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Probing the QCD vacuum with overlap fermions *

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We use low lying eigenvectors of the overlap-Dirac operator as a probe of the QCD vacuum. If instantons play a significant role one would expect the low lying eigenmodes of the overlap-Dirac operator to consist mainly of the mixed "would be zero modes". Then, the eigenmodes should exhibit local chirality. Studying a recently introduced local chirality parameter, we find evidence supporting this picture.

1. Introduction

Understanding the QCD vacuum has been a goal of lattice QCD for many years. This understanding, in turn, should illuminate the mechanisms of confinement and chiral symmetry breaking. A good candidate for the source of chiral symmetry breaking are topological fluctuations of the gauge fields associated with instantons and anti-instantons. Through mixing, the "would be zero modes" from the instantons and anti-instantons become the small modes that lead, when present with a finite density, to chiral symmetry breaking, since $\langle\bar{\psi}\psi\rangle = \pi\rho(0^+)$, with $\rho(0^+)$ the density of the small modes. Many of the low energy properties of QCD can be explained phenomenologically by the interactions of the fermions with instantons and anti-instantons [1,2]. To our knowledge, confinement, though, is not one of those properties.

There is a problem, though, with investigating the topological content of Monte Carlo generated gauge configurations. Ultraviolet fluctuations (noise) dominate these configurations. Pure gauge probes of the vacuum therefore need filters, like smearing or cooling, to get rid of the UV noise and make it possible to uncover the underlying topological fluctuations [3]. But do such filters change the physics? Fermions, on the other hand, should provide a more physical filter, provided lattice artifacts do not impede the chiral and topological properties. Since a few years such fermions are known, the overlap fermions of Neuberger [4].

Global topology, *i.e.* the topological susceptibility χ_t has already been successfully computed from the number and chirality of the zero modes. This was actually done, completely equivalently, from studying the spectral flow of the underlying Wilson Dirac operator [5].

Here we want to address local properties. If instanton and anti-instanton like fluctuations are important, then the low-lying non-zero eigenmodes should have significant contributions from mixed "would-be zero modes". Locally, in their peaks, these low eigenmodes should therefore be chiral, since zero modes are chiral, even though the integrated chirality of all non-zero modes is exactly zero. This can be measured with the "local chirality" parameter, $X(x)$, defined by [6]

$$\tan\Big(\frac{\pi}{4}(1+X(x))\Big) = \left(\frac{\psi_L^\dagger(x)\psi_L(x)}{\psi_R^\dagger(x)\psi_R(x)}\right)^{1/2}, \quad (1)$$

where $\psi_L(x)$ and $\psi_R(x)$ are the left- and right-handed components of the eigenmode.

It should be noted, however, that Witten pointed out an inconsistency between instanton based phenomenology and large-N_c QCD [7]. Instantons would produce an η' mass that vanishes exponentially for large N_c, while considerations based on large-N_c chiral dynamics suggest that the η' mass should be of order $1/N_c$. The topological charge fluctuations then should

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be associated not to instantons but rather some other, confinement-related vacuum fluctuations. And the strong attraction produced by these confinement-related fluctuations would also induce the breaking of chiral symmetry. Then there is no reason that the low-lying modes should be chiral in their peaks. Using non-chiral Wilson fermions at a lattice spacing of $a \simeq 0.17$ fm Horváth *et al.* [6] found that the local chirality $X(x)$ was not peaked near ± 1. We use chiral overlap fermions instead to avoid the potentially large $\mathcal{O}(a)$ lattice artifacts from the explicit chiral symmetry breaking term of Wilson fermions.

2. Setup

For our study we consider the standard massless overlap-Dirac operator [4]

$$D(0) = \frac{1}{2}\big[1 + \gamma_5 \epsilon(H_w(M))\big] \tag{2}$$

with $H_w(M) = \gamma D_w(-M)$ and $D_w(M)$ the usual Wilson-Dirac operator. Then, $H^2(0) = D^\dagger(0)D(0)$ commutes with γ_5 and can be simultaneously diagonalized [8,9]. $H^2(0)$ can have zero eigenvalues with chiral eigenmodes, which are also eigenmodes of $D(0)$, due to global topology. The non-zero eigenmodes of $H^2(0)$ are doubly degenerate and have opposite chirality,

$$H^2(0)\psi_{\uparrow,\downarrow} = \lambda^2 \psi_{\uparrow,\downarrow} \ , \qquad \gamma_5\psi_\uparrow = \psi_\uparrow \ , \gamma_5\psi_\downarrow = -\psi_\downarrow \ , \tag{3}$$

with $0 < \lambda \leq 1$. The non-zero (right) eigenmodes of $D(0)$ are then easily obtained as

$$\psi_\pm = \frac{1}{\sqrt{2}}\Big(\psi_\uparrow \pm i\psi_\downarrow\Big) \tag{4}$$

with eigenvalues

$$\lambda_\pm = \lambda^2 \pm i\lambda\sqrt{1-\lambda^2} \ . \tag{5}$$

The "chirality" parameter of eq. (1) is then given by

$$\tan\Big(\frac{\pi}{4}(1+X(x))\Big) = \left(\frac{\psi_\downarrow^\dagger(x)\psi_\downarrow(x)}{\psi_\uparrow^\dagger(x)\psi_\uparrow(x)}\right)^{1/2} \ , \tag{6}$$

Table 1
The gauge ensembles. Listed are the volume in lattice units and in units of fm^4, the number of configurations analyzed, the average number of zero modes and the average of the square of the global topological charge as obtained from the number of zero modes.

β	V [a^4]	V [fm^4]	N	$\langle\lvert Q\rvert\rangle$	$\langle Q^2\rangle$
2.2872	$6^3 \times 12$	2.2	100	1.0(1)	1.7(2)
2.2872	$8^3 \times 16$	7.0	69	2.0(2)	5(1)
2.2872	$12^3 \times 16$	23.6	10	4.2(9)	25(8)
2.45	$8^3 \times 16$	2.0	197	1.0(1)	1.8(2)
2.45	$12^3 \times 16$	6.7	47	2.3(4)	8(1)
2.65	$12^3 \times 16$	2.1	50	1.3(2)	2.4(6)

and is equal for the two related modes $\psi_\pm$. In the following they will be therefore counted as one mode. The exact zero modes, of course, have $X(x) \equiv \pm 1$.

We computed low-lying eigenmodes $\psi_{\uparrow,\downarrow}$ of $H^2(0)$ with the Ritz functional algorithm of Ref. [10]. For the sign function $\epsilon(H_W)$ in eq. (2) we used the optimal rational approximation of [8,9] with "projection of low-lying eigenvectors of H_w" to ensure sufficient accuracy.

3. Results

For a more extensive and detailed discussion of our results see [11]. Here, we concentrate on the results obtained on gauge field configurations generated with the Iwasaki action [12]. At comparable lattice spacing, the implementation of overlap fermions on such configurations is about a factor 3 faster than on Wilson action configurations [11]. We computed the 20 eigenmodes $\psi_{\uparrow,\downarrow}$ of $H^2(0)$ with smallest λ, with the Wilson-Dirac mass in the overlap-Dirac operator set to $M = 1.65$. Six ensembles were considered corresponding to three different lattice spacings and three different volumes, in physical units (see Table 1).

The chirality histograms of the two lowest non-zero modes at the 2.5% of sites with largest $\psi^\dagger\psi(x)$ for all six ensembles are shown in Fig. 1. The physical volume of the systems shown in the left column are approximately the same, 2

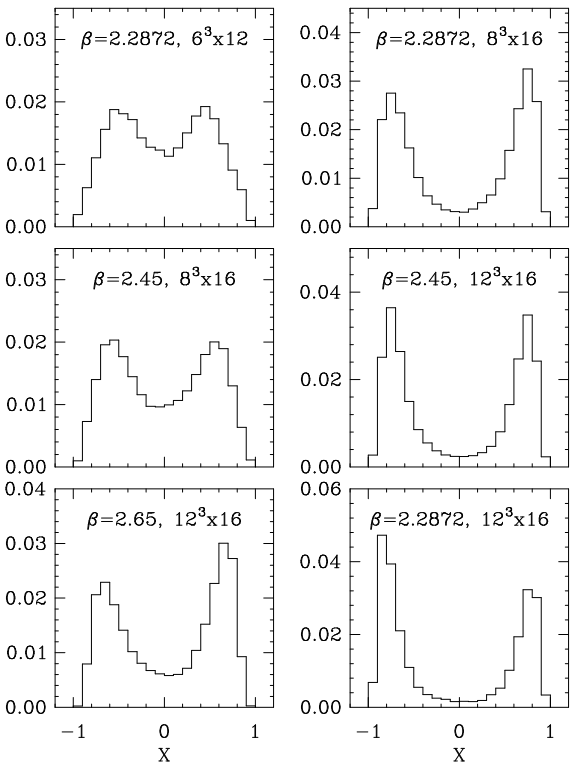

Figure 1. Chirality histograms for the lowest two non-zero modes of the overlap-Dirac operator at the 2.5% sites with the largest $\psi^\dagger\psi(x)$.

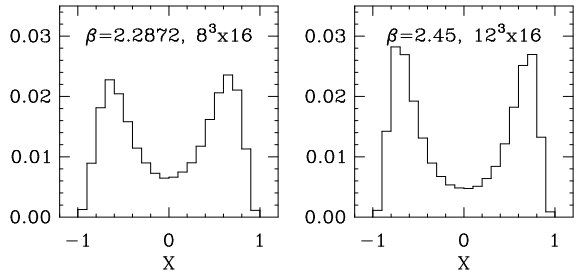

Figure 2. Chirality histogram for the lowest six non-zero modes at the 2.5% sites with the largest $\psi^\dagger\psi(x)$ on the two lattices with volume of about 7 fm^4.

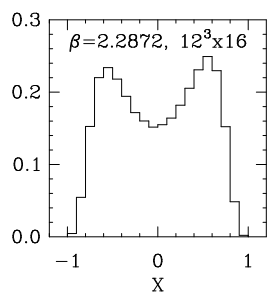

Figure 3. Chirality histogram for all the non-zero modes out of the 20 lowest modes at the 20% sites with the largest $\psi^\dagger\psi(x)$ on the lattice at $\beta = 2.2872$ with the largest volume.

fm^4. We see that the lowest non-zero modes tend to become more chiral in their peaks as the lattice spacing is decreased. This strongly suggests that the behavior observed here will survive the continuum limit. Comparing the three histograms for $\beta = 2.2872$ having equal lattice spacing, we see a dramatic dependence on the physical volume. In an instanton liquid picture of the vacuum, the number of instantons and anti-instantons, and hence the number of almost zero modes grows linearly with the volume. Then it is not surprising that the lowest modes become increasingly chiral in their peaks with increasing volume.

In Fig. 2 we show the chirality histogram of the lowest six non-zero modes on the two systems with physical volume of about 7 fm^4 and in Fig. 3 the chirality histogram from all non-zero modes out of the 20 lowest modes that we computed (there are between 11 and 20 non-zero modes per configuration) for the system with the largest physical volume. Here we kept the 20% sites with largest $\psi^\dagger\psi(x)$. Comparing with the histograms in the left column of Fig. 1 indicates that the number of non-zero modes with similar chirality histograms grows roughly like the physical volume so that there is a finite density of modes which are chiral in their peaks. This is good evidence that we are not observing a finite volume effect that would disappear in the infinite volume limit.

In the chirality histograms shown we typically considered the contribution from the 2.5% sites

Table 2
The average of the sum of $\psi^\dagger\psi(x)$ over the 2.5% of sites kept for the chirality histograms as a percentage of the sum over all sites for some low-lying eigenvectors for the various gauge field ensembles.

β	V $[a^4]$	ev 1	ev 2	ev 20
2.2872	$6^3 \times 12$	8.9(2)	7.8(2)	5.1(1)
2.2872	$8^3 \times 16$	11.5(3)	10.6(2)	6.2(2)
2.2872	$12^3 \times 16$	12.9(5)	13.4(6)	11.3(5)
2.45	$8^3 \times 16$	9.9(2)	8.7(1)	6.0(1)
2.45	$12^3 \times 16$	12.4(3)	11.5(3)	7.8(3)
2.65	$12^3 \times 16$	10.4(3)	9.2(2)	6.9(3)

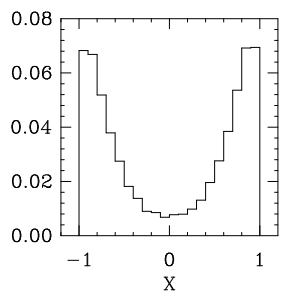

Figure 4. Chirality histogram for the lowest ten non-zero modes of the overlap-Dirac operator at the 6% sites with the largest $\psi^\dagger\psi(x)$ on 24^2 U(1) configurations at $\beta = 1.9894$.

with the largest $\psi^\dagger\psi(x)$. In Table 2 we list the sum of $\psi^\dagger\psi(x)$ over those sites as a percentage of the sum over all sites for some low-lying eigenvectors for all the gauge field ensembles considered. The more chiral the eigenvectors are on those 2.5% sites, the higher is the percentage of the eigenvector contained on those sites.

4. Abelian gauge theories

To contrast the results for quenched QCD of the previous section we considered the chirality parameter also for quenched U(1) theories in two and four dimensions. In 2d topology plays a crucial role and one expects local chirality in the peaks of low-lying modes. This is clearly seen in Fig. 4, where the 10 lowest non-zero modes have been kept.

In 4d U(1), in the confined phase, chiral symmetry is spontaneously broken, and a finite density of near zero modes exists. But there are no instantons, and so one would not expect the near zero modes to be dramatically chiral even in their peak regions. There do exist exact zero modes of the overlap-Dirac operator, which again, of course, are chiral, although their origin is not completely understood [13]. We analyzed, stored eigenmodes from Ref. [13]. The chirality histogram for the lowest two non-zero modes is shown in Fig. 5. We note that while there is some mild indication of chirality peaking, the proportion of sites showing this behavior is much reduced compared to the SU(3) case and appears to be of a qualitatively different nature than seen before. Also, the sum of $\psi^\dagger\psi(x)$ over the sites kept for the histograms of Fig. 5 where almost identical for all 12 lowest eigenmodes (4.0% and 1.20%, respectively; for the 2.5% of sites with largest $\psi^\dagger\psi(x)$ the sum is about 8.4% for all eigenmodes as compared to the non-abelian case in Table 2). In the U(1) case the (near) lack of peaking could conceivably come about from the scenario outlined by Horváth, *et. al.* – namely from the confinement inducing vacuum fluctuations.

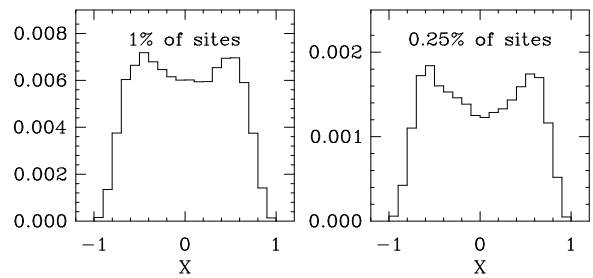

Figure 5. Chirality histogram for the lowest two non-zero modes of the overlap-Dirac operator at the 1% (left panel) and 0.25% (right panel) sites with the largest $\psi^\dagger\psi(x)$ on 8^4 U(1) configurations at $\beta = 0.9$.

5. Conclusions

By studying the local chirality parameter, introduced in Ref. [6], for low-lying non-zero eigen-

modes of the overlap-Dirac operator, we investigated the topological content of the QCD vacuum. Considering several volumes and lattice spacings, we found convincing evidence for chirality of the low lying modes in their peak region. Our results give evidence that the number of modes which are chiral in their peaks grows linearly with the volume so that there is a *finite density* of such modes, and that the chirality of the modes becomes more pronounced as the lattice spacing is decreased. Therefore, our observations should remain valid in the continuum limit. Our observations are consistent with the topological fluctuations being instanton dominated and the low-lying eigenmodes being the mixed "would be zero modes" due to the instantons and anti-instantons.

Our findings confirm the results of Ref. [14], where a different version of overlap fermions coupled to smoothened gauge fields were used, and extend them by a careful study of finite lattice spacing and volume effects. Our results, on the other hand, contradict the conclusions of Horváth *et al.* [6] who studied the chirality parameter with Wilson fermions at a large lattice spacing. That the results of Ref. [6] are strongly affected by lattice artifacts was confirmed in a recent paper of Hip *et al.* [15]. These authors considered an improved chirality parameter for Wilson fermions, clover improved Wilson fermions, and lattices at smaller lattice spacing than Ref. [6]. With these reductions of lattice artifacts, they conclude that instanton dominance of topological charge fluctuations is not ruled out by the response of (improved) Wilson fermions. Together with the results from overlap fermions presented here and the lack of significant chirality enhancement in the 4-d U(1) model where instantons should not exist, the case for instanton domination in 4-d SU(3) gauge theory, as measured by the chirality parameter, becomes even more compelling. However, for an opposite point of view see Ref. [16].

Acknowledgements

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