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TWO DIMENSIONAL SIMULATIONS OF MULTIPASS BEAM BREAKUP

G. A. Krafft and J. J. Bisognano
Continuous Electron Beam Accelerator Facility
12070 Jefferson Avenue
Newport News, VA 23606

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Summary

A vectorized two dimensional beam breakup code, TDBBU, is used to simulate bunch dynamics in recirculating accelerators. First, the code is briefly described, including effects needed to accurately model the CEBAF accelerator. Next, various code tests are sketched. Finally, the multipass beam breakup threshold current is determined to be from 10-20 mA for the CEBAF accelerator, a result in agreement with other calculations of the threshold current.

Description of TDBBU

TDBBU incorporates several characteristics of the CEBAF accelerator[1]. The following options are examples:

- Multipass accelerators may be simulated.
- Accelerators with several linac segments may be simulated.
- The simulations include effects due to differing path lengths between linac segments of the different energy beams.
- The current of successive bunches may be varied in a programed manner.

A two dimensional calculation is undertaken to test whether a 90° rotation of the beam during recirculation can appreciably affect the beam breakup thresholds.

The bunch phase space vector is denoted by

$$\mathbf{V}_i^I(k) = \begin{pmatrix} x_i^I(k) \\ p_{x_i}^I(k) \\ y_i^I(k) \\ p_{y_i}^I(k) \end{pmatrix}$$

where $x_i^I(k)$ and $y_i^I(k)$ are the coordinates of bunch k after the ith cavity on the Ith pass and likewise for the momentum. The symbol T_{ml}^{JI} represents the transport matrix between cavity l on the Ith pass and cavity m on the Jth pass.

First consider the case of a single pair of dipole deflecting modes of angular frequency ω in each cavity. One mode has a magnetic field directed along the y-axis yielding a deflection along the x-axis, and the other mode causes a deflection in the y-direction. The fundamental equations of the dynamics are[2,3]

$$\mathbf{V}_i^I(k) = T_{i,i-1}^{II}\mathbf{V}_{i-1}^I(k) + \begin{pmatrix} 0 \\ I\frac{eZ''\ell\tau}{2Q}\sum_{l=1}^{k+t_i(I)-1} s_{k+t_i(I)-l}\overline{x}_i(l) \\ 0 \\ I\frac{eZ''\ell\tau}{2Q}\sum_{l=1}^{k+t_i(I)-1} s_{k+t_i(I)-l}\overline{y}_i(l) \end{pmatrix}$$

$$\mathbf{V}_1^I(k) = T_{1N}^{I,I-1}\mathbf{V}_N^{I-1}(k) + \begin{pmatrix} 0 \\ I\frac{eZ''\ell\tau}{2Q}\sum_{l=1}^{k+t_1(I)-1} s_{k+t_1(I)-l}\overline{x}_1(l) \\ 0 \\ I\frac{eZ''\ell\tau}{2Q}\sum_{l=1}^{k+t_1(I)-1} s_{k+t_1(I)-l}\overline{y}_1(l) \end{pmatrix}$$

where N is the number of cavities, I is the average current of the beam, $Z''\ell$ is the transverse coupling impedance of the modes, τ is the bunch repetition time,

$$s_m = e^{-\frac{m\omega\tau}{2Q}}\sin(m\omega\tau)$$

and the exciting current is for example

$$\overline{x}_i(l) = \sum_{J=1}^{N_p} x_i^J(l - t_i(J))$$

where N_p is the number of passes and $t_i(I)$ gives the recirculation time as follows

$$t_i(1) = 0$$

$t_i(2)$ = number of RF periods until 2nd crossing of cavity i

$t_i(3)$ = number of RF periods until 3rd crossing of cavity i

etc.

In general $t_i(I)$ depends on i because different bunches coincide at different cavities due to path length effects in the optics between linac segments. However, within a linac segment the time delays do not depend on cavity number.

In TDBBU one iteration simulates the machine bunch dynamics through one RF cycle of the machine. This means that in one iteration the following calculations must be accomplished:

- move a bunch to the next machine element
- update all cavity excitation levels based on the transverse position of the bunches entering the cavity

At present the possible transport matrices within a linac segment are those for a thin lens and for a drift. The transport matrix between linac segments is arbitrary within the standard limitations.

At the end of each iteration, the bunches leaving a linac segment during that iteration are considered separately. To pass a bunch to the next linac segment TDBBU multiplies the coordinates of a bunch leaving the previous linac segment by the correct transport matrix and uses the results as injection coordinates of the bunch in the next linac segment. Recirculation is accomplished by multiplying the coordinates of the bunch leaving the final linac segment by a transport matrix and using the results for injecting the bunch into the first linac segment. The $t_i(I)$s are set by introducing a time delay giving the number of iterations that the coordinates of a bunch reside in a buffer before being injected into the next linac segment. In order to model bunch current variations, the charge of the injected bunches is recorded and traced through the machine in the same manner as the other bunch coordinates.

TDBBU has diagnostics giving bunch position and momentum at any location, diagnostics giving phase space plots at any location, a cavity excitation diagnostic, and a bunch orbit diagnostic.

Tests of TDBBU

The main tests of TDBBU have been from comparisons to threshold predictions based on the formalism of Gluckstern and Bisognano[3,4]. Following these authors, the stability of the

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system is investigated by assuming a solution of the form

$$\mathbf{V}_i^I(k) = \hat{\mathbf{V}}_i^I e^{2\pi i\nu k}$$

where ν is the (complex) frequency of the normal mode. When the transport matrices do not couple the two directions, one may determine the threshold in the two directions separately. In this case a matrix equation for

$$D_{xi} = \sum_{J=1}^{N_p} \hat{x}_i^J e^{-2\pi i\nu t_i(J)}$$

is obtained

$$\begin{aligned} D_{xi} = &\sum_{I=2}^{N_p} \sum_{J<I} \sum_{l=1}^{N} (T_{il}^{IJ})_{12} e^{2\pi i\nu(t_l(J)-t_i(I))} h_l(\nu) D_{xl} \\ &+ \sum_{I=1}^{N_p} \sum_{l=1}^{i-1} (T_{il}^{II})_{12} e^{2\pi i\nu(t_l(I)-t_i(I))} h_l(\nu) D_{xl} \end{aligned} \qquad (1)$$

where

$$h_i(\nu) = I\left(\frac{eZ''\ell\tau}{2Q}\right)_i \frac{H_i(\nu)\sin(\omega_i\tau)}{1 + H_i(\nu)^2 - 2H_i(\nu)\cos(\omega_i\tau)}$$

and

$$H_i(\nu) = e^{-\frac{\omega_i\tau}{2Q_i}} e^{-2\pi i\nu}.$$

The equation for

$$D_{yi} = \sum_{J=1}^{N_p} \hat{y}_i^J e^{-2\pi i\nu t_i(J)}$$

is similar to Eqn.(1) but in this case $(T)_{34}$ is the significant matrix element.

It is important to note that $\hat{p}_{xi}^I$ does not appear in Eqn.(1). This means that a $N \times N$ matrix determinant must be solved to obtain ν, instead of a $2N \times 2N$ matrix determinant which might be anticipated.

For the single cavity case, analytic solution of Eqn.(1) is possible[3]. **Figure 1** gives a comparison between the analytic results and the simulation results in a four-pass case. The two curves are results with different choices of time delay. One observes an effect on both the threshold currents and on the growth rates above threshold. The agreement between the simulation and the stability analysis is good.

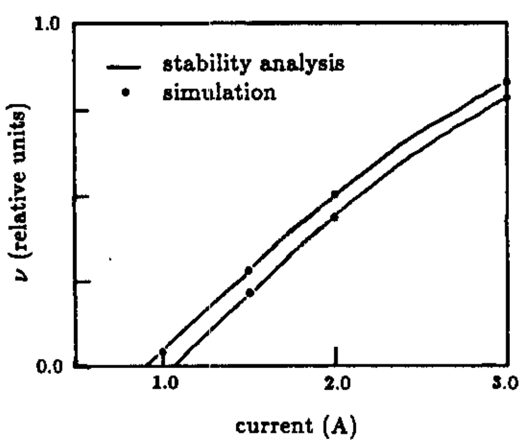

Figure 1 Instability Growth Rate vs. Current for Two Sets of Timing Choices

The code has also been checked by comparing to other simulations[5] and by comparing to analytic calculations of the single pass case[2].

Simulations of the CEBAF Accelerator

The CEBAF design consists of two side-by-side linacs connected by recirculation arcs which yield four passes through each linac. The FODO lattice of the linac has constant focal length for the first pass[1]. There are two hundred CEBAF/Cornell superconducting cavities in each linac, separated by 1.2 m. After every eighth cavity there is a quadrupole lens of focal length ±5.4 m. A single impulse is used to model the effect of a cavity pair. At each cavity it is assumed there are eight modes with the parameters shown in **Table 1**. The resonance freqencies of the cavity pairs are chosen to be random between f and $f+1$ MHz to mimic the expected frequency spread in the cavities. The threshold currents for several different cavity frequency seeds are tabulated in **Table 2**. The thresholds are largely independent of the seed.

f (MHz)	polarization	R/Q (Ω)	Q
1890	x	25.0	32000
1890	y	25.0	32000
1969	x	54.2	4000
1969	y	54.2	4000
2086	x	14.7	10000
2086	y	14.7	10000
2110	x	28.7	13000
2110	y	28.7	13000

Table 1 Cavity Parameters in CEBAF Accelerator Simulation

seed	2 GeV	4 GeV
1	11	21
5	13	22
3	12	22
4	14	19
5	12	24

Table 2 Threshold Currents of the CEBAF Accelerator for Several Frequency Seeds

Examples of simulation results using the first seed are shown in **Figures 2-4**, where the initial excitation is provided by a displacement of the injected beam. The figures show the displacement of the bunch leaving the accelerator as a function of bunch number (i.e. time). In **Figure 2** the current is 10 mA, below the threshold current of 21 mA. In **Figure 3** the current is 20 mA, at approximately the threshold current. The envelope of the oscillations (which are due to the RF fluctuations) is roughly constant in time. In **Figure 4** the current is 30 mA and the envelope is growing due to the instability.

Simulations were undertaken to determine whether a 90° rotation of the beam during recirculation improved the threshold current. The results of the simulations are shown in **Table 3**. In these simulations the rotation is applied only after the first pass as doing the rotation at higher energy becomes technically formidable. This technique yields more than 60% gains in threshold current. Further increases are possible by varying the recirculation optics to achieve a better match.

The CEBAF accelerator is designed so that up to three different current beams may be extracted. This is accomplished by

seed	2 GeV	4 GeV
1	22	34
2	23	36
3	24	38
4	21	33
5	21	36

Table 3 Threshold Currents of the CEBAF Accelerator Including a Beam Rotator After the First Pass

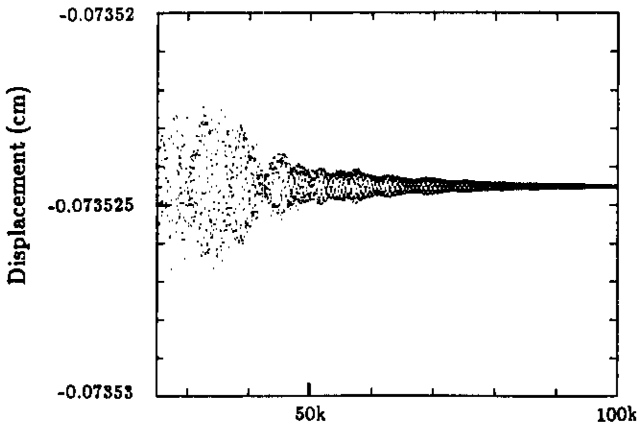

Figure 2 Bunch Displacement vs. Bunch Number for $I = 10$ mA, below threshold

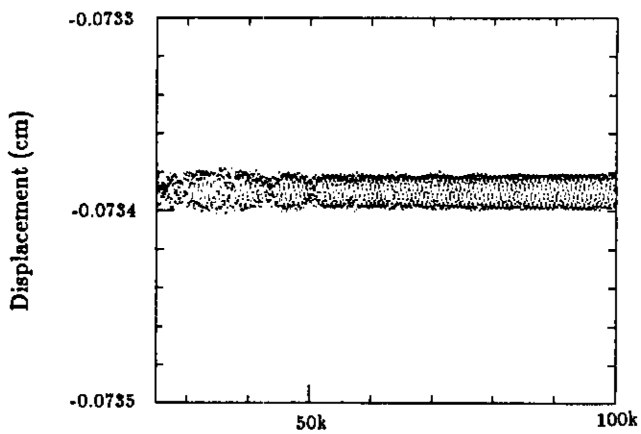

Figure 3 Bunch Displacement vs. Bunch Number for $I = 20$ mA, approximately at threshold

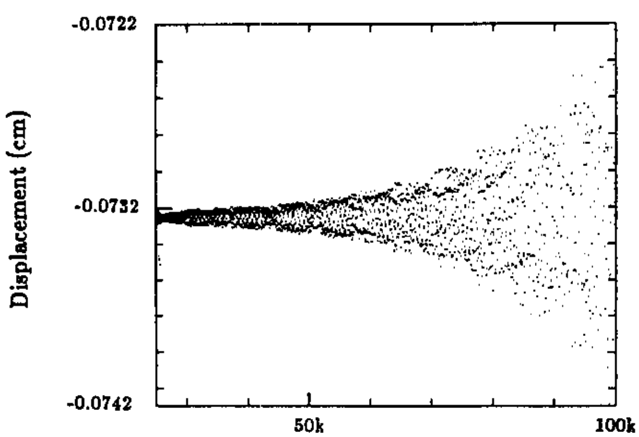

Figure 4 Bunch Displacement vs. Bunch Number for $I = 30$ mA, above threshold

loading successive RF buckets with varying amounts of charge, the repetition frequency being 3 RF periods[1]. This differential loading of the bunches can produce signals at harmonics of $1500/3 = 500$ MHz which excite the cavities. The question is whether the induced RF fluctuations can cause significant deflection of the beam. **Figure 5** gives an example of a worst-case estimate obtained by assuming the frequency of the deflecting modes falls on a harmonic of 500 MHz (2 GHz). The phase space plot gives the coordinates of successive bunches as they leave the linac after a time long enough that all the transients have decayed away. One of the extracted beams had 200 μA average current, the next had 100 μA average current, and the final beam had 1 μA average current, a typical operating scenario for the accelerator. The deflections are small enough that no correction is needed.

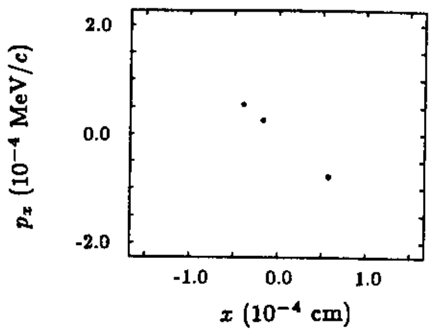

Figure 5 Phase Space Coordinates of Bunches Leaving the Linac When Beam Differentially Loaded

Runs of duration up to .03 sec (real time) have been done in order to check that no slowly growing phenomenon is found.

Conclusions

A vectorized two dimensional code, TDBBU, has been written to simulate beam breakup in RF linacs. Aside from simulating multipass accelerators, the code may be used to model devices with multiple linac segments with energy varying time delays between the segments. The major conclusions resulting from code work of pertinence to the CEBAF accelerator are

- the CEBAF accelerator threshold currents are 19 mA at 4 GeV and 11 mA at 2 GeV
- beam rotation after the first pass can increase the beam breakup threshold current by more than 60%
- fluctuations in the transverse coordinates of the beam due to excitation of the cavities from differential loading are small.

The code has been tested in detail against several analytic models.

Acknowledgement

S. Laubach is responsible for developing several diagnostic routines in TDBBU.

References

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[3] R. L. Gluckstern, "Beam Breakup in a Multi-section Recirculating Linac", *Proc. 1986 Linac Conference*, pg. 543
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