The Thomas Jefferson National Accelerator Facility

Theory Group Preprint Series

Additional copies are available from the authors.

The Southeastern Universities Research Association (SURA) operates the Thomas Jefferson National Accelerator Facility for the United States Department of Energy under contract DE-AC05-84ER40150.

JLAB-THY-01-11
April 20, 2001

QCD Calculations of Pion Electromagnetic and Transition Form Factors

A.V. RADYUSHKIN

Physics Department, Old Dominion University, Norfolk, VA 23529, USA

and

Theory Group, Jefferson Lab, Newport News, VA 23606, USA

The transition $\gamma\gamma^* \to \pi^0$ in which a real and a virtual photon produce a pion is the cleanest exclusive process for testing QCD predictions. In the lowest order of perturbative QCD, the asymptotic behavior of the relevant form factor $F_{\gamma\gamma^*\pi^0}(Q^2)$ is given by[1]

$$F^{LO}_{\gamma^*\gamma^*\pi^0}(Q^2) = \frac{4\pi}{3}\int_0^1 dx \frac{\varphi_\pi(x)}{xQ^2} + O(\alpha_s/\pi) + O(1/Q^4) \; , \qquad (1)$$

where $\varphi_\pi(x)$ is the pion distribution amplitude describing the projection of the pion onto the quark-antiquark pair carrying the momenta xp and $(1-x)p$. The nonperturbative information is accumulated here by the integral

$$I_0 = \int_0^1 \frac{\varphi_\pi(x)}{x} dx \; . \qquad (2)$$

Its value depends on the shape of $\varphi_\pi(x)$. In particular, using the asymptotic form[2, 1] $\varphi_\pi^{as}(x) = 6f_\pi x(1-x)$ one obtains $I_0 = 3f_\pi$ which gives the $F^{as}_{\gamma\gamma^*\pi^0}(Q^2) = 4\pi f_\pi/Q^2$ prediction for the large-Q^2 behavior[1]. Brodsky and Lepage[1] proposed the interpolation formula $F^{BL}_{\gamma\gamma^*\pi^0}(Q^2) = 1/[\pi f_\pi(1+Q^2/s_0)]$ which reproduces, for $s_0 = 4\pi^2 f_\pi^2$, both the $Q^2 = 0$ value and the high-Q^2 behavior. The same result follows from the model[3] based on local quark-hadron duality, in which $F_{\gamma\gamma^*\pi^0}(Q^2)$ is obtained by calculating the amplitude of the transition of the pion into a $\bar{q}q$ pair with the invariant mass s, with subsequent integration over the pion duality interval $0 < s < s_0$.

Adding the one-loop pQCD corrections[4, 5] and assuming the asymptotic distribution amplitude (DA) one obtains

$$F^{NLO}_{\gamma\gamma^*\pi^0}(Q^2)\Big|_{\varphi=\varphi^{as}} = \frac{4\pi f_\pi}{Q^2}\left\{1 - \frac{5}{3}\frac{\alpha_s}{\pi}\right\} \; . \qquad (3)$$

Another frequently used model $\varphi_\pi^{CZ}(x) = 6f_\pi x(1-x)(1-2x)^2$ was proposed by Chernyak and Zhitnitsky[7]. It gives an essentially larger value $I_0 = 5f_\pi$.

Comparison with recent CLEO data[6] favors the distribution amplitudes close to the asymptotic one.

The asymptotic behavior of the pion electromagnetic form factor can be also calculated in pQCD[7, 2, 1]

$$F_\pi^{LO}(Q^2) = \frac{8\pi\alpha_s(Q^2)}{9}\int_0^1 dx\int_0^1 dy\,\frac{\varphi_\pi(x)\varphi_\pi(y)}{xyQ^2} = \frac{8\pi\alpha_s(Q^2)}{9Q^2}I_0^2 . \qquad (4)$$

It involves the same integral I_0 of the pion DA. However, taking $I_0 \approx 3f_\pi$ (as suggested by the $\gamma\gamma^*\pi^0$ data) and $\alpha_s \sim 0.3$ gives the result which is too small. It is instructive to rewrite the pQCD term (for the asymptotic DA) as $2(\alpha_s/\pi)(s_0/Q^2)$. Since $s_0 = 4\pi^2 f_\pi^2 \approx 0.67$ GeV$^2 \sim m_\rho^2$, the pQCD term has an extra factor $2(\alpha_s/\pi) \sim 0.2$ compared to the m_ρ^2/Q^2 behavior suggested by the VMD model $F_\pi^{VMD}(Q^2) = 1/(1+Q^2/m_\rho^2)$. Note, that the $O(\alpha_s/\pi)$ factors are the standard penalty for each extra loop in Feynman diagrams. Hence, the natural way out is to add the contribution which has the zeroth order in α_s. It corresponds to overlap of the soft parts of the pion wave functions. This nonperturbative contribution can be estimated using the local quark-hadron duality model. Calculating the amplitude for the transition $\bar{q}q\gamma^* \to \bar{q}'q'$, with the initial $\bar{q}q$ pair having mass s_1 while the final $\bar{q}'q'$ pair having mass s_2, and integrating over the pion duality region $0 < s_1, s_2 < s_0$ one obtains[8]

$$F_\pi^{LD,soft}(Q^2) = 1 - \frac{1+6s_0/Q^2}{(1+4s_0/Q^2)^{3/2}} .$$

Asymptotically, this contribution decreases as $1/Q^4$. Within the local duality approach, the α_s/Q^2 term is obtained from the $O(\alpha_s)$ contribution to the $\bar{q}q\gamma^* \to \bar{q}'q'$ amplitude. Fortunately, the $Q^2 = 0$ limit of this contribution is fixed by the Ward identity resulting in $F_\pi^{LD,\alpha_s}(Q^2=0) = \alpha_s/\pi$. The simple interpolation formula (analogous to the Brodsky-Lepage expression) gives

$$F_\pi^{LD,\alpha_s}(Q^2) = \left(\frac{\alpha_s}{\pi}\right)\frac{1}{1+Q^2/2s_0} .$$

The sum $F_\pi^{LD,soft}(Q^2) + F_\pi^{LD,\alpha_s}(Q^2)$ is in a full agreement (for $\alpha_s/\pi = 0.1$) with the recent Jefferson Lab data[9]. Similar results for the pion form factor have been obtained within the light-cone QCD sum rule approach[10].

The pQCD radiative corrections to the asymptotic term are known[11]. In case of the asymptotic DA, the result in the $\overline{MS}$ scheme is[11, 12]

$$F_\pi^{pQCD,NLO}(Q^2) = \frac{8\pi f_\pi^2\alpha_s(Q^2)}{Q^2}\left\{1+\frac{\alpha_s}{\pi}\left[\frac{7}{6}\beta_0 - 3.91\right]\right\} ,$$

where $\beta_0 = 11 - 2N_f/3$ is the lowest coefficient of the QCD β function. The $O(\beta_0)$ term can be absorbed into the redefinition of the argument of the QCD running coupling $\alpha_s(Q^2) \to \alpha_s(Q^2 e^{-14/3})$, which indicates that the average virtuality of the exchanged "hard" gluon is much smaller than Q^2. Numerically, for all accessible Q^2, the scale $Q^2 e^{-14/3}$ is well below the typical hadronic scales like m_ρ^2, so one should treat $\alpha_s(Q^2 e^{-14/3})$ as an effective constant ~ 0.4 corresponding to α_s taken in the "infrared" limit, below which α_s does not run[12]. The remaining negative correction has the same nature as the $O(\alpha_s)$ term in the expression for the $\gamma\gamma^* \to \pi^0$ form factor. They both are due to the Sudakov effects[5] which sqeeze the effective transverse size of $\bar{q}q$ pairs[13].

Summarizing, both the perturbative and nonperturbative aspects of the Q^2 dependence of the $\gamma\gamma^* \to \pi^0$ form factor and of the pion electromagnetic form factor are rather well understood in quantum chromodynamics. However, new experimental data at higher Q^2 would be extremely helpful for detailed tests of the transition to the regime where the pQCD hard contribution plays the dominant role.

This work was supported by the US Department of Energy under contract DE-AC05-84ER40150.

References

[1] G. P. Lepage and S. J. Brodsky, *Phys. Rev.* **D22**, 2157 (1980); *ibid.* **D24**, 1808 (1981)

[2] A. V. Efremov and A. V. Radyushkin, *Phys.Lett.* **94B**, 245 (1980).

[3] A. V. Radyushkin, *Acta. Phys. Polonica*, **26**, 2067 (1995).

[4] E. Braaten, *Phys. Rev.* **D28**, 524 (1983); E. P. Kadantseva, S. V. Mikhailov and A. V. Radyushkin, *Sov.J.Nucl.Phys.* **44**, 326 (1986).

[5] I. V. Musatov and A. V. Radyushkin, *Phys.Rev.* **D56**, 2713 (1997).

[6] CLEO Collaboration (J. Gronberg et al.), *Phys.Rev.* **D57**, 33 (1998).

[7] V. L. Chernyak, A.R. Zhitnitsky and V. G. Serbo, *JETP Lett.* **26**, 594 (1977).

[8] V. A. Nesterenko and A. V. Radyushkin, *Phys.Lett.* **115B**, 410 (1982).

[9] J. Volmer *et al.* [The Jefferson Lab F(pi) Collaboration], Phys. Rev. Lett. 86, 1713 (2001).

[10] V. M. Braun, A. Khodjamirian and M. Maul, *Phys. Rev.* **D61**, 073004 (2000).

[11] F. M. Dittes and A. V. Radyushkin, *Sov. J. Nucl. Phys.* **34**, 293 (1981); R. D. Field *et al. Nucl. Phys.* **B186**, 429 (1981);

[12] S. J. Brodsky, C. Ji, A. Pang and D. G. Robertson, *Phys. Rev.* **D57**, 245 (1998).

[13] H. Li and G. Sterman, *Nucl. Phys.* **B381**, 129 (1992).