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NONPERTURBATIVE QCD AND ELASTIC PROCESSES AT CEBAF ENERGIES

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Abstract

We outline how one can approach nonperturbative aspects of the QCD dynamics studying elastic processes at energies accessible at upgraded CEBAF. Our point is that, in the absence of a complete theory of the nonperturbative effects, a possible way out is based on a systematic use of the QCD factorization procedure which separates theoretically understood ("known") short-distance effects and nonperturbative ("unknown") long-distance ones. The latter include hadronic distribution amplitudes, soft components of hadronic form factors *etc.* Incorporating the QCD sum rule version of the QCD factorization approach, one can relate these nonperturbative functions to more fundamental obects, vacuum condensates, which accumulate information about the nonperturbative structure of the QCD vacuum. The emerging QCD sum rule picture of hadronic form factors is characterized by a dominant role of essentially nonperturbative effects in the few GeV region, with perturbative mechanisms starting to show up for momentum transfers Q^2 closer to 10 GeV^2 and higher. Thus, increasing CEBAF energy provides a unique opportunity for a precision study of interplay between the perturbative and nonperturbative phenomena in the QCD description of elastic processes.

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I. HIERARCHICAL STRUCTURE OF NPQCD DESCRIPTION OF ELASTIC PROCESSES

"What can we learn about nonperturbative QCD from studying elastic processes at CEBAF energies?" In some sense, *all* we will learn from studying elastic processes at any energy will involve nonperturbative aspects of QCD because the very nature of initial and final state hadrons is essentially nonperturbative. In the few GeV energy range, the nonperturbative effects are especially important and, in fact, play a crucial role. However, it is also true that to learn something about nonperturbative QCD one should formulate first a theoretical framework adequately describing the nonperturbative aspects of quantum chromodynamics. The problem is that the nonperturbative QCD (NpQCD) is not a completed theory yet, and there is no formalism which allows one to calculate nonperturbative effects from the first principles of QCD. Still, because of asymptotic freedom, there are regions of momentum space where QCD is under theoretical control, and these regions can be used as a starting point for a step by step penetration into the nonperturbative regions of QCD. This step by step strategy is based on a generic idea that one should make first a full use of what is known, isolate the structures which are unknown, parameterize them in some way and then study the parameterizing functions as separate objects. In fact, this is exactly the procedure developed to study strong interaction physics long before the invention of QCD.

A. Form factors

Consider, *e.g.*, a typical CEBAF experiment on electron-proton elastic scattering. The electron properties are known from QED, so one can readily write down the matrix elements for interaction between the electron and electromagnetic field. However, a similar matrix element $\langle P'|J^\mu|P\rangle$ for the proton is not immediately known because the strong interaction physics is involved which cannot be directly approached in a straightforward perturbative framework. Introducing elastic form factors $G_E^p(Q^2), G_M^p(Q^2)$, we accumulate unknown information about the strong interactions (and, hence, about NpQCD) in these two functions. In fact, many models were proposed to calculate $G_E^p(Q^2), G_M^p(Q^2)$ and other hadronic form factors. However, most of these models have a very remote (if any) connection with the basic principles of QCD and QCD Lagrangian. As a result, experimental success or failure of such models usually tells us very little about the nonperturbative aspects of QCD. In other words, to get an informative answer from experiment about NpQCD, the question must be cast in a "QCD language" based on concepts which have a clear and direct relation to the QCD Lagrangian.

B. Wave functions

Introducing hadronic form factors, we incorporated only a part of what we know now, namely, the theory of electromagnetic interactions. However, as emphasized above, QCD is not a total *terra incognita*: its short-distance behaviour can be calculated using perturbation theory. In particular, one can use perturbative QCD to calculate the Q^2-dependence of hadronic form factors [1–3] at large momentum transfer Q (most probably, very large). According to the factorization theorem [4–6], $G^p(Q^2)$ is given by a convolution of the hard scattering amplitude $T(\{x\},\{x'\};Q^2)$ and proton distribution amplitudes (sometimes also called wave functions) $\varphi(x_1,x_2,x_3)$ specifying the probability amplitude to find the proton composed of its three valence quarks carrying the fractions x_1, x_2, and x_3 of its large longitudinal momentum P (of course, $x_1+x_2+x_3=1$). Formallly, the distribution amplitude $\varphi(x_1,x_2,x_3)$ can be defined as a function whose moments are given by proton-to-vacuum matrix elements of some local composite operators

$$\int_0^1 \varphi(x_1,x_2,x_3)x_1^{N_1}x_2^{N_2}x_3^{N_3}\delta(1-\sum_1^3 x_i)\,dx_1dx_2dx_3 \sim \langle 0|(D^{N_1}u)(D^{N_2}u)(D^{N_3}d)|P\rangle. \tag{1}$$

The hard scattering amplitude $T(\{x\},\{x'\};Q^2)$ is calculable from perturbative QCD and, in this sense, is known. On the other hand, the distribution amplitudes $\varphi(x_1,x_2,x_3)$ or matrix elements $\langle 0|O_{\{N\}}|P\rangle$ are unknown functions/parameters describing nonperturbative properties of the nucleons in QCD. These functions are more universal than form factors: the same distribution amplitude can be used to calculate different types of hard elastic amplitudes in perturbative QCD.

C. Condensates

As emphasized above, matrix elements $\langle 0|O_{\{N\}}|P\rangle$ are not known in pQCD. However, one can again incorporate asymptotic freedom in the form of the operator product expansion to calculate the large-p^2 behaviour of the correlator

$$\Pi_{\{N\}}(p^2) = i\int \langle 0|T(O_{\{N\}}(x)O_{\{0\}}(0)|P\rangle e^{ipx}d^4x \tag{2}$$

of the relevant composite operators.

The operator product expansion represents a two-point correlator as a sum of terms, each of which is a product of perturbatively calculable coefficient function and a vacuum-to-vacuum matrix elements of local operators - condensates [7]. These matrix elements describe/parameterize nonperturbative properties of the QCD vacuum. In some cases, it is convenient to introduce nonlocal condensates [8,9], *e.g.* $\langle 0|\bar{q}(0)q(z)|0\rangle$, which generate the local condensates $\langle 0|\bar{q}(0)q(0)|0\rangle \equiv \langle\bar{q}q\rangle$, $\langle qD^2q\rangle$, *etc.* after Taylor expansion in z.

Incorporating the fact that the two-point correlator can be written *via* a dispersion relation as an integral over the hadronic spectrum, one can get a QCD sum rule [10,6]

$$\langle 0|O_{\{N\}}|P\rangle\langle P|O_{\{0\}}|0\rangle e^{-m_p^2/M^2} + \frac{1}{\pi M^2}\int_{threshold}^{\infty} \rho_N(s)e^{-s/M^2} = \Pi^{pert}_{\{N\}}(M^2) + f_q(M^2)\langle\bar{q}q\rangle + f_G(M^2)\langle GG\rangle + \ldots , \tag{3}$$

in which the proton-to-vacuum matrix element $\langle 0|O_{\{N\}}|P\rangle$ is related to condensates, *i.e.*, to a set of universal numbers (or functions, if one uses the nonlocal condensates) specifying the properties of the nonperturbative QCD vacuum. Knowing these functions, one can calculate $\langle 0|O_{\{N\}}|P\rangle$ from the QCD sum rule, and *vice versa*, knowing matrix elements like $\langle 0|O_{\{N\}}|P\rangle$, one can get information about vacuum condensates. The condensates are more universal than distribution amplitudes: the same set of condensates appears in QCD sum rules for different form factors, distribution amplitudes, decay constants, *etc.*

D. Λ_{QCD}

In principle, all the condensates $\langle\bar{q}q\rangle$, $\langle GG\rangle$, *etc.* are determined by the magnitude of the fundamental QCD parameter Λ_{QCD}, *e.g.*, $\langle\bar{q}q\rangle = -a\Lambda^3_{QCD}$, $\langle GG\rangle = b\Lambda^4_{QCD}$, *etc.* However, at present stage of NpQCD, the coefficients $a, b, etc.$ are not known. When (and if) the NpQCD will become a complete theory, then Λ_{QCD} will become the only unknown parameter to be extracted from experiment.

E. Hierarchy in NpQCD description

To summarize, the nonperturbative aspects of QCD can be described/parameterized by a hierarchy of functions having more and more fundamental nature:

1. Form factors - hadron to hadron matrix elements of various currents, *e.g.*, $\langle P'|J^\mu|P\rangle$.
2. Distribution amplitudes (wave functions) - hadron to vacuum matrix elements of local (for moments) or nonlocal (for distribution amlitudes) composite operators constructed from quark and gluonic fields, *e.g.*, $\langle 0|O_{\{N\}}|P\rangle$.
3. Condensates - vacuum to vacuum matrix elements of local or nonlocal composite operators, *e.g.*, $\langle\bar{q}q\rangle$, $\langle\tilde{G}G\rangle$, $\langle\bar{q}(0)q(z)\rangle$, *etc.*
4. Λ_{QCD} - fundamental scale governing all other mass, momentum, distance *etc.* scales in QCD.

II. THE SIMPLEST FORM FACTOR: $F_{\gamma\gamma^*\pi^0}(Q^2)$

A. Preliminaries

The distribution amplitudes $\varphi_H(\{x_i\}\ )$ are universal functions describing a hadron H in various hard elastic processes. Studying the latter experimentally, one can hope to fix a set of distribution amplitudes which will be in agreement with all the data. Adhering to our strategy of a step by step expansion from simpler situations to more complicated ones, let us consider the simplest case when the hadron has only two constituents. For the pion, the simplest and directly measurable $\langle 0|O|P\rangle$-type object is the matrix element of the axial current

$$\langle 0|\bar{d}\gamma_5\gamma^\mu u|P\rangle = if_\pi P^\mu. \tag{4}$$

It appears in the amplitude for the $\pi \to \mu\nu$ decay, which involves one known weak interaction vertex and one unknown $\bar{q}q\pi$ vertex parameterized by a single constant f_π. Experimentally, $f_\pi \approx 133\,MeV$. By definition, this constant also fixes the integral normalization of the pion distribution amplitude

$$\int_0^1 \varphi_\pi(x)\,dx = f_\pi\ . \tag{5}$$

Next in complexity are the amplitudes involving three vertices. The problem is simpler when the extra vertex has a known structure, *i.e.*, when it corresponds to an EM or weak current. An example of such an amplitude is the form factor $F_{\gamma\gamma^*\pi^0}(q_1, q_2)$ describing the coupling of the neutral pion to two photons with momenta q_1 and q_2. To apply perturbative QCD, one should be sure that virtuality of at least one of the photons is large. Experimentally, it is difficult to arrange two highly virtual photons, so a reasonable compromise is the situation when one of the photons is real $q_1^2 = 0$ or slightly virtual $-q_1^2 << m_\rho^2$ and another one has a large virtuality $q_2^2 \equiv -Q^2 \gtrsim m_\rho^2$.

The cleanest experimental realization for this form factor is on a e^+e^--machine, *i.e.*, one should study the process $e^+e^- \to e^+e^-\pi^0$ implying the lowest order QED subprocess $e^+e^- \to e^+e^-\gamma\gamma^*$ and subsequent fusion of $\gamma\gamma^*$ into π^0. In this case, the origin of the photons is most clean theoretically, so one directly measures $F_{\gamma\gamma^*\pi^0}(Q^2)$.

For a fixed-target machine like CEBAF, one can imagine two possibilities for the subprocess:

a) $\gamma\gamma^* \to \pi^0$, with an almost real Primakoff photon coming from the hadronic target, or

b) $\gamma^*\pi^0 \to \gamma$, now with a pion extracted from the target and a real photon detected in the final state. To detect the second subprocess, one should exclude

the pomeron exchange. This is a very difficult task though polarization measurements can make this more feasible. With the second subprocess, one would also face the standard set of problems related to extrapolation to the pion pole. Feasibility of the $F_{\gamma\gamma^*\pi^0}(Q^2)$ experiments at CEBAF is under study [12].

B. Theory

In perturbative QCD, the $\gamma\gamma^* \to \pi^0$ form factor is described by a simple diagram similar to the handbag diagram for deep inelastic scattering. It gives [5]:

$$F^{pQCD}_{\gamma\gamma^*\pi^0}(Q^2) = \frac{4\pi}{3Q^2}\int_0^1 \frac{\varphi_\pi(x)}{x}\,dx + O(1/Q^4), \tag{6}$$

where the unknown nonperturbative information is accumulated by the integral

$$I = \int_0^1 \frac{\varphi_\pi(x)}{x}dx\ . \tag{7}$$

A few remarks are in order here.

1. The pQCD expansion starts with the term of zeroth order in the QCD coupling constant α_s, *i.e.*, the asymptotically leading term has no suppression.This situation is identical to that in deep inelastic scattering. Hence, we have all the reasons to expect that the pQCD description in this case must work at Q^2 as low as $1\,GeV^2$.

2. The value of $F_{\gamma\gamma^*\pi^0}(Q^2)$ at $Q^2 = 0$ reduces to the amplitude for the two-photon decay $\pi^0 \to \gamma\gamma$, and the latter is fixed by the ABJ-anomaly [11].

Thus, despite the fact that the process $\pi^0 \to \gamma\gamma$ for real photons does not involve any large momentum transfer, this amplitude is calculable

$$F^{pQCD}_{\gamma\gamma^*\pi^0}(0) = \frac{1}{\pi f_\pi}, \tag{8}$$

though the result involves another nonperturbative parameter - the pion decay constant f_π. However, its value is well known experimentally: $f_\pi \simeq 133\,MeV$.

On the large-Q^2 side, assuming the so-called asymptotic form [1,4,3] for the pion distribution amplitude

$$\varphi_\pi^{as}(x) = 6 f_\pi x(1-x)\,, \tag{9}$$

one obtains an absolute prediction for the asymptotic behaviour [5]

$$F^{as}_{\gamma\gamma^*\pi^0}(Q^2) = \frac{4\pi f_\pi}{Q^2} + O(1/Q^4). \tag{10}$$

Brodsky and Lepage [13] proposed an interpolation formula

$$F_{\gamma\gamma^*\pi^0}(Q^2) = \frac{1}{\pi f_\pi \left(1 + \frac{Q^2}{4\pi^2 f_\pi^2}\right)} \tag{11}$$

which reproduces both the $Q^2 = 0$ value and the high-Q^2 asymptotics.

The BL-interpolation formula has a monopole form $F_{\gamma\gamma^*\pi^0}(Q^2) \sim 1/(1 + Q^2/s_0)$ with the scale $s_0 = 4\pi^2 f_\pi^2 \approx 0.7\, GeV^2$ being numerically very close to the ρ-meson mass squared: $m_\rho^2 \approx 0.6\, GeV^2$. So, the BL-interpolation suggests a form similar to that based on the VMD expectation $F_{\gamma\gamma^*\pi^0}(Q^2) \sim 1/(1 + Q^2/m_\rho^2)$. In fact, as it will be explained later, the combination $s_0 = 4\pi^2 f_\pi^2$ plays an important role in the QCD sum rule analysis of the axial current channel: s_0 gives the pion duality interval or the effective threshold for higher states in this channel, and the BL-interpolation formula exactly coincides with the result based on the so-called local duality prescription [14].

Comparison with existing experimental data [15], shown in Fig.1, shows a good agreement with the BL-interpolation formula and local duality. It should be emphasized here that these approaches assume/require the asymptotic form for the pion distribution amplitude. On the other hand, the curve based on the unmodified lowest-twist pQCD prediction with asymptotic wave function (eq.(10)) goes marginally higher than the data. The situation is even worse if one takes the Chernyak-Zhitnitsky form

$$\varphi_\pi^{CZ}(x) = 30 f_\pi x(1-x)(1-2x)^2 \tag{12}$$

which increases the I-integral by the factor $5/3 \approx 1.67$. A simple monopole-type interpolation to the CZ asymptotics is also excluded. Thus, the $F_{\gamma\gamma^*\pi^0}(Q^2)$ is

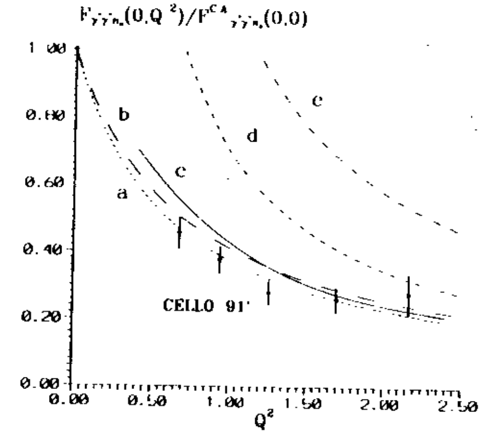

FIG. 1. Comparison of experimental data with theoretical predictions for $F_{\gamma\gamma^*\pi^0}(Q^2)$. *a*) VMD, *b*) BL-interpolation & QCD local duality, *c*) Finite-M^2 QCD sum rules, *d*) Leading-twist pQCD with asymptotic distribution amplitude, *e*) Leading-twist pQCD with CZ distribution amplitude.

rather sensitive to the form of the pion distribution amplitude $\varphi_\pi(x)$. In fact, if one agrees that there are no reasons why the interpolated pQCD prediction for this process should not work with, say, 10 or 20 % accuracy for Q^2 between 1 and 2 GeV^2, then one should also agree that existing data exclude the CZ form for the distribution amplitude. On the other hand, the data are absolutely consistent with predictions based on the asymptotic form of $\varphi_\pi(x)$. The data also agree with the VMD-curve and the results of a more detailed ("finite-M^2") QCD sum rule analysis [14].

For further applications, it is also very important to note that the BL-interpolation formula implies a suppression of the hard quark propagator in the region of small virtualities

$$\int_0^1 \frac{\varphi(x)}{xQ^2} dx \to \frac{3 f_\pi}{Q^2 + s_0}\,. \tag{13}$$

A possible interpretation is that in the regions where the quark virtuality xQ^2 is small, one should take into account effects due to the transverse momentum: $xQ^2 \to (xQ^2 + k_\perp^2)$. Then s_0 may be treated as an average value of the combination $k_\perp^2/x$. This value is not small: $s_0 \approx 0.7\, GeV^2$, and one should expect sizable predasymptotic effects for $Q^2 \simeq 1\,GeV^2$. On the other hand, one should expect that the higher $1/Q^2$-corrections do not strongly affect the pQCD result above $Q^2 \gtrsim 2-3\,GeV^2$.

C. Lessons

Thus, the analysis of $F_{\gamma\gamma^*\pi^0}(Q^2)$, which is the simplest form factor from both the pQCD and NpQCD (QCD sum rule) points of view, shows that

- The experimentally measured magnitude of this form factor agrees well with the prediction assuming the asymptotic form of the pion distribution amplitude.
- Extrapolating the pQCD prediction $F_{\gamma\gamma^*\pi^0}(Q^2) = 4\pi f_\pi I/Q^2$ into the region of moderate and small Q^2, one should substitute I by a smaller value $I(Q^2) \simeq I/(1+s_0/Q^2)$ with s_0 being a "typical hadronic scale" $\approx m_\rho^2$.

III. PION ELECTROMAGNETIC FORM FACTOR

Now we can make the next step and consider a more complicated case when only one vertex of a 3-point function is known and two are unknown. So, let us discuss the pion electromagnetic form factor $F_{\gamma^*\pi\pi}(Q^2)$ and compare its physics to that of $F_{\gamma\gamma^*\pi^0}(Q^2)$.

The perturbative QCD prediction [1-3] is that, at (asymptotically) high Q^2, the pion EM form factor is dominated by a hard gluon exchange

$$F_\pi^{pQCD}(Q^2) = \frac{8}{9}\pi\alpha_s\frac{I^2}{Q^2} + O(1/Q^4) \qquad (14)$$

where I is the same integral of the pion distribution amplitude we encountered studying $F_{\gamma\gamma^*\pi}(Q^2)$.

It is instructive to rewrite the pQCD formula as

$$F_\pi^{pQCD}(Q^2) = \frac{4\pi^2 f_\pi^2}{Q^2}\left(\frac{\alpha_s}{\pi}\right)\frac{2}{9}\left(\frac{I^2}{f_\pi^2}\right) = \frac{2}{9}\left(\frac{I^2}{f_\pi^2}\right)\frac{s_0}{Q^2}\left(\frac{\alpha_s}{\pi}\right). \qquad (15)$$

For the asymptotic wave function, one has $I/f_\pi = 3$ and $F_\pi^{pQCD,as}(Q^2) = \frac{2s_0}{Q^2}(\alpha_s/\pi)$, or, numerically, $Q^2 F_\pi^{pQCD,as} \to (\alpha_s/\pi)(1.4\,GeV^2)$. "Experimentally", (α_s/π) is close to 1/10. So, this is the high-Q^2 prediction. On the other hand, the pion form factor has an absolute normalization at $Q^2 = 0$: $F_\pi(0) = 1$. To extrapolate the one-gluon-exchange (OGE) term to low Q^2 values, one can try to use the prescription $I \to I/(1+s_0/Q^2)$ discussed above. In this case, the OGE formula extrapolates into zero at $Q^2 = 0$, indicating that the low-Q^2 behaviour might have nothing to do with the OGE. One can try another extrapolation prescription $I^2 \to I^2/(1+2s_0/Q^2)$ which coincides with the original one in the first $1/Q^2$-correction at large Q^2, but gives an overall monopole formula $F_\pi^{OGE}(Q^2) \sim 1/(Q^2+2s_0)$. Then $F_\pi^{OGE}(0) = (I/3f_\pi)^2(\alpha_s/\pi)$. Thus, with the asymptotic wave function $(I/3f_\pi = 1)$, favored by the $F_{\gamma\gamma^*\pi^0}(Q^2)$ data, this extrapolation gives a nonzero, but small $(\alpha_s/\pi) \sim 1/10$ part of the total pion form factor value $F_\pi(0) = 1$. This also suggests that the low-Q^2 behaviour of the pion form factor is dominated by contributions different from the OGE.

One can try also to arrange a brute force interpolation $F_\pi(Q^2) = 1/(1+Q^2/M^2)$ between the $Q^2 = 0$ value and the asymptotic pQCD prediction. Then one should take $M^2 = 2s_0(\alpha_s/\pi)$ which is $0.14\,GeV^2$ for the "standard" value $(\alpha_s/\pi) \approx 0.1$. To make M^2 closer to m_ρ^2, one should take $\alpha_s \sim 1.5$ which seems too large. In any case, pQCD is not applicable if $\alpha_s \gtrsim 1$. If one takes Chernyak-Zhitnitsky wave function, then $I/f_\pi = 5$, and the interpolation parameter M^2 is equal to $\frac{50}{9}\alpha_s/\pi$, and $\alpha_s \simeq 0.5$ puts M^2 close to the desired value around $0.6\,GeV^2$. However, as discussed earlier, the CZ distribution amplitude is ruled out by the data on the $F_{\gamma\gamma^*\pi}(Q^2)$ form factor.

An alternative explanation, suggested by our original interpolation attempt (and by QCD sum rules, as we will see later) is that "1" (*i.e.*, $F_\pi(0)$) does not interpolate into the one-gluon-exchange term, simply because "1" does not and cannot contain the (α_s/π) factor. The very presence of the (α_s/π) factor means that the one-gluon-exchange diagram is a higher-order correction to a diagram which has the zeroth order in α_s. Such a diagram exists and it corresponds to the so-called soft contribution. It describes the situation when the active quark is only slightly virtual both before and after scattering. It is this diagram which provides the pion form factor normalization to unity at zero momentum transfer. The soft contribution F_π^{soft} is totally nonperturbative, it definetely dominates the form factor at low Q^2, and though one may question the range of Q^2 in which this dominance persists, there is no question that the soft contribution plays a very important role at CEBAF energies [16].

IV. QCD SUM RULES: A KEY(HOLE) TO NPQCD PHYSICS

Ideally, one should be able to calculate the NpQCD functions $F_\pi^{soft}(Q^2)$, $F_{\gamma\gamma^*\pi}(Q^2)$, $\varphi_\pi(x)$, $G_N(Q^2)$, $\varphi_N(x_1, x_2, x_3)$, *etc.* from a theory. However, bad news is that there is no such theory yet. Of course, there are some good news coming from the progress of lattice QCD, chiral perturbation theory, QCD sum rules and other approaches going beyond the standard pQCD boundaries. Among these approaches, the QCD sum rule method [7] has the widest range of applications and many impressive results. An attractive feature of the method is that it is based exactly on the idea to use the well-understood properties of QCD in the asymptotic freedom region to probe its nonperturbative dynamics.

A. Basics

One of the fundamental ideas of the QCD sum rule approach is that the applicability of pQCD is limited *not* by an uncontrollable increase of $\alpha_s(k^2)$ at small momenta k. Rather, the predictive power of pQCD is limited by nonperturbative effects which appear at momentum transfers for which α_s is still small. Consider, *e.g.*, the one-gluon-exchange diagram discussed above. If momentum k flowing through the gluon line is, say, only 500 MeV, one cannot rely on pQCD not because $\alpha_s(500\,MeV)$ is too large (in fact, it is still reasonably small), but because one cannot trust the pQCD formula $D(k) \sim 1/k^2$ for the gluon propagator. Just like the $F_{\gamma\gamma^*\pi}(Q^2)$ form factor has no $1/Q^2$ singularity at small Q^2, the "real-life" gluon propagator will not blow up in small-k^2 region. In other words, the "exact" gluon propagator $D(k)$ (which is a T-product of two gluonic fields averaged over the "physical" vacuum) differs from its perturbative counterpart obtained by averaging the T-product over the perturbative vacuum, and the difference is concentrated in the small-momentum region. At large momenta, the difference vanishes and one can rely on pQCD.

Within the QCD sum rule approach, the objects parameterizing the difference between "real" ("exact", "physical", *etc.*) and perturbative vacua of QCD play a crucial role. Examples include quark condensate $\langle\bar{q}q\rangle$, gluon condensate $\langle GG\rangle$, mixed condensate $\langle\bar{q}(\sigma G)q\rangle$, *etc.*

Another basic concept is the quark-hadron duality. A good example is provided by the e^+e^--annihilation into hadrons. In the low-energy region, $\sigma^{tot}(s, e^+e^- \to hadrons)$, the total cross section contains pronounced resonances and has no similarity to the smooth curve given by a perturbative QCD calculation of $\sigma^{tot}(s, e^+e^- \to q\bar{q})$. However, despite the fact that locally in energy the difference between the theoretical perturbative calculation and experimental nonperturbative result is drastic, if one integrates both cross sections over a large enough energy range $(0, S)$, the results will be very close:

$$\int_0^S \sigma^{tot}(s, e^+e^- \to q\bar{q})\, ds \approx \int_0^S \sigma^{tot}(s, e^+e^- \to hadrons)\, ds\,. \tag{16}$$

In fact, the version of quark-hadron duality incorporated within the QCD sum rule approach is more sophisticated. A typical QCD sum rule has the form

$$\int_0^\infty \rho^{hadron}(s) e^{-s/M^2} ds = \int_0^\infty \rho^{quark}(s) e^{-s/M^2} ds + a\frac{\langle GG\rangle}{M^2} + b\frac{\langle\bar{q}q\rangle^2}{M^2} + \ldots \tag{17}$$

Thus, the difference between the idealistic perturbative quark spectrum and the "real-life" hadronic one is measured with an adjustable weight function e^{-s/M^2} (M is usually referred to as the "Borel parameter"), and the result is expressed in terms of quark, gluon, *etc.* condensates $\langle\bar{q}q\rangle, \langle GG\rangle, \ldots$.

Of course, having just a few first condensates, it is possible to reconstruct only the gross features of the physical spectrum. The usual approximation is to represent the physical spectrum by a "first resonance plus continuum" model, with the "continuum" coinciding with the perturbative prediction and starting at some effective threshold $s = s_0$. The value of s_0 and parameters f_R, m_R specifying the first resonance contribution are extracted from the sum rule

$$f_R^2 e^{-m_R^2/M^2} = \int_0^{s_0} \rho^{quark}(s) e^{-s/M^2}\, ds + a\frac{\langle GG\rangle}{M^2} + b\frac{\langle\bar{q}q\rangle^2}{M^2} + \ldots \tag{18}$$

by requiring the best agreement between its left- and right-hand sides.

B. Local duality

Now, if one formally takes $M = \infty$, one obtains the local duality relation

$$f_R^2 = \int_0^{s_0} \rho^{quark}(s)\, ds \tag{19}$$

which states that one can get the lowest resonance contribution by just integrating the perturbative density over the appropriate duality interval s_0 (the bonus of the finite-M^2 QCD sum rules with condensates is that it allows one to calculate s_0 from the condensate values rather than input it as a free parameter).

In the axial-current channel, one has an infinitely narrow pion peak $\rho_\pi = \pi f_\pi^2 \delta(s - m_\pi^2)$, a rather wide peak at $s \approx 1.6\, GeV^2$ corresponding to A_1 and then "continuum" at higher energies. The simplest model is to treat A_1 also as a part of the continuum. For the continuum, the lowest-order perturbative QCD calculation gives an energy independent result $\rho^{pert} = 1/4\pi$. Incorporating the local duality, we get the relation between the pion decay constant and the pion duality interval:

$$s_0 = 4\pi^2 f_\pi^2\,. \tag{20}$$

For the pion distribution amplitude, the local duality prescription gives

$$\varphi_\pi^{LD}(x) = \frac{3 s_0}{2\pi^2 f_\pi} x(1-x)\,, \tag{21}$$

which coincides with the asymptotic wave function if one takes $s_0 = 4\pi^2 f_\pi^2$.

As we have seen earlier, the combination $4\pi^2 f_\pi^2$ naturally appears in the Brodsky-Lepage interpolation formula. In fact, the local duality prescription

$$\int_0^{s_0} \rho^{quark}(s)\, ds = \int_0^{s_0} \rho^{hadron}(s)\, ds \tag{22}$$

applied to the relevant spectral density gives the expression [14]

$$F^{(LD)}_{\gamma\gamma^*\pi^0}(Q^2) = \left(\frac{1}{\pi f_\pi}\right) \frac{1}{1 + Q^2/s_0} \tag{23}$$

which coincides with the BL formula if $s_0 = 4\pi^2 f_\pi^2$, *i.e.* if the pion duality interval for the 3-point function is equal to that extracted from the basic 2-point function.

Local duality can also be used to get an estimate for the soft contribution to the pion form factor [17]:

$$F_\pi^{soft(LD)}(Q^2) = 1 - \frac{1 + \frac{6 s_0}{Q^2}}{\left(1 + \frac{4 s_0}{Q^2}\right)^{3/2}}\,. \tag{24}$$

For large Q^2, this soft contribution behaves like $6 s_0^2/Q^4$, which is one power of s_0/Q^2 down compared to the one-gluon-exchange term.

To get the local duality estimate for the one-gluon-exchange (OGE) contribution, one should perform a two-loop calculation for a three-point function, which is rather involved. However, the $Q^2 = 0$ value can be easily extracted from existing result for the two-point function: $F_\pi^{OGE(LD)}(Q^2 = 0) = \alpha_s/\pi$. The high-$Q^2$ local duality result coincides with that obtained in pQCD with the asymptotic wave function $F_\pi^{OGE(LD)}(Q^2)|_{Q^2\to\infty} = F_\pi^{pQCD,as}(Q^2) = \frac{2 s_0}{Q^2}(\frac{\alpha_s}{\pi})$, and the simplest interpolation is

$$F_\pi^{OGE(LD)}(Q^2) = \left(\frac{\alpha_s}{\pi}\right) \frac{1}{1 + Q^2/2s_0} \tag{25}$$

Combining the $O(1)$ (soft) and the $O(\alpha_s)$ ("hard") terms produces the total local duality estimate for the pion form factor

$$F_\pi^{(LD)}(Q^2) = \frac{F_\pi^{soft(LD)}(Q^2) + F_\pi^{OGE(LD)}(Q^2)}{1 + \frac{\alpha_s}{\pi}}. \tag{26}$$

The resulting curve is in good agreement with existing data (see Fig.2).

Local duality can be also applied to the nucleon form factors [18]. For the proton magnetic form factor the result is [19]

$$G_M^p(Q^2) = \frac{8}{3}\sqrt{T^2 - 1}\left\{(4T^2 - 1)(T^2 - 1) + (4T^2 - 3)T\sqrt{T^2 - 1}\right\}^{-1}, \tag{27}$$

where $T = 1 + Q^2/2s_0$ and the duality interval s_0 for the nucleon [20] is $2.3\, GeV^2$. The agreement with experimental data shown in Fig.2 is also good. The theoretical curve starts to systematically deviate from the data only in the region $Q^2 \gtrsim 15\, GeV^2$.

It should be noted, that though local duality works very well in many cases, it has a less fundamental status compared to the original QCD sum rules involving

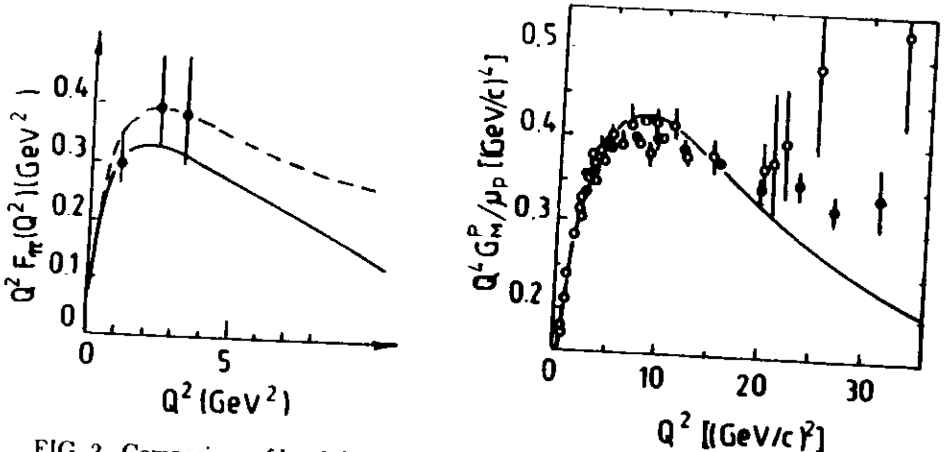

FIG. 2. Comparison of local duality predictions with experimental data for the pion form factor (left): $Q^2 F_\pi^{soft(LD)}(Q^2)$ (solid curve), $Q^2 F_\pi^{(LD)}(Q^2)$ (dashed curve) and proton magnetic form factor (right).

condensates and finite values of the Borel parameter M^2. In fact, only a thorough study of the Borel sum rules can tell whether the duality interval s_0 is really the same in different situations, in particular, if it depends on Q^2 or not in the form factor calculations. Absence of such a dependence is not obvious, because the coefficients characterizing the magnitude of the condensate corrections in a form factor sum rule, usually depend on Q^2. Such a dependence is especially important at large and small Q^2.

C. Low Q^2

At small Q^2, the perturbatively calculated coefficient functions contain singularities $\ln(Q^2)$, $1/Q^2$, *etc*. Just like in all cases considered earlier, these singularities simply signalize that one cannot extrapolate the perturbative result into the nonperturbative region. More formally, these singularities are generated by long-distance propagation in the channel related to small momentum Q. As a result, one obtains a new type of NpQCD objects - two-point correlators $\Pi(Q^2)$ taken at low values of Q^2. To evaluate them, one can use again the same old tactics: take the correlator $\Pi(Q^2)$ in the asymptotic freedom region of large Q^2 where one can rely on the operator product expansion (with condensates *etc.*), construct the model hadronic spectrum and then, using this spectrum, calculate

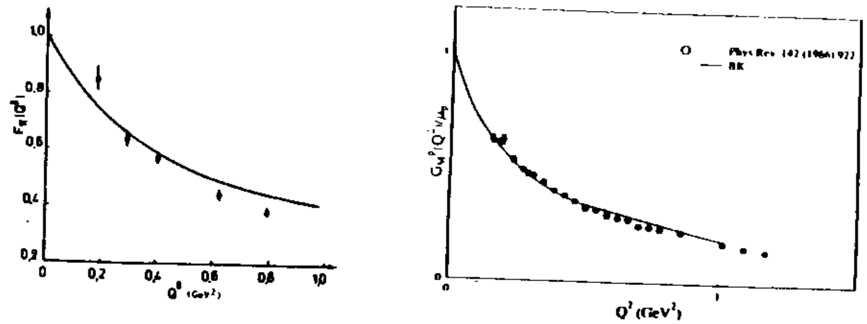

FIG. 3. Comparison of QCD sum rule calculations in the low-Q^2 kinematics with experimental data for the pion form factor (left) and proton magnetic form factor (right).

$\Pi(Q^2)$ in small-Q^2 region. As a result, one would get $1/(Q^2+m_\rho^2)$ instead of $1/Q^2$, and there will be no singularities for $Q^2=0$, just as expected. In this way, QCD sum rules were used to calculate the low-Q^2 behaviour of pion [21], kaon [22], proton and neutron [23] form factors.

D. High Q^2 and nonlocal condensates

At high Q^2, some of the coefficients in front of higher condensates contain factors like Q^2, Q^4, *etc*. and one should resum OPE in some way. It was demonstrated [24] that the Q^2 growth results from the Taylor expansion of the original nonlocal condensates over the local ones. To get rid of this problem, it was suggested [9] to avoid the Taylor expansion and work directly with nonlocal condensates. It is convenient to parameterize the nonlocal condensate $\langle \bar{q}(0)q(z)\rangle$ as

$$\langle \bar{q}(0)q(z)\rangle = \langle \bar{q}q\rangle \int_0^\infty e^{\nu z^2/4} f(\nu)\, d\nu\,. \tag{28}$$

The ν^N-moment of the function $f(\nu)$ is given by matrix element $\langle \bar{q}(D^2)^N q\rangle$, and for this reason one can interpret $f(\nu)$ as the distribution function of vacuum quarks in virtuality.

In particular, for the lowest two moments one has

$$\int_0^\infty f(\nu)\,d\nu = 1\ , \quad \int_0^\infty \nu f(\nu)\,d\nu = \frac{1}{2}\frac{\langle\bar{q}(D^2)q\rangle}{\langle\bar{q}q\rangle} \equiv \frac{\lambda_q^2}{2}, \tag{29}$$

with λ_q^2 having the meaning of the average virtuality of vacuum quarks. Some earlier QCD sum rule studies [20] give the estimate $\lambda_q^2 = 0.4\,GeV^2$. Constructing models of nonlocal condensates, one should satisfy also some other constraints. For instance, if one assumes that the vacuum matrix element $\langle\bar{q}(D^2)^{N_0}q\rangle$ exists, then $f(\nu)$ should vanish faster than $1/\nu^{N_0+1}$ as $s\to\infty$. The opposite, small-ν limit of $f(\nu)$ is governed by the large-$|z|$ properties of the function $Q(z^2)$, *i.e.*, by its behaviour at large space separations or at large values of the imaginary time variable $\tau = iz_0$. The latter case can be easily assessed using the QCD sum rule for the heavy-light meson spectrum in the heavy quark effective theory (HQET). In the HQET, the heavy quark has a trivial propagator $S_Q(z)\sim\delta^3(\mathbf{z})\theta(z_0)$ and, hence, the time dependence of the correlator of two heavy-light currents is determined by the light quark propagator [25]. At large imaginary time τ, the correlator is dominated by the lowest state contribution $\sim e^{-\tau\bar{\Lambda}}$ where $\bar{\Lambda} = (M_Q - m_Q)|_{m_Q\to\infty}$ is the lowest energy level of the mesons in HQET. This means that $Q(z^2)\sim e^{-|z|\bar{\Lambda}}$ for large Euclidean z and $f(\nu)\sim e^{-\bar{\Lambda}^2/\nu}$ in the small-ν region [26]. Numerically, $\bar{\Lambda}$ is around $0.45\,GeV$. As a result, $f(\nu)$ is a function peaked around $\nu\approx\lambda_q^2/2\sim 0.2\,GeV^2$. Choosing a different value of the average virtuality λ_q^2 changes the peak location. Calculating $F_\pi^{soft}(Q^2)$ within the QCD sum rules approach and using simple models for the nonlocal condensates, we obtained the curves showing a rather strong dependence on the parameters characterizing the nonlocal condensates [24]. This, in principle, gives a possibility of extracting these parameters from the data. The qualitative observation is that the bigger the average virtuality parameters λ_q^2, λ_G^2, the smaller the soft contribution.

The nonlocal condensates were also used [9] in QCD sum rules for the pion distribution amplitude $\varphi_\pi(x)$. The trend observed in these studies is that when the average virtuality parameter λ_q^2 increases, the resulting distribution amplitude becomes narrower, changing from the Chernyak-Zhitnitsky form for $\lambda_q^2 = 0$ to the asymptotic form for $\lambda_q^2\sim 1\,GeV^2$.

From above observations, one can expect that there may be a correlation between the form of the pion distribution amplitude and the size of the soft contribution to the pion form factor, namely, that broad distribution amplitudes, *e.g.*, those of the CZ type imply a large soft contribution, while the narrow ones

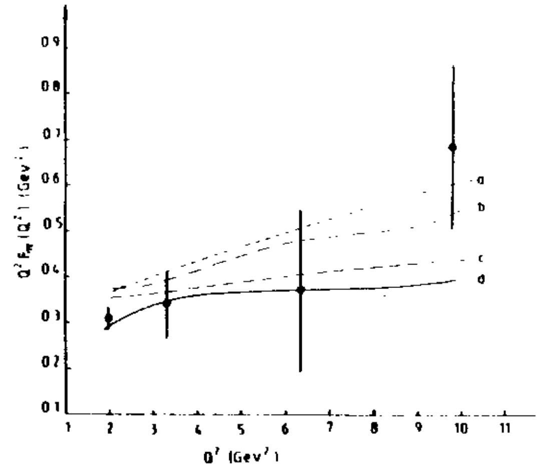

FIG. 4. Dependence of $Q^2F_\pi(Q^2)$ on Q^2 for various virtuality parameters of the quark and gluon condensates a) $\lambda_q^2 = 0.6\,GeV^2, \lambda_g^2 = 0.3\,GeV^2$; b) $\lambda_q^2 = 0.6\,GeV^2, \lambda_g^2 = 0.5\,GeV^2$; c) $\lambda_q^2 = 0.8\,GeV^2, \lambda_g^2 = 0.3\,GeV^2$; d) $\lambda_q^2 = 0.8\,GeV^2, \lambda_g^2 = 0.5\,GeV^2$.

are accompanied by smaller soft terms. In fact, this correlation really takes place because the QCD sum rules for the moments of the pion distribution amplitude and those for the pion form factor have a very similar structure [19,27].

E. Light-cone sum rules

Recently, another way of extending the applicability range of the QCD sum rules to higher Q^2 was proposed [28]. It is based on the analysis of the hadron-to-vacuum matrix elements for the correlator of two currents, *e.g.*, for the pion form factor one should consider

$$T_{\mu\nu}(p_1,p_2) = \int\langle 0|T\{J_\mu(0)j_\nu^5(x)\}|\pi(p_1)\rangle e^{ip_2x}d^4x \tag{30}$$

and apply, at large $Q^2\equiv -(p_1-p_2)^2$, the operator product expansion

$$T\{J(0)j^5(x)\}\sim\sum_n C_n(x^2)\mathcal{O}_n\ . \tag{31}$$

In this approach, one of the pions (which is in the bracket $|\pi(p_1)\rangle$) appears explicitly and is treated like in pQCD, *i.e.*, decsribed by matrix elements $\langle 0|\mathcal{O}_n|\pi(p)\rangle$

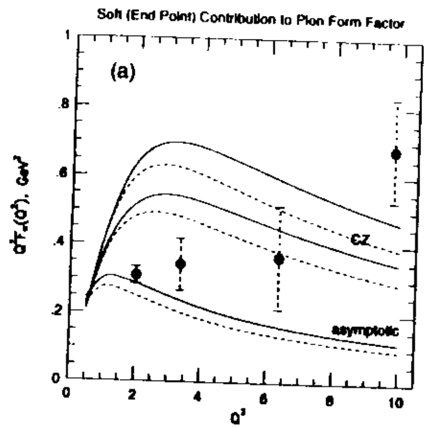

FIG. 5. Combination $Q^2F_\pi(Q^2)$ calculated within the light-cone sum rule approach for different choices of the pion distribution amplitude.

or distribution amplitudes, while another (generated by the current $j_\nu^5(x)$) is extracted from the correlator using the QCD sum rule technique.

As a result, one can explicitly study sensitivity of the soft term to the choice of the pion distribution amplitude. Again, assuming the CZ amplitude, one gets a very large soft term (see Fig.5). One should also remember, that using the CZ form, one obtains a large OGE contribution as well. Hence, in the CZ-case, adding the soft and OGE terms marginally overshoots the data. On the other hand, using the asymptotic form, one obtains a curve comfortably close to experiment.

The light-cone QCD sum rules were also applied to the $\gamma^*\pi \to A_1$ transition form factor [29]. Future applications of the light-cone sum rules should include baryon form factors $G^{e,p}_{E,M}(Q^2)$, transition form factors for $\gamma N \to \Delta, \gamma N \to N^*$, *etc.* transitions, all measurable at CEBAF. The kaon form factor can be calculated both in the light-cone sum rule approach and with standard 3-point sum rules. The results for $F_K(Q^2)$ are sensitive both to the strange-quark mass m_s and to the difference between the strange-quark condensate $\langle \bar{s}s \rangle$ and the "ordinary" $\langle \bar{u}u \rangle$. It should be mentioned, that the results of the QCD sum rule calculations of the kaon form factor in the low-Q^2 region [22] are in good agreement with existing data.

V. CONCLUSIONS

Our goal here was to describe a modern picture of the nonperturbative QCD physics behind the elastic electron-hadron scattering processes at CEBAF energies. This picture, formulated directly in terms of quark and gluon fields, is based on a systematic use of the QCD factorization of the short-distance (perturbative) and long-distance (nonperturbative) effects within the framework of the QCD sum rule method. This method relates the nonperturbative dynamics of individual hadrons to nontrivial properties of the QCD vacuum, thus providing a unified description of nonperturbative effects in QCD. The QCD sum rule analysis shows that elastic processes at CEBAF energies are dominated by nonperturbative effects, with possible signatures of the perturbative QCD dynamics in $10\,GeV$ energy region. On several examples, we demonstrated sensitivity of the QCD sum rules predictions for elastic processes to parameters and/or functions describing/parameterizing the nonperturbative aspects of the QCD dynamics. This opens a possibility of experimental determination of these parameters. Such a determination, however, would require both a high precision of CEBAF experimental data and a wider range of energies and momenta involved in these experiments. The increase of CEBAF energy is especially crucial for a detailed study of the interplay between the nonperturbative and perturbative QCD effects in elastic processes.

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