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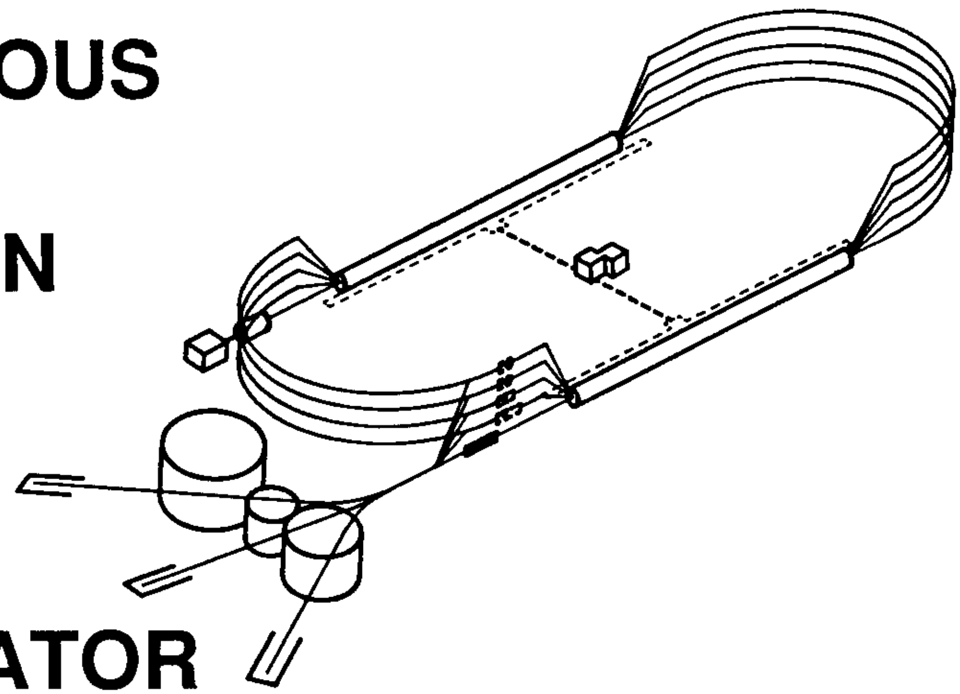

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Correction of Dipole and IR Quadrupole Nonlinear Content in Large Colliders

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Abstract

The concept of quasilocal (F, C, D) correction of nonlinearities is reviewed. Correction by two or more orders of magnitude is obtained; in addition, separated-function control of the horizontal, coupled and vertical motion becomes possible. Quasi-local correction of dipole nonlinearities is planned for LHC, and will greatly increase the linear aperture. The 1990 SSC Site-Specific Conceptual Design includes quasilocal correction in the 20 TeV Collider, but restrictions may limit correction capability. Quasilocal (F, C, D) correction may also be useful in the SSC 2-TeV High Energy Booster (HEB). In high-luminosity collider mode, the nonlinear effects of the interaction-region (IR) quadrupoles may be dominant; these errors may also be compensated quasilocally. Extensions of the (F, C, D) concept for IR correction, which exploit the restricted IR symmetry, are also suggested.

1.0 Introduction

Large colliders use high-field conductor-dominated superconducting magnets; these magnets have relatively large nonlinear (multipole) fields. The greatly increased circumferences of the highest energy machines magnify the nonlinear effects, while forcing the designs toward smaller aperture, more nonlinear magnets. Beam stability for billion turn collider cycles requires highly linear motion. Previously, beam dynamics was dominated by dipole, quadrupole and first-order sextupole effects, and corrector elements near focusing (F) and defocussing (D) quads were adequate. However correctors near the quads are ineffective for higher orders. Including correctors in the center (C) of accelerator half-cells permits enormous improvements.[1]

Figure 1 shows the correction method as applied in its simplest form in a FODO cell of a large collider with correctors in the center (C) of the half-cell, as well as near the F and D quads. (In a first approximation, a large collider consists simply of hundreds of such cells.) On the half-cell level, the correctors form a three-point (F, C, D) system. Application of basic physical principles to this system provides accurate compensation and control of all nonlinearities.

The (F, C, D) correctors can form an optimal, quasi-local cancellation of the continuous multipole content of the dipoles. ("Quasi-local" correction means that the nonlinear field is compensated within the same optical unit, the FODO cell.) The magnetic fields in the dipoles may be expressed as

$$B_y + iB_x = B_0[1 + \sum(b_n + ia_n)(x + iy)^n] \tag{1}$$

where b_n and a_n are the normal and skew multipole strengths. For the case of constant (systematic) multipole content, the optimum corrector strengths S_i are close to Simpson's Rule values: $(S_F,\ S_C,\ S_D) = -(1/6,\ 4/6,\ 1/6)\ B_o b_n L$. That simple solution reduces all nonlinear effects (tune shifts, resonance widths, distortion functions) by two or more orders of magnitude.

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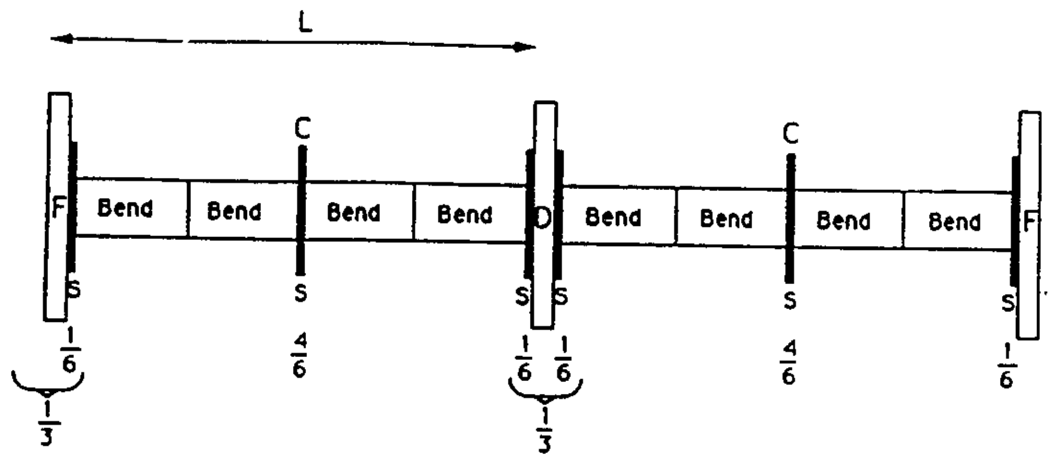

Figure 1. A sample collider cell. The element labels are: F and D – quads, S – slots for correctors, C – center slot. Correction strengths on opposite sides of the thin quads would be combined in units on either side; there are only two correctors per half-cell.

The accuracy of the solution can be understood by noting that any nonlinear effect can be expressed in terms of integrals over the lattice. For example, one corrected octupole (b_3) tune shift in a half-cell of length L can be written as:

$$\Delta\nu_3 = \int_0^L b_3(s)\beta_x^2(s)ds + S_{3,F}\beta_x(0)^2 + S_{3,C}\beta_x(L/2)^2 + S_{3,D}\beta_x(L)^2 \tag{2}$$

where $S_{F,C,D}$ are octupole corrector strengths. There are six first-order tune shift terms, which include differing powers of the lattice functions β_x, β_y, η. The same Simpson's Rule solution reduces all of these effects by two orders of magnitude. While initially developed for tune shift correction, the method greatly reduces all other nonlinear effects. (For example, the Collins sextupole distortion functions[2] are exactly reduced to zero at the half-cell level by (F, C, D) sextupole correctors.)

The general accuracy of correction indicates that the (F, C, D) correctors are fully equivalent to the continuous distribution at the 1% level. A similar algorithm has been developed to compensate varying (random) multipole content; similar cancellations are obtainable.[3] However, in this note we will emphasize the application to the correction of systematic multipole content. The systematic effects most severely restrict linear and long-term dynamic apertures, and are also most easily corrected.

The (F, C, D) correctors are also at optimal locations for separated-function control of horizontal-, coupled-, and vertical-motion parameters, and these are precisely the operational observables. The C corrector adds the ability to control coupled-motion parameters independently of horizontal and vertical motion parameters. This tunability can be used in improving correction from initial approximations. For instance, (F, C, D) octupoles are appropriate elements for control of all amplitude-dependent and second-order chromatic tune shifts. The (F, C, D) elements permit exact control of the motion through 10-pole order.

The approach was described and discussed thoroughly at the second advanced ICFA beam dynamics workshop (Lugano, Switzerland, 1988)[4] and in international physics journals.[5,6,7]

2.0 Application to the Large Hadron Collider (LHC)

Quasi-local (F, C, D) correction of sextupole, octupole and 10-pole components (b_2, b_3, b_4) is included in the current design of the LHC.[8] The application is described in detail by Scandale in these proceedings[9] and in CERN reports by Scandale and co-workers.[10,11] A graphical representation of the effectiveness of the correction in the LHC is shown in figure 2, from reference[8].

In figure 2b), the uncorrected horizontal and vertical tune shifts as a function of horizontal and vertical amplitudes due to nonlinear fields are shown. In figure 2c) the corrected values are shown; the nonlinearity is nearly completely removed. Similar reductions in momentum-dependent tuneshifts and "smear" are also obtained.

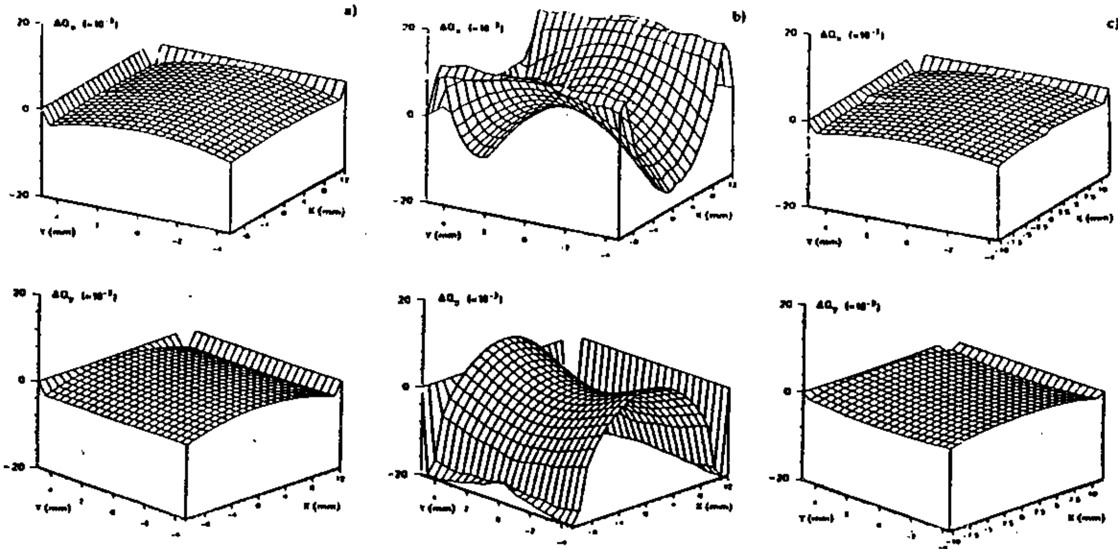

Figure 2. Tune shift with the amplitude for on-momentum particles in LHC (from ref. 8).
a) bare machine chromaticity corrected
b) machine with systematic errors in the dipoles, chromaticity corrected
c) machine with systematic errors in the dipoles and lumped (F, C, D) correctors in the arcs

The correction greatly increases the linear and long-term dynamic apertures in the LHC, and will greatly improve its operational capability. It also simplifies the difficult central task of developing a reliable high-field dipole, by relaxing field quality requirements.

3.0 Application to the SSC (Superconducting Super Collider) Collider

Nonlinear correction is also extremely desirable in the SSC Collider. The 1990 SSC Site-specific Conceptual Design (SCD)[12] includes a modified version of quasilocal (F, C, D) correction. Figure 3 shows an SSC cell from that design. The cell has five dipoles per half-cell, which forces an unsymmetrical, less desirable location for the C correctors. A more serious concern is the very short space (3.5 m at F, D and 0.5 m at C locations) currently allotted for correction elements, which must also include linear correctors, beam monitors and chromatic correction, none of which have been built or even fully designed. It is uncertain whether adequate space is available for the immediate and future needs of the SSC, including extensions such as low-beta IR chromaticity correction and skew quad correction at C locations.

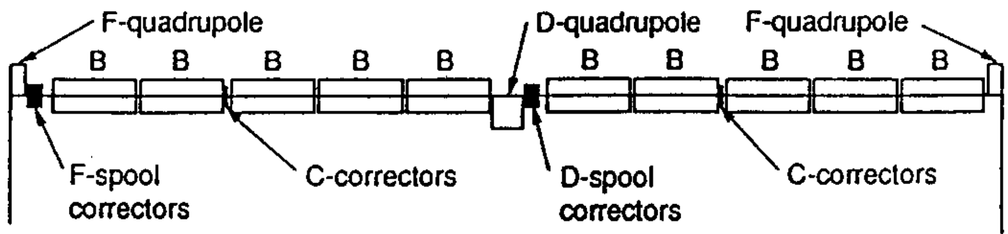

Figure 3. A cell in the SSC lattice (180 m long) showing locations for F, C, D correctors.

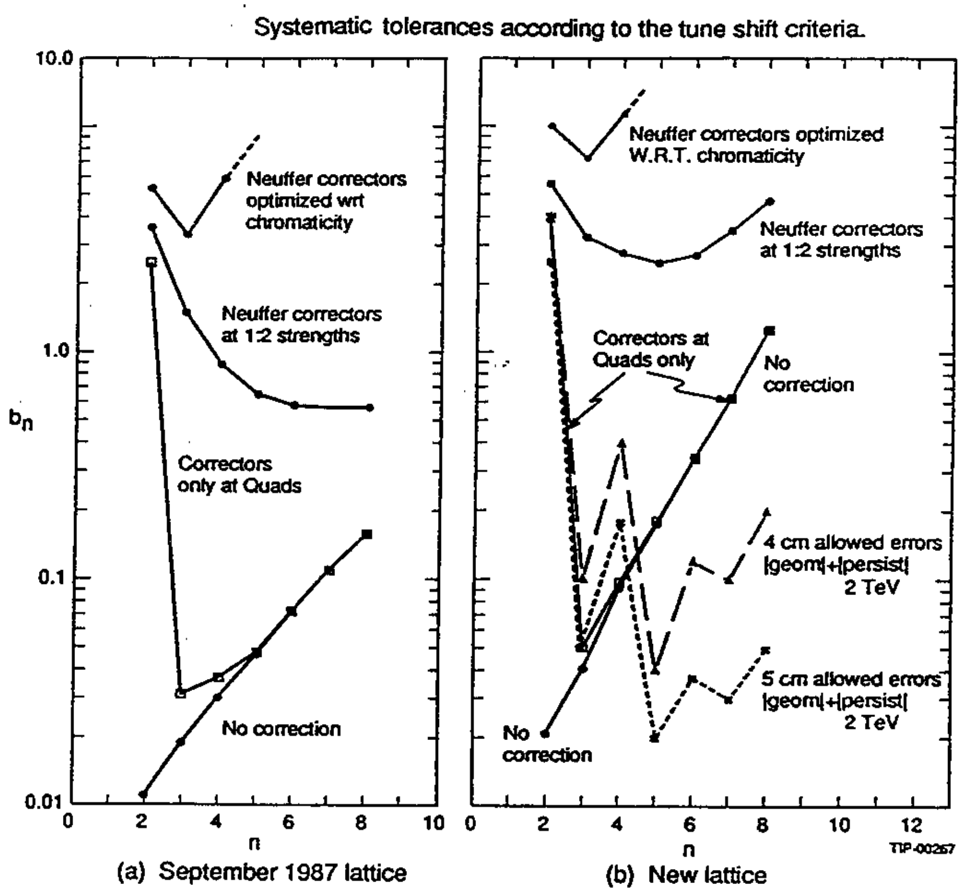

Figure 4. A graphical representation of error correction in the SSC (from ref. 12, corrected version). The tolerances shown are the maximum values of multipole strengths b_n (in "units" of $10^{-4}\ cm^{-n}$) permitted under tune shift criteria: $\Delta\nu \lesssim \pm 0.005$ for amplitudes A_x, A_y $\lesssim$ 4.5 mm and $\Delta P/P \lesssim \pm 0.001$. (F, C, D) correction increases "tolerances" for b_2, b_3, b_4 by $\gtrsim$ 100×, which means the multipole nonlinearity can be corrected by these factors. Expected multipole content for 4 cm and 5 cm are also shown.

Figure 4 is an unmodified version of a graph developed by J. Peterson for the SCD. The dashed lines show the design specifications for systematic magnet errors for SSC dipoles of 4 and 5 cm aperture; the specifications are based on Tevatron data plus calculated persistent current effects at 2 TeV injection. These errors are compared with tolerances with or without correction; the tolerances are based on the requirement that nonlinear tune shifts be less than 0.005 within the SSC design aperture of 4.5 mm amplitude, $\Delta p/p < \pm 0.001$. Correction of b_2, b_3, and b_4 is indicated; quasilocal (F, C, D) correction can correct these by more than two orders of magnitude and the design included these. The correction would greatly increase the useable aperture beyond the design values. We also note that experience shows that it is likely that production magnets will have some multipole component (probably b_4 or a_1) substantially larger than expected; a prudent design will include capacity for correction.

Since then, unphysical considerations have caused the SSC to consider modifying the correction by leaving nonlinear corrector slots empty, except for placement of (F, C, D) octupole and 10-pole correctors in every fifth cell. Strong correctors at these locations can be used to cancel the global tune shifts, and these may be the dominant terms, if nonlinear fields are not too large. However, the modified correction would no longer be quasi-local, and therefore will not provide a universal correction of all nonlinear effects, may magnify some resonances, and will be modified by ring perturbations. Thus, the modification would add substantial risk and complication to future operation of the SSC.

Implementation of optimal physics for the SSC arcs requires unmodified use of quasilocal (F, C, D) correction and an evaluation of corrector space and needs, using proven elements.

4.0 Application to the SSC High Energy Booster

More serious nonlinear effects may be expected in the SSC High Energy Booster. In order to avoid the complication of designing another superconducting dipole, the SSC currently plans to use the same 5-cm aperture magnet in the 2 TeV HEB as in the Collider. However, the lower energy HEB beam is twice as large as the Collider beam and will therefore experience much stronger nonlinear fields. Even after optimistically assuming much smaller nonlinear fields than those extrapolated from Tevatron experience (half of SCD values and much smaller than prototypes), studies by A. Chao *et al.*[13] indicate an inadequate dynamic and linear aperture within the HEB, implying that "it may be necessary to increase the dipole aperture," unless "a simple and effective correction scheme" is implemented. The correction scheme which could be implemented is, of course, a simple version of quasi-local (F, C, D) correction.

Results of some calculations showing the need for nonlinear correction and the effectiveness of simple Simpson's Rule (F, C, D) correction are shown in Table 1. In that table we have assumed that the HEB requires a linear aperture only 50% larger than the collider (maximum amplitude of 7 mm and $\Delta p/p \leq \pm 0.0015$), even though the beam is twice as large (closed orbit error and lifetime requirements may be smaller). We then set a linearity tolerance level on the multipole strengths by requiring that the nonlinear tune shift due to each multipole be less than 0.005. This tolerance level is compared with the expected 5 cm systematic multipole strengths from the SCD; sextupole, octupole and 10-pole (b_2, b_3, b_4) strengths are well above tolerance and should be corrected. With (F, C, D) correctors at Simpson's rule values, all corrected multipoles are well within tolerances. Correction by about two orders of magnitude is obtained. A linear aperture much larger than specified is obtained.

Quasi-local (F, C, D) correction, particularly of octupole and 10-pole, is recommended for the HEB and will obtain the safety margin needed for reliable operation. The (F, C, D) octupoles also add the ability to control the amplitude-dependent tune shifts; this is a capability which is extremely useful for slow extraction, a desired HEB function.

Table 1. Nonlinear Correction in the HEB

Multipole strengths are in "units" of 10^{-4} at 1 cm. Tolerances are set for each multipole by the requirement that $\Delta\nu < \pm 0.0015$ for all amplitudes less than 0.7 cm and $\delta p/p < \pm 0.005$. The simple correction method used is chromatic sextupole correction with (F, C, D) correctors set at Simpson's Rule values. Only (b_2, b_3, b_4) are corrected.

Multipole	Expected Strength (SCD, 5cm)	Tolerance (no Correction)	Tolerance (F, D) Chromatic Correction only	Tolerance with Simple (F, C D) Correction (Simpson's Rule)
b_2	2.6	0.02	3.0	6.5
b_3	0.05	0.023	0.023	2.1
b_4	0.18	0.029	0.029	0.82
b_5	0.02	0.035	–	–
b_6	0.037	0.043	–	–

5.0 Interaction Region Nonlinear Effects

High luminosity operation requires focussing of the beam to very small sizes in collider interaction regions (IRs), using strong triplets (or doublets) of quadrupoles. The direct and indirect nonlinear effects of these IR quads dominate the linear aperture constraints in collider mode; these effects can be much larger than arc nonlinearities. Simulations confirm the dominance of these effects in setting the dynamic aperture.[14] However, the restricted symmetry and localization of the IR region permits techniques for reduction of these effects. Intelligent application of accelerator physics optimizations can enhance operations.

The dominant first-order effect of IR quads is a large addition to the chromaticity. Linear chromaticity ($d\nu/dp$) correction can be done with sufficiently strong arc sextupoles. However, the nonlinear chromaticity ($d^2\nu/dp^2$, etc.) is greatly magnified by the IR quads. The contributions to the higher-order chromaticity can be greatly reduced by modifying the global lattice. It has been shown that if the phase advance between adjacent IRs is near a 1/4-integer and the superperiod tune is near a 1/4-integer, nonlinear chromatricity is reduced by an order of magnitude.[15] Reductions can also be obtained by using multiple sextupole corrector families in the arcs;[16] however, this adds sextupole nonlinearities. Also (F, C) (not D) arc octupoles can be used to minimize second-order chromaticity, and (F, C, D) octupoles can control the first-order amplitude-dependent tune shifts. These global lattice optimizations can reduce these chromatic effects to a point where the nonlinear fields within the IR quads are dominant.

It is known that the linear aperture can be greatly restricted by nonlinear fields in the IR quads, because of their relatively large strengths and because of the very large values of β_x, β_y at the quads, which magnify the nonlinear effects. Also, it is not possible to rely on statistical cancellation of random multipole components, even in nonallowed multipoles, because of the small number of IR quads. In the 1986 Conceptual Design Report,[17] it was found necessary to correct b_2, b_3, b_4, b_5, a_2, a_3, a_4, a_5, by at least an order of magnitude, and the existence of corrector trim-coils within each quad for all of these multipoles was postulated.

We note that a variation of quasilocal (F, C, D) correction can provide adequate correction. For an initial example we consider a sample SSC IR triplet; betatron functions are displayed in figure 5. The IR center has $\beta = 0.5$ m and a drift of 20 m leads into the triplet, where betatron functions vary up to maxima of ~ 8000 m. To set up a corrector configuration, we split each of the F, D, F quad units into half-length magnets and place short corrector elements in the gaps between the magnets as well as at the ends of the (F, D, F) units (see figure 6). The correctors can be powered to cancel the total nonlinear field quasilocally in quad units following Simpson's Rule: $S_{n,i} = -(1/6, 4/6, 1/6)B'b_nL_Q$.

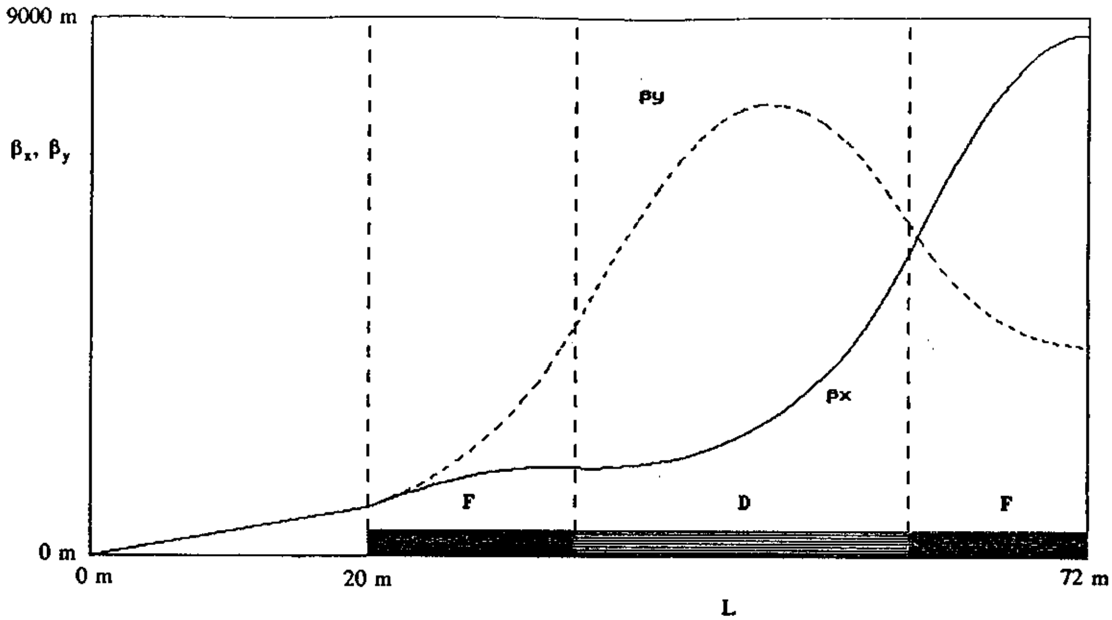

Figure 5. Betatron functions β_x, β_y within an SSC-type final focussing IR triplet.

For a numerical example, we consider a 12-pole systematic multipole error (b_5) of unit strength (10^{-4} of the focussing field at 1 cm). b_5 is chosen since its allowed in quad symmetry, and therefore may have a large systematic value, it directly causes amplitude dependent tune shifts, and it is a higher order term, and therefore difficult to correct. There are four first-order amplitude dependent tune shifts driven by b_5. Table 2 summarizes tune shift and correction calculations. The (F, C, D) system corrects tune shifts by two orders of magnitude.

The calculations can be inverted to illustrate the uncorrected aperture restriction. A moderate amplitude particle ($\epsilon = 10^{-7}$ m-R or amplitude $A_x = 1.8$ mm in the arcs) has an amplitude of 9 mm in the IR triplet and would have a tune shift of ~ 0.1 per IR triplet at $b_5 = 1$ unit. Requiring that that amplitude be within the linear aperture ($\Delta\nu < 0.005$) for a machine with 2 IRs requires that b_5 be less than ~ 0.02 units, unless corrected, a difficult constraint. That amplitude is $\sim 14\sigma$ of the design rms beam size at 20 TeV, and may be an acceptable minimal aperture requirement for well-tuned collider runs (It is only 40% of the 4.5 mm aperture spec in the arc discussion above.).

Figure 6a) shows seven correctors over the IR triplet. However, since β is not greatly magnified in the quad nearest the IR, its correctors are not needed and at least one of the other correctors is redundant. A 3 or 4 point correction would be as effective in locally cancelling the nonlinearity and would be simpler. We have demonstrated that it is possible to correct any IR multipole quasilocally; it remains to integrate the correction into a practical system.

6.0 IR Region Optimization for Nonlinearity Reduction

Correction of ($b_2, \ldots a_5$) in every quad would require 32 independent correctors per triplet, which appears impractical. Reduction to a manageable subset of most important corrections is therefore desirable.

It can be hoped that the small number of IR quads can be built to relatively high field quality, and this will reduce the number of required corrections. Close monitoring of as-built field quality will be necessary to identify these, and the monitored field strengths can also be used to set corrector strengths. (Also, operational closed orbit amplitudes can be fine-tuned to small amplitudes within the IRs; this reduces feed-down nonlinearity effects.)

Table 2. Tune Shift Terms in IR Triplet

Element, Term	$\Delta\nu_{x,y}(\lvert b_5\rvert = 10^{-4}, \epsilon_y = \epsilon_x = 10^{-8})$	$\Delta\nu$ (corrected)
$F_1,\ \beta_x^3$	0.00002	-0.000000
$F_1,\ \beta_x^2\beta_y$	0.0017	0.000013
$F_1,\ \beta_x\beta_y^2$	0.0022	-0.000043
$F_1,\ \beta_y^3$	0.0001	-0.000014
D, β_x^3	0.00025	-0.00006
$D, \beta_x^2\beta_y$	0.021	-0.00026
$D, \beta_x\beta_y^2$	0.057	0.00086
D, β_y^3	0.005	-0.00042
$F_2,\ \beta_x^3$	0.0022	0.000000
$F_2,\ \beta_x^2\beta_y$	0.050	-0.000043
$F_2,\ \beta_x\beta_y^2$	0.033	0.000038
$F_2,\ \beta_y^3$	0.0007	-0.000009

ϵ_y and ϵ_x are the emittance in m-R and are related to the amplitudes and actions by $A_{x,y} = \sqrt{\beta_{x,y}\epsilon_{x,y}}$ and $\epsilon_{x,y} = 2\ I_{x,y}$, respectively. 10^{-8} m-R corresponds to $A = 1.7$ mm at $\beta = 300$ m, a typical maximum value in the arcs. In the uncorrected $\Delta\nu$ calculations, signs have been ignored. A negative sign in the corrected $\Delta\nu$ numbers indicates the particular term is overcorrected by the 3-point correctors. The tune shifts are calculated from

$$\Delta\nu_x = \frac{1}{2\pi B\rho}\left[\frac{5}{4}I_x^2\int B'b_5\beta_x^3 ds - \frac{15 I_x I_y}{2}\int B'b_5\beta_x^2\beta_y ds + \frac{15}{4}I_y^2\int B'b_5\beta_x\beta_y^2 ds\right]$$

with a symmetric expression for $\Delta\nu_y$.

It is likely that the most important multipoles in limiting the dynamic aperture are those with non-vanishing zero-harmonic tune shifts (b_3 and b_5 in on-axis orbits). It is therefore possible that correcting only these elements will provide the aperture gains needed for low-β. This hypothesis should, of course, be thoroughly tested with accurate tracking and analytical studies.

The situation is complicated by the design feature that both beams pass through the same IR quads displaced from the centers, with crossing at an angle. Because of the displacements, feed-down multipoles also have large zero-harmonic tune shifts (b_2 and b_4 for horizontal and a_2 and a_4 for vertical crossings, respectively) and must also be considered for correction.

It is therefore desirable to change the IR geometry to separate the beams before they reach the IR quads, so that they can be centered in those quads. Early separation would have the added advantage of improving the linearity by greatly reducing the long-range beam-beam force, and also would permit separate control of focussing and correction of both beams. It however forces a more constrained geometry on the IRs, and possibly lengthens them unacceptably. The advantages/disadvantages of separated beams should be reevaluated, considering nonlinearity and aperture constraints.

The correction system can be somewhat simplified by considering the restricted symmetry of betatron functions in IR regions, as was also done in the chromaticity optimizations outlined in the previous section. Because β_x, β_y are so large within the IR quads, particle phases ψ_x, ψ_y change very little within a triplet ($\psi = \int ds/\beta$). Resonance driving terms J can be expressed as integrals of nonlinear field strengths b_n times betatron functions times phase factors of the form:

$$J \propto \int b_n(s)\ \beta_x^{\frac{n_x}{2}}\ \beta_y^{\frac{n_y}{2}}\ e^{i(N_x\psi_x + N_y\psi_y)}\ ds$$

This form includes zero harmonic terms ($N_x = N_y = 0$), such as tune shifts. Since phase factors vary little over the IR triplet, they can removed from the integration and local cancellation of all resonant terms can be obtained by simply requiring that the integral over the IR triplet of the nonlinearity strengths (magnets plus correctors) weighted by the appropriate powers of β_x, β_y be zero. For multipoles through 10-pole, a three-family corrector system is sufficient to correct all such terms: an "X" corrector where $\beta_x >> \beta_y$, a "C" corrector where $\beta_x \simeq \beta_y$, and a "Y" corrector where $\beta_x << \beta_y$. For 12-pole a fourth corrector might be needed. The optimal locations for correctors would be at large values of β_x, β_y (see figure 6b).

In these last two sections, we have discussed techniques for the reduction of nonlinear effects from IR focussing systems. Implementation of an optimal system requires much further study. It is expected that magnet optimization and accurate analyses will reduce nonlinear correction to a few most dangerous multipoles (such as b_3 and b_5), for which 3 elements (X, C, Y) are sufficient. It may also be preferable to split the colliding beams into separate focusing systems; further studies are needed. High luminosity will depend on optimal resolution of the IR correction issues.

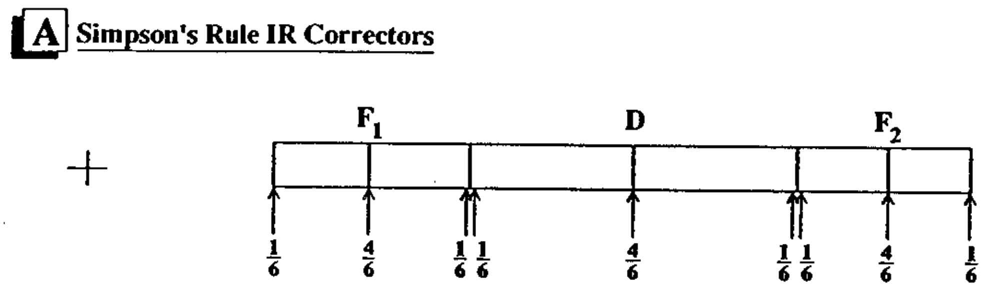

Figure 6a. Layout for "Simpson's Rule" Correctors in an IR final focussing triplet.

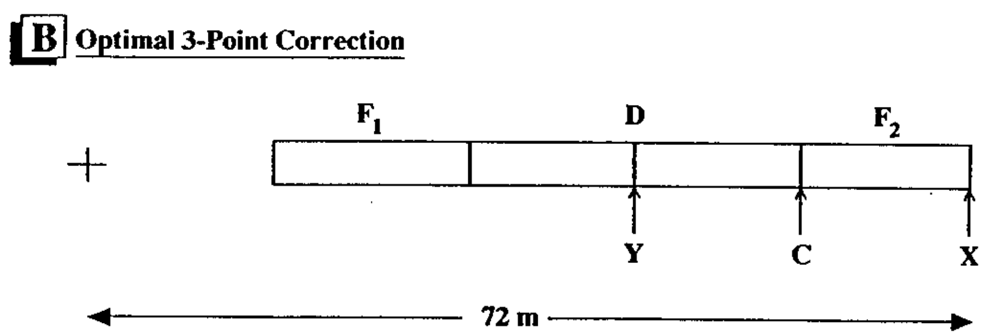

Figure 6b. Layout for optimal 3-point correction in an IR triplet: X-corrects horizontal motion predominantly, C-corrects coupled motion, Y-corrects vertical motion (F-D-F lattice of Fig. 5 assumed.)

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