s}{16\pi} \left(1 - \frac{4m_\pi^2}{s} \right)^{3/2} (s - m_\pi^2) \sin^2 \theta_\pi \quad (2.44)$$

$$\sigma_{\gamma\pi \rightarrow \pi\pi}(s) = \frac{(F^{3\pi}(s, t', u))^2}{128\pi} \cdot \frac{s}{6} \left(1 - \frac{4m_\pi^2}{s} \right)^{3/2} (s - m_\pi^2) \quad (2.45)$$

Since we do not have a free pion to use as a target for the $\gamma\pi \rightarrow \pi\pi$ reaction, we will assume that the photon interacts with the pion cloud of the associated nucleon. The measured cross section for the experiment will be the cross section with a proton target. Then we need to relate $\sigma_{\gamma p \rightarrow \pi^+ \pi^0 n}$ to $\sigma_{\gamma\pi \rightarrow \pi\pi}$. The general form of $\sigma_{\gamma p \rightarrow \pi^+ \pi^0 n}$ near the pion pole was suggested by Chew and Low [26],

$$\frac{d^2\sigma}{ds\,dt} = 2 \left(\frac{2f^2}{4\pi} \right) \frac{1}{m_\pi^2} \cdot \frac{-t}{(t - m_\pi^2)^2} \cdot \frac{s}{E_\gamma^2} \sqrt{1 - \frac{4m_\pi^2}{s}} \sigma_{\gamma\pi \rightarrow \pi\pi} \quad (2.46)$$

where f^2 is the $\pi^0 p$ coupling constant and $f^2 = 0.08$. The coupling constant for the charged pion is therefore $2f^2$. The additional factor of two in front of the equation is to account for the photon polarization average in $\sigma_{\gamma\pi \rightarrow \pi\pi}$. The Chew-Low expression was originally derived for spinless particles. We see from Eq. (2.46) that there is a pole at $t = m_\pi^2$. An extrapolation from the physical region to non-physical region is needed in order to extract the $\gamma\pi$ cross section from the γp cross section. We shall discuss about this extrapolation in Chapter 6.

CHAPTER 3

EXPERIMENTAL APPARATUS

This chapter describes the functions of each detector system used in this experiment followed by the running conditions. A brief description of the accelerator will be discussed as well.

3.1 CEBAF

The Continuous Electron Beam Accelerator Facility(CEBAF)[27] is a unique 100% duty factor accelerator using super-conductive radio-frequency cavities. It provides a high-intensity continuous electron beam from 0.8 GeV up to 4 GeV and will be upgraded to higher beam energy in near future.It can deliver up to three different electron beam energies to three end stations, called Hall A, Hall B and Hall C, simultaneously.

Figure 3.1 shows the accelerator configuration. Pre-accelerated electrons from the injector are sent to the North Linac which increased the electrons energies to 0.4 GeV then enter to the Recirculation Arcs. The electrons then again enter the South Linac and get accelerated up another 0.4 GeV. At this point the electron beams can either be extracted by the RF separators which are time dependent deflecting devices that allow one out of every three passing bunches to be extracted and sent to the experimental hall or kept recirculating until reaching desired energies up to 5 passes or 4 GeV then extracted into the halls. Since the beams are continuous, there will be up to 5 different electron beam energies presented simultaneously in

Figure 3.1 Accelerator configuration.

the same linac. Upon exiting the linac, a beam spreader will separate and transport the beams to appropriate beam lines in the recirculation arc corresponding to different beam energies. These different paths compensate for the energies losses due to synchrotron radiations and relative phases for different energies. The beam recombiner then combines the beams from different beam lines and transports to the other linac. The process then continues until the RF separators, located downstream of the south linac spreader, extract the beams to the destination halls. A summary of accelerator specifications is listed in Table 3.1.

Table 3.1 Summary of accelerator specifications

Beam characteristics	
Electron energy (GeV)	$0.8 \leq E \leq 4$
Average current (μA)	200
Transverse emittance (πm)	$< 10^{-8}$
Energy spread ($\Delta E/E$)	$< 2 \times 10^{-4}$
Duty factor	100 %
Simultaneous beams	3
Simultaneous correlated energies	3
Linac parameters	
Concept	Recirculating CW linac
Structure type	Superconducting RF cavity
Number of linac segments	2
Number of beam passes	5
Energy gain per pass	0.8 GeV
Frequency (GHz)	1.5
Design gradient (MV/m)	5.0
Design residual (Q)	3×10^9
Nominal injection energy (MeV)	50
Arc average radius (m)	80

3.2 Hall B

Experimental Hall B is one of the three end stations dedicated to physics researches on:

- Baryon and meson structure and spectroscopy
- Gluonic degrees of freedom
- Multinucleon correlations in nuclei

Figure 3.2 Hall B configuration. In this picture the electron beam from the accelerator enters Hall B from the right-handed side, passes through the tagger, interacts with the target and exits to the beamdump on the left-handed side.

Hall B houses two major detector systems, the CEBAF Large Acceptance Spectrometer (CLAS) and the Bremsstrahlung tagging system (see Figure 3.2). The wide range of momentum acceptance and large solid angle of CLAS make it an appropriate tool for these searches. To fulfill these scientific goals, the detectors were designed to achieve the following:

1. A large angular and momentum range for charged particles, photons and neutrons
 - angular range for particle detection $8^\circ \leq \theta \leq 140^\circ$ and $0^\circ < \phi < 360^\circ$
 - momentum range for charged particles $0.1 \leq p \leq 4 \text{ GeV}/c$
 - energy range for photons detection $E \geq 50 \text{ MeV}$
2. Good momentum and angular resolutions
 - momentum resolution $\delta p/p \approx 0.5\%$ at small θ
 - momentum resolution $\delta p/p \approx 1.0\%$ at large θ
 - angular resolution $\delta\theta \approx 1.0 \text{ mrad}$
3. Good particle identification for charged particles such as electrons, protons, pions, kaons and deuterons as well as neutral particles *i.e.* photons and neutrons.
4. High luminosity and high counting rate
 - electron beam luminosity $L \approx 10^{34} \text{ cm}^{-2}\text{sec}^{-1}$
 - tagged photon beam intensity $N \approx 10^7/\text{sec}$
5. Symmetric configuration around the beam axis

6. Homogeneous magnetic field

7. Allow the use of polarized target and future upgrades

As opposed to an electro-production experiment in which electrons beam directly hit the target and go through the beam dump, a photo-production experiment like this one produces Bremsstrahlung photons which interact with the target and use the tagging system to measure the energies of tagged electrons, thus giving the tagged photon energy(see Table 3.2).

Table 3.2 Electro- and Photo-production Comparisons

	Electro-production	Photo-production
Incident particles	Electrons	Photons
Incident particle energy measured by	Accelerator	Hall B Tagging system
Incident energy	E_0	$0.20E_0 - 0.95E_0$
Flux normalization	Faraday cup	Downstream devices

3.3 CLAS

CLAS is the main apparatus for Hall B. It is a six-fold geometric detector which consists of 3×6 Drift Chambers, 1×6 Cerenkov Counters, 4×6 panels of Time-of-Flight systems, 1×6 Forward Angle Electromagnetic Calorimeters and 1×2 Large Angle Electromagnetic Calorimeters(see Figure 3.3 and 3.4). For photo-production experiments, there is also a Start Counter for event start time measurement. The detail of each detector systems are described below.

3.3.1 CLAS Toroidal Magnets

The magnetic field was generated by six superconducting toroidal coils, which define six sectors. The magnetic field in the forward direction is about 5 times stronger

Figure 3.3 Various detector systems of CLAS.

Figure 3.4 Midplane cross section view of CLAS.

Figure 3.5 Sector definition of CLAS. The beam is along Z-axis which points inward to the paper.

than the fields at large angles. The full field is about 3 T. In this experiment the magnetic field was set to 50% of the full field, and was set to bend positively charged particles away from the beam axis.

3.3.2 Drift Chambers

The tracking system[28] consists of three regions — Region 1, 2 and 3. Each region consists of six drift chambers(DC), one for each sector. Figure 3.5 shows the sector definition of CLAS and Figure 3.6 shows a cross section view of the Drift Chambers. Each drift chamber provides a polar angle coverage($\Delta\theta$) of 134° (from 8° – 142°) and an azimuthal angle coverage($\Delta\phi$) of approximately 50° . There are approximately 35,000 hexagonal drift cells. Each cell consists of one sense wire surrounded by six field wires.

In the radial direction, Region 1 chambers(R1) are located between the target and the toroidal magnets in a low magnetic field region. Region 2 chambers(R2) are

in the high magnetic field area of the torus — sandwiched between the cryostats. Region 3 chambers(R3) are beyond the toroidal magnets.

Figure 3.6 Cross section view of the drift chambers.

The wires in each drift chamber are grouped into two superlayers: axial and stereo. Each superlayer consists of six layers of sense wires surrounded by field wires in a hexagonal arrangement, except for the stereo superlayer of Region 1, which has only four layers of sense wires. Axial wires are perpendicular to the midplane of each sector and are approximately parallel to the magnetic field, while the stereo wires are canted by 6° with respect to the axial wires to provide the azimuthal information.

Each sense wire layer consists of 128 wires in Region 1 and 192 wires in Regions 2 and 3. Gold-plated tungsten wires (diameter of $20\ \mu\text{m}$) are used for sense wires and gold-plated aluminum wires (diameter of $140\ \mu\text{m}$) are used for field wires. The wire tensions are about 20 g and 140 g for sense and field wires, respectively.

Argon-CO₂ gas (90:10 by volume) which has a drift velocity of about 4 cm/ μ s, was used in the chambers. The cell size for Region 1, 2 and 3 are about 1.6, 2.7 and 4.3 cm with the whole-cell resolution of 310, 315 and 380 μ m, respectively.

3.3.3 Start Counter

The Start Counter(ST)[29] is a small detector surrounded the target region. It consists of 3 pairs of thin scintillators folded half way back to meet the other end. Photomultiplier tubes are attached at each end(see Figure 3.7). The purpose of the start counter was to determine the event start time relative to the trigger time and was part of the G1 trigger as well. Since the electron beam buckets were sent to Hall B in the time interval of 2 ns apart, the start counter was used to pinpoint which beam bucket gave rise to the trigger event. The time resolution for the start counter was 350 ps. The more accurate start time was obtained from the tagger which had a better resolution.

3.3.4 Time-of-Flight

The Time-of-Flight system(TOF) or Scintillation Counters(SC)[30] were used for triggering and for particle identification. They are located downstream of the target, next to the drift chambers and the Cerenkov counters(this experiment did not require the Cerenkov counters). The TOF system consists of 4 panels of scintillation counters per sector which has the same polar angle coverage as the drift chambers. Each sector consists of layers of 48 scintillation counters(20 cm wide and 5 cm thick) with a photomultiplier tube at each end(see Figure 3.8). The scintillation counters are parallel to the axial wires of the drift chambers and cover an area of 206 m². The timing resolution varies from about 80 ps for the short counters to 160 ps for the longer ones.

Figure 3.7 Diagrammatic drawing of the Start Counter.

Figure 3.8 Scintillation counters arrangement of the Time-of-Flight system.

3.3.5 Electromagnetic Calorimeter

The Electromagnetic Shower Calorimeter was used to measure the energy deposits and the positions of the two photons from the π^0 decay. It consists of the forward angle ($8^\circ < \theta < 45^\circ$) and the large angle ($45^\circ < \theta < 90^\circ$) components — angles were measured with respect to the beam line.

The forward angle calorimeters consist of 6 sectors and are positioned beyond the TOF. Each sector consists of 13 identical submodules, each submodule has 3 layers of 2.3 mm thick lead plates interleaved with 10 mm thick, 10 cm wide scintillator. Each submodule is 1.14 radiation lengths thick. Scintillator strips are positioned parallel to each side of the triangular region (U-, V- and W-strips). Figure 3.9 shows the Electromagnetic Calorimeters configuration and a cross section view is shown in Figure 3.10.

Figure 3.9 Electromagnetic Calorimeters configuration.

Figure 3.10 Cross section view of Electromagnetic Calorimeters.

The large angle calorimeter covers only two sectors and is positioned in sector 1 and sector 2. Each sector has a rectangular shape which covers an area of $3.6 \times 3.9 \text{ m}^2$. The scintillator strips in each layer are perpendicular to scintillator strips in the next layer. The total thickness is 12 radiation lengths. The total energy resolution is about $10\%/\sqrt{E}$.

3.4 Bremsstrahlung Tagging System

The bremsstrahlung tagging system (Tagger)[31] is essential for Hall B photo-production experiments in order to tag the photons that interact with the target. This equipment is by passed for electro-production experiments. Tagged photons produced by the bremsstrahlung tagging system have energies range between 20% and 95% of the primary electron beam energy. Basically the Tagger is a big dipole

magnet, located downstream next to the radiator, with two arrays of scintillation counters focal planes. The primary electron beams are bent 30° vertically to the Tagger dump. When a relativistic electron passes through a thin gold foil radiator, a bremsstrahlung photon is emitted in the forward direction, the electron loses some fraction of its initial energy to the photon. An electron then passes through the Tagger dipole magnet which deflects the electron trajectory onto the focal planes. The greater its energy lost, the more it deflects. By measuring the position where the electron hits along the focal planes, one can deduce the corresponding tagged photon energy. Figure 3.11 shows a sketch of the Photon Tagger configuration.

Figure 3.11 Sketch of the Photon Tagger configuration.

There are two types of scintillation counters focal planes. One is for energy measurement(E-counters) and the other is for timing measurement(T-counters). There are 384 E-counters and 61 T-counters. Each E-counter has only one photomultiplier at one end however there are 2 photomultipliers at each ends of T-counter.

The tagged photon rate is about $10^7/\text{sec}$ with a fractional photon energy resolution about 0.3% and the timing resolution about 150 ps.

3.5 Downstream Devices

The downstream devices consist of three types of detectors—Pair Spectrometer(PS), Pair Counter(PC), and Total Absorption Shower Counter(TASC). The purpose of these detectors is to measure the photon flux normalization. Figure 3.12 shows a sketch of general layout of the downstream devices.

Figure 3.12 Downstream Devices.

3.5.1 Pair Spectrometer

The Pair Spectrometer consists of a dipole magnet and four scintillation counters on each side of the beamline. A photon incident on a thin radiator produces an electron-positron pair which are swept away from the beamline by the magnetic field and detected in the scintillation counters. By observing a coincidence between the Tagger and the Pair Spectrometer during the physics run, one can estimate the number of incident photons.

3.5.2 Pair Counter

The Pair Counter is located between the PS and the TASC. It consists of a veto scintillator, a thin converter and four quadrant-wise positioned scintillators. The electron-positron pairs, produced by a converter, are detected by the four scintillators in coincidence with the veto scintillator which eliminates charged particles background coming from the CLAS.

3.5.3 Total Absorption Shower Counter

The Total Absorption Shower Counter consists of four lead-glass blocks which are used as a primary absolute measure of the photon flux. The TASC has a detection efficiency close to 100%. However due to higher photon beam rates ($\approx 10^5$ Hz) limitation, this device cannot be placed in the beamline during the physics run in which the PS and the PC will be used as a photon flux normalization monitor. The TASC was used only during the normalization run which had lower beam intensity to determine the tagging efficiency.

Figure 3.13 Sketch of target cell geometry.

3.6 Target

This experiment used a 18 cm long liquid hydrogen target. The target cell itself is made of 170 μm thick of mylar and 59.68 mm in diameter. The total volume is 447.88 cm^3 . The liquid hydrogen target was operated at 20.40 K and had a target density approximately of 0.071 g/cm^3 . A sketch of the target cell geometry is shown in Figure 3.13.

3.7 Experimental Setup

This section describes some specific setup for the G1 experiments. This includes the prescaling scheme for the tagged photons and the trigger setup.

3.7.1 Prescaling

Since the number of bremsstrahlung photons are proportional to the inverse of the photons energies as

$$\frac{dN}{dE_\gamma} \propto \frac{1}{E_\gamma} \left[\left(1 + \epsilon^2 - \frac{2}{3}\epsilon \right) \log \left(\frac{183}{Z^{1/3}} \right) + \frac{\epsilon}{9} \right] \quad (3.1)$$

where $E_0 = E + E_\gamma$ and $\epsilon = E/E_0$. E_0 is the incident electron energy. Z is atomic number of radiator material.

In order to obtain an approximately equal number of photons across the range of tagged photons energies, the G1 group came up with a prescaling scheme which reduced the frequency of low energy photon events going into the trigger logic. There were four sections of prescale for this experiment as described in Table 3.3.

Table 3.3 Tagger Prescaling Sections

T-counters	Prescale factor	Range of photon energy
1 – 15	1	$0.95E_0 - 0.80E_0$
16 – 29	1/2	$0.80E_0 - 0.55E_0$
30 – 45	1/4	$0.55E_0 - 0.33E_0$
46 – 61	1/8	$0.33E_0 - 0.20E_0$

Figure 3.14 shows the number of tagged photons as functions of E-counter bins for with and without prescaling. One can obviously see the effect of prescaling from the plot. The higher end of E-counter bins corresponding to lower tagged photon energies

Figure 3.14 Energy spectrum with and without prescale.

3.7.2 Trigger

As mentioned before, this experiment was among the other five approved experiments to be run at the same time so a common trigger was used during this period. There were 2 types of triggers used in this experiment, one for physics runs and the other for flux normalization runs.

- **Physics Trigger** was a hadronic trigger which triggered on charged particles and required triple coincidence of tagger, start counter and time-of-flight system. The tagger trigger is so-called Master-OR(MOR) which was set to trigger on a coincidence between the E- and T-counters of the tagger and veto by the prescaling gate. The start counter trigger triggers on any signal which exceeds the threshold on each pair. The time-of-flight trigger has 200 mV threshold for each paddle included adjacent paddles. The gate widths for the MOR, ST and TOF trigger signals are 7 ns, 15 ns and 100 ns respectively thus the tagger defines the event start time.
- **Flux Normalization Trigger** was a MOR only. This trigger was used at lower beam intensity around 100 pA. The purpose of this trigger is to determine the tagging efficiency and cross calibrate the Downstream Devices for flux normalization determination during physics runs. A detail of photon flux normalization is discussed in Chapter 4.

Table 3.4 Configuration for physics runs and normalization runs

	Physics Runs	Normalization Runs
Beam current	7–8 nA	100 pA
Radiator	A(10^{-4} g/cm ²)	B(2×10^{-5} g/cm ²)
Trigger	MOR · ST · TOF	MOR

3.8 Data Acquisition System

The CEBAF Online Data Acquisition(CODA) system was used to acquire experimental data at the Jefferson Lab. When a trigger is registered, the Trigger Supervisor(TS) will send a command to each Fastbus crate's Read Out Controller(ROC a.k.a. IOC) to execute its trigger routine. The first read-out list will read the raw data from the TDCs and the ADCs. This data is then sent to the secondary read-out list in which the raw data will be translated and formatted. For example, for the drift chambers, the secondary read-out list will translate each hit to wire number, layer number and time. After accumulating one token interval's worth of events, each ROC will send one token interval's worth of event fragments to the Event Builder(EB). The EB will concatenate the event fragments and turn them into a complete event. These events are then sent to the Data Distribution(DD) ring. From the DD ring, any client such as the CLAS Event Display(CED) or the scaler display, can monitor the events online while they are taken. Eventually, the data from DD ring will be sent to the Event Recorder(ER) and will be written to a file for offline analysis.

The event rate during this experiment was about 500 Hz with a dead time about 30%. The typical event size is 2 kbytes.

CHAPTER 4

EVENT RECONSTRUCTION

This chapter describes the $p(\gamma, \pi^+ \pi^0)n$ event selection criteria and reconstruction procedure of the three final states, π^+ , π^0 and neutron. The photon flux normalization is also discussed.

4.1 Event Selection and Reconstruction

The event reconstruction of the $p(\gamma, \pi^+ \pi^0)n$ reaction involves the identification of π^+ and π^0 , and then identifying the neutron using missing mass technique. Since the π^+ is the only charged particle in this reaction, and the neutron and π^0 which decays into two photons are neutral particles, the candidate for this reaction is the one that has only one positively charged particle detected in the drift chambers and the time-of-flight scintillators and has 2 or 3 cluster hits detected in the calorimeters which do not correspond to any track in the drift chambers. Various cut parameters were also applied to this reconstruction which will be discussed later.

The event reconstruction programs called RECSIS(REConstruction and analysis) and SEB(Simple Event Builder) were used for this purpose. Within RECSIS there are different reconstruction program modules for each detector system used to convert the ADC/TDC channels into the physical quantities such as energy or momentum. These program modules are:

- DC package: Drift Chamber reconstruction to obtain the momentum and trajectory of a charged particle
- EC package: Electromagnetic Calorimeter reconstruction to obtain the energy and position of the non-charged particle
- SC package: Scintillation Counter reconstruction to obtain the flight time of a charged particle
- ST package: SStart counter reconstruction to obtain the event start time
- Tag package: Tagger reconstruction to obtain the energy of the incident photon and event start time
- Scaler package: Scaler summation to determine the photon flux

The SEB then linked this information from each detector system to determine the four-momentum for each charged particles in that event. For the neutral particles, the energy deposited, position and flight time will be used for further reconstruction and analysis.

4.1.1 Event Start Time Determination

Since the trigger coincident gate width of the Tagger is the narrowest one among the Start Counter's and the Time-of-Flight's, the event start time can be determined by the Tagger alone in principle. However most of the time there is more than one hit in the Tagger due to the accidental coincidence between the T- and E-counters. Thus one cannot pick the right start time by using the Tagger information only. The start time obtained from the Start counter is very useful in order to filter out the accidental hits in the Tagger by considering the time difference between the Tagger start time and the Start counter start time. Since the period of the electron beam sent to Hall B is 2 ns, the start time difference between the Tagger

and the Start Counter should be within 2 ns as well. Figure 4.1 shows the 2 ns structure of the time difference between the start time obtained from the Tagger and the Start Counter. The big peak in the middle corresponds to the hits in the Tagger that have the start time matched with the start time from the Start Counter within 2 ns. The small satellite peaks correspond to the hits from different beam buckets. By requiring the time difference between the Tagger time and the Start Counter time should be within ± 1 ns, one can obtain the right event start time. The corresponding tagged photon energy is obtained by

$$E_\gamma = E_e - E_{e'} \quad (4.1)$$

where E_γ is the tagged photon energy, E_e is the incident electron beam energy and $E_{e'}$ is the tagged electron energy measured by the Tagger.

Figure 4.1 The difference between the start time obtained from the Start Counter and the Tagger.

4.1.2 π^+ Reconstruction

The π^+ was reconstructed by using the time-of-flight technique. By measuring the particle momentum with the Drift Chambers, path length and the time when it hit the Start Counter and the Time-of-Flight system, one can calculate the mass squared of this particle by

$$m^2 = p^2 \cdot \left(\frac{1}{\beta^2} - 1 \right) \quad (4.2)$$

where m is the particle mass, p is the magnitude of measured momentum and β is the particle velocity (in units of light velocity, c) which is determined by

$$\beta c = \frac{R_{SC} - R_{ST}}{t_{SC} - t_{ST}} \quad (4.3)$$

where R_{SC}, t_{SC} and R_{ST}, t_{ST} are the path length and time from the target to the Scintillation Counter plane and to the Start Counter plane respectively. Figure 4.2 shows a nice separation between π^+ and proton by plotting the particle's velocity versus its momentum. As shown in the plot, at the momentum up to about 2 GeV/ c it is quite straight forward to separate pions from protons. Another way to do the pion-proton separation is to plot a mass squared distribution and place a cut where it corresponds to a pion mass. The mass of the positively charged particles are plotted in Figure 4.3 as a linear scale and a logarithmic scale. The small bump between the pion peak and proton peak in Figure 4.3 corresponds to the kaon. Thus one can easily separate π^+ from proton and kaon by requiring a mass squared of that particle less than 0.2 GeV²/ c^4 . Figure 4.4 shows some plots of reconstructed quantities for π^+ .

Figure 4.2 Velocity vs momentum of positively charged hadrons.

Figure 4.3 Mass of positively charged hadrons.

Figure 4.4 Example histograms of π^+ reconstruction.

4.1.3 π^0 Reconstruction

Since the neutral pion decays into two photons which deposit their energies in the Electromagnetic Calorimeter, the position and energy deposit of these hits are used to reconstruct the π^0 by using

$$m_{\gamma\gamma}^2 = 2E_1E_2(1 - \cos \theta) \quad (4.4)$$

where θ is the opening angle between the two photons, E_1 and E_2 are the energies of the two photons in the laboratory system as reconstructed by the Electromagnetic Calorimeter. The reconstruction of a π^0 involves the identification of two photons detected in the Electromagnetic Calorimeter. However the timing calibration of the calorimeter at the present time is not good enough to distinguish a photon from a neutron. Thus at first hand all the hits in the calorimeter that do not correspond to any track in the Drift Chambers are assumed to be photons and are used to calculate the invariant mass Eq. (4.4) to determine whether they are the product of π^0 decay. For an event which has 3 hits in the forward Electromagnetic Calorimeter, a combination of these 3 hits are used and the 2 hits that correspond to π^0 mass are chosen as a π^0 candidate. Due to a limitation of the Large Angle Calorimeter calibration at the present time, they are not used in π^0 reconstruction. Figure 4.5 shows the invariant mass of the two photons from π^0 decay along with the background. The mass resolution is about 20 MeV. The invariant mass of the two photons between 0.08 and 0.2 GeV/ c^2 is considered to be a π^0 .

Figure 4.5 Example histograms of π^0 reconstruction.

4.1.4 Neutron Reconstruction

Once the π^+ and π^0 were identified, the missing particle(which is a neutron in this case) can be reconstructed straightforwardly by considering the missing energy(E_X) and missing momentum($\mathbf{p}_X$) of the missing particle.

$$E_X = E_\gamma + E_p - E_{\pi^+} - E_{\pi^0} \quad (4.5)$$

$$\mathbf{p}_X = \mathbf{p}_\gamma + \mathbf{p}_p - \mathbf{p}_{\pi^+} - \mathbf{p}_{\pi^0}, \quad (4.6)$$

where $E_\gamma, E_p, E_{\pi^+}, E_{\pi^0}$ are the energies and $\mathbf{p}_\gamma, \mathbf{p}_p, \mathbf{p}_{\pi^+}, \mathbf{p}_{\pi^0}$ are the momenta of the incident photon, proton target, π^+ and π^0 , respectively. The missing mass squared(M_X^2) can be calculated using

$$M_X^2 = E_X^2 - \mathbf{p}_X^2 \quad (4.7)$$

Figure 4.6 shows a neutron peak on top of background. The width of the neutron peak in this plot is quite broad about 90 MeV. A kinematic constraint can be used to improve the π^0 resolution, and thus eventually improve the neutron resolution. This optimization procedure will be discussed in chapter 6.

Figure 4.6 Missing mass of $\gamma p \rightarrow \pi^+ \pi^0 X$ reaction.

4.2 Photon Flux Normalization

In order to determine a cross section for a photoproduction experiment, one needs to know the number of incident photons that interact with the target. In this experiment, the Pair Spectrometer(PS) and the Total Absorption Shower Counter(TASC) were used for this purpose. Since the TASC cannot tolerate the high intensity of the photon flux for a long period of time, it was not inserted into the beamline during the physics runs. Hence one cannot use the TASC to determine the number of photons directly. However one can relate the detection efficiency of the TASC to the PS during the normalization runs, then use the PS to determine the number of photons during the regular physics runs[32].

In general the number of photons reaching the target is proportional to the number of the electrons tagged in the Tagger,

$$N_\gamma(T_i) = \varepsilon_{tag} \mathcal{N}(T_i) \quad (4.8)$$

where $N_\gamma(T_i)$ is the number of photons in a given T-counter i , $\mathcal{N}(T_i)$ is the number of tagged electron in a given T-counter i measured by a scaler and ε_{tag} is the tagging efficiency. One can relate the number of photons measures during the normalization run to the one during the physics run and take into account a variation of the tagging efficiency during the run period as

$$N_\gamma^{phys}(T_i) = N_\gamma^{norm}(T_i) \frac{\mathcal{N}^{phys}(T_i)}{\mathcal{N}^{norm}(T_i)} \cdot \frac{\varepsilon_{tag}^{phys}(T_i)}{\varepsilon_{tag}^{norm}(T_i)} \quad (4.9)$$

where the superscript *phys* and *norm* represent the quantity obtained from the physics run and normalization run, respectively.

The photon flux normalization was determined by taking 2 different kind of runs— a Normalization run to determine ε_{tag}^{norm} and ε^{PS} and a Physics run to calculate the number of photons for any given E-counters, $N_\gamma^{phys}(E_k)$.

4.2.1 Normalization Run: N_γ^{norm} , ε_{tag}^{norm} and ε^{PS}

This step is to determine the tagging efficiency using the TASC then determine the detection efficiency of the PS relative to the TASC. There are only 3x61 scaler channels available for this purpose eventhough one needs the total of 4x61 channels to cover all 61 channels of T-counters, 61 channels of the coincidence between each T-counter and the PC (T·PC), 61 channels of the coincidence between each T-counter and the PS (T·PS) and 61 channels of the coincidence between each T-counter and the TASC (T·TASC). The arrangement was made so that there were

only 31 channels of T·PC correspond to the odd T-counter number and 30 channels of T·PS correspond to the even T-counter number.

The tagging efficiency(ε_{tag}^{norm}) is determined using the normalization runs and is defined as the number of detected photons in the TASC coincidentally with the tagger divided by the number of electrons detected in the tagger.

$$\varepsilon_{tag}^{norm}(T_i) = \frac{\mathcal{N}^{norm}(T_i \cdot TASC)}{\mathcal{N}^{norm}(T_i)} \quad (4.10)$$

where $\mathcal{N}^{norm}(T_i)$ is the number of scaler events of a given T-counter i and $\mathcal{N}^{norm}(T_i \cdot TASC)$ is the number of scaler events of the coincidence between a given T-counter i and the TASC obtain during normalization runs. The detection efficiency of the TASC is about 100%.

Figure 4.7 Tagging efficiency for each T-counter.

The PS efficiency relative to the TASC(ε^{PS}) is defined as

$$\varepsilon_i^{PS} = \frac{\mathcal{N}^{norm}(T_i \cdot PS)}{\mathcal{N}^{norm}(T_i \cdot TASC)} \quad (4.11)$$

where $\mathcal{N}^{norm}(T_i \cdot PS)$ is the number of scaler events of the coincidence between a given T-counter i and the PS with accidentals subtracted obtained during the normalization runs. For the odd T-counter numbers, an interpolation was used to determine the PS efficiency for these counters.

The number of tagged photons, N_γ^{norm} , is the coincidence between the Tagger TDCs and the TASC TDCs measured during the normalization run.

4.2.2 Physics Run: $N_\gamma(E_k)$

The goal here is to obtain the number of photons for any E-counter. However, there is no scaler information for the E-counters. One can see from Figure 4.8 that for a given T-counter bin there are several associated E-counter bins. Thus one needs to distribute the number of photons from a given T-counter to the interested E-counter. The number of photons in a given T-counter i can be determined using Eq. (4.9). To obtain the number of photons for a given E-counter k , one multiplies Eq. (4.9) with a distribution factor; η_{ik}

$$N_\gamma^{phys}(E_k) = N_\gamma^{norm}(T_i) \frac{\mathcal{N}^{phys}(T_i)}{\mathcal{N}^{norm}(T_i)} \cdot \frac{\varepsilon_{tag}^{phys}(T_i)}{\varepsilon_{tag}^{norm}(T_i)} \cdot \eta_{ik} \quad (4.12)$$

The factor η_{ik} are considered using the hits information of the E- and T-counters in coincidence and are defined as the number of electrons that coincidentally hit T-counter i and E-counter k divided by the total number of electrons that hit T-counter i .

Figure 4.8 Tagger occupancy between T-counter bins and E-counter bins.

Now we need to determine the tagging efficiency during the physics runs, ε_{tag}^{phys} . As mentioned earlier the TASC cannot be used to determine the tagging efficiency for the physics run, however, we already obtained the PS efficiency relative to the TASC from the normalization run, ε_{PS} . Thus ε_{tag}^{phys} and Eq. (4.12) can be rewritten as

$$\varepsilon_{tag}^{phys}(T_i) = \frac{\mathcal{N}^{phys}(T_i \cdot PS)}{\varepsilon_{PS}} \cdot \frac{1}{\mathcal{N}^{phys}(T_i)} \quad (4.13)$$

$$N_{\gamma}^{phys}(E_k) = N_{\gamma}^{norm}(T_i) \frac{\mathcal{N}^{phys}(T_i \cdot PS)}{\mathcal{N}^{norm}(T_i \cdot PS)} \cdot \eta_{ik} \quad (4.14)$$

4.3 Summary

The number of analysed triggers and the reconstructed final states are summarized in Table 4.1. The π^+ is identified by requiring the reconstructed mass

squared less than $0.2 \text{ GeV}^2/c^4$. The invariant mass of the two neutral particles reconstructed using the calorimeter between 0.08 and $0.2 \text{ GeV}/c^2$ is identified as a π^0 . The number of neutrons are estimated by requiring the missing mass between 0.83 and $1.03 \text{ GeV}/c^2$ with background subtraction.

Table 4.1 Summary of event reconstruction.

	Number of Events
Analysed triggers	1.3×10^8
π^+, π^0 and n	1.9×10^5
Tagged photons	4.1×10^{11}

CHAPTER 5

MONTE CARLO SIMULATION

This chapter describes the procedure to generate simulated events for the $\gamma p \rightarrow \pi^+ \pi^0 n$ reaction in order to determine the CLAS acceptance for this reaction in the kinematic region of interest. The simulated events were produced using the CLAS Geant Simulation program (GSIM) with a model independent event generator. The magnetic field used in this simulation was set to 50% of the full field. An accepted event was determined by the detection of π^+ along with the two photons which arise from the π^0 decay with the missing mass corresponding to a neutron.

5.1 Event Generator

The $\gamma p \rightarrow \pi^+ \pi^0 n$ events were generated uniformly in s and t phase space (also known as Chew-Low Plot) where s is the invariant mass squared of the $\pi^+ \pi^0$ system, and t is the four-momentum transfer squared to the nucleon. The ranges of s and t are $4m_\pi^2 \leq s \leq 50m_\pi^2$ ($\approx 1 \text{ GeV}^2/c^4$) and $0 \leq -t \leq 50m_\pi^2$ with the incident photon energies (E_γ) between 0.875 and 2.125 GeV. For a given t and E_γ , the range of s can be obtained from

$$s_{min} = 4m_\pi^2 \tag{5.1}$$

$$s_{max} = W^2 + m_n^2 - \frac{2W(E_p(T_n + m_n) - p_p \sqrt{T^2 + 2Tm_n})}{m_p} \tag{5.2}$$

$$W = \sqrt{m_p^2 + 2E_\gamma m_p} \quad (5.3)$$

$$E_p = \frac{W^2 + m_p^2}{2W} \quad (5.4)$$

$$p_p = \sqrt{E_p^2 - m_p^2} \quad (5.5)$$

$$T_n = \frac{-t + (m_p - m_n)^2}{2m_p} \quad (5.6)$$

where W is the total center-of-mass energy, E_p and p_p are the proton target energy and momentum in the center of mass system respectively, and T_n is the kinetic energy of the recoil neutron[33, 34].

The values of s, t and E_γ were drawn randomly to fulfill the above conditions and energy-momentum conservation. First a pair of s and t were randomly selected, followed by the E_γ and was compared with the calculated s_{min} and s_{max} . If the value of s was between the s_{min} and s_{max} , one then accepted this pair of s and t . Otherwise a new pair of s and t was randomly drawn again and the process kept continuing. The cosine of the π^+ polar angle ($\cos \theta_{\pi^+}$) relative to the incident photon direction and the azimuthal angle of the π^+ (ϕ_{π^+}) in the $\pi^+\pi^0$ center-of-mass system were generated uniformly. Figure 5.1 shows the Chew-Low plots between s and $-t$ up to $50m_\pi^2$ for 5 different incident photon energy bins. The last panel is summed for all 5 energy bins. As the photon energy increases, the phase space also increases. The reason one does not get a uniformly populated distribution over this phase space even though a uniform distribution for s and t were used is because different photon energies give different s_{max} and t_{max} thus smear out around the boundary. Figure 5.2 also shows the generated $s, -t, \cos \theta_{\pi^+}$ and ϕ_{π^+} distributions summed over $0.875 \leq E_\gamma \leq 2.125$ GeV. A cut on the event that was generated within the shadow of the Torus coil was applied. This shows up as a gap regions in the ϕ_{π^+} plot.

Figure 5.1 The Chew-Low plot for different incident photon energy bins.

Figure 5.2 The generated distributions for s , t , $\cos \theta_{\pi^+}$ and ϕ_{π^+} .

The 4-momenta of the final states were determined by the following:

1. The energy, magnitude of momentum and scattering angle of the $\pi^+\pi^0$ system in the laboratory system are

$$E_h = E_\gamma + E_p - E_n \quad (5.7)$$

$$p_h = \sqrt{E_h^2 - s} \quad (5.8)$$

$$\cos \theta_h = \frac{t + 2E_\gamma E_h - s}{2E_\gamma p_h} \quad (5.9)$$

2. The energy and momentum of the scattered nucleon in the laboratory system are

$$E_n = m_n + T_n \quad (5.10)$$

$$p_{x_n} = -p_h \sin \theta_h \cos \phi_h \quad (5.11)$$

$$p_{y_n} = -p_h \sin \theta_h \sin \phi_h \quad (5.12)$$

$$p_{z_n} = E_\gamma - p_h \cos \theta_h \quad (5.13)$$

3. In the $\pi^+\pi^0$ system, the $\cos \theta_{\pi^+}$ distribution and the azimuthal angle were generated using a flat distribution.

$$E_{\pi^+}^{rest} = \frac{s - m_{\pi^+}^2 + m_{\pi^0}^2}{2\sqrt{s}} \quad (5.14)$$

$$p_{\pi^+}^{rest} = \sqrt{(E_{\pi^+}^{rest})^2 - m_{\pi^+}^2} \quad (5.15)$$

$$p_{x_{\pi^+}}^{rest} = p_{\pi^+}^{rest} \sin \theta \cos \phi \quad (5.16)$$

$$p_{y_{\pi^+}}^{rest} = p_{\pi^+}^{rest} \sin \theta \sin \phi \quad (5.17)$$

$$p_{z_{\pi^+}}^{rest} = p_{\pi^+}^{rest} \cos \theta \quad (5.18)$$

$$E_{\pi^0}^{rest} = \sqrt{(p_{\pi^+}^{rest})^2 + m_{\pi^0}^2} \quad (5.19)$$

$$\mathbf{p}_{\pi^0}^{rest} = -\mathbf{p}_{\pi^+}^{rest} \quad (5.20)$$

4. Boost the π^+ and π^0 to the laboratory frame

$$\beta = \frac{p_h}{E_h} \quad (5.21)$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad (5.22)$$

$$E_{\pi^+} = \gamma(E_{\pi^+}^{rest} + \beta p_{z_{\pi^+}}^{rest}) \quad (5.23)$$

$$p_{x_{\pi^+}} = p_{x_{\pi^+}}^{rest} \quad (5.24)$$

$$p_{y_{\pi^+}} = p_{y_{\pi^+}}^{rest} \quad (5.25)$$

$$p_{z_{\pi^+}} = \gamma(p_{z_{\pi^+}}^{rest} + \beta E_{\pi^+}^{rest}) \quad (5.26)$$

$$E_{\pi^0} = \gamma(E_{\pi^0}^{rest} + \beta p_{z_{\pi^0}}^{rest}) \quad (5.27)$$

$$p_{x_{\pi^0}} = p_{x_{\pi^0}}^{rest} \quad (5.28)$$

$$p_{y_{\pi^0}} = p_{y_{\pi^0}}^{rest} \quad (5.29)$$

$$p_{z_{\pi^0}} = \gamma(p_{z_{\pi^0}}^{rest} + \beta E_{\pi^0}^{rest}) \quad (5.30)$$

5. Rotate from helicity axes back to normal laboratory axes (beam along z-axis)

$$E_{\pi^+}^{lab} = E_{\pi^+} \quad (5.31)$$

$$p_{x_{\pi^+}}^{lab} = p_{x_{\pi^+}} \cos \phi_h - p_{y_{\pi^+}} \cos \theta_h \sin \phi_h + p_{z_{\pi^+}} \sin \theta_h \sin \phi_h \quad (5.32)$$

$$p_{y_{\pi^+}}^{lab} = p_{x_{\pi^+}} \sin \phi_h + p_{y_{\pi^+}} \cos \theta_h \cos \phi_h - p_{z_{\pi^+}} \sin \theta_h \cos \phi_h \quad (5.33)$$

$$p_{z_{\pi^+}}^{lab} = p_{y_{\pi^+}} \sin \theta_h + p_{z_{\pi^+}} \cos \theta_h \quad (5.34)$$

$$E_{\pi^0}^{lab} = E_{\pi^0} \quad (5.35)$$

$$p_{x_{\pi^0}}^{lab} = p_{x_{\pi^0}} \cos \phi_h - p_{y_{\pi^0}} \cos \theta_h \sin \phi_h + p_{z_{\pi^0}} \sin \theta_h \sin \phi_h \quad (5.36)$$

$$p_{y_{\pi^0}}^{lab} = p_{x_{\pi^0}} \sin \phi_h + p_{y_{\pi^0}} \cos \theta_h \cos \phi_h - p_{z_{\pi^0}} \sin \theta_h \cos \phi_h \quad (5.37)$$

$$p_{z_{\pi^0}}^{lab} = p_{y_{\pi^0}} \sin \theta_h + p_{z_{\pi^0}} \cos \theta_h \quad (5.38)$$

Figure 5.3 Drift chamber occupancy plot from the real data.

Figure 5.4 Drift chamber occupancy plot from the simulated data with dead wires subtraction.

Before these simulated events were reconstructed by RECSIS, they were passed through another program called GSIM Post Processor(GPP) in order to reject the event that fell within the dead regions of the Drift Chambers. These dead regions were caused by the dead wires in the Drift Chambers. Some of them were pulled out individually before starting the experiment. The others were discovered later and could not be pulled out at that time. Thus the high voltage connectors corresponding to these dead wires were disconnected. This unfortunately resulted in some good wires being disconnected as well. This can be seen in Figure 5.3 which shows the drift chamber occupancy plot from the data taken during the experiment. Figure 5.4 shows the simulated data with dead wires subtraction from GPP. One can see that after dead wire subtraction the simulated data looks similar to the real data.

Figure 5.5 shows an example of an accepted event from GSIM. There is only one positively charged particle(π^+) which is bent outward from the beamline. The two photons are detected in the Electromagnetic Calorimeter. Figure 5.6, 5.7 and 5.8 show the reconstructed event histograms for π^+ , π^0 and neutron, respectively.

Figure 5.9 shows the resolutions of s , t , t' and u for $E_\gamma = 2$ GeV. One see that the t resolution is about $1.3m_\pi^2$ and since the anomaly signal is sensitive at low t region thus we need to improve these resolutions which will be discussed in the next chapter.

5.2 Summary

The Monte Carlo simulation about 40 million events were generated in order to study the CLAS acceptance. The events were generated with a flat distribution in s , t , $\cos \theta_{\pi^+}$ and ϕ_{π^+} . The range of s and t were generated up to $1 \text{ GeV}^2(50m_\pi^2)$ with $1 < E_\gamma < 2 \text{ GeV}$.

Figure 5.5 An example of accepted event.

Figure 5.6 Example histograms of π^+ reconstruction.

Figure 5.7 Example histograms of π^0 reconstruction.

Figure 5.8 A histogram of missing mass reconstruction.

Figure 5.9 Resolutions of s , t , t' and u for $E_\gamma = 2$ GeV before optimization.

CHAPTER 6

DATA ANALYSIS

The previous two chapters describe the procedures to handle the real data as well as the simulated data. In this chapter details of data analysis are discussed. This includes the optimization procedure to improve the s and t resolutions, acceptance determination and the cross section measurement and finally the $F^{3\pi}$ extraction.

6.1 Optimization

The physics of the chiral anomaly is sensitive to the low s and t regions. However, the resolution of the reconstructed s and t are quite poor, as can be seen from the previous chapter. One can use the kinematic constraint of the $\pi^0 \rightarrow 2\gamma$ reaction and neutron missing mass to improve the missing mass resolution. By considering that the resolution of the two photon opening angle is better than the resolution of measured photon energies, E_1 and E_2 , one can select the best pair of energies E'_1 and E'_2 which minimize the function f_{c1}

$$f_{c1} = \frac{(E'_1 - E_1)^2}{\sigma_{E_1}^2} + \frac{(E'_2 - E_2)^2}{\sigma_{E_2}^2} + \frac{(m_{\pi^0} - \sqrt{(E'_1 + E'_2)^2 - (E'_1 \hat{\mathbf{n}}_1 + E'_2 \hat{\mathbf{n}}_2)^2})^2}{\sigma_{\pi^0}^2} \quad (6.1)$$

where E_1 , E_2 , $\hat{\mathbf{n}}_1$, $\hat{\mathbf{n}}_2$, σ_{E_1} , and σ_{E_2} are the measured photon energies, unit vectors and energy resolution of the calorimeter, $\sigma_E = 0.1\sqrt{E}$, respectively. σ_{π^0} is the

weighting factor for this optimization[35, 36] which is set to $1 \text{ MeV}/c^2$ *i.e.* we require that the two photons reconstruct to the π^0 . This optimization is performed on events in which the invariant mass of the two photons is between 0.08 and 0.2 GeV/c^2 . Figure 6.1 shows a comparison of the missing mass plot for the $\gamma p \rightarrow \pi^+ \pi^0 X$ reaction before and after π^0 optimization. Note that the width of the missing mass is now much narrower with the resolution about 40 MeV as compared to 90 MeV before the optimization.

Figure 6.1 Missing mass before(*left*) and after(*right*) π^0 optimization.

The sideband subtraction technique was used to estimate the background under the neutron peak by defining the following regions:

sideband 1: $0.73 < M_X < 0.83 \text{ GeV}^2/c^4$

peak: $0.83 \leq M_X \leq 1.03 \text{ GeV}^2/c^4$

sideband 2: $1.03 < M_X < 1.13 \text{ GeV}^2/c^4$

The number of neutrons were estimated by subtracting these 2 sidebands from the peak assuming a linear background under the peak. In order to further im-

prove the resolution, the second optimization was applied to the peak region by minimizing the function f_{c2}

$$f_{c2} = \frac{(E'_1 - E_1)^2}{\sigma_{E_1}^2} + \frac{(E'_2 - E_2)^2}{\sigma_{E_2}^2} + \frac{(m_{\pi^0} - \sqrt{(E'_1 + E'_2)^2 - (E'_1 \hat{\mathbf{n}}_1 + E'_2 \hat{\mathbf{n}}_2)^2})^2}{\sigma_{\pi^0}^2} + \frac{(m_n - \sqrt{(E_\gamma + E_p - E_{\pi^+} - E'_1 - E'_2)^2 - (\mathbf{p}_\gamma + \mathbf{p}_p - \mathbf{p}_{\pi^+} - E'_1 \hat{\mathbf{n}}_1 - E'_2 \hat{\mathbf{n}}_2)^2})^2}{\sigma_n^2} \quad (6.2)$$

This function f_{c2} guaranteed that one got the best pair of E'_1 and E'_2 that have been optimized to give the masses of π^0 and neutron. σ_{π^0} and σ_n were taken as 1 MeV/ c^2 . Thus by using the background estimation from the first optimization and applying the second optimization one obtains the neutron yield with improved resolution. The improved resolution of s , t , t' and u for $E_\gamma = 2$ GeV can be seen in Figure 6.2. We see that the resolutions are now much better than the ones without the optimization as shown in Figure 5.9 in the previous chapter. Note that the lower photon energies give better resolutions.

The background reactions to be considered in this experiment are $\gamma p \rightarrow \Delta \pi \rightarrow \pi^+ \pi^0 n$ and $\gamma p \rightarrow \rho^+ n \rightarrow \pi^+ \pi^0 n$. Since the anomaly signal has a $\sin^2 \theta_{\pi^+}$ distribution as in Eq. (2.44), we can put a cut on this angular distribution to reduce the background. For high photon energies, one might consider putting a cut on the pion-nucleon invariant mass to reduce the background contributed from Δ production.

Figure 6.2 Resolution of s , t , t' and u for $E_\gamma = 2$ GeV.

6.2 Acceptance Calculation

Since the $\gamma p \rightarrow \pi^+ \pi^0 n$ reaction has 3 final state particles, there are 5 degrees of freedom. In order to determine the model independent acceptance, the bin sizes and ranges of these 5 kinematic variables are defined below. Different bin sizes are also used when plotting the differential cross section as functions of s and t . Two six-dimensional arrays for s and t distributions are defined as follows:

1. s distribution

- 46 bins in s , $4m_\pi^2 \leq s \leq 50m_\pi^2$, $\Delta s = 1m_\pi^2$
- 25 bins in t , $0 \leq -t \leq 50m_\pi^2$, $\Delta t = 2m_\pi^2$
- 6 bins in $\cos \theta_{\pi+}$, $|\cos \theta_{\pi+}| \leq 0.6$, $\Delta \cos \theta_{\pi+} = 0.2$
- 9 bins in $M_{n\pi+}^2$, $0 \leq M_{n\pi+}^2 \leq 4.5 \text{ GeV}^2/c^4$, $\Delta M_{n\pi+}^2 = 0.5 \text{ GeV}^2/c^4$
- 6 bins in $\phi_{\pi+}$, $\pm 25^\circ$ from midplane in each sector, $\Delta \phi_{\pi+} = 50^\circ$
- 5 bins in E_γ , $0.875 \text{ GeV} \leq E_\gamma \leq 2.125 \text{ GeV}$, $\Delta E_\gamma = 0.25 \text{ GeV}$

2. t distribution

- 11 bins in s , $4m_\pi^2 \leq s \leq 48m_\pi^2$, $\Delta s = 4m_\pi^2$
- 100 bins in t , $0 \leq -t \leq 100m_\pi^2$, $\Delta t = 0.5m_\pi^2$
- 6 bins in $\cos \theta_{\pi+}$, $|\cos \theta_{\pi+}| \leq 0.6$, $\Delta \cos \theta_{\pi+} = 0.2$
- 9 bins in $M_{n\pi+}^2$, $0 \leq M_{n\pi+}^2 \leq 4.5 \text{ GeV}^2/c^4$, $\Delta M_{n\pi+}^2 = 0.5 \text{ GeV}^2/c^4$
- 6 bins in $\phi_{\pi+}$, $\pm 25^\circ$ from midplane in each sector, $\Delta \phi_{\pi+} = 50^\circ$
- 5 bins in E_γ , $0.875 \text{ GeV} \leq E_\gamma \leq 2.125 \text{ GeV}$, $\Delta E_\gamma = 0.25 \text{ GeV}$

The range of $\cos \theta_{\pi+}$ was chosen to be between -0.6 and 0.6 to enhance the anomaly signal since the $\gamma\pi \rightarrow \pi\pi$ cross section increases as a function of $\sin^2 \theta_{\pi+}$.

Figure 6.3 and 6.4 show the Monte Carlo simulation of the s distributions for all 25 bins of t at $E_\gamma = 2$ GeV bin from the “accepted” events and “thrown” events, respectively. The similar plots of the t distributions for different bins of s are also shown in Figure 6.5 and 6.6.

For a given E_γ bin, the acceptance was calculated by

$$Acceptance([s], [t], [\cos \theta_{\pi^+}], [M_{n\pi^+}^2], [\phi_{\pi^+}]) = \frac{N_{Accepted}([s], [t], [\cos \theta_{\pi^+}], [M_{n\pi^+}^2], [\phi_{\pi^+}])}{N_{Thrown}([s], [t], [\cos \theta_{\pi^+}], [M_{n\pi^+}^2], [\phi_{\pi^+}])} \quad (6.3)$$

where $[]$ represents the bin of the enclosed quantity. $N_{Accepted}$ is the number of reconstructed and accepted events in CLAS and N_{Thrown} is the number of generated events for given bins. Figure 6.7 – 6.11 show the calculated acceptances for s distribution and Figure 6.12 – 6.16 show the calculated acceptances for t distribution. On average the acceptances are about 10% – 20%. This mainly due to the limited space of detecting the two photons from π^0 decay in the forward calorimeters which suspend a solid angle approximately about 25%.

Figure 6.3 s distribution for accepted events, $E_\gamma = 2$ GeV.

Figure 6.4 s distribution for thrown events, $E_\gamma = 2$ GeV.

Figure 6.5 t distribution for accepted events, $E_\gamma = 2$ GeV.

Figure 6.6 t distribution for thrown events, $E_\gamma = 2$ GeV.

Figure 6.7 Fractional Acceptances for s distributions, $E_\gamma = 1$ GeV.

Figure 6.8 Fractional Acceptances for s distributions, $E_\gamma = 1.25$ GeV.

Figure 6.9 Fractional Acceptances for s distributions, $E_\gamma = 1.5$ GeV.

Figure 6.10 Fractional Acceptances for s distributions, $E_\gamma = 1.75$ GeV.

Figure 6.11 Fractional Acceptances for s distributions, $E_\gamma = 2$ GeV.

Figure 6.12 Fractional Acceptances for t distributions, $E_\gamma = 1$ GeV.

Figure 6.13 Fractional Acceptances for t distributions, $E_\gamma = 1.25$ GeV.

Figure 6.14 Fractional Acceptances for t distributions, $E_\gamma = 1.5$ GeV.

Figure 6.15 Fractional Acceptances for t distributions, $E_\gamma = 1.75$ GeV.

Figure 6.16 Fractional Acceptances for t distributions, $E_\gamma = 2$ GeV.

6.3 Cross Section Determination

The differential cross section $\frac{d^2\sigma}{ds dt}$ was calculated by summed over the other three variables. The number of events observed in each bin is corrected for the trigger efficiency(as discussed in Appendix A) and acceptance. The trigger efficiency correction results in increasing of the number of events by about 15%. For a given incident photon energy range, the differential cross section in units of $\mu b/\text{GeV}^4$ was calculated using

$$\frac{d^5\sigma}{ds dt d\cos\theta_{\pi^+} dM_{n\pi^+}^2 d\phi_{\pi^+}} = \frac{N_{ev}([s], [t], [\cos\theta_{\pi^+}], [M_{n\pi^+}^2], [\phi_{\pi^+}]) \cdot 10^{30}}{N_\gamma N_0 \rho M_H^{-1} l \epsilon_A([s], [t], [\cos\theta_{\pi^+}], [M_{n\pi^+}^2], [\phi_{\pi^+}]) \Delta s \Delta t \Delta \cos\theta_{\pi^+} \Delta M_{n\pi^+}^2 \Delta \phi_{\pi^+}} \quad (6.4)$$

$$\frac{d^2\sigma}{ds dt} = \int_{-0.6}^{0.6} \int_0^{4.5} \int_{-25^\circ}^{25^\circ} \frac{N_{ev} \cdot 10^{30} d\phi_{\pi^+} dM_{n\pi^+}^2 d\cos\theta_{\pi^+}}{N_\gamma N_0 \rho M_H^{-1} l \epsilon_A \Delta s \Delta t \Delta \cos\theta_{\pi^+} \Delta M_{n\pi^+}^2 \Delta \phi_{\pi^+}} \quad (6.5)$$

$$\frac{d^2\sigma}{ds dt} = \frac{N'_{ev} \cdot 10^{30}}{N_\gamma N_0 \rho M_H^{-1} l \Delta s \Delta t} \quad (6.6)$$

where N_{ev} is the number of $\pi^+\pi^0n$ events in each bin after corrected for trigger efficiency,

N'_{ev} is the number of events for a given s and t bin summed over the $\cos\theta_{\pi^+}$, $M_{n\pi^+}^2$ and ϕ_{π^+} with acceptance correction,

N_γ is the number of incident tagged photon for a given energies range,

N_0 is the Avogadro Number; $6.022 \times 10^{23} \text{ mol}^{-1}$,

ρ is the target density; 0.0708 g/cm^3 ,

M_H is the molecular weight of hydrogen; 1.08 g/mol ,

l is the target length; 18 cm,
 Δs is the bin size of s ,
 Δt is the bin size of t , and
 ϵ_A is acceptance correction fraction.

Note that the limits of integration on the ϕ_{π^+} are considered in each sector. To reduce the statistical uncertainty, different bin sizes for Δs and Δt were used when calculating $\frac{d^2\sigma}{ds dt}$ as a function of s and t . This assumes the cross section changes smoothly within these bin sizes. For $\frac{d^2\sigma}{ds dt}$ as a function of s , $\Delta s = m_\pi^2$ and $\Delta t = 2m_\pi^2$ were used. On the other hand, for $\frac{d^2\sigma}{ds dt}$ as a function of t , $\Delta s = 4m_\pi^2$ and $\Delta t = 0.5m_\pi^2$ were used.

Figure 6.17 - 6.21 show the $\gamma p \rightarrow \pi^+ \pi^0 n$ yields with trigger efficiency correction for the s distribution at different photon energy bins, and Figure 6.22 - 6.26 show the yields of the t distribution.

Figure 6.27 - 6.36 show the acceptance corrected yields of the s and t distributions, respectively. The results of the $\gamma p \rightarrow \pi^+ \pi^0 n$ differential cross section are shown in Figure 6.37 - 6.46.

Figure 6.17 The s distribution yield without acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $0.875 \leq E_\gamma < 1.125$ GeV.

Figure 6.18 The s distribution yield without acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.125 \leq E_\gamma < 1.375$ GeV.

Figure 6.19 The s distribution yield without acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.375 \leq E_\gamma < 1.625$ GeV.

Figure 6.20 The s distribution yield without acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.675 \leq E_\gamma < 1.875$ GeV.

Figure 6.21 The s distribution yield without acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.875 \leq E_\gamma < 2.125$ GeV.

Figure 6.22 The t distribution yield without acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $0.875 \leq E_\gamma < 1.125$ GeV.

Figure 6.23 The t distribution yield without acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.125 \leq E_\gamma < 1.375$ GeV.

Figure 6.24 The t distribution yield without acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.375 \leq E_\gamma < 1.625$ GeV.

Figure 6.25 The t distribution yield without acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.675 \leq E_\gamma < 1.875$ GeV.

Figure 6.26 The t distribution yield without acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.875 \leq E_\gamma < 2.125$ GeV.

Figure 6.27 The s distribution yield with acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $0.875 \leq E_\gamma < 1.125$ GeV.

Figure 6.28 The s distribution yield with acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.125 \leq E_\gamma < 1.375$ GeV.

Figure 6.29 The s distribution yield with acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.375 \leq E_\gamma < 1.625$ GeV.

Figure 6.30 The s distribution yield with acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.675 \leq E_\gamma < 1.875$ GeV.

Figure 6.31 The s distribution yield with acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.875 \leq E_\gamma < 2.125$ GeV.

Figure 6.32 The t distribution yield with acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $0.875 \leq E_\gamma < 1.125$ GeV.

Figure 6.33 The t distribution yield with acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.125 \leq E_\gamma < 1.375$ GeV.

Figure 6.34 The t distribution yield with acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.375 \leq E_\gamma < 1.625$ GeV.

Figure 6.35 The t distribution yield with acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.675 \leq E_\gamma < 1.875$ GeV.

Figure 6.36 The t distribution yield with acceptance correction for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $1.875 \leq E_\gamma < 2.125$ GeV.

Figure 6.37 Differential cross sections for $\gamma p \rightarrow \pi^+ \pi^0 n$ as functions of s for $0.875 \leq E_\gamma < 1.125$ GeV.

Figure 6.38 Differential cross sections for $\gamma p \rightarrow \pi^+\pi^0 n$ as functions of s for $1.125 \leq E_\gamma < 1.375$ GeV.

Figure 6.39 Differential cross sections for $\gamma p \rightarrow \pi^+ \pi^0 n$ as functions of s for $1.375 \leq E_\gamma < 1.625$ GeV.

Figure 6.40 Differential cross sections for $\gamma p \rightarrow \pi^+ \pi^0 n$ as functions of s for $1.625 \leq E_\gamma < 1.875$ GeV.

Figure 6.41 Differential cross sections for $\gamma p \rightarrow \pi^+\pi^0 n$ as functions of s for $1.875 \leq E_\gamma < 2.125$ GeV.

Figure 6.42 Differential cross sections for $\gamma p \rightarrow \pi^+ \pi^0 n$ as functions of t for $0.875 \leq E_\gamma < 1.125$ GeV.

Figure 6.43 Differential cross sections for $\gamma p \rightarrow \pi^+ \pi^0 n$ as functions of t for $1.125 \leq E_\gamma < 1.375$ GeV.

Figure 6.44 Differential cross sections for $\gamma p \rightarrow \pi^+ \pi^0 n$ as functions of t for $1.375 \leq E_\gamma < 1.625$ GeV.

Figure 6.45 Differential cross sections for $\gamma p \rightarrow \pi^+\pi^0 n$ as functions of t for $1.625 \leq E_\gamma < 1.875$ GeV.

Figure 6.46 Differential cross sections for $\gamma p \rightarrow \pi^+\pi^0 n$ as functions of t for $1.875 \leq E_\gamma < 2.125$ GeV.

6.4 Estimated Differential Cross Section for $\gamma p \rightarrow \rho^+ n$

As shown in Figure 6.29 for $E_\gamma = 1.5$ GeV and $-t > 4m_\pi^2$, one starts to see the ρ^+ production in the s distribution plot, and it becomes much clearer for $E_\gamma = 2$ GeV. Recall that s is the invariant mass squared of the π^+ and π^0 , and the $\rho^+ \rightarrow \pi^+\pi^0$ branching ratio is about 100%. The mass of ρ^+ , m_ρ , is about 770 MeV which corresponds to s around $31m_\pi^2$. Thus by roughly estimating the $\gamma p \rightarrow \rho^+ n$ cross section in the low t region and comparing with previous measurements, one can evaluate the acceptance calculation and the measured cross sections. Unfortunately there is no previous measurements of the $\gamma p \rightarrow \rho^+ n$ cross section available for $E_\gamma \approx 2$ GeV. However one experiment measured the $\gamma n \rightarrow \rho^- p$ cross section for E_γ average of 1.85 GeV ($1.2 < E_\gamma < 2.5$ GeV)[37]. There is also a theoretical calculation for both channels[38]. The calculation agrees quite well with the data for the $\rho^- p$ channel as shown in Figure 6.47.

Figure 6.47 Differential cross sections for $\gamma n \rightarrow \rho^- p$ at $E_\gamma = 1.85$ GeV.

To estimate the number of ρ^+ , one fits the s distribution of $\frac{d^2\sigma}{ds dt}$ with a Breit-Wigner function which the full width at half maximum corresponding to the ρ width(150 MeV). A flat background is assumed for simplicity. Figure 6.48 shows the results of the s distribution with a flat background subtraction fitted with a Breit-Wigner function. The differential cross section, $\frac{d\sigma}{dt}$ for the ρ^+ production is estimated by integrating the $\frac{d^2\sigma}{ds dt}$ in Figure 6.48 for a given t bin. The theoretical prediction for the ρ^+n channel is shown as the solid line in Figure 6.49. The data points in Figure 6.49 are the estimated differential cross sections from this experiment. The results are consistent with the theoretical calculation and thus give us some confidence in our measured cross sections.

Figure 6.48 The s distribution fitted with a Breit-Wigner function. The width corresponds to the ρ^+ .

Figure 6.49 Differential cross sections for $\gamma p \rightarrow \rho^+ n$ at $E_\gamma = 1.85$ GeV.

6.5 Pion Pole Extrapolation

The $\gamma\pi \rightarrow \pi\pi$ cross section can be obtained from the $\gamma p \rightarrow \pi^+\pi^0 n$ cross section by using the model independent Chew-Low extrapolation technique[26, 39, 40] to extract the cross section to the pion pole, $t \rightarrow m_\pi^2$. This technique involves fitting the measured cross section to a polynomial of the form

$$\begin{aligned}
 (t - m_\pi^2) \frac{d^2\sigma}{ds dt} &\equiv F(t) \\
 &= -|p_0| + (-|p_0| - |p_1| + p_2 + \dots + (-)^n p_n)(t - m_\pi^2) + \\
 &\quad p_2(t - m_\pi^2)^2 + \dots + p_n(t - m_\pi^2)^n
 \end{aligned} \tag{6.7}$$

where $p_0, p_1, \dots, p_n$ are fitted parameters. There are 2 constraints used in the fitting procedure. The first one is $F(t = m_\pi^2) = -|p_0|$. This gives the cross section at the pion pole. The second constraint requires that $F(t = 0) \geq 0$. The cross section at the pion pole $F(t = m_\pi^2)$ can be obtained from Eq. (2.46).

$$F(t = m_\pi^2) = -|p_0| = \left(\frac{-f^2}{4\pi} \right) \frac{4s}{E_\gamma^2} \sqrt{1 - \frac{4m_\pi^2}{s}} \sigma_{\gamma\pi \rightarrow \pi\pi} \quad (6.8)$$

Note that Eq. (6.8) applies to the total cross section, however, in this analysis we applied a cut of $|\cos\theta_{\pi+}| \leq 0.6$ and a cut on $\phi_{\pi+}$ corresponds to shadow regions of the Drift Chamber. Thus a correction to this extracted cross section is needed when we use it to extract the $F^{3\pi}$ in the next chapter.

CHAPTER 7

RESULTS AND CONCLUSION

7.1 Results of Chew-Low Extrapolation

A Chew-Low extrapolation of $\frac{d^2\sigma}{ds dt}$ to the pion pole was performed at $E_\gamma = 2$ GeV for the $20 \leq s < 24m_\pi^2$ and $16 \leq s < 20m_\pi^2$ data. Figure 7.1 shows differential cross sections for the $\gamma p \rightarrow \pi^+\pi^0 n$ reaction as a function of t for these two s bins. One can see a peak at low t from the t -channel meson exchange. The statistical uncertainty is about 20% with some additional uncertainty from the acceptance calculation. The theoretical cross section $\frac{d^2\sigma}{ds dt}$ has a pole corresponding to $t = m_\pi^2$ as seen in Eq. (2.46). In order to obtain the associated $\gamma\pi \rightarrow \pi\pi$ cross section, Chew and Low suggested that one should multiply the differential cross section $\frac{d^2\sigma}{ds dt}$ by $(t - m_\pi^2)^2$ then fit the data with an appropriate function *e.g.* a polynomial function. The $\gamma\pi \rightarrow \pi\pi$ cross section is then extracted by extrapolating the fitted function to the pion pole, $t = m_\pi^2$. In this analysis, we fit the data with a polynomial function Eq. (6.7). The results are shown in Figure 7.2 – 7.13 for polynomials of degree 2 up to degree 9. The effect of a high- t cutoff on the fitting was also studied. Note that for all the Chew-Low extrapolation plots Figure 7.2 – 7.13, the y-axis is scaled by a factor of 0.139^{-4} since the cross sections are multiplied by $(t - m_\pi^2)^2$ in units of $(m_\pi^2)^2$. This larger scale also helps give a fit with more accuracy. To obtain the $\gamma\pi \rightarrow \pi\pi$ cross section, $|p_0|$, we then multiply by a factor of 0.139^4 .

Figure 7.1 Differential cross sections for $\gamma p \rightarrow \pi^+ \pi^0 n$ at $E_\gamma = 2$ GeV for $20 \leq s < 24 m_\pi^2$ and $16 \leq s < 20 m_\pi^2$.

Figure 7.2 Chew-Low extrapolation fitted with 2nd and 3rd order polynomial up to $-t < 50m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.3 Chew-Low extrapolation fitted with 4th and 5th order polynomial up to $-t < 50m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.4 Chew-Low extrapolation fitted with 6th and 7th order polynomial up to $-t < 50m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.5 Chew-Low extrapolation fitted with 8th and 9th order polynomial up to $-t < 50m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.6 Chew-Low extrapolation fitted with 2nd and 3rd order polynomial up to $-t < 30m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.7 Chew-Low extrapolation fitted with 4th and 5th order polynomial up to $-t < 30m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.8 Chew-Low extrapolation fitted with 6th and 7th order polynomial up to $-t < 30m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.9 Chew-Low extrapolation fitted with 8th and 9th order polynomial up to $-t < 30m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.10 Chew-Low extrapolation fitted with 2nd and 3rd order polynomial up to $-t < 10m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.11 Chew-Low extrapolation fitted with 4th and 5th order polynomial up to $-t < 10m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.12 Chew-Low extrapolation fitted with 6th and 7th order polynomial up to $-t < 10m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.13 Chew-Low extrapolation fitted with 8th and 9th order polynomial up to $-t < 10m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $20 \leq s < 24m_\pi^2$.

Figure 7.14 Chew-Low extrapolation fitted with 2nd and 3rd order polynomial up to $-t < 50 m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20 m_\pi^2$.

Figure 7.15 Chew-Low extrapolation fitted with 4th and 5th order polynomial up to $-t < 50 m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20 m_\pi^2$.

Figure 7.16 Chew-Low extrapolation fitted with 6th and 7th order polynomial up to $-t < 50 m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20 m_\pi^2$.

Figure 7.17 Chew-Low extrapolation fitted with 8th and 9th order polynomial up to $-t < 50m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20m_\pi^2$.

Figure 7.18 Chew-Low extrapolation fitted with 2nd and 3rd order polynomial up to $-t < 30m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20m_\pi^2$.

Figure 7.19 Chew-Low extrapolation fitted with 4th and 5th order polynomial up to $-t < 30m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20m_\pi^2$.

Figure 7.20 Chew-Low extrapolation fitted with 6th and 7th order polynomial up to $-t < 30m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20m_\pi^2$.

Figure 7.21 Chew-Low extrapolation fitted with 8th and 9th order polynomial up to $-t < 30m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20m_\pi^2$.

Figure 7.22 Chew-Low extrapolation fitted with 2nd and 3rd order polynomial up to $-t < 10m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20m_\pi^2$.

Figure 7.23 Chew-Low extrapolation fitted with 4th and 5th order polynomial up to $-t < 10 m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20 m_\pi^2$.

Figure 7.24 Chew-Low extrapolation fitted with 6th and 7th order polynomial up to $-t < 10 m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20 m_\pi^2$.

Figure 7.25 Chew-Low extrapolation fitted with 8th and 9th order polynomial up to $-t < 10m_\pi^2$ for $E_\gamma = 2 \text{ GeV}$ and $16 \leq s < 20m_\pi^2$.

The cross section at the pion pole can be extracted from the fitted parameter $|p_0|$ by using Eq. (7.1).

$$F(t = m_\pi^2) = -|c_0| = 2 \left(\frac{-2f^2}{4\pi} \right) \frac{s}{E_\gamma^2} \sqrt{1 - \frac{4m_\pi^2}{s}} \sigma'_{\gamma\pi \rightarrow \pi\pi} \quad (7.1)$$

where $\sigma'_{\gamma\pi \rightarrow \pi\pi}$ is the effective cross section due to cuts on $|\cos \theta_\pi| \leq 0.6$ and $|\phi_\pi| < 25^\circ$. Recall that

$$\frac{d\sigma_{\gamma\pi \rightarrow \pi\pi}}{d\Omega} = \frac{(F^{3\pi}(s, t', u))^2}{128\pi} \cdot \frac{s}{16\pi} \left(1 - \frac{4m_\pi^2}{s} \right)^{3/2} (s - m_\pi^2) \sin^2 \theta_\pi \quad (7.2)$$

and

$$\sigma'_{\gamma\pi \rightarrow \pi\pi} = \int_{-25^\circ}^{25^\circ} 6 d\phi_\pi \int_{-0.6}^{0.6} \frac{d\sigma}{d\Omega} d\cos \theta_\pi \quad (7.3)$$

$$\approx 0.8 \left(\frac{5}{6} \right) \sigma_{\gamma\pi \rightarrow \pi\pi} \quad (7.4)$$

The fitted value of p_0 varies with the polynomial degree. However on average the fitted values are bounded between 40 and 60 for $s = 22m_\pi^2$. Within this range the reduced chi squared changes little, thus indicating stability and quality of the fit. The results of fitting the data up to $-t = 50m_\pi^2$ are consistent with the fits with t cut off around $-t = 30m_\pi^2$. As for the cutoff around $-t = 10m_\pi^2$, these fits are consistent with the previous two cases up to the first few degrees of the polynomial function. This implies that for fewer data points one needs to fit with fewer parameters.

Data fits for $s = 18m_\pi^2$ are also shown in Figure 7.14 – 7.25. In this case one might consider restricting the data fits up to around $-t = 30m_\pi^2$ since the data fluctuate a great deal beyond this point. This high- t cutoff should not affect the extracted cross section since the fitting is well constrained in the low t region. The uncertainty in the fit is large, $\approx 30\%$. A detailed study of the fitting procedure should help reduce this uncertainty.

7.2 Result of Extracted $F^{3\pi}$

The value $F^{3\pi}$ can be extracted from the $\gamma p \rightarrow \pi^+ \pi^0 n$ cross section by extrapolating to the pion pole. Recall that

$$\sigma_{\gamma\pi\rightarrow\pi\pi} = \frac{(F^{3\pi}(s, t', u))^2}{128\pi} \cdot \frac{s}{6} \left(1 - \frac{4m_\pi^2}{s}\right)^{3/2} (s - m_\pi^2) \quad (7.5)$$

For $1.875 < E_\gamma < 2.125$ GeV, preliminary results for $F^{3\pi}(s, t', u)$ are shown in Table 7.1 for $16 \leq s < 20m_\pi^2$ and for $20 \leq s < 24m_\pi^2$ in units of $F^{3\pi}(0, 0, 0)$. Here we use $F^{3\pi}(0, 0, 0) = 9.5 \text{ GeV}^{-3}$. The uncertainty shown here is dominated by the systematic uncertainty from the Chew-Low extrapolation.

Table 7.1 Results of the $F^{3\pi}(s, t', u)$ from this experiment.

$s(m_\pi^2)$	$F^{3\pi}(s, t', u)/F^{3\pi}(0, 0, 0)$
18	$1.91 \pm 0.325(ys)$
22	$2.44 \pm 0.249(ys)$

Figure 7.26 A comparison between the $F^{3\pi}(s, t', u)$ extracted from this experiment and theoretical calculation.

7.3 Comparison with Theoretical Calculations

A comparison of the results with various theoretical calculations are shown in Figure 7.26. The results show a strong momentum dependence of the $F^{3\pi}$ amplitude for $s > 10m_\pi^2$. They disagree completely with the model of Bijmens *et.al* which is nearly constant over the entire range of s . As for the model of Alkofer and Roberts, it fails to reproduce the $s = 22m_\pi^2$ data point. The model of Terent'ev, Rudaz and, Ivanov and Mizutani have a pole at the ρ and therefore it is difficult to compare their calculations with our results.

Holstein's model which incorporates Rudaz's and Bijnen *et.al*'s calculations and takes into account the final states interaction among the pions is consistent with our results within 1σ . Note that the uncertainty here is limited by the uncertainty of the Chew-Low extrapolation. This model error should be reduced when the extrapolation method is explored in more detail. It is also interesting to see whether our data will exhibit the peak near $s = 28m_\pi^2$ predicted in Holstein's model.

7.4 Future Work

Ongoing analyses of extracting $F^{3\pi}$ for $s = 10, 14, 26, 30, 34$ and $38m_\pi^2$ are underway. The preliminary results are promising. A procedure for analysing the data with a cut on the pion-nucleon invariant mass *i.e.* $M_{\pi+n} > 1.4 \text{ GeV}/c^2$ and $M_{\pi^0 n} > 1.4 \text{ GeV}/c^2$ is under development. Figure 7.27 shows the invariant mass plot of the $n\pi^+$ and the $n\pi^0$ for $E_\gamma = 1.5 \text{ GeV}$. With these invariant mass cuts, one can reduce the background contributed from the $\pi^+\Delta^0$ and $\pi^0\Delta^+$ final states. This will enhance the anomaly signal at lower s bins. With the current data set we expect to obtain the values of $F^{3\pi}$ over a range of $10 < s < 40m_\pi^2$.

Additional data with tagged photon energies up to 3.2 GeV were acquired during fall 1999. This data set will increase the statistics by a factor of 20 and

Figure 7.27 Invariant mass of $n\pi^+$ (left) and $n\pi^0$ (right).

enable us to analyze the data with smaller bins in s , *e.g.* $1m_\pi^2$ as opposed to the current bin size of $4m_\pi^2$. With greater statistics in the low s region, we should be able to extract the $F^{3\pi}$ down to $s = 6m_\pi^2$. The increased photon energy also makes the extrapolation less model dependent at high s since $t_{min} \rightarrow 0$ with increasing photon energy. Therefore we expect to be able to extract the $F^{3\pi}$ over a range of $6 < s < 50m_\pi^2$.

There are approved experiments(G8)[41, 42, 43] using linearly polarized photons at CLAS which are scheduled to run in spring of 2001. With a polarized photon beam we can study the chiral anomaly with the $\vec{\gamma}p \rightarrow \pi^+\pi^0n$ reaction. The π^+ angular distribution in the center of mass system is given by[44]

$$W(\cos\theta_{\pi^+}, \phi_{\pi^+}) = \frac{3}{8\pi} \sin^2\theta_{\pi^+}(1 - \epsilon \cos 2\phi_{\pi^+}) \quad (7.6)$$

for pseudo-scalar(*i.e.* pion) exchange, where ϵ is the photon polarization, θ_{π^+} and ϕ_{π^+} are the polar and azimuthal angles of the π^+ relative to the photon direction and polarization. For pseudo-scalar exchange the pions are preferentially emitted

at 90° to the direction of the photon's electric vector. In general, this will not be the case for the $\gamma p \rightarrow \pi \Delta$ and $\gamma p \rightarrow \rho^+ n$ reactions.

On the theoretical side, in order to compare with experimental measurements near the ρ region, modifications are needed for some of the model calculations. This is because the models of Terent'ev, Rudaz and, Ivanov and Mizutani have a pole at the ρ mass. This will enable us to compare the measured momentum dependence of $F^{3\pi}$ near the ρ region with the calculations directly.

7.5 Conclusion

This experiment was one of the first photoproduction experiments performed at Jefferson Lab using the CLAS and the Photon Tagger. The event reconstruction and the photon flux determination procedures have been developed and were proven to work well as we can see from the cross section measurement of the $\gamma p \rightarrow \pi^+ n$ reaction. The preliminary results at CLAS for this reaction agree very well with previous world data. The analysis procedure has been developed to analyze the double-pion photoproduction. The differential cross sections for the $\gamma p \rightarrow \pi^+ \pi^0 n$ reaction have been measured with incident photon energies between 1 and 2 GeV. The Chew-Low extrapolation technique was used to extract the associated $\gamma \pi \rightarrow \pi \pi$ cross sections from the differential cross sections. The extrapolation procedure of extracting the pole cross section has been explored. $F^{3\pi}$ was obtained from the $\gamma \pi \rightarrow \pi \pi$ cross sections. The results show a momentum dependence of the $F^{3\pi}$ amplitude in which they agree with Holstein's calculation. These measurements test fundamental predictions of low energies QCD. Future work on this analysis will help reduce the uncertainty in $F^{3\pi}$, and extend the measurements to the lower and higher s regions.

APPENDIX A

TRIGGER EFFICIENCY

A.1 Introduction

The G1 group was the first group taking data with the CLAS and the Photon Tagger. During this period there were several factors which eventually contributed to a result in the trigger inefficiency[45]. The first factor was the background from the Tagger beam dump. It was not properly shielded during this period and contributed to the high event rate in the backward angle part of the TOF. The other factor was due to the dead time of the data acquisition system. With this high background rate and high dead time, we did raise the TOF threshold from nominal 100 MeV to 200 MeV in order to reduce the dead time and still achieve a desire event rate. This in turn was causing us an inefficient in the trigger since the TOF was part of the trigger and was affected directly. We ended up with the event rate about 500 Hz with 30% dead time.

The G1 trigger inefficiency was studied by using the $\gamma p \rightarrow \pi^+ n$ reaction. Since the $\gamma p \rightarrow \pi^+ \pi^0 n$ reaction is triggered by the π^+ , the $\gamma p \rightarrow \pi^+ n$ reaction is the appropriate reaction for studying any trigger effect that might affect the $\gamma p \rightarrow \pi^+ \pi^0 n$ reaction of interest. The π^+ was identified using tracking and Time-of-Flight. The neutron was identified by missing mass in the $\gamma p \rightarrow \pi^+ X$ reaction. By observing the angular distribution of the π^+ in the laboratory system, or plotting the TOF hit occupancies, the problem showed up as dips in those histograms. We would normally expect a smooth continuum for the angular distribution, however we see some dips in Figure A.1. Figure A.2 shows the similar behavior in the TOF occupancy.

Figure A.1 Angular distributions of the π^+ from exclusive $\gamma p \rightarrow \pi^+ n$ in each sector.

One might suspect the dead regions in the drift chambers could cause those dips. A simulation test which included all the known dead wires in the drift chambers was studied but could not reproduce what has been observed in the real data. Figure A.3 shows the hit occupancies along the scintillation counters of the π^+ from exclusive $\gamma p \rightarrow \pi^+ n$ in each sector. One can see that they are not uniformly populated along the scintillator paddles. Some paddles are missing hits on one side or the other and there are depletion regions about $L/4$ and $3L/4$ in every sector, where L is the length of a given scintillation counter.

Figure A.2 The TOF occupancy for the π^+ from exclusive $\gamma p \rightarrow \pi^+ n$ in each sector.

Figure A.3 Hit positions along the scintillation counters of the π^+ from exclusive $\gamma p \rightarrow \pi^+ n$ in each sector.

A.2 Trigger Simulation

To understand the cause of these observations, a trigger simulation was set up to test the assumption that what has been observed in the real data was caused by an inefficiency of the G1 trigger setup. There are 2 hypotheses:

1. The depletion regions around $L/4$ and $3L/4$ are caused by the TOF discrimination threshold are set too high.
2. Missing hits on one side or the other are caused by missing signals in the pretrigger board.

The G1 trigger is a coincidence between the Tagger, the Start Counters and the Time-of-Flight system. The assumption was that the trigger setup of the Tagger and the Start Counters worked as expected and the inefficiencies were caused by the Time-of-Flight trigger setup. To simulate a pulse shape, a typical TOF phototube pulse shape measured by a digital oscilloscope was used as a prototype as shown in Figure A.4.

Figure A.4 Pulse shape from TOF photomultiplier tube.

The energy loss in the scintillator was generated according to the Landau distribution and normalized to the measured minimum ionizing peak for each counter as calibrated from the real data. The hit position along a given scintillator was generated with a flat distribution and the pulse was attenuated using

$$P_L = P e^{(\frac{L/2-x}{\lambda})} \quad (\text{A.1})$$

$$P_R = P e^{-(\frac{L/2-x}{\lambda})} \quad (\text{A.2})$$

where P is the energy loss converted to ADC channels. P_L , P_R are the attenuated pulses seen by left PMT and right PMT, respectively. L is the length of the scintillator paddle, λ is the attenuation length, and x is the random hit position along a given scintillator paddle.

The resulting attenuated pulses from the left and right PMTs are summed together to simulate the function of the pretrigger board. If the amplitude of the summed pulse is above a setting threshold then one has a successful trigger. Otherwise there is no trigger. In this simulation the timing response of the pretrigger board circuitry[46] was ignored. We did not expect this to affect the general conclusions.

Three threshold settings(150 mV, 200 mV and 250 mV) were studied and each PMT was assigned a set of status words as nm as following:

- $n = 0, m = 0 \Rightarrow$ signal to pretrigger board OK, ADC/TDC OK
- $n = 0, m = 1 \Rightarrow$ signal to pretrigger board OK but no readout ADC
- $n = 0, m = 2 \Rightarrow$ signal to pretrigger board OK but no readout TDC
- $n = 0, m = 3 \Rightarrow$ signal to pretrigger board OK but no readout ADC/TDC
- $n = 1, m = 0 \Rightarrow$ no signal to pretrigger board from Left PMT but ADC/TDC OK

- $n = 2, m = 0 \Rightarrow$ no signal to pretrigger board from Right PMT but ADC/TDC OK
- $n \neq 0, m \neq 0 \Rightarrow$ combination of above case

The status words used in this study are shown in Table A.1. The first row of each sector represents the left PMT and second row the right PMT. The number in front of the parentheses represents the TOF paddle ID(1-48) and the number inside parentheses is the status word. All the other paddles which are not shown in this table have status words 0. These status words were determined by analysing the TDC and ADC of each scintillator paddle.

Table A.1 TOF status words used in this study.

Sector	Status Words
1	18(10),33(10),34(10) 16(23)
2	- 18(20),24(20),27(20),29(20),35(20)
3	36(3) 16(3),18(2),36(3)
4	14(10),25(10) 21(22)
5	29(10),33(10),37(10) 15(20),16(23),31(20)
6	- 3(1),25(20),35(2)

A.3 Results and Discussion

The results of the summed pulse which indicated a successful or failed trigger are shown in Figure A.5.

Figure A.5 Simulated pulse output from the pretrigger board.

The red(dash) pulse is the attenuated pulse from left PMT and the blue(solid) pulse is the attenuated pulse from right PMT. The green(thick) is the sum of the left and the right pulses. The horizontal axis shows the relative time between the left and the right pulses. There are 4 cases: the first 2 cases contributed to a successful trigger and the last 2 cases contributed to a failure trigger.

- *Top-Left*: This plot shows the normal situation in which both the pulses from left and right PMTs contributed to the trigger. The summed pulse is above the threshold of 200 mV.
- *Top-Right*: In this case there is only one signal from the left PMT but it is high enough to reach the threshold. The success or failure of the trigger depends on the hit position along the scintillator. If it hits near the active PMT and deposits enough energy to reach the threshold limit then one has a successful trigger.
- *Bottom-Left*: In this case the readout TDC/ADC from both left and right PMTs are OK but the signal to the pretrigger board of the right PMT is lost. Eventhough the signal from the right PMT is above the threshold, it does not take part in the pulse summation only the left one does. The summed pulse is exactly the same as the pulse from left PMT and never reach the threshold so the trigger fails in such a situation.
- *Bottom-Right*: When the hit position is around $L/4$ or $3L/4$, despite the readout TDC/ADC and signals to pretrigger board of the left and right PMTs are working properly, the failure of the trigger is because the threshold of 200 mV is too high. The scintillator light reaches the left and right PMTs are not in time with each other so the summed pulse is below the threshold thus one has a failure trigger.

Figure A.6, A.7 and A.8 show the results of this simulation with threshold 150 mV, 200 mV and 250 mV respectively. One starts to see the threshold problem in 200 mV case and more clearly for 250 mV. The lost signals in pretrigger board also show up as empty space on one side or the other.

One can see from Figure A.6, A.7 and A.8 and Table A.2 that the efficiencies drop when the threshold is increased and the longer scintillator is affected more severely than the shorter one.

A.4 Summary

This study shows that the trigger inefficiency during the G1 data taking period was caused by

1. The Time of Flight discrimination threshold was set too high(200 mV).
2. The missing signals either caused by bad cable connections to pretrigger board or malfunction of the pretrigger board itself.

This trigger efficiency study was also used as a correction when one determined the $\gamma p \rightarrow \pi^+ \pi^0 n$ cross section.

Figure A.6 Trigger simulation result: Threshold = 150 mV.

Figure A.7 Trigger simulation result: Threshold = 200 mV.

Figure A.8 Trigger simulation result: Threshold = 250 mV.

Table A.2 π^+ trigger efficiencies for each scintillator paddles in 6 sectors.

Paddle	Threshold (mV)	S1	S2	S3	S4	S5	S6
1	150	99	98	99	99	99	99
	200	99	98	99	83	99	99
	250	99	97	96	38	97	99
2	150	100	100	100	99	99	99
	200	100	100	100	99	98	99
	250	99	99	94	93	89	92
3	150	99	99	98	98	99	75
	200	99	99	98	98	98	33
	250	98	99	93	87	81	18
4	150	100	100	99	99	99	99
	200	99	97	97	98	97	97
	250	99	81	86	98	77	82
5	150	98	100	99	99	100	99
	200	82	100	92	99	99	99
	250	52	99	64	98	82	97
6	150	98	98	98	99	99	99
	200	81	96	90	98	90	99
	250	51	77	67	79	65	83
7	150	98	98	98	99	99	99
	200	98	97	97	99	95	96
	250	77	76	69	97	68	72
8	150	98	100	99	99	99	99
	200	98	100	84	78	98	95
	250	89	100	56	48	95	68
9	150	99	99	98	99	99	99
	200	99	90	97	98	98	98
	250	88	59	95	98	90	84
10	150	99	99	99	100	99	99
	200	98	99	93	99	99	98
	250	85	78	60	83	81	76

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Table A.2: (continue)

Paddle	Threshold (mV)	S1	S2	S3	S4	S5	S6
11	150	99	99	100	99	98	99
	200	98	98	99	99	98	98
	250	97	96	86	97	96	77
12	150	99	98	99	99	99	98
	200	98	98	98	88	75	97
	250	77	95	79	55	74	76
13	150	99	98	99	99	98	99
	200	98	92	98	98	85	97
	250	53	56	80	88	76	80
14	150	99	99	99	97	99	98
	200	95	95	98	70	99	97
	250	67	70	71	60	94	84
15	150	99	99	99	98	98	100
	200	89	99	99	97	70	97
	250	61	92	89	88	59	73
16	150	63	99	99	98	98	98
	200	40	99	40	98	60	96
	250	31	92	89	80	40	70
17	150	98	98	99	98	99	98
	200	98	98	98	98	99	98
	250	54	93	94	80	98	92
18	150	81	77	99	99	98	99
	200	50	30	98	98	98	98
	250	45	20	80	80	93	89
19	150	99	99	98	99	99	99
	200	98	99	96	98	99	99
	250	85	99	80	80	97	99
20	150	99	98	98	99	99	99
	200	99	98	98	99	98	98
	250	84	97	94	90	96	88

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Table A.2: (continue)

Paddle	Threshold (mV)	S1	S2	S3	S4	S5	S6
21	150	99	99	99	60	97	98
	200	98	99	99	40	97	98
	250	93	98	98	30	96	96
22	150	98	98	98	99	99	98
	200	98	81	88	94	97	98
	250	97	58	61	70	79	96
23	150	99	99	46	100	98	99
	200	50	98	19	90	98	99
	250	34	88	12	65	94	96
24	150	99	59	99	98	99	89
	200	77	38	96	95	98	44
	250	43	25	68	60	88	22
25	150	99	99	99	70	98	70
	200	86	95	83	50	83	40
	250	54	69	51	30	51	24
26	150	99	98	99	96	99	99
	200	80	88	82	69	98	76
	250	48	56	48	37	87	44
27	150	99	61	99	97	99	99
	200	78	30	90	72	90	82
	250	46	22	58	40	55	47
28	150	99	99	98	98	99	99
	200	86	79	86	70	97	91
	250	51	47	58	40	75	58
29	150	98	66	100	99	77	99
	200	91	80	95	83	53	98
	250	64	30	67	48	37	77
30	150	98	99	99	99	99	98
	200	77	95	98	77	98	86
	250	45	63	79	45	89	52

Continued next page

Table A.2: (continue)

Paddle	Threshold (mV)	S1	S2	S3	S4	S5	S6
31	150	98	99	97	98	70	99
	200	93	97	96	86	40	99
	250	68	75	70	54	30	98
32	150	99	99	99	97	97	98
	200	87	96	95	66	97	92
	250	57	67	70	35	84	62
33	150	68	98	99	96	81	98
	200	41	50	99	70	61	87
	250	19	39	79	39	40	50
34	150	63	99	99	97	98	99
	200	38	97	94	95	97	97
	250	21	80	67	85	81	72
35	150	99	68	97	97	98	100
	200	96	30	67	95	95	100
	250	74	20	35	85	67	98
36	150	100	99	0	99	99	99
	200	95	95	0	98	99	98
	250	66	71	0	90	89	87
37	150	98	98	100	99	98	99
	200	89	89	99	90	90	95
	250	61	59	81	62	80	71
38	150	99	100	99	98	99	98
	200	90	89	93	95	97	98
	250	59	57	67	60	74	76
39	150	98	99	99	98	98	99
	200	90	99	99	95	96	98
	250	61	88	88	60	75	85
40	150	99	97	100	98	98	99
	200	82	91	95	95	96	91
	250	50	64	65	64	75	64

Continued next page

Table A.2: (continue)

Paddle	Threshold (mV)	S1	S2	S3	S4	S5	S6
41	150	99	99	99	98	99	99
	200	80	95	95	95	95	92
	250	42	71	69	60	60	60
42	150	98	98	99	99	99	99
	200	93	97	94	93	94	96
	250	67	75	63	65	62	68
43	150	95	99	98	98	99	99
	200	52	80	85	70	95	94
	250	27	45	51	38	60	65
44	150	98	98	98	98	99	99
	200	80	91	93	97	96	85
	250	52	59	64	75	76	49
45	150	99	98	98	98	99	99
	200	91	89	73	87	96	97
	250	62	53	41	55	68	75
46	150	98	98	98	99	99	98
	200	78	74	87	90	97	92
	250	45	42	48	58	69	60
47	150	100	99	99	99	99	99
	200	99	96	98	98	98	99
	250	92	69	97	90	82	95
48	150	99	100	100	98	100	99
	200	98	98	91	93	100	99
	250	75	81	55	64	99	87

APPENDIX B

$$\gamma p \rightarrow \pi^+ n$$

B.1 Introduction

The purpose of this study is to determine the reliability of photon flux normalization and detector calibration and reconstruction. In order to do that the differential cross section for $\gamma p \rightarrow \pi^+ n$ was measured using minimum corrections. These cross sections were compared with the previous measurements which are available up to about 2 GeV of incident photon energy. The π^+ was reconstructed using the tracking and identified using the time of flight. The neutron was identified by missing mass in the $\gamma p \rightarrow \pi^+ X$ reaction[47].

B.2 Event Selection and Analysis

This study used the same data set and the same photon flux normalization as for the $\gamma p \rightarrow \pi^+ \pi^0 n$ measurement. However the event selection criterias are different as following

1. Select events with only one positively charged particle
2. Cut on mass square less than $0.2 \text{ GeV}^2/c^4 \Rightarrow \text{pion cut}$
3. Cut on time coincidence between Start Counter and Tagger within $\pm 1 \text{ ns}$
4. Cut on π^+ azimuthal angle ± 5 degrees from midplane
5. Cut on TOF paddle number greater than 6
6. Cut on missing mass between 0.88 and $1.0 \text{ GeV}^2/c^4 \Rightarrow \text{neutron cut}$

By applying the azimuthal cut and TOF cut, the CLAS acceptance was assumed to be about 100% in this region. However as discussed in Appendix A about the π^+ trigger efficiency, one needs to correct this efficiency for each TOF paddle by using Table B.1.

Table B.1 π^+ trigger efficiencies for scintillator paddles for tracks within ± 50 cm (or ± 5 degrees from midplane) in 6 sectors. TOF threshold = 200 mV.

Paddle ID	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6
1	99	99	99	83	99	99
2	100	100	100	99	98	99
3	99	99	99	98	96	33
4	99	97	99	98	97	97
5	80	100	90	99	99	99
6	83	97	92	96	92	99
7	98	97	97	99	95	96
8	98	100	89	83	99	95
9	99	94	97	98	98	98
10	98	98	94	100	98	98
11	99	99	98	99	98	98
12	97	98	98	90	99	97
13	87	89	98	98	97	86
14	95	93	95	70	99	97
15	88	99	98	98	98	97
16	33	99	98	98	80	96
17	85	98	98	98	99	98
18	50	30	98	98	98	98
19	97	99	97	98	99	99
20	98	98	98	99	98	98
21	98	99	99	40	97	98
22	98	81	91	94	97	98
23	94	98	16	97	98	99
24	87	40	98	96	98	35

Continued next page

Table B.1: (continue)

Paddle ID	Sector 1	Sector 2	Sector 3	Sector 4	Sector 5	Sector 6
25	92	98	95	40	95	35
26	91	95	95	85	98	90
27	93	27	98	87	96	94
28	90	91	96	87	97	96
29	97	50	98	90	46	98
30	90	98	99	94	98	95
31	98	98	98	89	40	99
32	90	99	98	88	97	98
33	50	50	99	84	54	96
34	28	99	98	99	97	99
35	98	30	87	99	98	100
36	98	97	0	98	99	98
37	97	98	98	98	90	98
38	98	98	99	98	99	98
39	96	100	99	98	91	98
40	96	97	99	90	98	98
41	92	98	98	98	91	98
42	97	98	98	97	98	99
43	77	96	97	91	97	98
44	93	98	97	98	99	94
45	95	96	88	95	99	97
46	94	95	96	97	98	98
47	99	98	99	99	99	99
48	99	99	97	98	100	99

Figure B.1 $\gamma p \rightarrow \pi^+ X$ missing mass.

Figure B.1 shows a missing mass plot from the $\gamma p \rightarrow \pi^+ X$ reaction in which the missing particle is identified as a neutron or a Δ^0 .

For a given incident photon energy, the differential cross section in units of $\mu b/Sr$ was calculated using

$$\frac{d\sigma}{d\Omega} = \frac{N_{ev} \cdot 10^{30}}{N_\gamma N_0 \rho M_H^{-1} l (\Delta \cos \theta_{cm}) \Delta \phi \epsilon_A} \quad (\text{B.1})$$

where N_{ev} is the number of $\pi^+ n$ events in each bin after corrected

for trigger efficiency,

N_γ is the number of incident tagged photon for a given energies range,

N_0 is the Avogadro Number; $6.022 \times 10^{23} \text{ mol}^{-1}$,

ρ is the target density; 0.0708 g/cm^3 ,

M_H is the molecular weight of hydrogen; 1.08 g/mol ,

l is the target length; 18 cm ,

$\Delta \cos \theta_{cm}$ is the polar angle bin size; 0.1 ,

$\Delta\phi$ is the azimuthal angle bin size; 0.1745 radians, and
 ϵ_A is acceptance correction fraction; 1.0.

B.3 Results and Summary

Figure B.2 shows the results from this study with 2 ranges of incident photon energies. Only statistical errors have been considered here. The differential cross sections measured with CLAS agree very well with the previous measurements. The world data were taken from the Scattering Analyses Interactive Dial-in(SAID) database[48].

The preliminary results show good agreement between the CLAS measurement and previous world data. This indicates that the calculation of photon flux normalization is well established and the detector calibrations and reconstructions are reliable.

PRELIMINARY $\gamma p \rightarrow \pi^+ n$ with CLAS

Figure B.2 Preliminary differential cross sections for $\gamma p \rightarrow \pi^+ n$

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