Intense, broadband far-IR / THZ radiation from the JLab FEL

Gwyn P. Williams

We calculate the spectral output from a dipole magnet in the energy recovery system of the Jefferson Laboratory Free Electron Laser[1] (FEL) in the 1 – 10,000 cm^{-1} (1 micron to 1cm) range. We show that due to multiparticle coherent enhancement[2] (coherent synchrotron radiation or CSR), the emission contains almost 1 watt/cm^{-1} of power emitted into the diffraction limit.

For the purpose of this illustrative calculation we assume some typical parameters for the machine, namely that we have 100 pico-coulomb electron bunches at a 37.4 MHz repetition rate. We also assumed full width half maximum (fwhm) horizontal and vertical beam sizes of 200 microns. We took the electron beam energy to be 40 MeV, the bending radius to be 1m, and we extracted 90 X 90 milliradians of light (approximately an f/11 beam).

The power emitted by an electron bunch as a function of frequency (ω) and solid angle (Ω), is derived by extending the classical electrodynamics[3] theory for a single electron, to a system of N electrons thus:

$$\frac{d^2I}{d\omega d\Omega} = \left[N[1-f(\omega)] + N^2 f(\omega)\right] \times \underbrace{\frac{e^2\omega^2}{4p^2c}\left(\int_{-\infty}^{\infty} \hat{n}\times\left(\vec{\beta}\times\hat{n}\right) e^{i\omega\left(t-\frac{\hat{n}\cdot\vec{r}(t)}{c}\right)} dt\right)^2}_{\zeta} \quad [1]$$

where e is the charge on an electron, β is the ratio of the velocity of the particle bunch to the velocity of light, n is a unit vector, r(t) is the position of the center of the electron bunch, and N is the number of particles in the bunch. f(ω) is the longitudinal particle distribution within the bunch, which is assumed to be Gaussian, and thus of the form:

$$f(\omega) = e^{-\frac{4\pi^2\sigma^2}{\lambda^2}} \quad [2]$$

Where ω is the frequency of the light at wavelength λ and we have written the expression in this form to show how the enhancement is proportional to the wavelength. There are practical ways to solve this. Kim[4] has shown that the term ζ of Eq. 1 can be written in practical units to give the ***time-averaged*** photon flux, F, as:

$$F_v(\theta_v) = 1.325\times 10^{16} E^2 \left(\frac{\lambda_c}{\lambda}\right)^2 \left(\left(\left(1+\gamma^2\theta_v^2\right) K_{\frac{2}{3}}(\xi) \right)^2 + \gamma^2\theta_v^2 \left(1+\gamma^2\theta_v^2\right) K_{\frac{1}{3}}^2(\xi) \right) \quad [3]$$

in units of photons/sec/mrad.(horiz)/mrad.(vert)/amp/0.1%bw. In the above equation, γ is the ratio of the electron mass to the rest mass, which in practical units is given by:

$$\gamma = 1957\times E(GeV) \quad [4]$$

λ_c is the critical wavelength which divides the output power in half and is given in practical units by:

$$\lambda_c(Angstroms) = \frac{5.59\rho(meters)}{E^3(GeV)} \quad [5]$$

while the K terms are modified Bessel functions of fractional order, whose argument, ξ, is given by:

$$\xi = \left(\frac{\lambda_c}{2\lambda}\right)\left(1+\gamma^2\vartheta_v^2\right)^{\frac{3}{2}} \quad [6]$$

The first term, in $K_{2/3}$, on the right represents light polarized parallel to the orbit plane (or horizontally polarized), while the second term involving $K_{1/3}$ corresponds to light polarized perpendicular to the orbit plane, (or vertically polarized). Clearly, we require the integral of $F_v(\theta_v)$ over θ_v in Eq. 3. Note that the expression is symmetrical about θ_v = 0.

The limits of the integration are set by the smallest of the physical opening angle of the extraction optics, or the "natural opening angle" θ_{nat}, of the synchrotron radiation, whose full width half-maximum value can be approximated in the infrared spectral region[5] by:

$$\theta_{nat}(mrads) = 1.66\left(\frac{1000 \times \lambda(\mu m)}{\rho(meters)}\right)^{\frac{1}{3}} \qquad [7]$$

Kostroun[6] has shown that the spherical Bessel functions $K_x(y)$, of fractional order x, can be readily evaluated using the following algorithm:

$$K_x(y) = 0.5\left(\frac{e^{-y}}{2} + \sum_{r=1}^{\infty} e^{-y\cosh(0.5r)}\cosh(0.5xr)\right) \qquad [8]$$

In Fig. 1 we present the results of this calculation in units of watts/cm^{-1} over the range 1-10,000 cm^{-1}, or 1 centimeter to 1 micrometer. In the same figure we compare a 2000K thermal source and a synchrotron radiation source, namely the vacuum ultraviolet ring of the National Synchrotron Light Source[7] at Brookhaven National Lab. The superiority of the JLab FEL in the THz range is clear.

Brightness

Having calculated the total flux emitted into the optical beamline as a function of wavelength, we now address the brightness, which is defined as the flux per unit source area per unit solid opening angle. This is the quantity that is conserved by any optical system and which ultimately limits the performance of any experimental system.

In practical terms, we assume that the brightness is the flux divided by the opening angle θ, and the area of the source. The angle is the smaller of the natural opening angle, θ_{nat}, or the physical angle defined by the extraction optics. The area of the source is determined practically by summing in quadrature the physical size of the electron bunch and the size determined by diffraction which is set equal to λ/θ.

In Fig.2 we present the result of these calculations, again comparing the JLab FEL with a thermal source and the synchrotron radiation source at Brookhaven National Laboratory. Once again the superiority in the THz region is clear. It is also interesting to observe that the synchrotron radiation source is superior to the thermal source in brightness across the entire spectrum.

Acknowledgements

This work was supported by U.S. DOE Contract No. DE-AC05-84-ER40150. The Free-electron Laser is supported by the Office of Naval Research, the Commonwealth of Virginia and the Laser Processing Consortium.

References

1. G.R. Neil, C.L. Bohn, S.V. Benson, G. Biallas, D. Douglas, H.F. Dylla, R. Evans, J. Fugitt, A. Grippo, J. Gubeli, R. Hill, K. Jordan, R. Li, L. Merminga, P. Piot, J. Preble, M. Shinn, T. Siggins, R. Walker and B. Yunn, Physical Review Letters **84**, 662 2000

2. C. J. Hirschmugl, M. Sagurton and G. P. Williams, Physical Review **A44**, 1316, (1991).

3. J.D. Jackson, Classical Electrodynamics, Wiley, New York 1975

4. K-J. Kim, Lawrence Berkeley Laboratory Pub. 490 (1986)

5. W.D. Duncan and G.P. Williams, Applied Optics **22**, 29l4 (1983).

6. V. Kostroun, Nuclear Instr. & Methods **172** 371 (1980).

7. A. van Steenbergen & NSLS Staff, Nuclear Instr. & Methods **172** 25 (1980).

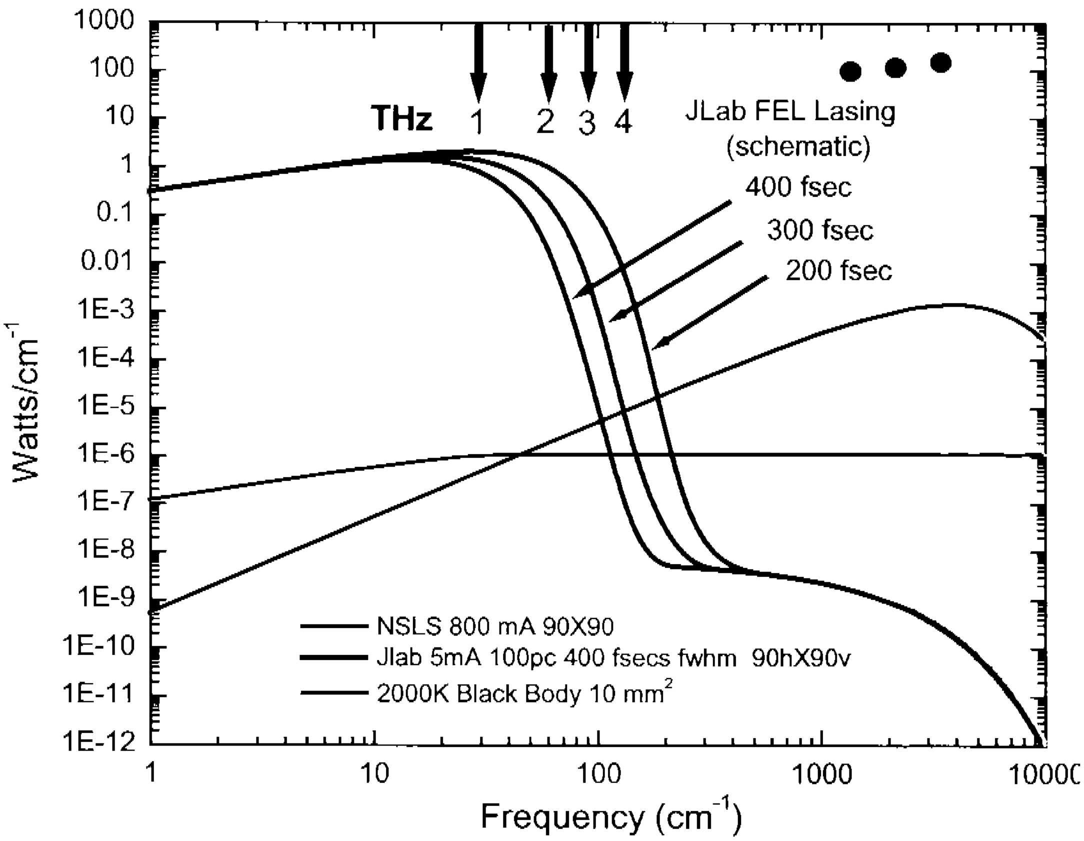

Fig. 1. Total power emitted by the JLab FEL for 3 pulse lengths, compared with the National Synchrotron Light Source, Brookhaven and a 2000K thermal source. Note that this is the time-averaged power, the peak power being ~10^2 times higher for the NSLS and ~10^5 times higher for the JLab FEL.

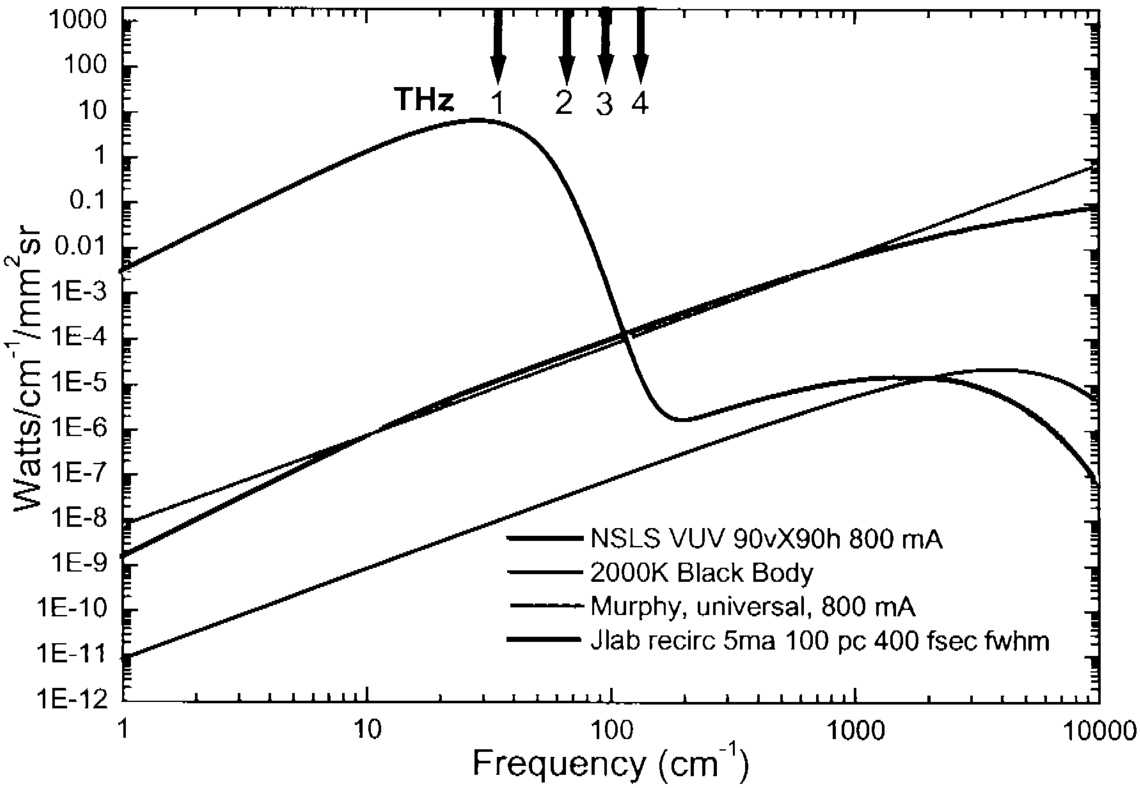

Fig. 2. Brightness of synchrotron radiation emitted from a dipole at the JLab FEL compared with the National Synchrotron Light Source, Brookhaven and a 2000K thermal source. Note that this is the time-averaged brightness, the peak brightness being ~10^2 times higher for the NSLS and ~10^5 times higher for the JLab FEL.