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# Quenched Chiral Log and Light Quark Mass from Overlap Fermions * †

Terrence Draper [a], Shao-Jing Dong [a], Ivan Horváth [a], Frank Lee [b c], Keh-Fei Liu [a], Nilmani Mathur [a], and Jianbo Zhang [d]

[a]Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, USA

[b]Center for Nuclear Studies, Dept. of Physics, George Washington Univ., Washington, DC 20052, USA

[c]Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA

[d]CSSM and Dept. of Physics and Math. Physics, Univ. of Adelaide, Adelaide, SA 5005, Australia

We study the quenched chiral behavior of the pion with mass as low as $\approx 180$ MeV. The calculation is done on a quenched lattice of size $16^3 \times 28$ and $a = 0.2$ fm with 80 configurations using overlap fermions and an improved gauge action. Using an improved constrained curve fitting technique, we find that the ground state pseudoscalar mass versus bare quark mass behavior is well controlled with small statistical errors; this permits a reliable fit of the quenched chiral log effects, a determination of the chiral log parameter ($\delta = 0.26(3)$), and an estimate of the renormalized mass of the light quark ($m^{\overline{MS}}(\mu = 2\,\text{GeV}) = 3.7(3)\,\text{MeV}$).

## 1. Simulation Details

Using a $\beta = 2.264$ renormalization-group-improved Iwasaki [1] gauge action, we study the chiral properties of hadrons on a $16^3 \times 28$ lattice with the overlap fermion [2,3] and massive overlap operator [4–6]

$$D(m_0) = (\rho + \frac{m_0 a}{2}) + (\rho - \frac{m_0 a}{2})\gamma_5 \epsilon(H)$$

where $\epsilon(H) = H/\sqrt{H^2}$, $H = \gamma_5 D_w$, and $D_w$ is the usual Wilson fermion operator, except with a negative mass parameter $-\rho = 1/2\kappa - 4$ in which $\kappa_c < \kappa < 0.25$; we take $\kappa = 0.19$ in our calculation which corresponds to $\rho = 1.368$.

We use the optimal partial fraction expansion with a 14th-order Zolotarev approximation of the matrix sign function [7]; the sign function approximated to better than 3 parts in $10^{10}$. As the conjugate-gradient inverter accommodates multi-mass [8] we obtain the quark propagator at 26 masses including 18 masses at or below the strange quark mass with less than 10% overhead.

## 2. Pion Decay Constant and $Z_A$

The renormalized pion decay constant is

$$f_\pi^{(R)}\sqrt{2} = Z_A f_\pi^{(U)}\sqrt{2} = \frac{f_P^{(U)}}{(m_\pi^2/2m_0)} = \frac{\sqrt{2m_\pi A_{PP}}}{(m_\pi^2/2m_0)}$$

where $m_0$ is the unrenormalized quark mass, $f_P^{(U)} = \sqrt{2m_\pi A_{PP}}$ is the unrenormalized pseudoscalar decay constant, and $A_{PP}$ is the amplitude of the two-point local-local correlator, $G_{PP}(t) \longrightarrow A_{PP}(e^{-m_\pi t} + e^{-m_\pi (T-t)})$. Since $f_\pi^{(R)}$ is free of quenched chiral logs, and since we determine it quite precisely, we use $f_\pi^{(R)}$ to set the scale and obtain $a^{-1} = 0.978(5)\,\text{GeV}$.

As a bonus, we obtain the axial-vector renormalization constant, $Z_A$, via the axial Ward identity $Z_A \partial_\mu A_\mu = 2Z_m m_0 Z_P P$ and the relations $Z_m^{-1} = Z_P = Z_S$ protected by the chiral symmetry of the overlap fermion. Thus $Z_A = \frac{2m_0}{m_\pi}\sqrt{\frac{A_{PP}}{A_{A_4P}}}$ which is plotted in Fig. 1.

A covariant polynomial fit yields

$$Z_A = 1.85(1) - 0.60(12) m_0 \Lambda_{\text{QCD}} a^2 + 0.59(4) m_0^2 a^2 \quad ; \Lambda_{\text{QCD}} a = 0.25$$

so $\mathcal{O}(m_0 \Lambda_{\text{QCD}} a^2)$ and $\mathcal{O}(m_0^2 a^2)$ terms are small.

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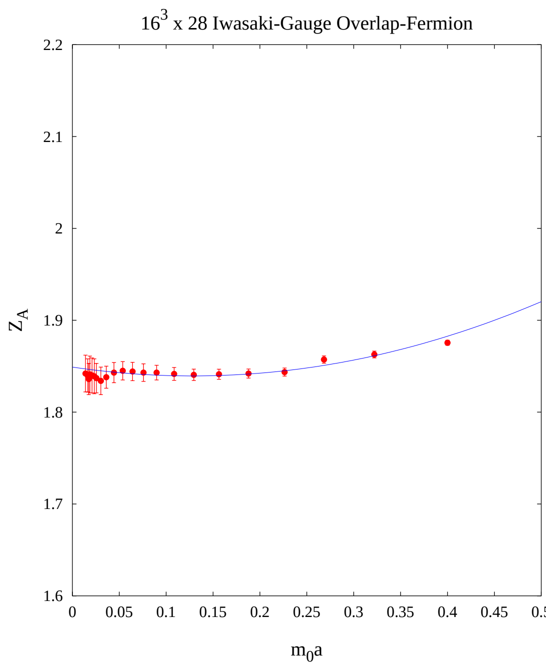


Figure 1. $Z_A$ *vs.* bare mass, $m_0 a$.

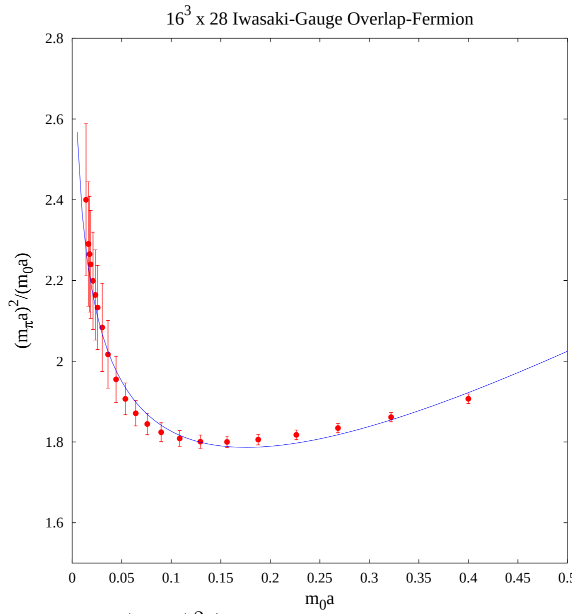


Figure 2. $(m_\pi a)^2/m_0 a$ *vs.* bare quark mass, $m_0 a$.

## 3. Pion Mass and Chiral Logs

Our plot of $(m_\pi a)^2/m_0 a$ in Fig. 2, with 18 data points at or below the strange quark mass, dramatically reveals the presence of the quenched chiral log. We fit $m_\pi^2 a^2$ to the form [9,10]

$$m_\pi^2 a^2 \quad = \quad A m_0 a \left\{ 1 - \delta \left[ \ln \left( \frac{A m_0 a}{\Lambda_\chi^2 a^2} \right) + 1 \right] \right\}$$

using a single multi-parameter (weakly) constrained fit (the constraint lifts the degeneracy of the three-parameter fit). We obtain simultaneously (the preliminary results) $\delta = 0.26(3)$ and $\Lambda_\chi a = 1.1(1)$, which *are quite stable* against changing the window, $[m_\pi^{\min}, m_\pi^{\max}]$, of masses in the fit provided (a) $m_\pi^{\min} < 250\,\mathrm{MeV}$; thereafter, $\delta$ monotonically decreases rapidly with increasing $m_\pi^{\min}$ (e.g. 0.14 for 300 MeV, 0.06 for 400 MeV), and (b) $m_\pi^{\max} < 350\,\mathrm{MeV}$; adding a term $B m_0^2 a^2$ to the fit formula increases the region of stability to $m_\pi^{\max} > 700\,\mathrm{MeV}$.

The pseudoscalar decay constant $f_P$ has similar behavior and fit results to $(m_\pi a)^2/m_0 a$. (This is automatic for overlap fermions since $Z_m^{-1} = Z_P = Z_S$ implies $f_P = f_\pi^{(R)} \sqrt{2} m_\pi^2 / 2 m_0$.)

Our value of $\delta$ is significantly higher than other recent values [10–14] most of which are well below the prediction of $\delta = 0.183$ from the quenched $\eta'$ loop (using the Witten-Veneziano model of the $\eta'$ mass) in chiral perturbation theory; however, we have exact chiral symmetry and, to date, use the largest spatial volume and probe the lightest pion masses. Indeed, the effect of the chiral log makes the data points rise as the pion mass drops below $m_\pi \approx$ 400–450 MeV. Notice that if one restricts the data to pion masses in the range of 350–400 MeV to 700 MeV, then the curve can be well approximated by a constant within errors. Equivalently, a plot of $m_\pi^2$ versus $m_0$ over this range would show no deviation from linearity, as is also seen in [6,15]. The effects of chiral logs are seen only at lower pion mass; one is unlikely to see the effect at all without data below $m_\pi \approx 400\,\mathrm{MeV}$. With adequate spatial volume, the extraction of $\delta$ would give a low value without data below $m_\pi \approx 250\,\mathrm{MeV}$. Our spatial lattice size is $L = 3.2\,\mathrm{fm}$ so finite-volume effects are expected to be small; our lightest (unrenormalized) quark mass of $\approx 14\,\mathrm{MeV}$ corresponds to a pion of about 180 MeV. With a lattice scale of $a = 0.202(1)\,\mathrm{fm}$ (set from $f_\pi$) then $L = 3.2\,\mathrm{fm}$ and $(L m_\pi)_{\text{lightest}} \approx 2.9$.

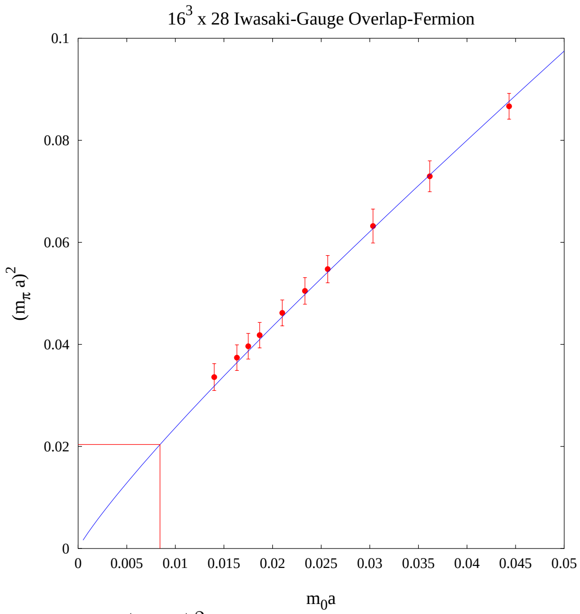


Figure 3. $(m_\pi a)^2$ *vs.* pion mass, $m_\pi a$. The solid curve is the the chiral log fit. The solid lines indicate the intercepts at the physical values.

## 4. Quark Masses

To renormalize our estimates of the quark mass, we follow [6] to determine the renormalization constants $Z_m^{-1} = Z_S = Z_P$ and thus $m^{RGI} = Z_m(g_0)m_0(g_0)$: The value of the bare quark mass $m_0 a$ that reproduces a given fiducial pseudoscalar mass (through $x_{\rm ref} = (r_0 m_\pi)^2$, with $r_0/a = 2.885(18)$ for our lattice) is obtained by interpolation of our data for the pion mass. From this we obtain $Z_m^{-1} = Z_S$ at three values of quark mass (the lowest two quark masses are roughly half-strange and strange). The operators and action are $\mathcal{O}(a)$ improved. For $Z_A$ we find small $\mathcal{O}(m_0\Lambda_{\rm QCD}a^2)$ and $\mathcal{O}(m_0^2a^2)$ dependence as well; $Z_A$ is well approximated (to $\approx 1\%$) by a constant up through the strange mass region. Thus we fit $Z_m^{-1} = Z_S$ to a constant and obtain $Z_S = 1.61(7)$ as a first approximation.

Our lightest pion mass is $\approx 180\,\text{MeV}$ so we needn't extrapolate far to obtain $m_0 a$ at the physical pion mass (Fig. 3). Furthermore, exact chiral symmetry (zero residual mass) and the form of the fit function as determined by quenched ChPT provide an end point at zero mass.

Using $m^{RGI} = m_0/Z_S$ and, from the 4-loop calculation with $N_f = 0$ [16], $Z_S^{\overline{MS}} = Z_S/0.72076$, we obtain (very preliminary results) for $(m_u + m_d)/2$: $m^{RGI} = 5.1(4)\,\text{MeV}$ and

$$m^{\overline{MS}}(\mu = 2\,\text{GeV}) \quad = \quad 3.7(3)\,\text{MeV}$$

Last year's result [17] from a $20^4$ lattice with a Lüscher-Weisz gauge action was $4.5(3)\,\text{MeV}$. (The result quoted in the proceedings was an average of this number with an attempt to subtract out the effects of the chiral log.) A recent estimate [18] of the world average is 4.5(6)MeV.

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