The Solar *hep* Process in Effective Field Theory

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Using effective field theory, we calculate the S-factor for the *hep* process in a totally parameter-free formulation. The transition operators are organized according to chiral counting, and their matrix elements are evaluated using the realistic nuclear wave functions obtained in the correlated-hyperspherical-harmonics method. Terms of up to next-to-next-to-next-to-leading order in heavy-baryon chiral perturbation theory are considered. Fixing the only parameter in the theory by fitting the tritium β-decay rate, we predict the *hep* S-factor with accuracy better than ~ 20 %.

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The *hep* process

$$p + {}^3\mathrm{He} \to {}^4\mathrm{He} + e^+ + \nu_e \tag{1}$$

figures importantly in astrophysics and particle physics; it has much bearing upon issues of great current interest such as the solar neutrino problem, non-standard physics in the neutrino sector, etc. The *hep* reaction produces the highest-energy solar neutrinos even though their flux is much smaller than that of the ^{8}B neutrinos. At some level, therefore, there can be a significant distortion of the higher end of the ^{8}B neutrino spectrum due to the *hep* neutrinos. This change can influence the interpretation of the results of a recent Super-Kamiokande experiment that have generated many controversies related to the solar neutrino problem and neutrino oscillations [1,2]. To address these issues, a reliable estimate of the *hep* cross section is indispensable. Its accurate evaluation, however, presents a challenge for nuclear and hadron physics [3]. The degree of this difficulty may be appreciated by noting that theoretical estimates of the *hep* astrophysical S-factor have varied by orders of magnitude in the literature. The reason for this great variation is multifold. Although the primary *hep* amplitude is formally of the Gamow-Teller (GT) type ($\Delta J = 1$, same parity), the single-particle GT matrix element for the *hep* process is strongly suppressed due to the symmetries of the initial and final state wave functions. Furthermore, the main two-body corrections to the "leading" one-body GT term generically come with opposite sign causing a large cancellation. Therefore, it is necessary to calculate these corrections with great accuracy, which is a highly non-trivial task. The nature of the specific challenge involved here can be further elucidated in terms of the *chiral filter* picture. If meson-exchange current (MEC) contributions to a specific transition amplitude are dominated by the one-soft-pion exchange diagram, then one can take advantage of the fact that the soft-pion-exchange piece is uniquely dictated by chiral symmetry and that there is a mechanism (called the chiral filter mechanism) that suppresses higher chiral-order terms [4,5]. We refer to a transition amplitude to which the chiral filter mechanism is applicable (not applicable) as a chiral-protected (chiral-unprotected) case. It is known that the space component of the vector current and axial-charge are chiral-protected, whereas the GT transition is chiral-unprotected. This means that, in contrast to isovector M1 and axial charge transitions, both of which are chiral-protected and hence easily calculated [6,7], the primary *hep* amplitude, i.e. the GT transition, needs to be calculated without invoking the chiral-filter mechanism and hence is more subtle.

In a highly successful method, which we call here the standard nuclear physics approach (SNPA) [8], one uses the potential picture supplemented with MEC contributions, the structures of which are obtained from one-boson-exchange. Many applications of SNPA to electroweak transitions in few-body systems are well documented. A detailed SNPA calculation for the *hep* cross section was recently carried out by Marcucci *et al.* [9]; this study has re-confirmed the substantial cancellation between the one-body and two-body terms for the GT transition [10,11]. It is therefore of primary importance to have a good theoretical control of short-distance physics. A "first-principle" approach based on effective field theory (EFT) will hopefully provide an insight into this issue. A possible approach that is formally consistent with systematic power counting is the pionless EFT (for a recent review, see [12]). This approach, however, cannot be readily extended to systems with $A \geq 3$. Apart from the basic problem of organizing chiral expansion for complex nuclei from "first-principles", a plethora of parameters involved would present a major obstacle. This difficulty is expected to be particularly pronounced for the *hep* reaction.

In this paper we calculate the *hep* astrophysical S-factor, adopting a variant of EFT, that has been proven to be extremely successful in estimating the S-factor for solar *pp* fusion [13]. The same method has also scored great success in predicting polarization observables in thermal *np* capture [14]. Our starting point is the observation that, to high accuracy, the leading-order single-particle matrix elements in SNPA and EFT are identical, and that they can be reliably estimated with the use of realistic SNPA wave functions for the initial and final nuclear states. Next, we note that in EFT the operators representing two-body corrections to the leading-order one-body term can be controlled by systematic chiral expansion in heavy-baryon chiral perturbation theory [15]. Then, since the ratio of a two-body matrix element to the leading-order one-body matrix element can be evaluated with sufficient accuracy with the use of the realistic SNPA wave functions, we are in a position to obtain a reliable estimate of the total (one-body + two-body) contribution. This method, which exploits the powers of *both* SNPA and EFT, may be referred to as a "*more effective* EFT (MEEFT)". Here we shall work out MEEFT up to next-to-next-to-next-to-leading order (N^{3}LO), adopting a cut-off regularization. Our method, which to this order preserves chiral symmetry and renormalization group invariance, enables us to predict the *hep* S-factor in a totally parameter-free manner.

In the present scheme it is sufficient to focus on "irreducible graphs" in Weinberg's classification [16]. Graphs are classified by the chiral power index ν given by $\nu = 2(A-C)+2L+\sum_i \nu_i$, where A is the number of nucleons involved in the process (A=4 in our case), C the number of disconnected parts, and L the number of loops. The chiral index, ν_i, of the i-th vertex is given by $\nu_i = d_i + e_i + n_i/2 - 2$, where d_i, e_i and n_i are respectively the numbers of derivatives, external fields and nucleon lines belonging to the vertex. The Feynman diagrams with a chiral index ν are suppressed by $(Q/\Lambda_\chi)^\nu$ compared with the leading-order one-body GT operator, with Q standing for the typical three-momentum scale and/or the pion mass, and $\Lambda_\chi \sim m_N \sim 4\pi f_\pi$ for the chiral scale. The physical amplitude is then expanded with respect to ν.

In this paper we shall limit ourselves to N^{3}LO, although it is possible to go to N^{4}LO without introducing new parameters. We write the current as

$$J^\mu(\boldsymbol{q}) = V^\mu(\boldsymbol{q}) + A^\mu(\boldsymbol{q}) = \int d\boldsymbol{x}\, \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{x}} J^\mu(\boldsymbol{x}), \qquad (2)$$

where $\boldsymbol{q}$ is the momentum carried by the lepton pair. The calculation is considerably simplified by the facts that we are dealing with small momentum transfers and that, as a consequence, both $q \equiv |\boldsymbol{q}|$ and the time component of the nucleon momentum are $\mathcal{O}(Q^2/\Lambda_\chi)$ rather than $\mathcal{O}(Q)$ as naive counting would suggest. For the vector and axial one-body current and charge operators, and also for the vector-current and axial-charge (chiral-protected) two-body operators, we can simply carry over the expressions given in Ref. [9], hereafter referred to as MSVKRB. The vector charge two-body operator does not appear up to the order considered, while the EFT axial-current two-body operator is given, in momentum space, in Ref. [13]. These two-body currents are valid only up to a cutoff Λ. This implies that, when we go to coordinate space, the currents must be regulated. This is a key point in our approach. We introduce a Gaussian cutoff regulator in performing Fourier transformation. The resulting r-space expressions of the currents in the center-of-mass (c.m.) frame are

$$\begin{aligned}
\boldsymbol{V}_{12}(\boldsymbol{r}) &= -\frac{g_A^2 m_\pi^2}{12 f_\pi^2}\tau_\times^- \,\boldsymbol{r}\, [\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\, y_{0\Lambda}^\pi(r) + S_{12}\, y_{2\Lambda}^\pi(r)] \\
&\quad - i\frac{g_A^2}{8f_\pi^2}\boldsymbol{q}\times\left[\boldsymbol{\mathcal{O}}_\times y_{0\Lambda}^\pi(r) + \left(\boldsymbol{\mathcal{T}}_\times - \frac{2}{3}\boldsymbol{\mathcal{O}}_\times\right) y_{1\Lambda}^\pi(r)\right], \\
A_{12}^0(\boldsymbol{r}) &= -\frac{g_A}{4f_\pi^2}\tau_\times^-\left[\frac{\boldsymbol{\sigma}_+\cdot\hat{\boldsymbol{r}}}{r} + \frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{r}}\,\boldsymbol{\sigma}_-\cdot\hat{\boldsymbol{r}}\right] y_{1\Lambda}^\pi(r), \\
\boldsymbol{A}_{12}(\boldsymbol{r}) &= -\frac{g_A m_\pi^2}{2m_N f_\pi^2}\Bigg[\\
&\qquad \left[\frac{\hat{c}_3}{3}(\boldsymbol{\mathcal{O}}_+ + \boldsymbol{\mathcal{O}}_-) + \frac{2}{3}\left(\hat{c}_4 + \frac{1}{4}\right)\boldsymbol{\mathcal{O}}_\times\right] y_{0\Lambda}^\pi(r) \\
&\qquad + \left[\hat{c}_3(\boldsymbol{\mathcal{T}}_+ + \boldsymbol{\mathcal{T}}_-) - \left(\hat{c}_4 + \frac{1}{4}\right)\boldsymbol{\mathcal{T}}_\times\right] y_{2\Lambda}^\pi(r)\Bigg] \\
&\quad + \frac{g_A}{2m_N f_\pi^2}\Bigg[\frac{1}{2}\tau_\times^-(\overline{\boldsymbol{p}}_1\,\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{r}} + \overline{\boldsymbol{p}}_2\,\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{r}})\frac{y_{1\Lambda}^\pi(r)}{r} \\
&\qquad + \delta_\Lambda(r)\,\hat{d}^R\boldsymbol{\mathcal{O}}_\times\Bigg],
\end{aligned} \qquad (3)$$

where $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$, S_{12} is the tensor operator, and

$$[\delta_\Lambda(r),\ y_{0\Lambda}^\pi(r)] \equiv \int \frac{d\boldsymbol{k}}{(2\pi)^3}\,\mathrm{e}^{-k^2/\Lambda^2}\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}\left[1,\ \frac{1}{\boldsymbol{k}^2+m_\pi^2}\right],$$

$y_{1\Lambda}^\pi(r) \equiv -r[y_{0\Lambda}^\pi(r)]'$ and $y_{2\Lambda}^\pi(r) \equiv (r/m_\pi^2)[[y_{0\Lambda}^\pi(r)]'/r]'$ (the $'$ symbols denote derivatives), $\boldsymbol{\mathcal{O}}_\odot^k \equiv \tau_\odot^-\sigma_\odot^k$, $\boldsymbol{\mathcal{O}}_\odot \equiv \tau_\odot^-\boldsymbol{\sigma}_\odot$, $\boldsymbol{\mathcal{T}}_\odot \equiv \hat{\boldsymbol{r}}\,\hat{\boldsymbol{r}}\cdot\boldsymbol{\mathcal{O}}_\odot - \frac{1}{3}\boldsymbol{\mathcal{O}}_\odot$, $\odot = \pm, \times$, $\tau_\odot^- \equiv (\tau_1\odot\tau_2)^- \equiv (\tau_1\odot\tau_2)^x - i(\tau_1\odot\tau_2)^y$ and $\boldsymbol{\sigma}_\odot \equiv (\boldsymbol{\sigma}_1\odot\boldsymbol{\sigma}_2)$. Finally, the parameter $\hat{d}^R$ is given by

$$\hat{d}^R \equiv \hat{d}_1 + 2\hat{d}_2 + \frac{1}{3}\hat{c}_3 + \frac{2}{3}\hat{c}_4 + \frac{1}{6}\,. \qquad (4)$$

The dimensionless parameters, $\hat{c}$'s and $\hat{d}$'s, are defined in Ref. [17]. The numerical values of $\hat{c}_3$ and $\hat{c}_4$ have been determined previously [18]: $\hat{c}_3 = -3.66 \pm 0.08$ and $\hat{c}_4 = 2.11 \pm 0.08$. The derivative operators, $\overline{\boldsymbol{p}}_i \equiv \frac{1}{2}(\boldsymbol{p}_i' + \boldsymbol{p}_i)$ ($i = 1, 2$) in Eq.(3), should be understood to act only on the wave functions.

The explicit degrees of freedom in our scheme are the nucleon and the pion, with all other degrees of freedom (ρ- and ω-mesons, $\Delta(1232)$, etc.) integrated out. Therefore, a reasonable range of the cutoff Λ would be somewhere around 500–800 MeV. We shall consider here three exemplary values, $\Lambda = 500,\ 600,\ 800$ MeV.

A crucial point is that $\boldsymbol{A}_{12}$ in Eq.(3) contains only one unknown parameter, $\hat{d}^R$, that needs to be fixed using an empirical input. The tritium β-decay rate, Γ_β, can be used for this purpose, since it is dominated by the *unsuppressed* GT term [19]. Thus, for each value of Λ, we adjust $\hat{d}^R$ to reproduce the experimental value of Γ_β and, using the value of $\hat{d}^R$ so determined, we evaluate the *hep* amplitude. To carry out this program, we must calculate both the tritium β-decay amplitude and the *hep* amplitude for the above-described currents, using realistic wave functions. We adopt here the method employed by MSVKRB. In particular, we use the correlated-hyperspherical-harmonics wave functions [20,21] obtained with the Argonne v_{18} (AV18) two-nucleon [22] and Urbana-IX three-nucleon [23] interactions.

In the notation of MSVKRB, the GT-amplitudes are given in terms of the reduced-matrix elements (RMEs) $\overline{L}_1(q;A)$ and $\overline{E}_1(q;A)$. Since these RMEs are related to each other as $\overline{E}_1(q;A) \simeq \sqrt{2}\,\overline{L}_1(q;A)$, with the exact equality holding at q=0, we consider here only one of them, $\overline{L}_1(q;A)$. For the three exemplary values of Λ, Table I gives the corresponding values of $\hat{d}^R$, as determined from Γ_β [13], and $\overline{L}_1(q;A)$ at q=19.2 MeV and zero c.m. energy. We see from the table that the variation of the two-body GT amplitude (row labelled "2B-total") is only $\sim$10 % for the range of Λ under study. The Λ-dependence in the total GT amplitude is made more pronounced by a strong cancellation between the one-body and two-body terms, but this amplified Λ-dependence still lies within acceptable levels.

Λ (MeV)	500	600	800
$\hat{d}^R$	1.00 ± 0.07	1.78 ± 0.08	3.90 ± 0.10
$\overline{L}_1(q;A)$	-0.032	-0.029	-0.022
1B	-0.081	-0.081	-0.081
2B (without $\hat{d}^R$)	0.093	0.122	0.166
2B ($\propto \hat{d}^R$)	-0.044	-0.070	-0.107
2B-total	0.049	0.052	0.059

TABLE I. Values of $\hat{d}^R$ and $\overline{L}_1(q;A)$ (in fm$^{3/2}$) calculated as functions of the cutoff Λ. The individual contributions from the one-body (1B) and two-body (2B) operators are also listed.

Table II shows the contribution to the S-factor, at zero c.m. energy, from each initial channel. For comparison we have also listed the MSVKRB results for the AV18/UIX interaction. It is noteworthy that for all the channels other than $^3S_1$, the Λ-dependence is very small ($\lesssim$ 2 %). While the GT terms are dominant, the contribution of the axial-charge term in the $^3S_1$ channel is sizable even though it is kinematically suppressed by the factor q. It is therefore reassuring that the chiral-filter mechanism allows a reliable evaluation of this amplitude.

Λ (MeV)	500	600	800	MSVKRB
$^1S_0$	0.02	0.02	0.02	0.02
$^3S_1$	7.00	6.37	4.30	6.38
$^3P_0$	0.67	0.66	0.66	0.82
$^1P_1$	0.85	0.88	0.91	1.00
$^3P_1$	0.34	0.34	0.34	0.30
$^3P_2$	1.06	1.06	1.06	0.97
Total	9.95	9.37	7.32	9.64

TABLE II. Contributions to the S-factor (in 10^{-20} keV-b) from individual initial channels calculated as functions of Λ. The last column gives the results obtained in MSVKRB.

Summarizing the results given in Table II, we arrive at a quantitative prediction for the *hep* S-factor:

$$S = (8.6 \pm 1.3) \times 10^{-20}\ \text{keV-b}\,, \qquad (5)$$

where the "error" spans the range of the Λ-dependence for Λ=500–800 MeV. This result should be compared to that obtained by MSVKRB [9], $S = 9.64 \times 10^{-20}$ keV-b. Note that the earlier studies [10,11] were based on less accurate variational wave functions than used here and in MSVKRB, and did not include P-wave capture contributions, which account for $\simeq$ 40 % of the total S-factor. To decrease the uncertainty in Eq.(5), we need to reduce the Λ-dependence in the two-body GT term. According to a general *tenet* of EFT, the Λ-dependence should diminish as we include higher order terms. A preliminary study [24] indicates that it is indeed possible to reduce the Λ-dependence significantly by including N^{4}LO corrections.

In order to better understand how the present scheme works, it is helpful to compare the *hep* reaction with the radiative np-capture. The polarization observables in $\vec{n} + \vec{p} \to d + \gamma$ are known to be sensitive to the isoscalar M1 matrix element, $M1S$, and this amplitude has been extensively studied in EFT [14,25]. The similar features of the *hep* GT amplitude and the $M1S$ matrix element are: (i) the leading one-body contribution is suppressed by the symmetries of the wave functions; (ii) there is no soft-pion exchange contribution; (iii) nonetheless, short-range physics can be reliably subsumed into a single contact term. In the $\vec{n}\vec{p}$ case the strength of this term can be determined from the deuteron magnetic moment (for a given value of the cutoff Λ). The calculation in Ref. [14] demonstrates that the Λ-dependence in the contact term and that of the remaining terms compensate each other so that the total $M1S$ is stable against changes in Λ. This suggests that, if we go to higher orders, the coefficient of the contact term in question will be modified, with part of its strength shifted to higher order terms; however, the total physical amplitude will remain essentially unchanged. These features are quite similar to what we have found here for the *hep* GT amplitude.

Evaluating the matrix element of the leading-order one-body operator in EFT with the use of realistic nuclear wave functions is analogous to fixing parameters in an EFT Lagrangian (at a given order) using empirical inputs. The realistic wave functions in SNPA can be regarded as a theoretical input that fits certain sets of observables [26]. In the present MEEFT scheme, we take the view that the same realistic wave functions also provide a framework for reliably calculating corrections to the leading-order one-body matrix element. While from a formal point of view the approach adopted here is, in certain cases, not in strict accordance with the systematic power-counting scheme of EFT proper, nevertheless the severity of this potential short-coming may depend on individual cases (see discussion in [27]). For the *hep* amplitude under consideration here, the moderate Λ-dependence exhibited by the numerical results suggests that the lack of rigorous power-counting cannot be too significative. Indeed, this type of “resilience” may also explain why the SNPA calculation in Ref. [9] gives a result very similar to the present one. It is true that the MSVKRB two-body terms are not entirely in conformity with the chiral counting scheme we are using here; some terms corresponding to chiral orders higher than N^{3}LO are included, while some others which are N^{3}LO in EFT are missing. This formal problem, however, seems to be largely overcome by the fact that also in MSVKRB a parameter (the axial $\pi N\Delta$ coupling strength) is adjusted to reproduce Γ_β.

The latest analysis of the Super-Kamiokande data [28] gives an upper limit of the solar *hep* neutrino flux, $\Phi(hep)^{\rm SK} < 40\times 10^3\ {\rm cm}^{-2}{\rm s}^{-1}$. The standard solar model [29] using the *hep* S-factor of Marcucci *et al.* [9] predicts $\Phi(hep)^{\rm SSM} = 9.4\times 10^3\ {\rm cm}^{-2}{\rm s}^{-1}$. The use of the central value of our estimate, Eq.(5), of the *hep* S-factor would slightly lower $\Phi(hep)^{\rm SSM}$ but with the upper limit compatible with $\Phi(hep)^{\rm SSM}$ in Ref. [29]. A concrete estimate of the theoretical uncertainty in Eq.(5) is expected to be useful for further discussion of the solar *hep* problem.

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