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Weak production of strangeness at threshold with polarization observables

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Abstract

The differential cross section for the charged current electroweak reaction $\vec{e} + p \to \vec{\nu}_e + \vec{\Lambda}$ at threshold with polarization observables is presented. The form of the cross section at threshold for the reaction is simplified compared to higher energy. An expression is given for the invariant matrix element appropriate for the reaction when the incident electron is polarized, and the final state hyperon polarization is determined. The energy dependence of the resulting cross section is shown near threshold. Under the right kinematic conditions, there can be a sizeable enhancement in the cross section, making an experimental measurement of the weak axial-vector form factor feasible.

Precision electroweak physics at intermediate energies has been the subject of intense study for some time [1,2]. Correlations between electrons and neutrinos in electroweak processes potentially provide new constraints on models of the interaction. These processes have been employed to even search for new phenomena beyond the Standard Model of particle physics [3]. Concurrently, hyperon production and decay in this energy regime has been investigated by several groups recently [4]. Studies of the electromagnetic production and decay of hyperons and hypernuclei is aided by the introduction of new strangeness degrees of freedom in the hadronic system [5]. Electroweak interaction studies have been confined to decay processes, by and large. However, with the high current, continuous-wave electron accelerators now in operation (JLAB, MIT-Bates, Mainz), it is possible to study electroweak production in different kinematic regimes [6–10]. There is a need for more phenomenological calculations to provide guidance for future experimental programs.

This Letter presents a brief description of the cross section for charged current weak production of strangeness

$$\vec{e} + p \to \vec{\nu}_e + \vec{\Lambda} \tag{1}$$

using polarization observables. This study complements those of several previous calculations that were made of the reaction where no polarization is considered [6,7].

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Consider the case of a polarized electron beam incident upon an unpolarized target proton. (The use of polarized targets is not generally feasible for the luminosities needed to carry out such a program.) In a reasonable experimental arrangement, the decay products of the hyperon will be detected; their invariant mass will determine the hyperon kinematics. The Λ-hyperon decays more than 60% of the time into a proton and negative pion. Since the Λ is self-analyzing, the detection of the decay proton may be used to determine its spin polarization P using the distribution

$$\frac{\mathrm{d}N}{\mathrm{d}\Omega_p} \sim (1 + \alpha P_\Lambda \hat{s}_\Lambda \cdot \hat{k}_P) = (1 + \alpha P_\Lambda \cos\theta_{ps_\Lambda}), \tag{2}$$

where k_p is the proton momentum, θ_{ps_Λ} is the angle between the hyperon spin vector and the proton momentum vector, α is the weak decay correlation coefficient (experimentally determined to be 0.642 ± 0.013), and P_Λ is the magnitude of the hyperon spin polarization in the given direction being considered.

Neutrino oscillation experiments over the past several years provide tantalizing new evidence that neutrinos may be massive fermions. In this case, the neutrino's helicity does not need to be an intrinsic property of the lepton. Nevertheless, all empirical evidence to date suggests that high energy neutrinos are left-handed (spin vector pointing in a direction opposite the momentum vector). In a (kinematically) fully constrained experiment, the neutrino momentum is precisely reconstructed and, at energies that are large compared to its rest mass, the neutrino spin polarization (its helicity) is known. The electron helicity determines the neutrino helicity.

The appropriate general expression for the matrix element that enters into the calculation of the cross section will include the fact that the incident electron as well as the final state neutrino and hyperon are polarized. The matrix element for reaction (1) is [6]

$$\langle \nu_e \Lambda | H_W | e^- p \rangle = \frac{G}{\sqrt{2}} \sin\theta_c \bar{u}_\nu \gamma^\lambda (1 - \gamma_5) u_e \langle \Lambda | J_\Lambda^+(0) | p \rangle, \tag{3}$$

where G is the weak coupling constant, θ_c is the Cabibo angle, the u's are the appropriate fermion spinors along with the standard gamma matrices.

As shown in [6,7], the hadronic matrix element may be expressed as

$$\langle \Lambda | J_\lambda^+(0) | p \rangle = \langle \Lambda | V_\lambda^+(0) - A_\lambda^+(0) | p \rangle, \tag{4}$$

where V_λ and A_λ are the vector and axial vector parts of the weak strangeness-changing hadronic current, respectively. The weak current matrix element is composed of several terms proportional to relevant hadronic form factors. Several authors have shown that only the axial-vector form factor is important for the kinematic region considered in this Letter [6,7]. Thus, the hadronic weak matrix element may be simplified to

$$\langle \Lambda | J_\lambda^+(0) | p \rangle = -\bar{u}_\Lambda \gamma_\lambda \gamma_5 F_A\big(q^2\big) u_p, \tag{5}$$

where u_Λ and u_p are the hyperon and nucleon spinors, and F_A is the axial-vector hadronic form factor.

Then, the matrix element M to be calculated is

$$M = F_A\big(q^2\big) \bar{u}_\nu \gamma^\lambda (1 - \gamma_5) u_e \bar{u}_\Lambda \gamma_\lambda \gamma_5 u_p. \tag{6}$$

The invariant matrix element squared takes the form

$$|M|^2 = \big|F_A\big(q^2\big)\big|^2 \bar{u}_\nu \gamma^\lambda (1 - \gamma_5) u_e \bar{u}_\Lambda \gamma_\lambda \gamma_5 u_p [\bar{u}_\Lambda \gamma_\lambda \gamma_5 u_p]^+ \big[\bar{u}_\nu \gamma^\lambda (1 - \gamma_5) u_e\big]^+ \tag{7}$$

$$= \frac{|F_A(q^2)|^2}{128 m_\nu m_e m_\Lambda m_p} \mathrm{Tr}\big[\gamma^\eta (\not{p}_\nu + m_\nu)(1 + \gamma_5 \not{s}_\nu) \gamma^\lambda (1 - \gamma_5)(\not{p}_e + m_e)(1 + \gamma_5 \not{s}_e)(1 + \gamma_5)\big]$$

$$\times \mathrm{Tr}\big[\gamma_\eta (\not{p}_\Lambda + m_\Lambda)(1 + \gamma_5 \not{s}_\Lambda) \gamma_5 \gamma_\lambda (\not{p}_p + m_p) \gamma_5\big]. \tag{8}$$

The traces are performed using standard techniques yielding

$$
\begin{aligned}
|M|^2 = \frac{|F_A(q^2)|^2}{2m_\nu m_e m_\Lambda m_p} \Big\{ & \big[(p_\nu \cdot p_\Lambda)(p_e \cdot p_p) + (p_e \cdot p_\Lambda)(p_\nu \cdot p_p)\big] \\
& - m_e\big[(p_\nu \cdot p_\Lambda)(p_p \cdot s_e) + (s_e \cdot p_\Lambda)(p_\nu \cdot p_p)\big] \\
& + m_\nu m_e\big[(s_\nu \cdot p_\Lambda)(p_p \cdot s_e) + (s_e \cdot p_\Lambda)(s_\nu \cdot p_p)\big] \\
& - m_\nu\big[(s_\nu \cdot p_\Lambda)(p_p \cdot p_e) + (p_e \cdot p_\Lambda)(s_\nu \cdot p_p)\big] \\
& + m_\Lambda m_p(p_\nu \cdot p_e) - m_e m_\Lambda m_p(p_\nu \cdot s_e) \\
& + m_\nu m_e m_\Lambda m_p(s_\nu \cdot s_e) - m_\nu m_\Lambda m_p(s_\nu \cdot p_e) \\
& + m_p\big[(p_\nu \cdot p_\Lambda)(p_e \cdot s_\Lambda) - (p_e \cdot p_\Lambda)(p_\nu \cdot s_\Lambda)\big] \\
& + m_e m_p\big[(s_e \cdot p_\Lambda)(s_\Lambda \cdot p_\nu) - (p_\nu \cdot p_\Lambda)(s_\Lambda \cdot s_e)\big] \\
& + m_\nu m_e m_p\big[(s_\nu \cdot p_\Lambda)(s_\Lambda \cdot s_e) - (s_e \cdot p_\Lambda)(s_\nu \cdot s_\Lambda)\big] \\
& + m_\nu m_p\big[(s_\nu \cdot s_\Lambda)(p_e \cdot p_\Lambda) - (p_e \cdot s_\Lambda)(s_\nu \cdot p_\Lambda)\big] \\
& + m_\Lambda\big[(p_e \cdot s_\Lambda)(p_p \cdot p_\nu) - (p_\nu \cdot s_\Lambda)(p_e \cdot p_p)\big] \\
& + m_e m_\Lambda\big[(p_\nu \cdot s_\Lambda)(p_p \cdot s_e) - (s_e \cdot s_\Lambda)(p_\nu \cdot p_p)\big] \\
& + m_\nu m_e m_\Lambda\big[(s_e \cdot s_\Lambda)(p_p \cdot s_\nu) - (s_\nu \cdot s_\Lambda)(s_e \cdot p_p)\big] \\
& + m_\nu m_\Lambda\big[(s_\nu \cdot s_\Lambda)(p_p \cdot p_e) - (p_e \cdot s_\Lambda)(s_\nu \cdot p_p)\big]\Big\},
\end{aligned} \tag{9}
$$

where use is made of the fermion spinor contractions such as

$$
u(s,p)\bar{u}(s,p) = \left(\frac{\not{p} + m}{2m}\right)\left(\frac{1 + \gamma_5 \not{s}}{2}\right) \tag{10}
$$

for a particle of mass m, spin s, and four-momentum p for the case where the final spin polarization of the particle is determined. When the particle spin polarization is not observed, the appropriate form of the contraction is obtained by setting s to zero and multiplying by 2 in Eq. (10). Following the formalism of [1], the fermion spin is written as $s = h(\Gamma\beta, \Gamma u_\parallel)$ where $\Gamma = E/m$ and $\beta = p/E$; h is the particle helicity, ± 1. The expression shown in (9) reduces to the same result given in [6,7] when no polarizations are determined for those terms proportional to the axial-vector hadronic form factor.

The general result (9) is simplified for the present study. If the neutrino spin is not observed in the final state, and there is no hyperon polarization in the final state, the squared invariant matrix element becomes

$$
\begin{aligned}
|M|^2 = 2\big|F_A\big(q^2\big)\big|^2 \Bigg\{ & \frac{[(E_\nu E_\Lambda - \vec{p}_\nu \cdot \vec{p}_\Lambda)E_e + (E_e E_\Lambda - \vec{p}_e \cdot \vec{p}_\Lambda)E_\nu]}{m_\nu m_e m_\Lambda} \\
& - h\frac{[(E_\nu E_\Lambda - \vec{p}_\nu \cdot \vec{p}_\Lambda)E_e + (p_e E_\Lambda - E_e p_\Lambda \cos\theta_{e\Lambda})E_\nu]}{m_\nu m_e m_\Lambda} \\
& + \frac{(E_\nu E_e - \vec{p}_\nu \cdot \vec{p}_e)}{m_\nu m_e} - h\frac{(E_\nu p_e - p_\nu \cos\theta_{e\nu} E_e)}{m_\nu m_e}\Bigg\}.
\end{aligned} \tag{11}
$$

The major difference in this result and that from [6,7] is that the incident electron helicity is explicitly shown here. It can rather easily be seen that for positive electron helicity scattering, there is no contribution to the matrix element while left-handed electron induced processes give maximum contribution as expected for the charged-current weak interaction. (Note that there is a small component that is proportional to $(1 - \beta_e)$ which is 10^{-4} of the other terms in the expression at threshold and negligible for the purposes of this analysis.)

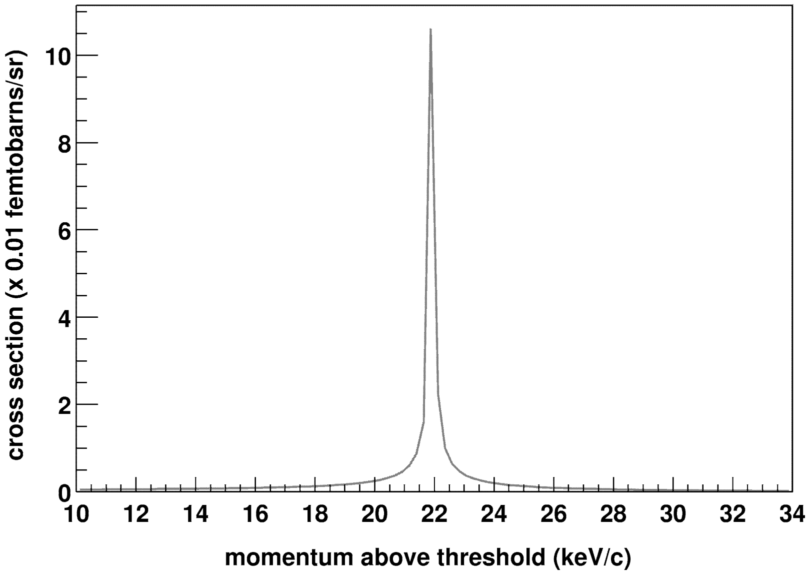

Fig. 1. The cross section (for negative helicity incident electrons) for weak production of the Λ hyperon as a function of the hyperon momentum (keV$/c$ above threshold).

The cross section for the reaction (1) can rather easily be derived using standard procedures [6–9]. It has the form for the case considered here (polarization observables included)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{G^2 m_\nu m_e m_\Lambda p_\Lambda |M|^2}{(2\pi)^2 E_e 8|m_p + E_e - E_e E_\Lambda \cos\theta_{e\Lambda}/p_\Lambda|}. \tag{12}$$

It is interesting to consider this weak production cross section near threshold. Shown in Fig. 1 is the cross section versus hyperon momentum (in units of keV$/c$ above the threshold for the reaction (1)) for a fixed beam energy of 194 MeV, and for the hyperon propagating along the incident beam direction, that is, for fixed angle $\theta_{e\Lambda}$. The momentum of the outgoing Λ is varied as shown with respect to the threshold energy (momentum) of the reaction. The resulting peak is rather narrow as can be seen from the figure. In an actual experiment, the incident electron beam will have a finite width in energy as opposed to a delta function profile assumed in the calculation; at Jefferson Lab, for example, this width, $\delta E/E$ is of the order 10^{-4}. This will broaden the width of the peak in an experimentally measured cross section. Additionally, the hyperons will emerge and be detected over a range of angles, and in the case of solid targets, there will be molecular motion of the target protons which together will also broaden the width of the peak in the cross section.

It has been shown in previous works [6,7,9] that this mild singularity in the expression for the cross section under certain kinematical conditions gives rise to the enhancement in shown in Fig. 1. The differential cross section can be made larger when the denominator of expression (12) gets small. For example, as the angle $\theta_{e\Lambda}$ approaches its maximum value, in cases where the final state hadron is detected but not the final state lepton, the denominator approaches zero and the cross section goes to infinity, theoretically. In actuality, the experimental resolution will be finite so that the denominator does not go exactly to zero and the cross section remains finite. Nevertheless there will be an enhancement in the resulting cross section as has been shown previously by several authors. The magnitude of any measured kinematical enhancement will depend on the experimental resolution. The calculation shown in Fig. 1 assumes an experimental resolution of roughly 10 keV$/c$. The calculation used the value for F_A given in [6]. As seen, reasonable cross sections (for high luminosities) can result making experimental studies feasible. The finite width of the peak of the cross section in the region of the kinematical enhancement should aid the experimental investigation. It should be possible to determine the axial-vector weak form factor in this reaction as a function of momentum transfer for the first time.

It is also interesting to consider the case of the matrix element for the difference of the two helicities:

$$|M|^2_{(h=-1)} - |M|^2_{(h=+1)} = \left|F_A\left(q^2\right)\right|^2 E_e E_\nu E_\Lambda \{2\beta_e - \beta_\Lambda \cos\theta_{e\Lambda} - \beta_e\beta_\Lambda \cos\theta_{e\Lambda} - \beta_\Lambda + \beta_e\beta_\Lambda\} + 2(E_\nu E_\Lambda E_e - E_e \vec{p}_\nu \cdot \vec{p}_\Lambda). \tag{13}$$

The difference in cross section

$$\Delta = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{(h=-1)}} - \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{(h=+1)}} \tag{14}$$

is then

$$\Delta = \frac{2|F_A(q^2)|^2 G^2 m_\nu m_e m_\Lambda p_\Lambda E_e E_\nu \beta_\nu p_\Lambda \cos\theta_{\nu\Lambda}}{(2\pi)^2 E_e 8|m_p + E_e - E_e E_\Lambda \cos\theta_{e\Lambda}/p_\Lambda|}. \tag{15}$$

In a properly designed weak production experiment, the incident electron and lambda hyperon can have their momentum vectors in the same direction (in the lab frame). In that case, there is a difference between the matrix elements when the neutrino velocity is very small ($\beta_\nu \to 0$) and when it is large ($\beta_\nu \to 1$), for nonzero neutrino mass. Under the right experimental conditions and with sufficient resolution, the weak production experiment with the results shown here may be used to place a more stringent limit on the electron neutrino mass. Any β_ν dependence (or dependence on the hyperon velocity) in the laboratory frame will be evidence for nonzero neutrino mass.

To summarize, it should be possible, under the right conditions, to select kinematics where the neutrino velocity in the laboratory frame is nearly zero so that a comparison of cross sections for the two different helicities will show a β_ν dependence, for sufficiently precise hyperon momentum determination. Additionally, in a high statistics experiment with sufficient momentum resolution (in incident beam energy and charged particle detection) it should be possible to get a measurement of the axial-vector weak form factor for the first time at these kinematics.

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