Parameter-Free Calculation of the Solar Proton Fusion Rate in Effective Field Theory

T.-S. Park[a], L.E. Marcucci[b,c], R. Schiavilla[d,e], M. Viviani[c,b], A. Kievsky[c,b], S. Rosati[b,c], K. Kubodera[a], D.-P. Min[f] and M. Rho[f,g,h]

[a] *Department of Physics and Astronomy, University of South Carolina, Columbia, SC 29208, USA*

[b] *Department of Physics, University of Pisa, I-56100 Pisa, Italy*

[c] *INFN, Sezione di Pisa, I-56100 Pisa, Italy*

[d] *Department of Physics, Old Dominion University, Norfolk, VA 23529, USA*

[e] *Jefferson Lab, Newport News, VA 23606, USA*

[f] *School of Physics and Center for Theoretical Physics, Seoul National University, Seoul 151-742, Korea*

[g] *Service de Physique Théorique, CEA Saclay, 91191 Gif-sur-Yvette Cedex, France*

[h] *Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea*

Abstract

Spurred by the recent complete determination of the weak currents in two-nucleon systems up to $\mathcal{O}(Q^3)$ in heavy-baryon chiral perturbation theory, we carry out a parameter-free calculation of the solar proton fusion rate in an effective field theory that *combines* the merits of the standard nuclear physics method and systematic chiral expansion. Using the tritium β-decay rate as an input to fix the only unknown parameter in the effective Lagrangian, we can evaluate with drastically improved precision the ratio of the two-body contribution to the well established one-body contribution; the ratio is determined to be (0.9 ± 0.1) %. This result is essentially independent of the cutoff parameter for a wide range of its variation (500 MeV $\leq \Lambda \leq$ 800 MeV), a feature that substantiates the consistency of the calculation.

1 Introduction

The principal role of effective field theories (EFT in short) in nuclear physics is two-fold. One is to describe nuclear dynamics starting from a "first-principle" theory anchored on QCD. As is well known, the standard nuclear physics approach (SNPA) based on phenomenological potentials has been enormously successful [1]; meanwhile, there is growing interest in establishing the foundation of SNPA, identifying nuclear physics as a *bona fide* element of the fundamental Standard Model. The second role of EFT in nuclear physics is to make precise model-independent predictions for nuclear observables with quantitative estimates of uncertainties attached to them. This second goal is particularly important in providing nuclear physics input needed in astrophysics. In focusing on the second objective, it has been emphasized in a series of recent articles [2] that a very promising approach is to combine the highly developed SNPA with an EFT based on chiral dynamics of QCD. This approach is intended to take full advantage of the extremely high accuracy of the wave functions achieved in SNPA while securing a good control of the transition operators via systematic chiral expansion[#1]. Such an approach–which is close in spirit to Weinberg's original scheme [4] based on the chiral expansion of "irreducible terms"–has been found to have an amazing predictive power for the $n + p \to d + \gamma$ process [5, 6].

In this paper we apply the same strategy to the solar proton fusion process

$$p + p \to d + e^+ + \nu_e \ . \tag{1}$$

The process (1) was previously analyzed in Ref. [7] (hereafter referred to as PKMR98) by four of the authors in heavy-baryon chiral perturbation theory (HBχPT). The calculation was carried out up to $\mathcal{O}(Q^3)$ in chiral order, viz., next-to-next-to-next-to-leading order (N^{3}LO). At N^{3}LO, two-body meson-exchange currents (MEC) begin to contribute, and there appears one unknown parameter in the chiral Lagrangian contributing to the MEC. This unknown constant, called $\hat{d}^R$ in [7], represents the strength of a contact interaction. One intuitively expects that zero-ranged terms of this sort are suppressed by hard-core correlations in the wave functions, and hence it is a common practice to drop their contributions altogether. This approximation–referred to as the hard-core cutoff scheme (HCCS)–can indeed be justified in cases where the "chiral filter mechanism" associated with pion dominance holds [5] (see [2] for details). It turns out, however, that the "chiral filter mechanism" does not apply to the Gamow-Teller (GT) matrix element that gives the dominant contribution to the process (1). Thus there is no good reason to argue away the contact term contribution in this case. For practical reasons, however, the $\hat{d}^R$-term contribution was simply ignored in [7] by invoking the HCCS and a "naturalness" argument. Let $\delta_{\rm 2B}$ stand for the ratio of the contribution of the two-body MEC to that of the one-body current. In [7], $\delta_{\rm 2B}$ for the case without the contact term is found to be $\delta_{\rm 2B} = (4.0 \pm 0.5)$ %, where the "errors" reflect changes in $\delta_{\rm 2B}$ as the hard core radius varies within the range, 0.55 fm $\le r_C \le$ 0.80 fm. To accommodate uncertainty associated with the contact term which was dropped by *fiat*, the total MEC contribution was assigned $\sim$ 100 % error. With this large uncertainty attached, an EFT prediction on the *pp* fusion rate reported in [7] was unable to corroborate or exclude the recent results of SNPA calculations [8], according to which $\delta_{\rm 2B} = 0.5 \sim 0.8$ %.

We argue here that the situation can be improved dramatically. The crucial point is that exactly the same combination of counter terms that defines the constant $\hat{d}^R$ enters the

[#1] Attempts were made in Ref. [3] to calculate the *pp* fusion rate with the use of another EFT scheme, called the power divergence subtraction scheme.

GT matrix elements that feature in *pp* fusion, tritium β-decay, the *hep* process, μ-capture on a deuteron, and ν–d scattering. This means that, if the value of $\hat{d}^R$ can be fixed using one of these processes, then it is possible to make a totally parameter-free prediction for the GT matrix elements of the other processes. The existence of accurate experimental data for the tritium β-decay rate, Γ_β, indeed allows us to carry out this program; here we are specifically interested in the model-independent determination of the *pp* fusion rate. The availability of extremely well tested, realistic wave functions for the A=3 nuclear systems enables us to eliminate ambiguities related to nuclear many-body problems. A particularly favorable aspect pertaining to tritium β-decay as well as *pp* fusion is that both of them are dominated by the GT operator to a very high degree. In order for our result to be physically acceptable, however, its cutoff dependence must be under control. In our scheme, for each value of $\Lambda \sim 1/r_C$ that defines the energy/momentum cutoff scale of EFT, $\hat{d}^R$ is determined to reproduce Γ_β; thus $\hat{d}^R$ is a function of Λ. The premise of EFT is that, even if $\hat{d}^R$ itself is Λ-dependent, physical observables (in our case the *pp*-fusion rate) should be independent of Λ as required by renormalization-group invariance. We shall show below that our method indeed gives an essentially Λ-independent result, $\delta_{2\mathrm{B}} \simeq (0.9 \pm 0.1)$ %. With this refined estimate of the two-body correction to the well established one-body contribution, we are in a position to make a parameter-free prediction for the astrophysical S factor for *pp* fusion with drastically improved precision.

It is worth emphasizing that the above EFT prediction for $\delta_{2\mathrm{B}}$ is in line with the latest SNPA results obtained in Ref. [8] (and mentioned earlier). There too, the short range behavior of the axial MEC was constrained by reproducing the GT matrix element of tritium β-decay. The inherent model dependence of such a procedure within the SNPA context was shown to be very weak simply because at small inter-particle separations, where MEC contributions are largest, the pair wave functions in different nuclei are similar in shape and differ only by a scale factor [9]. As a consequence, the ratios of GT and *pp*-capture matrix elements of different two-body current terms are nearly the same, and therefore knowledge of their sum in the GT matrix element is sufficient to predict their sum in the *pp*-capture matrix element [8].

2 Gamow-Teller matrix elements

For the solar *pp*-fusion process and tritium β-decay, it is sufficient to consider the limit in which the nuclear system receives no momentum transfer. In this limit, the iso-component a of the one- and two-body axial-vector currents read[#2]

$$\vec{A}^a_{1\mathrm{B}} = g_A \sum_{l=1,2} \frac{\tau^a_l}{2} \left[\vec{\sigma}_l + \frac{\vec{\bar{p}}_l\, \vec{\sigma}_l \cdot \vec{\bar{p}}_l - \vec{\sigma}_l\, \vec{\bar{p}}_l^{\,2}}{2m_N^2} \right], \tag{2}$$

[#2] We use here the same definitions and notations as in PKMR98. In particular, the hadronic weak current is written as $J^\mu = V^\mu - A^\mu$, which defines the sign convention of the axial-vector current. The dimensionless parameters $\hat{c}_i$ and $\hat{d}_i$ are defined as $c_i = \hat{c}_i/m_N$ and $d_i = (g_A/m_N f_\pi^2)\hat{d}_i$.

$$\vec{A}^a_{2\mathrm{B}} = -\frac{g_A}{2m_N f_\pi^2}\frac{1}{m_\pi^2+q^2}\Bigg[-\frac{i}{2}(\tau_1\times\tau_2)^a\,\vec{p}\,(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{q} + 2\,\hat{c}_3\,\vec{q}\,\vec{q}\cdot(\tau_1^a\vec{\sigma}_1+\tau_2^a\vec{\sigma}_2) + \left(\hat{c}_4+\frac{1}{4}\right)(\tau_1\times\tau_2)^a\,\vec{q}\times[(\vec{\sigma}_1\times\vec{\sigma}_2)\times\vec{q}\,]\Bigg] - \frac{g_A}{m_N f_\pi^2}\left[\hat{d}_1(\tau_1^a\vec{\sigma}_1+\tau_2^a\vec{\sigma}_2)+\hat{d}_2(\tau_1\times\tau_2)^a\vec{\sigma}_1\times\vec{\sigma}_2\right]\;, \tag{3}$$

with $\vec{p}\equiv(\vec{\bar{p}}_1-\vec{\bar{p}}_2)/2$, $\vec{\bar{p}}_l\equiv(\vec{p}_l+\vec{p}_l^{\,\prime})/2$. We emphasize that, up to N^{3}LO, these expressions completely exhaust every possibility; there are absolutely no other terms. Equation (2) includes a $1/m_N^2$ correction term which was ignored in PKMR98 as well as in all SNPA calculations we are aware of, with the exception of Ref. [10] . It should also be emphasized that, up to N^{3}LO, there are no loop contributions other than one-body radiative corrections that have already been included in Eq.(2). This frees our treatment from possible conflicts with chiral symmetry in adopting a momentum cutoff scheme for regularizing contact interactions.

We now describe the principal improvements in the present treatment over the one given in PKMR98. It is convenient to decompose the matrix element of the GT operator into one-body and two-body parts

$$\mathcal{M} = \mathcal{M}_{1\mathrm{B}} + \mathcal{M}_{2\mathrm{B}}\;, \tag{4}$$

and discuss them separately. In PKMR98, an extensive analysis was made of the leading-order (LO) one-body matrix element $\mathcal{M}_{1\mathrm{B}}^{C+N}$, making the connection between EFT and the effective range expansion. The results obtained with the Argonne v_{18} potential [11] (AV18) are

$$\mathcal{M}_{1\mathrm{B}}^{C+N} = (1\mp 0.02\ \%\mp 0.07\ \%\mp 0.02\ \%)\times 4.859\ \mathrm{fm}\;, \tag{5}$$

where the errors are due to uncertainties in the scattering length and effective ranges. The “full” one-body contribution in PKMR98 comprises the vacuum-polarization (VP) and two-photon-exchange (C2) contributions. Our new full one-body contribution contains an additional contribution due to the $1/m_N^2$ term. Although this term is required for formal consistency, its numerical role turns out to be quite minor, $\mathcal{M}_{1\mathrm{B}}^{1/m_N^2} = -0.006$ fm. The improved full one-body contribution can be expressed as

$$\mathcal{M}_{1\mathrm{B}} = (1+\delta_{1\mathrm{B}})\mathcal{M}_{1\mathrm{B}}^{C+N}, \tag{6}$$

$$\delta_{1\mathrm{B}} = \delta_{1\mathrm{B}}^{\mathrm{VP+C2}}+\delta_{1\mathrm{B}}^{1/m_N^2} = (-0.63-0.13)\ \% = -0.76\ \%\;. \tag{7}$$

In terms of Λ_{pp} defined in Ref. [12] #3 we have

$$\Lambda_{pp}^2 \equiv \frac{|a^C|^2\gamma^3}{2}\,A_S^2\,\mathcal{M}_{1\mathrm{B}}^2 = (1+\delta_{1\mathrm{B}})^2\times 7.02 = 6.91 \tag{8}$$

#3 The subscript pp has been added here to avoid confusion with the cutoff parameter Λ. The parameter a^C is the pp $^1\mathrm{S}_0$ scattering length, and γ and A_S are the wave number and S-wave normalization constant pertinent to the deuteron, respectively.

for the central value. This should be compared with 6.93 obtained in [7][#4].

In the two-body current (3), there are two constants, $\hat{d}_1$ and $\hat{d}_2$, that cannot be determined from theory and hence need to be fixed empirically. Thanks to Fermi-Dirac statistics, however, only one combination turns out to be relevant to the *pp* fusion process[#5]:

$$\hat{d}^R \equiv \hat{d}_1 + 2\hat{d}_2 + \frac{1}{3}\hat{c}_3 + \frac{2}{3}\hat{c}_4 + \frac{1}{6} \ . \tag{9}$$

Most fortunately, the same combination enters tritium β-decay, the *hep* process, μ-capture on deuteron, and ν–d scattering. Thus, as mentioned, if one of these processes is measured with sufficient precision, then all other processes can be calculated accurately. We use here the tritium β-decay rate, Γ_β, to determine $\hat{d}^R$. To this end, we calculate Γ_β from the matrix elements of the current operators in Eqs.(2) and (3) evaluated for appropriate A=3 nuclear wave functions; these wave functions must have sufficient accuracy to avoid any ambiguities associated with nuclear many-body problems. We employ here the wave functions obtained in Ref. [10] using the correlated-hyperspherical-harmonics (CHH) method. It is obviously important to maintain consistency between the treatments of the A=3 and A=2 systems. In our case the same AV18 potential is used (with the addition of the Urbana-IX three-nucleon potential [13] for the A=3 nuclei). Furthermore, we apply the same regularization method to both A=2 and A=3 systems to control short-range physics in a consistent manner. Specifically, in performing Fourier transformation to derive the r-space representation of a transition operator, we use the Gaussian regularization. This is equivalent to replacing the delta and Yukawa functions with the regulated ones,

$$\begin{aligned}
\delta_\Lambda^{(3)}(r) &\equiv \int \frac{d^3q}{(2\pi)^3}\, S_\Lambda^2(q^2)\, \mathrm{e}^{i\vec{q}\cdot\vec{r}} \ , \\
y_{0\Lambda}^\pi(r) &\equiv \int \frac{d^3q}{(2\pi)^3}\, S_\Lambda^2(q^2)\, \mathrm{e}^{i\vec{q}\cdot\vec{r}}\, \frac{1}{q^2+m_\pi^2} \ , \\
y_{1\Lambda}^\pi(r) &\equiv -r\frac{\partial}{\partial r} y_{0\Lambda}^\pi(r) \ , \\
y_{2\Lambda}^\pi(r) &\equiv \frac{1}{m_\pi^2} r \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} y_{0\Lambda}^\pi(r) \ ,
\end{aligned} \tag{10}$$

where $S_\Lambda(q^2)$ is defined as

$$S_\Lambda(q^2) = \exp\left(-\frac{q^2}{2\Lambda^2}\right). \tag{11}$$

The cutoff parameter Λ characterizes the energy-momentum scale of our EFT. The properly regularized two-body matrix elements read

$$\mathcal{M}_{2\mathrm{B}} \ = \ \frac{2}{m_N f_\pi^2}\int_0^\infty dr\, f_\Lambda(r) \ , \tag{12}$$

[#4] In Ref.[8], the contribution from the "C2:N" (in the terminology of PKMR98) has been omitted and resulted in a slightly bigger value of Λ_{pp}^2.

[#5] There was a sign error in the corresponding expression in PKMR98; as corrected here, the $\hat{d}_2$ term should have the positive sign. This error, however, does not affect the numerical results of PKMR98, since the resulting error gets absorbed into the definition of $\hat{d}^R$.

with

$$
\begin{aligned}
f_\Lambda(r) &= \frac{m_\pi^2}{3}\left(\hat{c}_3+2\hat{c}_4+\frac{1}{2}\right) y_{0\Lambda}^\pi(r)\, u_d(r)\, u_{pp}(r) \\
&- \sqrt{2}\frac{m_\pi^2}{3}\left(\hat{c}_3-\hat{c}_4-\frac{1}{4}\right) y_{2\Lambda}^\pi(r)\, w_d(r)\, u_{pp}(r) \\
&+ \frac{y_{1\Lambda}^\pi(r)}{12r}\left[\left[u_d(r)-\sqrt{2}w_d(r)\right] u'_{pp}(r)\right. \\
&- \left.\left[u'_d(r)-\sqrt{2}w'_d(r)\right] u_{pp}(r) + \frac{3\sqrt{2}}{r} w_d(r) u_{pp}(r)\right] \\
&- \hat{d}^R \delta_\Lambda^{(3)}(r)\, u_d(r) u_{pp}(r)\ , \qquad (13)
\end{aligned}
$$

where $u_d(r)$ and $w_d(r)$ are the S- and D-wave components of the deuteron wave function, and $u_{pp}(r)$ is the $^1\mathrm{S}_0$ pp scattering wave (at zero relative energy).

3 Results

As mentioned, we can associate the momentum cutoff parameter Λ in Eq.(11) to the cutoff scale of our EFT. We let Λ vary within a certain reasonable range and, for a given value of Λ, we deduce the value of $\hat{d}^R$ that reproduces Γ_β. With $\hat{d}_R$ so determined, we calculate the two-body matrix element $\mathcal{M}_{2\mathrm{B}}$. The results are given for three representative values of Λ in Table 1. (The results of the bottom row corresponding to the $\Lambda=\infty$ case will be discussed separately later.) The table indicates that, although the value of $\hat{d}^R$ is sensitive to Λ, $\mathcal{M}_{2\mathrm{B}}$ is amazingly stable against the variation of Λ over a wide range. In view of this high stability, we believe we are on the conservative side in adopting the estimate $\mathcal{M}_{2\mathrm{B}} = (0.041 \sim 0.048)$ fm. Since the leading single-particle term is independent of Λ, the total amplitude $\mathcal{M} = \mathcal{M}_{1\mathrm{B}} + \mathcal{M}_{2\mathrm{B}}$ is Λ-independent to the same degree as $\mathcal{M}_{2\mathrm{B}}$. The Λ-independence of the physical quantity $\mathcal{M}$, which is in conformity with the general *tenet* of EFT, is a crucial feature of the result in our present study. The relative strength of the two-body contribution as compared with the one-body contribution is

$$
\delta_{2\mathrm{B}} \equiv \frac{\mathcal{M}_{2\mathrm{B}}}{\mathcal{M}_{1\mathrm{B}}} = (0.9 \pm 0.1)\ \%, \qquad (14)
$$

where we have used $\mathcal{M}_{1\mathrm{B}} = 4.822$ fm, as obtained from Eqs.(5)–(7). We remark that the central value of $\delta_{2\mathrm{B}}$ here is considerably smaller than the corresponding value, $\delta_{2\mathrm{B}} = 4\%$, in PKMR98. Furthermore, the uncertainty of ± 0.1 % in Eq.(14) is drastically smaller than the corresponding figure, ± 4 %, in PKMR98.

We now turn to the threshold S factor, which is a key input for the solar model. In

Λ (MeV)	$\hat{d}^R$	$\mathcal{M}_{\rm 2B}$ (fm)
500	1.93 ± 0.07	$0.112 - 0.035\,\hat{d}^R \simeq 0.045 \pm 0.002$
600	3.09 ± 0.08	$0.142 - 0.031\,\hat{d}^R \simeq 0.046 \pm 0.002$
800	6.50 ± 0.10	$0.188 - 0.022\,\hat{d}^R \simeq 0.043 \pm 0.002$
∞	–	$0.256 - 0.001\,\hat{d}^R$

Table 1: The strength $\hat{d}^R$ of the contact term and the two-body GT matrix element, $\mathcal{M}_{\rm 2B}$, calculated for representative values of Λ.

the notation of PKMR98[#6],

$$S_{pp}(0) = (1+\delta_{\rm 2B})^2 \frac{6}{\pi}\, m_p\alpha\, G_V^2 g_A^2 \frac{\Lambda_{pp}^2}{\gamma^3}\, m_e^5 f(E_0). \tag{15}$$

PKMR98 used $g_A = 1.2601$ and $G_V = 1.136 \times 10^{-5}$ GeV^{-2}, but we adopt here the most recent values, $g_A = 1.2670 \pm 0.0035$ [14] and $G_V = (1.14939 \pm 0.00065) \times 10^{-5}$ GeV^{-2} [15]. This value of G_V has been deduced from the 0^+–0^+ nuclear β-decays with radiative corrections included. We then obtain

$$\begin{aligned} S_{pp}(0) &= 3.94 \left(\frac{1+\delta_{2B}}{1.01}\right)^2 \left(\frac{g_A}{1.2670}\right)^2 \left(\frac{\Lambda_{pp}^2}{6.91}\right)^2 \times 10^{-25}\text{MeV-barn} \\ &= 3.94 \times 10^{-25}(1 \pm 0.15\ \% \pm 0.2\ \% \pm \varepsilon)\text{MeV-barn}\ . \end{aligned} \tag{16}$$

Here the first error is due to uncertainties in the input parameters in the one-body part, while the second error represents the Λ dependence in the two-body part; ε denotes possible uncertainties due to higher chiral order contributions. To make a formally rigorous assessment of ε, we must evaluate loop corrections and higher-order counter terms. Although an N^{4}LO calculation would not involve any new unknown parameters, it is a non-trivial task. Furthermore, loop corrections necessitate a more elaborate regularization scheme since the naive cutoff regularization used here violates chiral symmetry at loop orders. (This difficulty, however, is not insurmountable.) These formal problems set aside, it seems reasonable to assess ε as follows. We first recall that both tritium β-decay and solar *pp* fusion are dominated by the one-body GT matrix elements, the evaluation of which is extremely well controlled from the SNPA as well as EFT points of view. Therefore, the precision of our calculation is governed by the reliability of estimation of small corrections to the dominant one-body GT contribution. Now, we have seen that the results of the present N^{3}LO calculation nicely fit into the picture expected from the general *tenet* of EFT: (i) the N^{3}LO contributions are indeed much smaller than the leading order term; (ii) the physical transition amplitude $\mathcal{M}$ does not depend on the cutoff parameter. Although these features do not constitute a formal proof of the convergence of the chiral expansion used here, it is *extremely unlikely*

[#6] The $f(E_0) = 0.1421$ is the phase volume, m_p (m_e) is the proton (electron) mass and $\alpha \simeq 1/137.036$.

that higher order contributions be so large as to completely upset the physically reasonable behavior observed in the N^{3}LO calculation. It should therefore be safe to assign to ε uncertainty comparable to the error estimate for the two-body part in Eq.(16); viz., $\varepsilon \approx 0.2\%$. In this connection we remark that an axial three-body MEC contribution to the ^{3}H GT matrix element was calculated explicitly in SNPA [10] and found to be negligible relative to the leading two-body mechanisms. This feature is consistent with the above argument since, in the context of EFT, the three-body MEC represents a higher-order effect subsumed in "ε" in Eq.(16).

Therefore our estimate of the threshold S-factor is

$$S_{pp}(0) = 3.94 \times 10^{-25}(1 \pm 0.15\ \% \pm 0.2\ \% \pm 0.2\ \%)\text{MeV-barn} \ . \tag{17}$$

4 Discussion

Apart from the notable numerical differences between the present work and PKMR98, it is important to point out that short-range physics is much better controlled here. In the conventional treatment of MEC, one derives the coordinate space representation of a MEC operator by applying ordinary Fourier transformation (with no restriction on the range of the momentum variable) to the amplitude obtained in momentum space; this corresponds to setting $\Lambda = \infty$ in Eq.(11). In PKMR98, where this familiar method is adopted, the $\hat{d}^R$ term appears in the zero-range form, $\hat{d}^R\delta(r)$. Since there were no known methods to fix the parameter $\hat{d}^R$ (except for rough estimates based on the naturalness argument), PKMR98 chose to introduce short-range repulsive correlation with hard-core radius r_C and eliminate the $\hat{d}^R\delta(r)$ term *by hand*. The remaining finite-range terms were evaluated as functions of r_C. $\mathcal{M}_{2\text{B}}$ calculated this way exhibited substantial r_C-dependence, indicating that short-range physics was not well controlled; if $1/r_C$ is interpreted as a typical value of the momentum cutoff Λ in EFT, the results in PKMR98 imply that the theory has significant dependence on Λ. Inclusion of the $\hat{d}^R$ term, with its strength renormalized as described here, eliminates this undesirable Λ-dependence to a satisfactory degree.

It is also of interest to examine the behavior of the $\hat{d}^R$ term in the case where $\Lambda = \infty$ but where the contribution of the $\hat{d}^R\delta(r)$ is estimated explicitly using the realistic wave functions (instead of invoking HCCS). The bottom row in Table 1 shows $\mathcal{M}_{2\text{B}}$ corresponding to this case. $\mathcal{M}_{2\text{B}} = 0.256 - 0.001\hat{d}^R$ for the $\Lambda = \infty$ case implies that contribution of the $\hat{d}^R\delta(r)$ term would be small if $\hat{d}^R(\Lambda = \infty)$ had a "natural" magnitude, i.e., of order of unity. This "natural" situation, however, does not seem likely to occur; to reproduce $\mathcal{M}_{2\text{B}} \sim 0.045$, we need an uncomfortably large value of $\hat{d}^R$. Since the $\Lambda = \infty$ case is unrealistic from the viewpoint of EFT[#7], we have not tried to determine $\hat{d}^R(\Lambda = \infty)$ from Γ_β–had we done so, we would have found the resulting $\hat{d}^R(\Lambda = \infty)$ to be ≈ 200, an unnaturally large number.

[#7] In EFT, Λ has a physical meaning as the cutoff scale and hence cannot be taken to be infinitely large.

As mentioned, the methodology employed in this work can be profitably used for the other related processes. An application to the *hep* process will be reported in a separate article [16].

Acknowledgments

TSP and KK thank S.-I. Ando and F. Myhrer for discussions. The work of TSP and KK is supported in part by the U.S. National Science Foundation, Grant No. PHY-9900756 and No. INT-9730847. The work of RS is supported by DOE contract No. DE-AC05-84ER40150 under which the Southeastern Universities Research Association (SURA) operates the Thomas Jefferson National Accelerator Facility. The work of DPM is supported in part by KOSEF Grant 1999-2-111-005-5 and KSF Grant 2000-015-DP0072. MR acknowledges the hospitality of the Physics Departments of SNU and Yonsei University, where his work was partially supported by Brain Korea21 in 2001. Some of the calculations were made possible by grants of computing time from the National Energy Research Supercomputer Center in Livermore.

References

[1] For a recent review, see J. Carlson and R. Schiavilla, Rev. Mod. Phys. **70**, 743 (1998).

[2] T.-S. Park, K. Kubodera, D.-P. Min, and M. Rho, "Effective field theory for nuclei: confronting fundamental questions in astrophysics," nucl-th/0005069, Nucl. Phys. **A684**, 101 (2001); "On making predictions with effective field theories in nuclear physics," nucl-th/9904053; G.E. Brown and M. Rho, "On the manifestation of chiral symmetry in nuclei and dense nuclear matter," hep-ph/0103102.

[3] X. Kong and F. Ravendal, Nucl. Phys. **A656**, 421 (1999); Nucl. Phys. **A665**, 137 (2000); Phys. Lett. **B470**, 1 (1999); nucl-th/0004038; M. Butler and J.-W. Chen, nucl-th/0101017.

[4] S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991).

[5] T.-S. Park, D.-P. Min, and M. Rho, Phys. Rev. Lett. **74**, 4153 (1995); Nucl. Phys. **A596**, 515 (1996).

[6] T.-S. Park, K. Kubodera, D.-P. Min, and M. Rho, Phys. Lett. **B472**, 232 (2000).

[7] T.-S. Park, K. Kubodera, D.-P. Min, and M. Rho, Astrophys. J. **507**, 443 (1998).

[8] R. Schiavilla, V.G.J. Stoks, W. Glöckle, H. Kamada, A. Nogga, J. Carlson, R. Machleidt, V.R. Pandharipande, R.B. Wiringa, A. Kievsky, S. Rosati, and M. Viviani, Phys. Rev. **C58**, 1263 (1998).

[9] J.L. Forest, V.R. Pandharipande, S.C. Pieper, R.B. Wiringa, R. Schiavilla, and A. Arriaga, Phys. Rev. C **54**, 646 (1996).

[10] L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky, and S. Rosati, Phys. Rev. Lett. **84**, 5959 (2000); L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky, S. Rosati, and J.F. Beacom, Phys. Rev. **C63**, 015801 (2000).

[11] R.B. Wiringa, V.G.J. Stocks, and R. Schiavilla, Phys. Rev. **C51**, 38 (1995).

[12] M. Kamionkowski and J.N. Bahcall, Astrophys. J. **420**, 884 (1994).

[13] B.S. Pudliner, V.R. Pandharipande, J. Carlson, and R.B. Wiringa, Phys. Rev. Lett. **74**, 4396 (1995).

[14] D.E. Groom *et al.* (Particle Data Group), Eur. Phys. Jour. **C15**, 1 (2000).

[15] J.C. Hardy, I.S. Towner, V.T. Koslowsky, E. Hagberg, and H. Schmeing, Nucl. Phys. **A509**, 429 (1990).

[16] T.-S. Park, L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky, S. Rosati, K. Kubodera, D.-P. Min, and M. Rho, in preparation.