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CHEMICAL RELAXATION TIMES IN A HADRON GAS
AT FINITE TEMPERATURE

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The relaxation times of particle numbers in hot hadronic matter with vanishing baryon number are estimated using the ideal gas approximation and taking into account resonance decays and annihilation processes as the only sources of particle number fluctuations. Near the QCD critical temperature the longest relaxation times turn out to be of the order of 10 fm and grow roughly exponentially to become of the order of 10^3 fm at temperatures around 100 MeV. As a consequence of such long relaxation times, a clear departure from chemical equilibrium must be observed in the momentum distribution of secondary particles produced in high energy nuclear collisions.

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I. INTRODUCTION

Experimental studies of QCD near and above the critical temperature for the hadron to quark-gluon phase transition are thought to be possible in ultra-relativistic nuclear collisions. After such a collision takes place, a hot spot of quark-gluon matter is expected to remain at central rapidity. This secondary system subsequently expands and cools until a phase transition to hadronic matter occurs. This first stage of the evolution is the most relevant one for these studies. Unfortunately, after the system enters the hadronic phase, thermal equilibrium is expected to persist, erasing strong interaction signatures of this first stage. The observed momentum distribution of stable hadrons corresponds to the freeze-out configuration, expected to correspond to temperatures well below the critical temperature. In present high energy nuclear collisions the final state of secondary particles shows as many as five hundred particles, mostly pions, stemming from a source whose radius is about 5 fm., as determined from pion interferometry. Under these conditions thermal equilibrium can persist down to temperatures of the order of 100 MeV [1-3]. It is assumed that freeze-out occurs suddenly, leading to a thermalized momentum distribution of the final state. The composition of the hadronic gas at freeze-out, however, will show an excess of stable particles if chemical equilibrium is lost in the course of the evolution. In fact, inelastic processes which can produce fluctuations in the number of particles and restore chemical equilibrium in this way are overcome by the expansion rate well before the stage of freeze-out is reached. This has been suggested [2-5] as a natural explanation for the enhancement of the pion spectrum at low transverse-momentum observed in present high energy nucleus collisions [6].

While the thermalization characteristic time can be well determined by considering elastic $\pi - \pi$ scattering as the only source of equilibration [1, 2], the determination of the characteristic times associated with chemical equilibrium involve a large number and variety of inelastic processes. Using the available information on inelastic processes, in this work we furnish estimates for these relaxation times.

The time evolution of the particle number densities per degree of freedom N_α of the different particles composing a system is determined by the following first order differential equation:

$$g_\alpha \dot{N}_\alpha = \sum_{\delta n=1}^{\infty} \delta n [P_\alpha(\delta n) - P_\alpha(-\delta n)], \quad (1)$$

where $g_\alpha = (2s_\alpha + 1)(2I_\alpha + 1)$ counts the number of degrees of freedom associated with the spin and the isospin of the particle and $P_\alpha(\delta n)$ is the probability per unit

time and unit volume for the elementary inelastic processes where the number of particles of type α change by δn . P_α is given by the following expression:

$$P_\alpha(\delta n) = \sum_{\{i\}, \{f\}} S(\{i\})S(\{f\}) \int \prod_{\beta_i \in \{i\}} \prod_{\beta_f \in \{f\}} d\tilde{k}_\beta, d\tilde{k}_{\beta_f} (2\pi)^4 \delta^4(K_i - K_f) \\ \times |T_{i,f}|^2 \sigma_{\beta_i}(k_{\beta_i}) \tilde{\sigma}_{\beta_f}(k_{\beta_f}), \quad (2)$$

where $\{i\}$ and $\{f\}$ denote respectively to the particle content of the initial and final states in the elementary process. Obviously, these states must satisfy the condition that $n_\alpha\{f\} - n_\alpha\{i\} = \delta n$. Here $T_{i,f}$ is the scattering amplitude, $S(\{i\})$ and $S(\{f\})$ are the initial and final state statistical factors, K_i and K_f are respectively the momenta of the initial and final state, the Lorentz invariant volume element in momentum space is as usual given by

$$d\tilde{k}_\beta \equiv \frac{d^3k_\beta}{(2\pi)^3 2E_\beta}, \quad (3)$$

and the density factors, which take into account the statistics of the particles, are given by:

$$\sigma_\beta(k) = N_\beta(k) \quad (4) \\ \tilde{\sigma}_\beta(k) = \begin{cases} 1 + N_\beta(k), & \text{if } \beta \text{ is a boson} \\ 1 - N_\beta(k), & \text{if } \beta \text{ is a fermion} \end{cases}$$

where $N_\beta(k)$ is the number density per degree of freedom of particles of type β with momentum k . In thermal equilibrium at temperature T this number density is given by:

$$N_\beta(k) = \begin{cases} \exp[-\epsilon_\beta(k)/2T] / (2 \sinh[\epsilon_\beta(k)/2T]), & \text{if } \beta \text{ is a boson} \\ \exp[-\epsilon_\beta(k)/2T] / [2 \cosh(\epsilon_\beta(k)/2T)], & \text{if } \beta \text{ is a fermion} \end{cases} \quad (5)$$

where $\epsilon_\beta(k) \equiv E_\beta(k) - \mu_\beta$, and μ_β is a chemical potential. A non-vanishing chemical potential can be associated with conserved and approximately conserved particle numbers. By the latter one means those numbers whose characteristic relaxation times are much greater than the thermal relaxation time of the system. A chemical potential be ascribed even to short lived resonances, since resonances are constantly regenerated in the collisions of stable states, and consequently, their lifetime is not directly related to the relaxation time of their chemical potential.

From eqs. (2, 4, 5), and in the limit of small chemical potentials, the following form for \dot{N}_α is obtained:

$$g_\alpha \dot{N}_\alpha = -\frac{1}{2T} \sum_{\{i\}} \sum_{\{f\}} \delta n_\alpha(i|f) \sum_\gamma \delta n_\gamma(i|f) \mu_\gamma \tilde{\Gamma}(i, f; T), \quad (6)$$

where γ runs over all particle species. The explicit expression for $\tilde{\Gamma}(i, f; T)$ is

$$\tilde{\Gamma}(i, f; T) = \prod_{\beta \in \{i\}} \prod_{\beta_f \in \{f\}} S(\{i\}) S(\{f\}) \int d\nu_\beta, d\nu_{\beta_f} (2\pi)^4 \delta^4(K_i - K_f) |T_{i,f}|^2, \quad (7)$$

with the definition

$$d\nu_\beta \equiv \frac{d\vec{k}_\beta}{2\chi_\beta} \quad (8)$$

$$\chi_\beta = \begin{cases} \sinh(E_\beta/2T), & \text{if } \beta \text{ is a boson} \\ \cosh(E_\beta/2T), & \text{if } \beta \text{ is a fermion} \end{cases}$$

For small chemical potential, we can write

$$\mu_\beta = T \frac{\Delta N_\beta}{N_\beta^0} = T \frac{(N_\beta - N_\beta^0)}{N_\beta^0}, \quad (9)$$

where N_β^0 is the density at vanishing chemical potential. Since we are considering the time evolution at constant temperature, $\dot{N}_\beta^0 = 0$, and eqn.(6) becomes a system of first order linear differential equations with constant coefficients. The characteristic times of this system of equations are the chemical relaxation times we need.

In the case of baryons, particle and antiparticle are taken into account separately, while mesons are considered as their own antiparticles by including all the members of each flavor multiplet. We are assuming that there is no net baryon number, and therefore the chemical potentials associated to a baryon and its antiparticle are equal.

In a realistic situation, eqn.(6) requires a large body of information about the multiple inelastic strong interaction processes. Only partial information is available from data: for instance, there are sufficient data on resonance decays and $N - \bar{N}$ annihilation. On the other hand, data on resonance production in hadronic collisions at low energy are poor, and more exotic inelastic processes are experimentally inaccessible. As a consequence, approximations are unavoidable. These approximations will consist in including in eqn. (6) only those processes for which data are already available and those whose rates can be estimated reliably enough.

For the sake of illustration, we first discuss the chemical relaxation time in a pion gas. In this case the relevant inelastic process is $\pi\pi \leftrightarrow \pi\pi\pi\pi$, for which an estimate within Chiral Perturbation Theory is possible. Later on, we consider a more realistic situation, where all hadronic levels, except those containing strange quarks, are included. The main conclusion is that chemical relaxation times are large, even at temperatures close to the phase transition, supporting the conviction that in a high energy heavy ion collision the evolution of the fire-ball in the hadronic phase will proceed away from chemical equilibrium.

II. PION GAS

At low temperatures and in the absence of net baryon number, a gas of pions gives a good approximation to the thermodynamics of a hadronic gas. As the temperature increases, heavier states become increasingly populated and eventually dominate the energy density [7]. Concerning the kinetic properties of the hadronic gas, it was found that the thermal relaxation time is reliably obtained within the pion gas approximation if the temperature is less than 130 MeV [9]. On the other hand, as we find in this work, the relaxation times associated with chemical equilibrium are poorly represented in this approximation if the temperature exceeds 70 MeV. Since the question of chemical equilibrium is turns out to be important at temperatures close to the critical temperature, for practical purposes the pion gas turns out to give an unrealistic picture. For the sake of making this claim clear, and also giving a simple illustration of the topic of this work, we discuss the pion gas case in detail.

We work in the dilute gas approximation ($E_\pi \gg 2T$), where according to eqn.(6) the time evolution of $\tilde{N}_\pi \equiv N_{\pi^0} + N_{\pi^+} + N_{\pi^-}$ becomes determined by

$$\frac{\partial}{\partial t} \Delta \tilde{N}_\pi \simeq -\frac{1}{12} \frac{\Delta \tilde{N}_\pi}{N_\pi^0} \int \prod_{i=1}^6 d\vec{k}_i (2\pi)^4 \delta^4(k_1 + k_2 - k_3 - k_4 - k_5 - k_6) \times \exp(-(E_\pi(k_1) + E_\pi(k_2))/T) \sum_{\text{isospin}} |T(k_1 k_2 \rightarrow k_3 k_4 k_5 k_6)|^2 \quad (10)$$

Performing the integrations over the final state phase space and the angular integrations in the initial state phase space, this equation takes the following form:

$$\Delta \dot{\tilde{N}}_\pi \simeq -\frac{1}{4\pi^4} \frac{\Delta \tilde{N}_\pi}{N_\pi^0} \int_{M_\pi}^{\infty} dE_1 \sqrt{E_1^2 - M_\pi^2} \int_{E_2^{\text{min}}}^{\infty} dE_2 \sqrt{E_2^2 - M_\pi^2} \times \exp(-(E_1 + E_2)/T) \sqrt{s(s - 4M_\pi^2)} \sum_{l_1, l_2} \sigma_{\pi\pi \rightarrow \pi\pi\pi\pi}(s, l_1, l_2) \quad (11)$$

where:

$$E_2^{\min} = \begin{cases} 7 E_1 - 4\sqrt{3}\sqrt{E_1^2 - M_\pi^2}, & \text{if } E_1 < 7 M_\pi \\ M_\pi, & \text{if } E_1 > 7 M_\pi \end{cases} \quad (12)$$

The cross section is estimated by using the amplitude obtained from the lowest order chiral Lagrangian. A practical simplification consists in neglecting the tree level contribution to the amplitude resulting from twice iterating the chiral Lagrangian term containing the four-leg vertices. We expect that this approximation will not significantly affect the results. In the chiral limit, and after summing over the initial state isospins I_1 and I_2 , the inelastic cross section, is:

$$\sum_{\text{isospin } I_1, I_2} \sigma_{\pi\pi \rightarrow \pi\pi\pi\pi}(s, I_1, I_2) = \frac{67}{2^{17} 3^4 \pi^5} \frac{s^3}{F_\pi^8} \quad F_\pi = 93 \text{ MeV} \quad (13)$$

In this limit, the integrations over the initial energies are readily performed, and the chemical relaxation time becomes:

$$\tau_\pi|_{M_\pi=0} \sim 4700 \frac{F_\pi^8}{T^9} \quad (14)$$

For the physical value of the pion mass we perform the integration numerically. Table I shows the results for the chemical relaxation time. For comparison, the thermal relaxation time of the pion gas [2] is also displayed.

T MeV	$\tau_\pi(M_\pi = 0)$ fm	$\tau_\pi(M_\pi = 138 \text{ MeV})$ fm	τ_{thermal} fm
100	5150	17000	17
120	1000	2300	7
140	250	450	3.3
160	75	120	1.7
180	26	40	0.9
200	10	15	0.5

Table I: Results for chemical relaxation time τ_π in a pure pionic gas. The last column shows the thermal relaxation time.

As s increases the lowest order chiral Lagrangian fails to give a good representation of the cross section, overestimating its value. For this reason, at temperatures above 150 MeV or so the chemical relaxation time of a pion gas ought to be larger than in the table.

III. HADRONIC GAS

In a hadronic gas a large variety of inelastic elementary processes can take place. These processes tend to maintain or restore chemical equilibrium as they can produce fluctuations of the different particle numbers. Due to the large multiplicity of hadronic states and the large variety of inelastic processes, it is clear that a precise analysis is impossible. Data are available only for resonance decays and $N - \bar{N}$ annihilation processes, and to a lesser extent, for resonance production in $\pi - N$ and $N - N$ collisions at low energy. For this reason, we will only take into account the following processes: (a) resonance decays, e.g. $\rho \rightarrow \pi\pi$, $N^* \rightarrow N\pi$, $N^* \rightarrow N\pi\pi$, etc.; (b) baryon-antibaryon and excited meson annihilation into pions, e.g. $\bar{N}N \rightarrow \pi\dots\pi$, $\omega\omega \rightarrow \pi\dots\pi$, etc. The purely pionic processes considered in the previous section turn out to give small corrections in this more realistic picture, and are therefore neglected. Strange hadrons are also neglected. We expect that by solely including these classes of processes, a reasonable estimate of the relevant chemical relaxation times can be obtained.

Within the dilute gas approximation to eqn.(6) one can then write down:

$$\theta_\alpha g_\alpha \dot{N}_\alpha = \sum_\beta A_{\alpha\beta} \mu_\beta \quad , \quad (15)$$

where $\alpha, \beta = \pi, N, N_i^*, \Delta_j$, and, $\theta_\alpha = 1(2)$ if α is a meson(baryon). The matrix $A_{\alpha\beta}$ is symmetric, and for the class of processes we take into account its matrix elements are given by the following expressions:

$$A_{\pi\pi} = -\frac{1}{T} \left(\sum_{\alpha=N_i^*, \Delta_j} \theta_\alpha N_\alpha g_\alpha \Gamma_\alpha \langle \delta n_\pi^2 \rangle_\alpha \right. \\ \left. + \sum_{\Delta_i, \Delta_j} g_{\Delta_i} g_{\Delta_j} \langle \delta n_\pi^2 \rangle_{\Delta_i, \Delta_j} \Omega_{\Delta_i, \Delta_j}(T) \right. \\ \left. + \sum_{N_i, \bar{N}_j} g_{N_i} (g_{N_j} - \delta_{ij}) \langle \delta n_\pi^2 \rangle_{N_i, N_j} \Omega_{N_i, N_j}(T) \right)$$

$$\begin{aligned}
A_{\pi N} &= -\frac{2}{T} \left(\sum_{N_i^*} N_{N_i^*} g_{N_i^*} \Gamma_{N_i^*} \langle \delta n_{\pi} \rangle_{N_i^*} \right. \\
&\quad \left. - \sum_{N_i} g_N (g_{N_i} - \delta_{i1}) \langle \delta n_{\pi} \rangle_{N N_i} \Omega_{N N_i}(T) \right) \\
A_{\pi \Delta} &= \frac{1}{T} \left(N_{\Delta_j} g_{\Delta_j} \Gamma_{\Delta_j} \langle \delta n_{\pi} \rangle_{\Delta_j} \right. \\
&\quad \left. + g_{\Delta_j} \sum_{\Delta_i} g_{\Delta_i} \langle \delta n_{\pi} \rangle_{\Delta_i \Delta_j} \frac{1}{2} \Omega_{\Delta_i \Delta_j}(T) \right) \\
A_{\pi N_i^*} &= \frac{2}{T} \left(N_{N_i^*} g_{N_i^*} \Gamma_{N_i^*} \langle \delta n_{\pi} \rangle_{N_i^*} \right. \\
&\quad \left. + g_{N_i^*} \sum_{N_j} (g_{N_j} - \delta_{ij}) \langle \delta n_{\pi} \rangle_{N_i^* N_j} \Omega_{N_i^* N_j}(T) \right) \\
A_{NN} &= -\frac{2}{T} \left(\sum_{N_i^*} N_{N_i^*} g_{N_i^*} \Gamma_{N_i^*} \right. \\
&\quad \left. + g_N \sum_{N_i} (g_{N_i} - \delta_{i1}) (1 + \delta_{i1}) \Omega_{N N_i}(T) \right) \\
A_{N\Delta} &= 0 \\
A_{NN_i^*} &= \frac{2}{T} \left(N_{N_i^*} g_{N_i^*} \Gamma_{N_i^*} - g_N g_{N_i^*} \Omega_{N N_i^*}(T) \right) \\
A_{\Delta\Delta} &= -\frac{1}{T} \left(\delta_{ij} N_{\Delta_i} g_{\Delta_i} \Gamma_{\Delta_i} + 4 g_{\Delta_i} g_{\Delta_j} \frac{1}{2} \Omega_{\Delta_i \Delta_j}(T) \right) \\
A_{\Delta N_i^*} &= 0 \\
A_{N_i^* N_j} &= -\frac{2}{T} \left(\delta_{ij} N_i^* g_{N_i^*} \Gamma_{N_i^*} + g_{N_i^*} (g_{N_j} - \delta_{ij}) \Omega_{N_i^* N_j}(T) \right) \quad (16)
\end{aligned}$$

where the following definitions have been used: $\langle \delta n_{\pi} \rangle_{\alpha}$ is the average number of pions produced in the decay of the state α , $\langle \delta n_{\pi} \rangle_{\alpha\beta}$ is the average number of pions produced in the annihilation of α with β , and analogously for the corresponding averages of the square of the number of pions. We have assumed that the annihilation averages are roughly energy-independent at low energy, so that they can be factored out. Here Γ_{α} is the width of the corresponding resonance, and $\Omega_{\alpha\beta}$ can be expressed in terms of the total annihilation cross section as follows:

$$\Omega_{\alpha\beta} = \int \frac{d^3 k_{\alpha} d^3 k_{\beta}}{(2\pi)^6} \exp(-(E_{\alpha} + E_{\beta})/T) |v(k_{\alpha}, k_{\beta})| \sigma_{\alpha\beta \rightarrow \text{pions}}, \quad (17)$$

where $|v(k_{\alpha}, k_{\beta})|$ the relative speed in the rest frame of one of the colliding partners. For $N - N$ annihilation the following approximation holds [8]:

$$\Omega_{NN}(T) \sim N_N^2 \frac{1}{M} (b + 4a \sqrt{MT/\pi}) \quad (18)$$

where M is the nucleon mass, $a \sim 17$ mb and $b \sim 39$ mb.GeV. We adopt here the following approximation: for all possible annihilation processes the form shown in (18) is used, with the densities replaced by the densities of the partners in the collision and the nucleon mass replaced by the average mass. This is a crude guess, which is required by the lack of experimental access to resonance annihilation processes. The following further approximations are performed: (a) the averages $\langle \delta n_{\pi} \rangle_{\alpha\beta}$ and $\langle \delta n_{\pi}^2 \rangle_{\alpha\beta}$ are taken to be the same for all annihilation processes and (b) resonances whose width is larger than 250 MeV are not included.

We first consider a simplified case, where only pions, nucleons and ρ mesons are taken into account. We find that the longest relaxation time corresponds to a configuration where $\mu_{\pi} \sim \frac{1}{2} \mu_{\rho}$ and, in the limit where $\langle \delta n_{\pi}^2 \rangle_{\text{annihil.}} \sim \langle \delta n_{\pi} \rangle_{\text{annihil.}}^2$, $\mu_N \sim \frac{1}{2} \langle \delta n_{\pi} \rangle_{\text{annihil.}} \mu_{\pi}$. In reality, $\langle \delta n_{\pi}^2 \rangle_{\text{annihil.}} > \langle \delta n_{\pi} \rangle_{\text{annihil.}}^2$, and the latter relation is substantially affected. The next longest relaxation time corresponds to a configuration where the chemical potentials of pions and ρ mesons are in the same relation as before, but the nucleon chemical potential is now similar to μ_{π} and of opposite sign. This is interpreted as follows: The longest relaxation time has to do with how fast the bulk excess of particles can be annihilated. The second longest time measures how fast an excess of nucleons plus antinucleons is annihilated.

When we include all hadronic levels with no strangeness content, a hierarchy of relaxation times results. Only those times longer than the time scale characterizing the evolution of the system are of relevance. It turns out that the longest times are similar to those obtained in the simplified case. Thus, the approach to chemical equilibrium is not substantially altered by the addition of more states. It however occurs that a larger number of configurations will have relaxation times which are long on this time scale. These configurations show different patterns for the chemical potentials. The only qualitative feature shared by these patterns is that at temperatures below 160 MeV the chemical potentials of mesonic resonances are all similar to each other, and approximately given by twice the pion chemical potential. The baryonic chemical potentials are also similar to each other, and much larger than μ_{π} , with $\mu_N \sim \kappa(T) \mu_{\pi}$, where $\kappa(T) \sim 10 \times 2^{(T_0 - T)/20 \text{ MeV}}$, $T_0 \sim 160$ MeV. Above that temperature the baryonic chemical potentials become of the same order as μ_{π} , and the mesonic ones

show values much larger than μ_π . This indicates that the approach to equilibrium of those configurations with the longest relaxation times are driven at high temperatures mainly by the baryons, and at lower temperatures by the meson resonances.

T MeV	$\tau^{(A)}$ fm	$\tau^{(B)}$ fm
100	2200; 1200 1300; 700	1060; 470; 20; 8 1030; 280; 20; 8
120	260; 130 200; 60	165; 65; 15; 8 140; 40; 15; 8
140	65; 25 50; 13	50; 17; 9; 7 35; 15; 8; 5.5
160	25; 9 17; 4	22; 10; 8; 4 12; 9; 8; 4
180	10; 4 7; 2	13; 9; 7; 4 9.5; 7; 5; 4
200	5; 2 4; 1	11; 9; 6; 4 9; 7; 4; 3

Table II: Results for chemical relaxation times in a hadronic gas. (A) refers to the case where only pions, nucleons and ρ -mesons are included. The two longest relaxation times are given. (B) corresponds to the inclusion of all relevant hadronic levels. The four longest relaxation times are given. The upper and lower rows correspond respectively to the cases $\langle \delta n_\pi^2 \rangle_{\text{annihil.}} = 30$ and 40. In all cases $\langle \delta n_\pi \rangle_{\text{annihil.}} = 5$.

We have checked that by removing some states the longest relaxation times are hardly affected. There are also minor effects in the composition of the gas corresponding to those longest times. We also observe modest changes when the annihilation cross section is changed by a factor of two or so. This suggests that the estimates given here should not be substantially altered by taking more precise values for the annihilation cross sections and by including other inelastic processes ignored in our approximations. Finally, comparison with the results for

the pion gas shows the inadequacy of the latter as an approximation (taking into account that the pion gas results obtained here are trustworthy below 150 MeV, as mentioned in section 2).

One can draw the following picture of a hadronic gas which drives out of chemical equilibrium in the course of the expansion. Suppose that the system is large enough so that initially equilibrium prevails. As the expansion rate increases and becomes larger than the longest relaxation time at the current temperature, a chemical potential associated to the configuration of the corresponding eigenvector starts to develop. As the expansion proceeds, it eventually becomes larger than the second longest relaxation time, and an additional chemical potential starts to develop. Thus, in the course of the expansion, at each stage the thermodynamic state of the hadronic gas will be characterized by T, and as many chemical potentials as the number of characteristic relaxation times which are longer than the current expansion rate. The evolution of these potentials in the course of the expansion can in principle be estimated. In practice, the typical radii of the highest multiplicity systems produced in high energy nuclear collisions are of the order of 10 fm at freeze-out. Thus, in this situation, the relaxation times ought to be compared with the size of the system rather than with the inverse expansion rate, as this is smaller. Clearly, chemical equilibrium cannot be sustained even in the early stages after hadronization occurs.

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