Quark-Hadron Duality and the Nuclear EMC Effect

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Abstract

Recent data on polarized proton knockout reactions off $^4$He nuclei suggest a small but nonzero modification of proton electromagnetic form factors in medium. Using model independent relations derived on the basis of quark-hadron duality, we relate the medium modification of the form factors to the modification at large $x$ of the deep-inelastic structure function of a bound proton. This places strong constraints on models of the nuclear EMC effect which assume a large deformation of the intrinsic structure of the nucleon in medium.
I. INTRODUCTION

The modification of hadron properties in the nuclear environment is of fundamental importance in understanding the implications of QCD for nuclear physics. Over the past few years there has been considerable interest in possible changes to masses, charge radii and other hadron properties in the nuclear medium. There is a significant constraint on the possible change in the “radius” of a bound nucleon based on $y$-scaling of a bound nucleon — especially in $^3$He [1]. On the other hand, the axial charge of the nucleon is known to be suppressed in nuclear $\beta$ decay, and a change in the charge radius of a bound proton provides a natural suppression of the Coulomb sum rule [2]. One of the most famous nuclear medium effects — the nuclear EMC effect [3], or the change in the inclusive deep-inelastic structure function of a nucleus relative to that of a free nucleon — has stimulated theoretical and experimental efforts for almost two decades now which seek to understand the dynamics responsible for the change in the quark-gluon structure of the nucleon in medium [4].

The EMC effect illustrates an inherent difficulty in identifying genuine nuclear quark-gluon effects in a background of purely hadronic physics, such as conventional nuclear binding and Fermi motion, associated with the nuclear bound state. Most features of the nuclear to nucleon structure function ratio can be (at least qualitatively) understood in terms of conventional nuclear physics [3]. On the other hand, some of these features can also be attributed to a modification of the intrinsic nucleon structure function in medium.

Recently the search for evidence of modification of nucleon properties in medium has been extended to electromagnetic form factors, in polarized ($e, e'p$) scattering experiments on $^{16}$O [5] and $^4$He [6] nuclei. These experiments measured the ratio of transverse to longitudinal polarization of the ejected protons, which for a free nucleon is proportional to the ratio of electric to magnetic elastic form factors [7],

$$\frac{G_E}{G_M} = \frac{-P_x' P_z}{P_z'} \frac{E + E'}{2M \tan(\theta/2)},$$  

(1)

where $P_x'$ and $P_z'$ are the transverse and longitudinal polarization transfer observables, $E$ and $E'$ the incident and recoil electron energies, and $\theta$ the electron scattering angle. Compared with the more traditional cross section measurements, polarization transfer experiments provide more sensitive tests of dynamics, especially of any in-medium changes in the form factors. The feasibility of this technique was first demonstrated in the commissioning experiment at Jefferson Lab on $^{16}$O [5] nuclei at $Q^2 = 0.8$ GeV$^2$. Unfortunately, the errors in this exploratory study were too large to draw firm conclusions about possible medium modification effects. In the subsequent experiment at MAMI on $^4$He [6] at $Q^2 \approx 0.4$ GeV$^2$, which had much higher statistics, the polarization ratio in $^4$He was found to differ by $\approx 10\%$ from that in hydrogen.

Conventional models using free nucleon form factors and the best phenomenologically determined optical potentials and bound state wave functions, as well as relativistic corrections, meson exchange currents, isobar contributions and final state interactions [5,11], fail to account for the observed effect in $^4$He [6]. Indeed, full agreement with the data was only obtained when, in addition to these standard nuclear corrections, a small change in the structure of the bound nucleon was taken into account [12,13]. Regardless of the microscopic origin of the nucleon structure modification, if there are density dependent effects
which modify the quark substructure of the nucleon, then these should leave traces in a variety of processes and observables, including structure functions and form factors.

Of course, one must caution that the study of off-shell nucleon effects is hampered with difficulties in unambiguously identifying effects associated with nucleon structure deformation [15]. In principle, one can reshuffle strength from off-shell corrections to meson exchange currents or interaction terms [16], so that “off-shell effects” can only be identified after specifying a particular form of the interaction of a nucleon with the surrounding nuclear medium. Nevertheless, within a given model of the nucleus, one can study the capacity to simultaneously describe form factors and structure functions as well as static nuclear properties. It is in this context that we proceed with the discussion of the possible modifications of nucleon properties in the nuclear medium.

There has recently been considerable interest in the interplay between form factors and structure functions in the context of quark-hadron duality. As observed originally by Bloom and Gilman [17], the $F_2$ structure function measured in inclusive lepton scattering at low $W$ (where $W$ is the mass of the hadronic final state) generally follows a global scaling curve which describes high $W$ data, to which the resonance structure function averages. Furthermore, the equivalence of the averaged resonance and scaling structure functions appears to hold for each resonance region, over restricted intervals of $W$, so that the resonance-scaling duality also exists locally. These findings were dramatically confirmed in recent high-precision measurements of the proton and deuteron $F_2$ structure function at Jefferson Lab [18,19], which demonstrated that local duality works remarkably well for each of the low-lying resonances, including surprisingly the elastic, to rather low values of $Q^2$.

In this paper we use the concept of quark-hadron duality to relate the medium dependence of nucleon electromagnetic form factors to the medium dependence of nucleon structure functions. To the extent that local duality is a good approximation, these relations are model independent, and can in fact be used to test the self-consistency of the models. We find that the recent form factor data for a proton bound in $^4$He [13] place strong constraints on the medium modification of inclusive structure functions at large Bjorken-$x$. In particular, they strongly disfavor models in which the bulk of the nuclear EMC effect is attributed to deformation of the intrinsic nucleon structure off-shell – see e.g. Ref. [20].

In Section II we discuss the modification of nucleon electromagnetic form factors as inferred from the recent polarization transfer experiments. As found in the analysis of the data in Ref. [13], amongst those models for which predictions were available, the modifications could only be understood within the context of the quark-meson coupling model [12,14]. We therefore use this model to calculate the density dependence of the bound nucleon electromagnetic form factors. In Section III quark-hadron duality is used to relate the observed form factor modification to that which would be expected in the deep-inelastic structure functions. We briefly review the relevant features of Bloom-Gilman duality and compare the results of models with and without large medium modifications of the intrinsic nucleon structure. Finally, in Section IV we make concluding remarks and discuss implications of our results for future experiments.
II. NUCLEAR MEDIUM MODIFICATION OF FORM FACTORS

Let us briefly review the medium modification of the electromagnetic form factors of the nucleon, as suggested in the recent quasi-elastic scattering experiment on $^4$He [6]. The data were analyzed using a variety of models, nonrelativistic and relativistic, based on conventional nucleon-nucleon potentials and well-established bound state wave functions, including corrections from meson exchange currents, final state interaction and other effects [8-11]. The conventional models with the free nucleon form factors could produce a deviation of at most one half of a percent in the nuclear transverse to longitudinal ratio, $P_x/P_z$, compared with that in hydrogen. The observed deviation, which was of order 10%, could only be explained by supplementing the conventional nuclear description with the effects associated with the modification of the nucleon internal structure. Even though the effect is currently only at the level of 1–2 standard deviations, it is of considerable interest and importance as the first relatively model independent indication of a change in the internal structure of the nucleon in a nuclear environment.

In the quark-meson coupling (QMC) model [13,14] the medium effects arise through the self-consistent coupling of phenomenological scalar ($\sigma$) and vector ($\omega_\mu, \rho_\mu$) meson fields to confined valence quarks, rather than to the nucleons, as in Quantum Hadrodynamics [21]. As a result, the internal structure of the bound nucleon is modified by the surrounding nuclear medium. The modification of the electromagnetic form factors of the bound nucleon has been studied using an improved cloudy bag model (CBM) [22,23], together with the QMC model [12]. The improved CBM includes a Peierls-Thouless projection to account for center of mass and recoil corrections, and a Lorentz contraction of the internal quark wave function. In this study we calculate the change of the nucleon electromagnetic form factor in a nuclear medium as in Ref. [12].

Because the average nuclear densities for all existing stable nuclei heavier than deuterium lie in the range $\frac{1}{2}\rho_0 \lesssim \rho \lesssim \rho_0$, where $\rho_0 = 0.15$ fm$^{-3}$ is the normal nuclear matter density, we consider two specific nuclear densities ($\rho = \frac{1}{2}\rho_0$ and $\rho = \rho_0$) to give the upper and lower bounds for the change of the electromagnetic form factors (and structure functions at large $x$) of the bound nucleon. Furthermore, for the isoscalar $^4$He and $^{16}$O nuclei we neglect the tiny amount of charge symmetry breaking (due to the Coulomb force and the $u$ and $d$ current quark mass differences).

The Lagrangian density of the QMC model for symmetric nuclear matter is given by [13,14]:

$$\mathcal{L}_{QMC} = \sum_q \bar{\psi}_q (i\gamma^\mu \partial_\mu - m_q)\psi_q \theta_V - B \theta_V$$

$$+ g_\sigma^q \bar{\psi}_q \sigma \psi_q - g_\omega^q \bar{\psi}_q \gamma^\mu \omega_\mu \psi_q - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_\mu \omega_\mu,$$

where $\psi_q$ is the quark field for a quark flavor $q$, $B$ is the bag constant, $g_\sigma^q$ and $g_\omega^q$ denote the quark-meson coupling constants, and $\theta_V$ is a step function equal to unity inside the confining volume and vanishing outside (the $\rho_\mu$ meson mean field vanishes in symmetric nuclear matter). In the mean field approximation, the meson fields are treated as classical fields, and the quark field inside the bag satisfies the equation of motion [12]:

$$\left[ i\gamma^\mu \partial_\mu - m_q^* - g_\omega^q \overline{\psi} \gamma^0 \right] \psi_q(x) = 0,$$
where $\overline{\sigma}$ and $\overline{\tau}$ denote the constant mean values of the scalar and the time component of the vector field, respectively, in symmetric nuclear matter, and $m_q^* \equiv m_q - g^2 \overline{\sigma}$ is the current quark mass in the nuclear medium (hereafter we denote the in-medium quantities by an asterisk $^*$). The electromagnetic current is given by the sum of the contributions from the quark core and the pion cloud,

$$ j^\mu(x) = \sum_q Q_q \overline{\psi}_q(x) \gamma^\mu \psi_q(x) - i e [\overline{\pi}(x) D^\mu \pi(x) - \pi(x) D^\mu \overline{\pi}(x)] , $$

(4)

where $Q_q$ is the charge operator for a quark flavor $q$, and $\pi(x)$ destroys a negatively charged (or creates a positively charged) pion.

In the Breit frame the quark core contribution to the electromagnetic form factors of the bound nucleon is given by [12]:

$$ G_E(Q^2) = \eta^2 G_E^{\text{sph}}(\eta^2 Q^2) , $$

$$ G_M(Q^2) = \eta^2 G_M^{\text{sph}}(\eta^2 Q^2) , $$

(5a)

(5b)

where $Q^2 \equiv -q^2 = \overline{q}^2$, and the scaling factor $\eta = M^*/E^*$, with $E^* = \sqrt{M^* + Q^2/4}$ the energy and $M^*$ the mass of the nucleon in medium, and $G_E^{\text{sph}}(Q^2)$ are the form factors calculated with the static spherical bag wave function:

$$ G_E^{\text{sph}}(Q^2) = \frac{1}{D} \int d^3r \ j_0(Qr) \ f_q(r) \ K(r) , $$

(6a)

$$ G_M^{\text{sph}}(Q^2) = \frac{1}{D} \frac{2 M}{Q} \int d^3r \ j_1(Qr) \ \beta_q \ j_0(\omega_0 r/R) \ j_1(\omega_0 r/R) \ K(r) . $$

(6b)

Here $f_q(r) = j_0^2(\omega_0 r/R) + \beta_q^2 \ j_1^2(\omega_0 r/R)$, where $R$ is the bag radius, $\omega_0$ the lowest eigenfrequency, and $\beta_q^2 = (\Omega_q - m_q R)/\Omega_q + m_q R$, with $\Omega_q = \sqrt{\omega_0^2 + m_q^2 R^2}$. The recoil function $K(r) = \int d^3x f_q(\vec{x}) f_q(-\vec{x} - \vec{r})$ accounts for the correlation of the two spectator quarks, and $D = \int d^3r \ f_q(r) \ K(r)$ is the normalization factor. The scaling factor $\eta$ in the argument of $G_E^{\text{sph}}$ arises from the coordinate transformation of the struck quark, and the prefactor in Eqs.(5) comes from the reduction of the integral measure of the two spectator quarks in the Breit frame.

The contribution from the pionic cloud is calculated along the lines of Ref. [12]. Although the pion mass would be slightly smaller in the medium than in free space, the pion field has little effect on the proton form factors, so that we use $m_\pi^* = m_\pi$. Furthermore, since the $\Delta$ isobar is treated on the same footing as the nucleon in the CBM, and because it contains three ground state light quarks, its mass should vary in a similar manner to that of the nucleon in the QMC model. As a first approximation we therefore take the in-medium and free space $N-\Delta$ mass splittings to be approximately equal, $M_\Delta^* - M^* \simeq M_\Delta - M$.

Including both the quark core and pion cloud contributions, the electric and magnetic form factors of the free and bound nucleons were calculated in Ref. [12]. One finds that the modification of the bound nucleon form factors is 1–2% for the magnetic and of order 8% for the electric form factor, respectively, at normal nuclear matter density ($\rho = \rho_0$), and for $Q^2 \sim 0.3$ GeV$^2$. The values of the current quark masses, $m_q \equiv m_u = m_d = 5$ MeV, and the nucleon bag radius in free space, $R = 0.8$ fm, are the same as those used in the earlier calculations which reproduce nuclear saturation properties, and which produced the
good agreement with the form factor data in Ref. [6]. None of the results for nuclear properties, however, depend strongly on the choice of parameters once the quark-meson coupling constants are fixed to reproduce the nuclear saturation properties. As shown in Ref. [14], the dependence of the properties of finite nuclei on $m_q$ and $R$ is relatively weak.

The change in the ratio of the electric to magnetic form factors of the proton,

$$R_{EM}^p(Q^2) = \frac{G_{E}^p(Q^2)}{G_{M}^p(Q^2)}$$

from free to bound, is illustrated in Fig. 1 for $Q^2$ up to 4 GeV$^2$, for $\rho = \rho_0$ and $\rho = \frac{1}{2}\rho_0$. Because of charge conservation, the value of $G_{E}^p$ at $Q^2 = 0$ remains unity for any $\rho$. On the other hand, the proton magnetic moment is enhanced in the nuclear medium, increasing with $\rho$, so that $R_{EM}^p < R_{EM}^p$ at $Q^2 = 0$. In fact, the electric to magnetic ratio is $\sim 5\%$ smaller in medium than in free space for $\rho = \frac{1}{2}\rho_0$, and $\sim 10\%$ smaller for $\rho = \rho_0$. The effect increases with $Q^2$ out to $\sim 2$ GeV$^2$, where the bound/free ratio deviates by $\sim 20\%$ from unity.

On the other hand, because nuclear density is not uniform throughout the nucleus, the $\approx 20\%$ change in the form factors produces only a few $\%$ effect in the polarization ratio [6]. The experiment not only probes the central region where $\rho$ is maximal, but also outer regions where $\rho$ is much smaller, so that integration over the entire nucleus dilutes the effect. Nevertheless, a form factor modification of this order of magnitude is needed to explain the observed effect [3]. In the next section we examine the implications of the modification of the form factors for the medium modification of structure functions at large $x$.

III. QUARK-HADRON DUALITY AND NUCLEON STRUCTURE FUNCTIONS IN MEDIUM

The relationship between form factors and structure functions, or more generally between inclusive and exclusive processes, has been studied in a number of contexts over the years. Drell & Yan [24] and West [25] pointed out long ago that, simply on the basis of scaling arguments, the asymptotic behavior of elastic electromagnetic form factors as $Q^2 \to \infty$ can be related to the $x \to 1$ behavior of deep-inelastic structure functions. In perturbative QCD language, this can be understood in terms of hard gluon exchange [26]: deep-inelastic scattering at $x \sim 1$ probes a highly asymmetric configuration in the nucleon in which one of the quarks goes far off-shell after the exchange of at least two hard gluons in the initial state; elastic scattering, on the other hand, requires at least two gluons in the final state to redistribute the large $Q^2$ absorbed by the recoiling quark [27].

More generally, the relationship between resonance (transition) form factors and the deep-inelastic continuum has been studied in the framework of quark-hadron, or Bloom-Gilman, duality: the equivalence of the averaged structure function in the resonance region and the scaling function which describes high $W$ data. The recent high precision Jefferson Lab data [15] on the $F_2$ structure function suggests that the resonance-scaling duality also exists locally, for each of the low-lying resonances, including surprisingly the elastic [19], to rather low values of $Q^2$.

In the context of QCD, Bloom-Gilman duality can be understood within an operator product expansion of moments of structure functions [28,29]: the weak $Q^2$ dependence of
the low $F_2$ moments can be interpreted as indicating that higher twist ($1/Q^2$ suppressed) contributions are either small or cancel. However, while allowing the duality violations to be identified and classified according to operators of a certain twist, it does not explain why some higher twist matrix elements are intrinsically small.

A number of recent studies have attempted to identify the dynamical origin of Bloom-Gilman duality using simple models of QCD [30-32]. It was shown, for instance, that in a harmonic oscillator basis one can explicitly construct a smooth, scaling structure function from a set of infinitely narrow resonances [30,31]. Although individual resonance contributions are suppressed by powers of $1/Q^2$, the number of states accessible increases with $Q^2$ so as to compensate the fall off, and as $Q^2 \to \infty$ quark-hadron duality arises from the summation over a complete set of hadronic states. At lower $Q^2$, however, the appearance of duality could in some cases be accidental, for example, because of a fortuitous cancellation of off-diagonal terms in the valence quark charges in the proton [33-35], allowing a coherent process (exclusive form factors) to be expressed in terms of incoherent scattering (structure functions). Whatever the ultimate microscopic origin of Bloom-Gilman duality, for our purposes it will be sufficient to note the empirical fact that local duality is realized in lepton-proton scattering down to $Q^2 \sim 0.5$ GeV$^2$ at the 10-20% level for the lowest moments of the structure function. In other words, here we are not concerned about why duality works, but rather that it works.

Motivated by the experimental verification of local duality, one can use measured structure functions in the resonance region to directly extract elastic form factors [28]. Conversely, empirical electromagnetic form factors at large $Q^2$ can be used to predict the $x \to 1$ behavior of deep-inelastic structure functions [17,26,36,37]. The assumption of local duality for the elastic case implies that the area under the elastic peak at a given $Q^2$ is equivalent to the area under the scaling function, at much larger $Q^2$, when integrated from the pion threshold to the elastic point [17]. Using the local duality hypothesis, de Rújula et al. [28], and more recently Ent et al. [19], extracted the proton’s magnetic form factor from resonance data on the $F_2$ structure function at large $x$, finding agreement to better than 30% over a large range of $Q^2$.

Applying the argument in reverse, one can formally differentiate the local elastic duality relation [17] with respect to $Q^2$ to express the scaling functions, evaluated at threshold, $x = x_{th} = Q^2/(W_{th}^2 - M^2 + Q^2)$, with $W_{th} = M + m_x$, in terms of $Q^2$ derivatives of elastic form factors. In Refs. [17,30] the $x \to 1$ behavior of the neutron to proton structure function ratio was extracted from data on the elastic electromagnetic form factors. (Nucleon structure functions in the $x \sim 1$ region are important as they reflect mechanisms for the breaking of spin-flavor SU(6) symmetry in the nucleon [38].) Extending this to the case of bound nucleons, one finds that as $Q^2 \to \infty$ the ratio of bound to free proton structure functions is:

$$\frac{F_{2n}^*}{F_2^p} \to \frac{dG_{M}^{p*}}{dQ^2}/\frac{dG_{M}^{p}}{dQ^2}. \quad (8)$$

At finite $Q^2$ there are corrections to Eq. (8) arising from $G_{E}^{p}$ and its derivatives, as discussed in Ref. [36]. (In this analysis we use the full, $Q^2$ dependent expressions [36,37].) Note that in the nuclear medium, the value of $x$ at which the pion threshold arises is shifted:
\[
x_{th} \rightarrow x_{th}^* = \left( \frac{m_\pi (2M + m_\pi) + Q^2}{m_\pi (2M + V + m_\pi) + Q^2} \right) x_{th},
\]
(9)

where \( V = 3g_\omega^2 \bar{\omega} \) is the vector potential felt by the nucleon and (consistent with chiral expectations and phenomenological constraints) we have set \( m_\pi^* = m_\pi \). However, the difference between \( x_{th} \) and \( x_{th}^* \) has a negligible effect on the results for most values of \( x \) considered.

Using the duality relations between electromagnetic form factors and structure functions, in Fig. 2 we plot the ratio \( F_2^{N*}/F_2^N \) as a function of \( x \), with \( x \) evaluated at threshold, \( x = x_{th} \) (solid lines). Note that at threshold the range of \( Q^2 \) spanned between \( x = 0.5 \) and \( x = 0.8 \) is \( Q^2 \approx 0.3-1.1 \) GeV\(^2\). Over the range \( 0.5 \lesssim x \lesssim 0.75 \) the effect is almost negligible, with the deviation of the ratio from unity being \( \lesssim 1\% \) for \( \rho = \frac{1}{2} \rho_0 \) and \( \lesssim 2\% \) for \( \rho = \rho_0 \). For \( x \gtrsim 0.8 \) the effect increases to \( \sim 5\% \), although, since larger \( x \) corresponds to larger \( Q^2 \), the analysis in terms of the QMC model is less reliable here. However, in the region where the analysis can be considered reliable, the results based on the bound nucleon form factors inferred from the polarization transfer data \[6\] and local duality imply that the nucleon structure function undergoes very little modification in medium.

It is instructive to contrast this result with models of the EMC effect in which there is a large medium modification of nucleon structure. For example, let us consider the model of Ref. \[20\], where it is assumed that for large \( x \) the dominant contribution to the structure function is given by the point-like configurations (PLC) of partons which interact weakly with the other nucleons. The suppression of this component in a bound nucleon is assumed to be the main source of the EMC effect. This model represents one of the extreme possibilities that the EMC effect is solely the result of deformation of the wave function of bound nucleons, without attributing any contribution to nuclear pions or other effects associated with nuclear binding \[39\].

The deformation of the bound nucleon structure function in the PLC suppression model is governed by the function \[20\]:
\[
\delta(k) = 1 - 2(k^2/2M + \epsilon_A)/\Delta E_A ,
\]
(10)
where \( k \) is the bound nucleon momentum, \( \epsilon_A \) is the nuclear binding energy, and \( \Delta E_A \sim 0.3-0.6 \) GeV is a nucleon excitation energy in the nucleus. For \( x \gtrsim 0.6 \) the ratio of bound to free nucleon structure functions is then given by \[20\]:
\[
\frac{F_2^{N*}(k, x)}{F_2^N(x)} = \delta(k) .
\]
(11)

The \( x \) dependence of the suppression effect is based on the assumption that the point-like configuration contribution in the nucleon wave function is negligible at \( x \lesssim 0.3 \) \( (F_2^{N*}/F_2^N = 1) \), and for \( 0.3 \lesssim x \lesssim 0.6 \) one linearly interpolates between these values \[20\]. The results for \(^4\)He and \(^{16}\)O are shown in Fig. 2 (dashed lines) for the average values of nucleon momentum, \( \langle k^2 \rangle \), in each nucleus. The effect is a suppression of order 20\% in the ratio \( F_2^{N*}/F_2^N \) for \( x \sim 0.6-0.7 \). In contrast, the ratios extracted on the basis of duality, using the QMC model constrained by the \(^4\)He polarization transfer data \[1\], show almost no suppression \( (\lesssim 1-2\%) \) in this region. Thus, for \(^4\)He, the effect in the PLC suppression model is an order of magnitude too large at \( x \sim 0.6 \), and has the opposite sign for \( x \gtrsim 0.65 \).
Although the results extracted from the polarization transfer measurements \cite{6} rely on the assumption of local duality, we stress that the corrections to duality have been found to be around \( \lesssim 20\% \) for \( Q^2 \gtrsim 0.5 \text{ GeV}^2 \) \cite{18}. The results therefore appear to rule out large bound structure function modifications, such as those assumed in the point-like configuration suppression model \cite{20}, and instead point to a small medium modification of the intrinsic nucleon structure, which is complemented by standard many-body nuclear effects.

As a consistency check on the analysis, one can also examine the change in the form factor of a bound nucleon that would be implied by the corresponding change in the structure function in medium. Namely, from the local duality relation \cite{28,37}:

\[
\left[ G_M^p(Q^2) \right]^2 \approx \frac{4 - 2\xi_0}{\xi_0^2} \frac{(1 + \tau)}{(\mu_p + \tau)} \int_{\xi_{th}}^1 d\xi \; F_2^p(\xi),
\]

one can extract the magnetic form factor by integrating the \( F_2^p \) structure function over the Nachtmann variable \( \xi = 2x/(1 + \sqrt{1 + x^2/\tau}) \), where \( \tau = Q^2/4M^2 \), between threshold, \( \xi = \xi_{th} \), and \( \xi = 1 \). Here \( \xi_0 = \xi(x = 1) \), and \( \mu_p \) is the proton magnetic moment. In Fig. 3 we show the PLC model prediction (solid curve) for the ratio of the magnetic form factor of a proton bound in \(^4\text{He} \) to that in vacuum, derived from Eqs.\((11)\) and \((12)\), using the parameterization for \( F_2^p(\xi) \) from Ref. \cite{19}, and an estimate for the in-medium value of \( \mu_p^* \) from Ref. \cite{12}. The result is a suppression of about 20\% in the ratio \( G_M^{p*}/G_M^p \) at \( Q^2 \approx 1\text{--}2 \text{ GeV}^2 \). In contrast, the QMC calculation, which is consistent with the MAMI \(^4\text{He} \) quasi-elastic data, produces a ratio which is typically 5\text{--}10\% larger than unity (dashed). Without a very large compensating change in the in-medium electric form factor of the proton (which seems to be excluded by \( y \)-scaling constraints), the behavior of the magnetic form factor implied by the PLC model + duality would produce a large enhancement of the polarization transfer ratio, rather than the observed small suppression \cite{4}.

\section*{IV. CONCLUSION}

In this paper we have examined the consequences of quark-hadron duality applied to nucleons in the nuclear medium. Utilizing the experimental results \cite{6} from polarized proton knockout reactions off \(^4\text{He} \) nuclei, which suggest a small but nonzero modification of the proton electromagnetic form factors in medium, we use local duality to relate model-independently the medium modified form factors to the change in the intrinsic structure function of a bound proton.

The analysis in Ref. \cite{6} found that the medium modifications observed in the \(^4\text{He} \) data could only be described (amongst published calculations) within the quark-meson coupling model \cite{12,14}. In the context of the QMC model, the change in nucleon form factors allowed by the data imply a modification of the in-medium structure function of \( \lesssim 1\text{--}2\% \) at \( 0.5 \lesssim x \lesssim 0.75 \) for all nuclear densities between nuclear matter density, \( \rho = \rho_0 \), and \( \rho = \frac{1}{2}\rho_0 \). While the results rely on the validity of quark-hadron duality, the empirical evidence suggests that for low moments of the proton’s \( F_2 \) structure function the duality violations due to higher twist corrections are \( \lesssim 20\% \) for \( Q^2 \gtrsim 0.5 \text{ GeV}^2 \) \cite{18}, and decrease with increasing \( Q^2 \).

The results place rather strong constraints on models of the nuclear EMC effect, especially on models which assume that the EMC effect arises from a large deformation of the
nucleon structure in medium. For example, we find that the PLC suppression model \cite{20} predicts an effect which is about an order of magnitude larger than that allowed by the data \cite{6}. The findings therefore strongly disfavor any model with large medium modifications of structure functions as a viable explanation for the nuclear EMC effect, although it would be desirable to have more data on a variety of nuclei and in different kinematical regions. The medium modifications will be further tested in upcoming experiments at Jefferson Lab on $^{16}$O \cite{40}, which will have about 15 times the statistics of the original commissioning experiment \cite{5} and will cover a much larger range of $Q^2$. This will enable a more thorough comparison of the medium dependence of form factors and structure functions for different nuclei.

These results have other important practical ramifications. For instance, the PLC suppression model was used recently \cite{41} to argue that the EMC effects in $^3$He and $^3$H differ significantly at large $x$, in contrast to calculations \cite{42,43} based on conventional nuclear physics using well-established bound state wave functions which show only small differences. Based on the findings presented here, one would conclude that the conventional nuclear physics description of the $^3$He/$^3$H system should indeed be a reliable starting point for nuclear structure function calculations, as the available evidence suggests little room for large off-shell corrections. Finally, let us stress that quark-hadron duality is a powerful tool with which to simultaneously study the medium dependence of both exclusive and inclusive observables, and thus provides an extremely valuable guide towards a consistent picture of the effects of the nuclear environment on nucleon substructure.

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REFERENCES

FIG. 1. Comparison of the ratio of electric to magnetic form factors of the proton, $R_{EM}^p = G_E^p / G_M^p$, in medium to that in free space in the QMC model [12]. The bound proton form factors are calculated at nuclear matter density, $\rho = \rho_0$ (dashed), and at $\rho = \frac{1}{2} \rho_0$ (solid).
FIG. 2. In-medium to free proton $F_2$ structure function ratio as a function of $x$ at threshold, $x = x_{\text{th}}$, extracted from the polarization transfer data [6] within the QMC model and local duality, at nuclear matter density, $\rho = \rho_0$, and at $\rho = \frac{1}{2}\rho_0$ (solid). For comparison the results of the PLC suppression model [20] are shown for $^4\text{He}$ and $^{16}\text{O}$ (dashed).
FIG. 3. Ratio of in-medium to free proton magnetic form factors, extracted from the PLC suppression model \cite{20} for the EMC ratio in $^4$He, using the $F_2^p$ data from Refs. \cite{18,19} and local duality (solid). The QMC model prediction (dashed) is shown for comparison.