

ORBITALLY EXCITED BARYON SPECTROSCOPY IN THE $1/N_c$ EXPANSION

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The discussion of the 70-plet of negative parity baryons illustrates the large N_c QCD approach to orbitally excited baryons. In the case of the $\ell = 1$ baryons the existing data allows to make numerous predictions to first order in the $SU(3)$ symmetry breaking. New relations between splittings are found that follow from the spin-flavor symmetry breaking. The $\Lambda(1405)$ is well described as a three-quark state and a spin-orbit partner of the $\Lambda(1520)$. Singlet states with higher orbital angular momentum ℓ are briefly discussed.

1 Introduction

In the $N_c \rightarrow \infty$ 't Hooft limit QCD¹ has a contracted dynamical spin-flavor symmetry² $SU(2F)_c$ for the ground state baryons³ (F is the number of light flavors). This is a consequence of unitarity in pion-nucleon scattering in that limit and at fixed energy of order $\mathcal{O}(N_c^0)$ ^{4,5}. In general $SU(2F)$ is broken at $\mathcal{O}(1/N_c)$ but for some observables only at $\mathcal{O}(1/N_c^2)$ ⁵, which makes the $1/N_c$ expansion around the $SU(2F)$ symmetric limit a powerful tool of analysis as it is shown in numerous works^{5,6,7,8,9}. The excited baryons are expected to reveal further the details of strong QCD and are therefore of current theoretical interest^{10,11,12} and also a central goal of lattice QCD studies¹³. In the context of the $1/N_c$ expansion^{6,14,15} this baryon sector is less well understood, the principal reason being that even in the $N_c \rightarrow \infty$ limit the spin-flavor symmetry is broken¹⁴. However, most of the known baryons of negative parity seem to fit very well in the $(3, 70)$ irreducible representation (irrep) of $O(3) \otimes SU(6)$. The $1/N_c$ operator expansion for the full 70-plet can be implemented along the lines developed for two flavors^{14,16} and shows that the leading order spin-flavor breaking ($\mathcal{O}(N_c^0)$) is indeed small, thus justifying $SU(2F)$ as an approximate symmetry useful for classifying excited baryons¹⁰.

The $1/N_c$ expansion is an appropriate tool for the study of some long-standing problems of the quark model in a model independent way. For a long time the quark model in its different versions has been the preferred framework for investigating the properties of baryons¹⁷. Despite the success of this model in reproducing general features of the spectrum, it is not a complete representation of QCD. One consequence of this incompleteness is that, in

those cases where the quark model does not agree with phenomenology, such as the problem of the mass splittings between spin-orbit partners in the negative parity baryons (spin-orbit puzzle), it is not clear whether the problem is due to the quark model itself or to specific dynamical properties of the states involved. This situation is clarified in the $1/N_c$ expansion where the presence of other operators, the leading one being of $\mathcal{O}(N_c^0)$, solves the contradictions that arise in the quark model when the spin-orbit interaction is considered.

2 The space of states

The states in the $(3, 70)$ of $O(3) \otimes SU(6)$ decompose into five octets (${}^{2S+1}d_J = {}^2\mathbf{8}_{1/2}, {}^2\mathbf{8}_{3/2}, {}^4\mathbf{8}_{1/2}, {}^4\mathbf{8}_{3/2}$ and ${}^4\mathbf{8}_{5/2}$, where S is the total spin, d the degeneracy of the $SU(3)$ irrep and J is the total angular momentum), two decuplets (${}^2\mathbf{10}_{1/2}$ and ${}^2\mathbf{10}_{3/2}$) and two singlets (${}^2\mathbf{1}_{1/2}$ and ${}^2\mathbf{1}_{3/2}$). An explicit representation of these states can be obtained from a tensor product of quark states. Coupling an orbitally excited quark with $\ell = 1$ to $N_c - 1$ s-wave quarks that constitute a spin-flavor symmetric core gives the following states with core spin S^c

$$\begin{aligned} & |J, J_z ; S ; (\lambda, \mu), Y, I, I_z ; S^c \rangle = \\ & = \sum \left(\begin{array}{c} S \quad \ell \\ S_z \quad m \end{array} \middle| \begin{array}{c} J \\ J_z \end{array} \right) \left(\begin{array}{c} S^c \quad \frac{1}{2} \\ S_z^c \quad s_z \end{array} \middle| \begin{array}{c} S \\ S_z \end{array} \right) \left(\begin{array}{c} (\lambda^c, \mu^c) \quad (1, 0) \\ (Y^c, I^c, I_z^c) \quad (y, \frac{1}{2}, i_z) \end{array} \middle| \begin{array}{c} (\lambda, \mu) \\ (Y, I, I_z) \end{array} \right) \\ & \quad \left| \begin{array}{c} S^c \quad (\lambda^c, \mu^c) \\ S_z^c \quad (Y^c, I^c, I_z^c) \end{array} \right\rangle \left| \begin{array}{c} \frac{1}{2} \quad (1, 0) \\ s_z \quad (y, \frac{1}{2}, i_z) \end{array} \right\rangle \left| \begin{array}{c} \ell \\ m \end{array} \right\rangle . \end{aligned} \quad (1)$$

The (λ, μ) labels indicate the $SU(3)$ irrep, Y is the hypercharge, I the isospin and J_z, I_z the obvious projections. For arbitrary N_c the $(3, 70)$ states are embedded in a larger multiplet and are taken to have strangeness of order N_c^0 . From the decomposition of the $SU(6)$ symmetric representation into irreps of $SU(2) \otimes SU(3)$ the relations $\lambda^c + 2\mu^c = N_c - 1$ and $\lambda^c = 2S^c$ follow. They are the generalization of the $I = J$ rule well known for two flavors. The $(3, 70)$ states are in the mixed symmetric irrep of $SU(6)$, which for the octets with $S = 1/2$ corresponds to a linear combination of states of the form of Eq.(1)

$$|{}^2\mathbf{8} \rangle = -\frac{\sqrt{3}}{2} \sqrt{1 - \frac{1}{N_c}} |S^c = 0 \rangle + \frac{1}{2} \sqrt{1 + \frac{3}{N_c}} |S^c = 1 \rangle , \quad (2)$$

where the coefficients can be obtained diagonalizing the quadratic Casimir operator of $SU(6)$

$$C_{SU(6)}^{(2)} = 2 G_{ha} G_{ha} + \frac{1}{2} C_{SU(3)}^{(2)} + \frac{1}{3} C_{SU(2)} . \quad (3)$$

The core state for the 48 and 210 irreps is $|1, (2, \frac{N_c-3}{2}) \rangle$, in the 21 irrep the core state is $|0, (0, \frac{N_c-1}{2}) \rangle$ and the corresponding states given by Eq.(1) are already in the mixed representation of $SU(6)$.

The physical states are in general a mixing of states with the same J . In the $SU(3)$ symmetric limit only the octets with $J = 1/2$ and $J = 3/2$ mix. The mixing angle θ_{2J} is defined as

$$\begin{pmatrix} 8_J \\ 8'_J \end{pmatrix} = \begin{pmatrix} \cos \theta_{2J} & \sin \theta_{2J} \\ -\sin \theta_{2J} & \cos \theta_{2J} \end{pmatrix} \begin{pmatrix} {}^28_J \\ {}^48_J \end{pmatrix}. \quad (4)$$

3 Construction of operators

A basis of mass operators can be built using the generators of $O(3) \otimes SU(2F)^{14}$. A generic n -body mass operator has the general structure

$$O^{(n)} = \frac{1}{N_c^{n-1}} O_\ell O_q O_c, \quad (5)$$

where the factors O_ℓ , O_q , and O_c can be expressed in terms of products of generators of orbital angular momentum (ℓ_i), spin-flavor of the excited quark (s_i, t_a and $g_{ia} \equiv s_i t_a$) and spin-flavor of the core (S_i^c, T_a^c and $G_{ia}^c \equiv \sum_{m=1}^{N_c-1} s_i^{(m)} t_a^{(m)}$), respectively. The explicit $1/N_c$ factors originate in the $n-1$ gluon exchanges required to give rise to an n -body operator. The matrix elements of operators may also carry a nontrivial N_c dependence due to coherence effects^{2,4}: for the states considered, G_{ia}^c ($a = 1, 2, 3$) and T_a^c have matrix elements of $\mathcal{O}(N_c)$, while the rest of the generators have matrix elements of higher order.

In the case of the $\ell = 1$ baryons the highest orbital angular momentum operator that contributes is the rank 2 tensor

$$\ell_{hk}^{(2)} = \frac{1}{2} \{\ell_h, \ell_k\} - \frac{\ell^2}{3} \delta_{hk}. \quad (6)$$

4 Counting the number of operators

For $N_c = 3$ and in the $SU(3)$ symmetric limit there are eleven independent quantities: nine masses (one for each $SU(3)$ multiplet) and two mixing angles θ_1 and θ_3 , which correspond to the mixing of the 28_J and 48_J octets with $J = 1/2$ and $J = 3/2$. This leads to the basis of eleven $SU(3)$ -singlet mass operators which are listed in Table 1. Further information about the structure of these operators can be obtained from the $SU(3)$ singlets in the

decomposition of

$$\overline{70} \otimes 70 = 4(1, 1) \oplus 5(1, 3) \oplus 2(1, 5) \oplus (1, 7) \oplus \dots \quad (7)$$

which shows that there are four operators with $\ell = 0$ (O_1, O_6, O_7, O_{10}), five operators with $\ell = 1$ ($O_2, O_4, O_5, O_9, O_{11}$), two operators with $\ell = 2$ (O_3, O_8) and one operator with $\ell = 3$ that does not contribute in the case of interest. In terms of $1/N_c$ one operator is of $\mathcal{O}(N_c)$, namely the identity, $O_{2,3,4}$ are of $\mathcal{O}(N_c^0)$, and the remaining seven $O_{5,\dots,11}$ are of $\mathcal{O}(1/N_c)$, one of which is the very important hyperfine operator. They are a simple generalization of those known for two flavors, although the calculation of their matrix elements is in general more involved.

When $SU(3)$ breaking is included with isospin conservation, the number of independent observables raises up to 50, of which 30 are masses and 20 are mixing angles. However, if $SU(3)$ symmetry breaking is restricted to linear order in quark masses only isosinglet octet operators can appear, and the number of independent observables is reduced to 35 (21 masses and 14 mixing angles) implying 24 linearly independent octet mass operators. As a consequence of this reduction several mass relations exist, among them there is a Gell-Mann Okubo relation for each octet and an equal spacing rule for each decuplet. The octet contributions are proportional to $\epsilon \propto (m_s - m_{u,d})/\nu_H$ where ν_H is a typical hadronic mass scale, for instance m_ρ ; for $N_c = 3$ the quantity ϵ counts as of the same order as $1/N_c$. Explicit construction shows that up to order $\mathcal{O}(\epsilon N_c^0)$ only a small subset of independent octet operators B_i appears. Since such octet operators are isospin singlets, it is possible to modify them by adding singlet operators so that the resulting operators vanish in the subspace of non-strange baryons. This procedure of improving the flavor breaking operators may change the $1/N_c$ counting: for instance, after improving T_8 with the identity operator O_1 the resulting operator is of order N_c^0 . Indeed, the improved operators give the splitting due to $SU(3)$ breaking with respect to the non-strange baryons in each multiplet, and they must be of zeroth order or higher in $1/N_c$ for states with strangeness of order N_c^0 . The four improved flavor breaking operators \bar{B}_1 through \bar{B}_4 that remain at $\mathcal{O}(\epsilon N_c^0)$ when $N_c = 3$ are shown in Table 1.

5 Fitting the data

As a result of the above analysis the 70-plet mass operator up to $\mathcal{O}(\epsilon N_c^0)$ has the most general form:

Table 1. Operator list and best fit coefficients¹⁰.

Operator	Fitted coef. [MeV]
$O_1 = N_c \mathbf{1}$	$c_1 = 449 \pm 2$
$O_2 = l_h s_h$	$c_2 = 52 \pm 15$
$O_3 = \frac{3}{N_c} l_{hk}^{(2)} g_{ha} G_{ka}^c$	$c_3 = 116 \pm 44$
$O_4 = \frac{4}{N_c+1} l_h t_a G_{ha}^c$	$c_4 = 110 \pm 16$
$O_5 = \frac{1}{N_c} l_h S_h^c$	$c_5 = 74 \pm 30$
$O_6 = \frac{1}{N_c} S_h^c S_h^c$	$c_6 = 480 \pm 15$
$O_7 = \frac{1}{N_c} s_h S_h^c$	$c_7 = -159 \pm 50$
$O_8 = \frac{1}{N_c} l_{hk}^{(2)} s_h S_k^c$	$c_8 = 6 \pm 110$
$O_9 = \frac{1}{N_c^2} l_h g_{ka} \{S_k^c, G_{ha}^c\}$	$c_9 = 213 \pm 153$
$O_{10} = \frac{1}{N_c^2} t_a \{S_h^c, G_{ha}^c\}$	$c_{10} = -168 \pm 56$
$O_{11} = \frac{1}{N_c^2} l_h g_{ha} \{S_k^c, G_{ka}^c\}$	$c_{11} = -133 \pm 130$
$\bar{B}_1 = t_8 - \frac{1}{2\sqrt{3}N_c} O_1$	$d_1 = -81 \pm 36$
$\bar{B}_2 = T_8^c - \frac{N_c-1}{2\sqrt{3}N_c} O_1$	$d_2 = -194 \pm 17$
$\bar{B}_3 = \frac{1}{N_c} d_{8ab} g_{ha} G_{hb}^c + \frac{N_c^2-9}{16\sqrt{3}N_c^2(N_c-1)} O_1 +$ $+ \frac{1}{4\sqrt{3}(N_c-1)} O_6 + \frac{1}{12\sqrt{3}} O_7$	$d_3 = -150 \pm 301$
$\bar{B}_4 = l_h g_{h8} - \frac{1}{2\sqrt{3}} O_2$	$d_4 = -82 \pm 57$

$$M_{70} = \sum_{i=1}^{11} c_i O_i + \sum_{i=1}^4 d_i \bar{B}_i \quad , \quad (8)$$

where c_i and d_i are unknown coefficients which are reduced matrix elements (of a QCD operator) that are not determined by the spin-flavor symmetry. Calculating these reduced matrix elements is equivalent to solve QCD in this baryon sector. Fortunately, the experimental data available in the case of the 70-plet is enough to obtain them by making a fit¹⁰. The resulting values are given in Table 1. The natural size of coefficients associated with the singlet operators is set by the coefficient of O_1 , and is about 500 MeV, while the natural size for the coefficients associated with octet operators is roughly ϵ times 500 MeV. The experimental masses (three or more stars status in the the Particle Data listing¹⁸) shown in Table 2 together with the two leading order mixing angles $\theta_1 = 0.61$, $\theta_3 = 3.04$ ^{19,20} are the 19 empirical quantities used in the fit. The resulting χ^2 per degree of freedom turns out to be $\chi^2/4 = 1.29$. The best fit masses and state compositions are displayed in Table 2.

6 Splitting relations

Because at $\mathcal{O}(\epsilon N_c^0)$ there are only four flavor breaking operators, it is possible to find new mass splitting relations which are independent of the coefficients d_i . These relations involve states in different $SU(3)$ multiplets. Of particular interest are the following five relations that result when the operator \bar{B}_3 is neglected (from the fit it is apparent that \bar{B}_3 gives very small contributions):

$$\begin{aligned}
9(s_{\Sigma_{1/2}} + s_{\Sigma'_{1/2}}) + 21s_{\Lambda_{5/2}} &= 17(s_{\Lambda_{1/2}} + s_{\Lambda'_{1/2}}) + 5s_{\Sigma_{5/2}} , \\
2(s_{\Lambda_{3/2}} + s_{\Lambda'_{3/2}}) &= 3s_{\Lambda_{5/2}} + s_{\Sigma_{5/2}} , \\
18(s_{\Sigma_{3/2}} + s_{\Sigma'_{3/2}}) + 33s_{\Lambda_{5/2}} &= 28(s_{\Lambda_{1/2}} + s_{\Lambda'_{1/2}}) + 13s_{\Sigma_{5/2}} , \\
9s_{\Sigma''_{1/2}} &= s_{\Lambda_{1/2}} + s_{\Lambda'_{1/2}} + 3s_{\Lambda_{5/2}} + 4s_{\Sigma_{5/2}} , \\
18s_{\Sigma''_{3/2}} + 3s_{\Lambda_{5/2}} &= 8(s_{\Lambda_{1/2}} + s_{\Lambda'_{1/2}}) + 5s_{\Sigma_{5/2}} . \tag{9}
\end{aligned}$$

Here $s_{\mathcal{B}_i}$ is the mass splitting between the baryon \mathcal{B}_i and the non-strange baryons in the $SU(3)$ multiplet to which it belongs. These relations are independent of mixings because they result from relations among traces of the octet operators. If \bar{B}_3 is not neglected there are instead four relations. The first relation in equation (9) predicts the $\Sigma_{1/2}$ to be 103 MeV above the $N_{1/2}$, consistent with the $\Sigma(1620)$, a two star state that is not included as an input to the fit. Each of the remaining relations makes a similar prediction for other states but requires further experimental data to be tested.

7 The singlet Lambdas

The singlet Lambdas are the two lightest states of the 70-plet, something that has its natural explanation in the dominant effect of the hyperfine interaction²¹. Although spin-flavor symmetry is broken at $\mathcal{O}(N_c^0)$, it is apparent from our fit that the $\mathcal{O}(N_c^0)$ operators are dynamically suppressed as their coefficients are substantially smaller than the natural size. It turns out that the chief contribution to spin-flavor breaking stems from the $\mathcal{O}(1/N_c)$ hyperfine operator O_6 , as in the ground state baryons. Since O_6 is purely a core operator, the gross spin-flavor structure of levels is determined by the two possible core states. In particular, the two singlet Λ s are not affected by O_6 , while the other states are moved upwards, explaining in a transparent way the lightness of these two states. Indeed, by keeping only O_1 and O_6 the 28 masses are 1510 MeV, the 48 and 210 masses are 1670 MeV, and the 21 masses are left at the bottom with 1350 MeV. This clearly shows the dominant pattern of spin-flavor breaking observed in the 70-plet.

Table 2. Masses and spin-flavor content as predicted by the large N_c analysis¹⁰. Also given are the empirical masses and those obtained in a quark model (QM) calculation²⁰.

State	Masses [MeV]			Spin-flavor content			
	Expt.	Large N_c	QM	² ₁	² ₈	⁴ ₈	² ₁₀
$N_{1/2}$	1538 ± 18	1541	1490		0.82	0.57	
$\Lambda_{1/2}$	1670 ± 10	1667	1650	-0.21	0.90	0.37	
$\Sigma_{1/2}$	(1620)	1637	1650		0.52	0.81	0.27
$\Xi_{1/2}$		1779	1780		0.85	0.44	0.29
$N_{3/2}$	1523 ± 8	1532	1535		-0.99	0.10	
$\Lambda_{3/2}$	1690 ± 5	1676	1690	0.18	-0.98	0.09	
$\Sigma_{3/2}$	1675 ± 10	1667	1675		-0.98	-0.01	-0.19
$\Xi_{3/2}$	1823 ± 5	1815	1800		-0.98	0.03	-0.19
$N'_{1/2}$	1660 ± 20	1660	1655		-0.57	0.82	
$\Lambda'_{1/2}$	1785 ± 65	1806	1800	0.10	-0.38	0.92	
$\Sigma'_{1/2}$	1765 ± 35	1755	1750		-0.83	0.54	0.17
$\Xi'_{1/2}$		1927	1900		-0.46	0.87	0.18
$N'_{3/2}$	1700 ± 50	1699	1745		-0.10	-0.99	
$\Lambda'_{3/2}$		1864	1880	0.01	-0.09	-0.99	
$\Sigma'_{3/2}$		1769	1815		0.01	(-0.57)	(-0.82)
$\Xi'_{3/2}$		1980	1985		-0.02	(-0.57)	(-0.82)
$N_{5/2}$	1678 ± 8	1671	1670			1.00	
$\Lambda_{5/2}$	1820 ± 10	1836	1815			1.00	
$\Sigma_{5/2}$	1775 ± 5	1784	1760			1.00	
$\Xi_{5/2}$		1974	1930			1.00	
$\Delta_{1/2}$	1645 ± 30	1645	1685				1.00
$\Sigma''_{1/2}$		1784	1810		-0.14	-0.31	0.94
$\Xi''_{1/2}$		1922	1930		-0.14	-0.31	0.94
$\Omega_{1/2}$		2061	2020				1.00
$\Delta_{3/2}$	1720 ± 50	1720	1685				1.00
$\Sigma''_{3/2}$		1847	1805		-0.19	(-0.80)	(0.57)
$\Xi''_{3/2}$		1973	1920		-0.19	(-0.80)	(0.57)
$\Omega_{3/2}$		2100	2020				1.00
$\Lambda''_{1/2}$	1407 ± 4	1407	1490	0.97	0.23	0.04	
$\Lambda''_{3/2}$	1520 ± 1	1520	1490	0.98	0.18	-0.01	

The long standing problem in the quark model of reconciling the large $\Lambda(1520) - \Lambda(1405)$ splitting with the splittings between the other spin-orbit partners in the 70-plet is resolved in the large N_c analysis. The singlet Λ s receive contributions to their masses from O_1 and $\ell \cdot s$ while the rest of the operators give vanishing contributions because the core of the singlets carries $S^c = 0$. The splitting between the singlets is, therefore, a clear display of the spin-orbit coupling. The problem with the splittings between spin-orbit

Table 3. Matrix elements of O_2 and O_4 .

	$8_{1/2}$	$8'_{1/2}$	$8_{1/2} - 8'_{1/2}$		
O_2	$-\frac{2N_c-3}{3N_c}$	$-\frac{5}{6}$	$-\frac{1}{3\sqrt{2}}\sqrt{1+\frac{3}{N_c}}$		
O_4	$\frac{2}{9}\frac{(N_c+3)(3N_c-2)}{N_c(N_c+1)}$	$\frac{5}{18}\frac{3N_c+1}{N_c+1}$	$\frac{1}{9\sqrt{2}}\frac{3N_c-5}{N_c+1}\sqrt{1+\frac{3}{N_c}}$		
	$8_{3/2}$	$8'_{3/2}$	$8_{3/2} - 8'_{3/2}$		
O_2	$\frac{2N_c-3}{6N_c}$	$-\frac{1}{3}$	$-\frac{\sqrt{5}}{6}\sqrt{1+\frac{3}{N_c}}$		
O_4	$-\frac{1}{9}\frac{(N_c+3)(3N_c-2)}{N_c(N_c+1)}$	$\frac{1}{9}\frac{3N_c+1}{N_c+1}$	$\frac{\sqrt{5}}{18}\frac{3N_c-5}{N_c+1}\sqrt{1+\frac{3}{N_c}}$		
	$8_{5/2}$	$10_{1/2}$	$10_{3/2}$	$1_{1/2}$	$1_{3/2}$
O_2	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$	-1	$\frac{1}{2}$
O_4	$-\frac{1}{6}\frac{3N_c+1}{N_c+1}$	$-\frac{1}{9}\frac{3N_c+7}{N_c+1}$	$\frac{1}{18}\frac{3N_c+7}{N_c+1}$	0	0

partners in the non-singlet sector, illustrated by the fact that the $\ell \cdot s$ operator gives a contribution to the $\Delta_{1/2} - \Delta_{3/2}$ splitting that is of opposite sign of what is observed, is now solved by the presence of the operators O_4 , O_5 , O_9 and O_{11} , with the contribution from O_4 being the dominant one in accordance with the $1/N_c$ counting. While O_2 and O_4 are of order N_c^0 separately, their sum $O_2 + O_4$ is of order $1/N_c$ for the non-singlet states, as can be seen from the explicit expressions for their matrix elements given in Table 3. O_4 is therefore the natural operator that cancels the effect of O_2 at large N_c . This also leaves O_3 as the dominant contribution to the leading mixing angles θ_1 , θ_3 . The analytic expressions for the rest of the operators will be given elsewhere.

In principle, a similar situation would be expected for states with one quark excited at higher angular momentum $\ell > 1$. It is interesting to note that the splittings of the observed states ($\Lambda(1405)\frac{1}{2}^-$, $\Lambda(1520)\frac{3}{2}^-$), ($\Lambda(1890)\frac{3}{2}^+$, $\Lambda(2110)\frac{5}{2}^+$), ($\Lambda(1830)\frac{5}{2}^-$, $\Lambda(2100)\frac{7}{2}^-$), ($\Lambda(2020)\frac{7}{2}^+$, $\Lambda(2350)\frac{9}{2}^+$) are in a relation $3.0 : 5.7 : 7.0 : 8.6$ while the $\ell \cdot s$ operator predicts $3.0 : 5.0 : 7.0 : 9.0$. Thus, the observed data also hints that c_2 may be of approximately the same size in different spin-flavor multiplets. Further support to this picture can be drawn from scaling down to the strange sector the mass splitting between the ($\Lambda_c(2593)\frac{1}{2}^-$, $\Lambda_c(2625)\frac{3}{2}^-$) as suggested by Isgur²².

8 Conclusions

The $1/N_c$ expansion provides a systematic approach to the spectroscopy of the excited baryons. In the case of the negative parity $\ell = 1$ baryons it successfully describes the existing data and, to the order considered, also makes numerous testable predictions. In addition to the well known Gell-Mann-Okubo and equal spacing relations, new splitting relations between different multiplets that follow from the spin-flavor symmetry have been found. The $\Lambda(1405)$ is well described as a three-quark state and the spin-orbit partner of the $\Lambda(1520)$. Available experimental data for higher ℓ states and extrapolations from the charmed sector also seem to hint at the presence of a spin-orbit interaction. Effective interactions that correspond to flavor quantum number exchanges, such as the ones mediated by the operators O_3 and O_4 , are apparently needed. Although the corresponding coefficients seem to be dynamically suppressed their relevance shows up in the well established finer effects, namely mixings and splittings between non-singlet spin-orbit partners. These interactions are not accounted for in the standard quark model based on one gluon exchange.

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