Large $\mathrm{N_c}$ Limit of Spin-Flavor Breaking in Excited Baryon Levels

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Abstract

Spin-flavor symmetry breaking in the levels of excited Baryons are studied to leading order in the $1/\mathrm{N_c}$ expansion. This breaking occurs at zeroth order. For non-strange Baryons with a single quark excited, it is shown that to first order of perturbation theory the breaking is given by one 1-body operator (spin-orbit), and three 2-body operators, all involving the orbital angular momentum of the excited quark. Higher-body operators can be reduced to that set of operators. As illustration, p-wave Baryons are briefly discussed.

In the large $\mathrm{N_c}$ limit, QCD admits an expansion in powers of $1/\mathrm{N_c}$. This is the only small expansion parameter of the gauge theory identifiable in the non-perturbative regime. The coupling constant α_s scales as $\alpha_0/\mathrm{N_c}$, where α_0 is of order unity. In general, the coefficients of the expansion are non-perturbative in α_0 and, therefore, difficult to calculate. Despite this general difficulty, the contributions to different orders in the expansion can be systematically identified as 't Hooft showed long ago [1]. The usefulness of the $1/\mathrm{N_c}$ expansion has been shown in the study of mesons in QCD in $1+1$ dimensions [2] where exact leading order solutions have been obtained [2, 3]. Similar expansions have been shown to be very successful in different field theory models [4]. In QCD in 3+1 dimensions no definite progress has been made in implementing the expansion at the quantitative level. However, some important

qualitative predictions can be drawn by identifying the leading scaling in $1/\mathrm{N_c}$ of various observables, for instance, $F_\pi \sim \sqrt{\mathrm{N_c}}$, $M_{\mathrm{Meson}} = \mathcal{O}(1)$, $M_{\mathrm{Baryon}} = \mathcal{O}(\mathrm{N_c})$, excited meson widths $\Gamma_{\mathrm{Meson}} = \mathcal{O}(1/\mathrm{N_c})$, excited Baryon widths $\Gamma_{\mathrm{Baryon}} = \mathcal{O}(1)$, OZI rule, validity of the valence quark picture up to $\mathcal{O}(1/\mathrm{N_c})$ corrections, classification of $\mathcal{O}(p^4)$ effective coupling constants in ChPT according to their large $\mathrm{N_c}$ behaviour, etc.

Many of the interesting questions in the large $\mathrm{N_c}$ limit can be found in the sector of Baryons, a sector of particular interest due to its complexity. Baryons in the large $\mathrm{N_c}$ limit consist of $\mathrm{N_c}$ valence quarks in the singlet representation of $\mathrm{SU(N_c)}$. It was shown by Witten [5] that Baryons in the ground state are described in terms of a constituent quark picture which can be implemented in the Hartree approximation neglecting spin-flavor dependent interactions. Moreover, these ground state Baryons, which are in the totally symmetric representation of the spin-flavor group $\mathrm{SU(2N_f)}$ ($\mathrm{N_f}$: number of flavors), can be described as Skyrme solitons as well [6, 7]. Both pictures turn out to be equivalent [8]. Always in the sector of ground state Baryons, important relations have been derived for couplings of pions [9, 10], SU(3) breaking in the Baryon masses [11], and magnetic moments [12].

In this paper we address a less explored sector, the excited Baryon sector. There are important questions one may ask concerning the spectroscopy and the different transitions.

Some have been considered, like the question of strong decays of p-wave Baryons [13], and the Adler-Weisberger sum rule in large $\mathrm{N_c}$ [14]. Here we study the leading corrections to the spectrum of non-strange Baryons, and the aim is to explore the spin-flavor violating pieces of the Baryon masses. The leading $\mathcal{O}(\mathrm{N_c})$ piece of the masses is spin-flavor symmetric, and the leading corrections which break this symmetry are of $\mathcal{O}(1)$. We only address Baryons where a single quark is excited, with the rest $\mathrm{N_c}-1$ quarks left in the ground state. In the limit where the symmetry breaking can be taken as a perturbation, we show that there are five different types of effective interactions that can give leading order spin-isospin symmetry breaking, namely, spin-orbit of Thomas precession type, spin-isospin tensor, and three operators involving spin, orbit and isospin components. Baryons with more than one excited quark or the case of three flavors can be studied along similar lines.

I. THE PICTURE OF EXCITED BARYONS IN LARGE $\mathrm{N_C}$

In large $\mathrm{N_c}$, Baryons can be pictured as a color singlet state of $\mathrm{N_c}$ valence (constituent) quarks, with a subleading admixture of sea quarks. The wave functions are required to be totally symmetric under the simultaneous interchange of coordinate and spin-flavor indices due to color antisymmetrization. The ground state Baryons form a tower of states described by a self-consistent Hartree wave function [5]:

$$\Phi(x_1,\ \xi_1; ...; x_{\mathrm{N_c}},\ \xi_{\mathrm{N_c}}) = \prod_{i=1}^{\mathrm{N_c}} \psi(x_i)\ \chi_S(\xi_1, ...,\ \xi_{\mathrm{N_c}}) \tag{1}$$

where $\psi(x_i)$ are $\ell = 0$ wave functions, x_i the position of the i^{th} quark and ξ_i its spin-flavor quantum numbers. $\chi_S(\xi_1, ...,\ \xi_{\mathrm{N_c}})$ is the totally symmetric tensor of rank $\mathrm{N_c}$ in spin-flavor space. For two flavors, the ground state Baryons form a tower satisfying the $I = J$ rule [15]. A typical state in the tower can be expressed as follows [18]:

$$\mid B\rangle = \frac{1}{\mathrm{N_c}!} \int \prod_{i=1}^{\mathrm{N_c}} \frac{d^3k_i}{(2\pi)^3}\ \tilde{\Phi}_B(k_1,\ \xi_1; ...; k_{\mathrm{N_c}},\ \xi_{\mathrm{N_c}}) \epsilon_{\alpha_{1,...,}\ \alpha_{\mathrm{N_c}}}\ \hat{a}^{\dagger\,\alpha_1}_{\xi_1}(k_1) ... \hat{a}^{\dagger\,\alpha_{\mathrm{N_c}}}_{\xi_{\mathrm{N_c}}}(k_{\mathrm{N_c}}) \mid 0\rangle \tag{2}$$

where $\hat{a}^{(\dagger)}$ are effective creation and annihilation operators of valence quarks, and α_i are color indices.

States with excitation energy of zeroth order in $1/\mathrm{N_c}$ correspond to having one or more quarks in an excited state [5], while a large number of order $\mathrm{N_c}$ remain in the ground state ("core"). In large $\mathrm{N_c}$ the core quarks are described by the same wave functions that describe the quarks in the ground state tower with the replacement of $\mathrm{N_c}$ by $\mathrm{N_c} - n$, n being the number of excited quarks. The classification of excited states in terms of spin-flavor $SU(2\mathrm{N_f})$ is very convenient, as this group is a dynamical symmetry group of the ground state sector. We consider states where only a single quark is excited. Such states are contained in the totally symmetric representation, corresponding to the $[\mathrm{N_c}]$ Young tableaux, and the mixed symmetry representation corresponding to the $[\mathrm{N_c}-1, 1]$ tableaux. As mentioned above, for two flavors the first representation contains the $I = J$ states with unit multiplicity, while the latter contains $I = J$ and $I = J \pm 1$ states also with unit multiplicity. In terms of the K-spin utilized in the induced representation method [9, 16, 17], the first representation corresponds to K=0 and the latter to K=1. The states in these representations are respectively given in terms of the wave functions:

$$
\begin{aligned}
\Phi_S(x_1,\ \xi_1; ...; x_{\mathrm{N_c}},\ \xi_{\mathrm{N_c}}) &= \frac{1}{2\sqrt{N_c}} \sum_j (\prod_{i \neq j} \psi(x_i))\, \phi(x_j)\ \chi_{_S}(\xi_1, ...,\ \xi_{\mathrm{N_c}}) \\
\Phi_M(x_1,\ \xi_1; ...; x_{\mathrm{N_c}},\ \xi_{\mathrm{N_c}}) &= \frac{1}{\sqrt{\mathrm{N_c}(\mathrm{N_c}-1)}} \sum_{i \neq j} (\prod_{k \neq i,j} \psi(x_k))\, \psi(x_i)\phi(x_j) - i \leftrightarrow j) \\
&\quad \times\ \chi_{_M}(\xi_i,\ \xi_j;\ \xi_1, ..., \xi_{i-1}, \xi_{i+1}, ..., \xi_{j-1}, \xi_{j+1}, ..., \xi_{\mathrm{N_c}})
\end{aligned}
\tag{3}
$$

where $\chi_{_M}$ is the irreducible rank $\mathrm{N_c}$ mixed symmetry tensor with the $[\mathrm{N_c} - 1, 1]$ tableaux, and $\phi(x)$ is the wave function of the excited quark. It is easy to show that the Hartree equations for the two different spin-flavor representations differ by exchange terms of $\mathcal{O}(1)$. The description of states with a larger number of excited quarks is by obvious generalization.

Although it does not play a crucial role in our analysis, the wave functions must be eigenfunctions of the total momentum. The following projection solves the CM motion problem present in the above wave functions:

$$
\Phi(x_1,\ \xi_1; ...; x_{\mathrm{N_c}},\ \xi_{\mathrm{N_c}}) \rightarrow \int \Phi(x_1 - \bar{x},\ \xi_1; ...; x_{\mathrm{N_c}} - \bar{x},\ \xi_{\mathrm{N_c}})\ \exp(-iP.\bar{x})\ d^3\bar{x} \tag{4}
$$

In this work we are interested in calculating matrix elements of spin-flavor breaking

Hamiltonians. Each Hamiltonian will be represented at the level of the effective theory by a sum of composite effective operators of different dimensions expressed in terms of the effective quark creation and annihilation operators and carrying the same relevant quantum numbers as the original QCD operator. These composite operators are classified in the $1/\mathrm{N_c}$ expansion according to their leading contributions to matrix elements. Operators which contain n creation and n annihilation effective quark operators are called n-body operators. Since in order to build an n-body operator one has to exchange at least $n-1$ gluons between quark lines, each n-body operator has a coefficient proportional to $\alpha_s^{n-1} = \mathcal{O}({\mathrm{N_c}}^{-n+1})$. The matrix elements of the n-body operator will in general bring back some factors of $\mathrm{N_c}$ which partially or totally eliminate that suppression. Static operators have been studied in the case of ground state Baryons [17, 18], where it is possible to establish reduction rules relating matrix elements of higher-body operators to lower-body operators, enormously simplifying the problem of obtaining the most general form of the matrix elements of a given QCD operator in the effective theory. A similar reduction applies to the static operators relevant to our analysis; higher-than-two-body operators can in fact be reduced to one- and two-body operators as shown later.

One-body color singlet static operators can be expressed as follows:

$$\hat{Q}_\Gamma^{(1)} = \int \frac{d^3k}{(2\pi)^3}\, \Gamma(\xi_1,\ \xi_2;\ k)\, \hat{a}^{\dagger\,\alpha}_{\xi_2}(-k)\, \hat{a}^{\xi_1}_{\alpha}(k), \tag{5}$$

where Γ is the kernel characterizing the operator. The matrix elements between excited states $\mid B\rangle$ and $\mid B'\rangle$ are then given by (assume the total momentum of the Baryon vanishes):

$$\begin{aligned} \langle B' \mid \hat{Q}_\Gamma^{(1)} \mid B\rangle = \mathrm{N_c} \int \prod_i \frac{d^3k_i}{(2\pi)^3}\, \Gamma(\xi_1,\ \xi_1';\ k_1)\delta^3(k_1 + ... + k_{N_c}) \\ \times\, \tilde{\Phi}^*_{B'}(k_1,\ \xi_1'; k_2,\ \xi_2; ...; k_{\mathrm{N_c}},\ \xi_{\mathrm{N_c}})\, \tilde{\Phi}_B(k_1,\ \xi_1; k_2,\ \xi_2; ...; k_{\mathrm{N_c}},\ \xi_{\mathrm{N_c}}) \end{aligned} \tag{6}$$

Similarly, two-body effective operators have the general form:

$$\hat{Q}_\Gamma^{(2)} = \int \prod_{i=1}^{4} \frac{d^3k_i}{(2\pi)^3}\, \Gamma(k_1,\ \xi_1; ...;\ k_4,\ \xi_4)\, \hat{a}^{\dagger\,\alpha_2}_{\xi_4}(k_4)\, \hat{a}^{\dagger\,\alpha_1}_{\xi_3}(k_3)\, \hat{a}^{\xi_2}_{\alpha_2}(k_2)\, \hat{a}^{\xi_1}_{\alpha_1}(k_1) \quad , \tag{7}$$

and their matrix elements are given by:

$$\langle B' \mid \hat{Q}_\Gamma^{(2)} \mid B\rangle = \mathrm{N_c}(\mathrm{N_c}-1)\int \prod_{i=1}^{\mathrm{N_c}-2} \frac{d^3k_i}{(2\pi)^3} \prod_{l=1}^{4} \frac{d^3q_l}{(2\pi)^3}\, \Gamma(q_1,\eta_1;\ldots;q_4,\eta_4)\; \delta^3(q_1+q_2-q_3-q_4)$$
$$\times\, \tilde{\Phi}^*_{B'}(k_1,\ \xi_1;\ldots;\ k_{\mathrm{N_c}-2},\ \xi_{\mathrm{N_c}-2};q_3,\ \eta_3;q_4,\ \eta_4)\; \tilde{\Phi}_B(k_1,\ \xi_1;\ldots;\ k_{\mathrm{N_c}-2},\ \xi_{\mathrm{N_c}-2};q_1,\ \eta_1;q_2,\ \eta_2). \quad (8)$$

It is not difficult to extend this to higher-body operators.

II. LEADING SPIN-FLAVOR SYMMETRY BREAKING

When spin-flavor dependent interactions are neglected, the excited Baryon spectrum shows degenerate spin-flavor towers. The masses of the towers are, as mentioned before, of order $\mathrm{N_c}$. Degeneracy is lifted at zeroth order in $1/\mathrm{N_c}$ by the effective Hamiltonian operators which we identify and study in this section. Of course, in order to lift the degeneracy we need operators which are non-trivial under spin- and/or flavor-transformations. Also, we are assuming here that the breaking of spin-flavor symmetry can be treated perturbatively, and therefore, we are only interested in taking matrix elements between states which are degenerate in the symmetry limit. This in particular means that one needs only consider matrix elements between states with the same orbital angular momentum ℓ of the excited quark; this eliminates some operators which are to be included beyond first order perturbations.

In the following we restrict the discussion to two flavors. States in the different towers can be expressed more conveniently in the basis $\{\mid I_c;\ I^T\ I^T_3;\ \ S^T\ S^T_3,\ \ell\ m\rangle\}$, where I_c and S_c are the isospin and spin quantum numbers of the core, I^T and S^T the total isospin and spin, and ℓ is the orbital angular momentum. States with $I^T = S^T \pm 1$ belong to $K=1$ towers, while for $I^T = S^T$ states one has for states with $I^T << \mathrm{N_c}/2$:

$$\begin{aligned}
\mid K=0, I^T=S^T, I^T_3, S^T_3\rangle &= \sqrt{\frac{I^T+1}{dim\, I^T}}\ \mid I_c = I^T+1/2;\ I^T\ I^T_3;\ \ S^T = I^T\ S^T_3\rangle \\
&+ \sqrt{\frac{I^T}{dim\, I^T}}\ \mid I_c = I^T-1/2;\ I^T\ I^T_3;\ \ S^T = I^T\ S^T_3\rangle \\
\mid K=1, I^T=S^T, I^T_3, S^T_3\rangle &= \sqrt{\frac{I^T}{dim\, I^T}}\ \mid I_c = I^T+1/2;\ I^T\ I^T_3;\ \ S^T = I^T\ S^T_3\rangle \\
&- \sqrt{\frac{I^T+1}{dim\, I^T}}\ \mid I_c = I^T-1/2;\ I^T\ I^T_3;\ \ S^T = I^T\ S^T_3\rangle
\end{aligned} \quad (9)$$

where $dim\ I \equiv 2I + 1$.

The matrix elements of static operators between states of a given spin-isospin tower can be expressed in terms of matrix elements of appropriate products of $SU(4) \times O(3)$ generators ($O(3)$ is generated by the orbital angular momentum operators). An illustration of this claim is the familiar case of matrix elements of the quadrupole moment operator between states of a given total angular momentum; they can be expressed in terms of the matrix elements of the operator $\{\hat{J}_i,\ \hat{J}_j\} - \frac{2}{3}\delta_{ij}\hat{J}^2$.

At the level of one-body operators there is only one type of Hamiltonian that can give leading order spin-flavor symmetry breaking, namely, the Thomas precession spin-orbit operator:

$$H_{LS} \propto\ \hat{a}^\dagger \vec{L}.\vec{\sigma}\ \hat{a} \quad , \tag{10}$$

where the proportionality factor is of order Λ_{QCD}. Here we assume that indices are contracted in the obvious way. This operator could be written in the most general form (4), but for our purpose we only need its spin-flavor structure. Two-body operators of the same LS type can be constructed by multiplying the form given above by a bilinear $\hat{a}^\dagger \hat{a}$ (which is the Baryon number operator if no momentum is exchanged), properly generalized to the form given in (6). These two-body operators can be taken into account by simply rescaling the matrix elements obtained with the one-body operator.

Symmetry breaking due to the interaction of the excited quark with the $\mathrm{N_c} - 1$ quarks in the core starts at the level of two-body operators. The leading operators are those that have coherent matrix elements ($\mathcal{O}(\mathrm{N_c})$ enhancement) between states with $I^T << \mathrm{N_c}/2$. The only coherent SU(4) generators are the axial currents

$$G^{ia} = \hat{a}^\dagger \sigma^i \otimes \tau^a\, \hat{a} \quad , \tag{11}$$

where σ^i and τ^a are the spin and isospin Pauli matrices. These operators naturally play a central role in the study of couplings between Baryons and pions [9]. Thus, the matrix elements of the leading operators of interest must include a factor of G^{ia} acting on the core.

There are four effective Hamiltonian structures with factors of G^{ia} we can write:

$$\begin{aligned}
H_T &\propto \frac{1}{\mathrm{N_c}}\, G^{ia} G_{ia} \\
H_1 &\propto \frac{1}{\mathrm{N_c}}\, \hat{a}^\dagger L^i \otimes \tau^a \,\hat{a}\, G_{ia} \\
H_2 &\propto \frac{i}{\mathrm{N_c}}\, \hat{a}^\dagger [L^i,\, L^j] \otimes \sigma_i \otimes \tau^a \,\hat{a}\, G_{ja} \\
H_3 &\propto \frac{1}{\mathrm{N_c}}\, \hat{a}^\dagger \{L^i,\, L^j\} \otimes \sigma_i \otimes \tau^a \,\hat{a}\, G_{ja}
\end{aligned} \tag{12}$$

where the proportionality factors are of order Λ_{QCD}. As we show in a moment, the operator H_T gives mass contributions that respect spin-flavor symmetry up to $1/\mathrm{N_c}$ terms, whereas the other operators break the symmetry at leading order.

Higher body operators can be reduced to one- and two-body operators. To show this, we notice that n-body operators will contain a total of (n-1) factors of the coherent operators G_{ia} (cases where a factor of the Baryon number operator appears are clearly equivalent to an (n-1) body operator, since this factor is equivalent to multiplying by a factor $\mathrm{N_c}$). Since the isospin piece associated with the matrix element between the excited quark states can carry at most one isospin index a, products of G_{ia} must have their isospin indices contracted in such a way that at most one such index is left. Using the relations [17]:

$$\begin{aligned}
G^{ia} G^{ja} &= N_c^2 \delta_{ij} \\
G^{ia} G^{ib} &= N_c^2 \delta_{ab} \\
\epsilon_{ijk} G^{ia} G^{jb} &= N_c \epsilon_{abc} G^{kc} \\
\epsilon_{abc} G^{ia} G^{jb} &= N_c \epsilon_{ijk} G^{kc} \\
\epsilon_{ijk} \epsilon_{abc} G^{ia} G^{jb} &= 2\, N_c G^{kc},
\end{aligned} \tag{13}$$

it is obvious how to carry out the reduction of the higher body operators acting on the core. We see that at leading order only one-body operators acting on the core and proportional to the axial current or Baryon number are left. Thus, the one- and two-body operators listed before are the complete set we need to discuss.

The matrix elements of G^{ia} between core states were obtained by Dashen, Jenkins and Manohar [17]:

$$\begin{aligned}
&\langle I'_c = S'_c,\ I'_{c3},\ S'_{c3} \mid G_{ia} \mid I_c = S_c,\ I_{c3},\ S_{c3} \rangle \\
&= \mathrm{N_c} \left[\frac{\dim I_c}{\dim I'_c} \right]^{1/2} \langle I_c\ I_{c3},\ 1\ a \mid I'_c\ I'_{c3} \rangle \ \langle I_c\ S_{c3},\ 1\ i \mid I'_c\ S'_{c3} \rangle + \mathcal{O}(1) \\
&\dim j \equiv 2\,j + 1 \quad ,
\end{aligned} \tag{14}$$

where the term of $\mathcal{O}(1)$ is actually proportional to the leading term. With this, the matrix elements of the operators (12) between degenerate states in the spin-flavor symmetry limit are straightforward to calculate. The explicit calculation in the case of H_T gives:

$$\begin{aligned}
&\langle I'_c;\ I^T\ I^T_3;\ \ S^{T\prime}\ S^{T\prime}_3,\ \ell\ m' \mid H_T \mid I_c;\ I^T\ I^T_3;\ \ S^T\ S^T_3,\ \ell\ m \rangle \\
&= \Lambda_T\ \delta_{S^{T\prime},S^T}\ \delta_{S^{T\prime}_3,S^T_3}\ \delta_{m,m'} (-1)^{1-S^T-I^T}\ \ [\dim I_c \dim I'_c]^{1/2} \\
&\times \begin{Bmatrix} S^T & I_c & 1/2 \\ 1 & 1/2 & I'_c \end{Bmatrix} \begin{Bmatrix} I_c & 1 & I'_c \\ \frac{1}{2} & I^T & \frac{1}{2} \end{Bmatrix} \quad ,
\end{aligned} \tag{15}$$

where I_c, $I'_c = I^T \pm 1/2$, J^T is the total angular momentum of the Baryon, and j is the total angular momentum of the excited quark.

Using (15) and (9), we readily check that H_T is diagonal in the basis of $K = 0$ and $K = 1$ states, and the eigenvalues are respectively $(-1)^{2S^T}/2$ and $(-1)^{2S^T+1}/6$. The contribution of order $\mathrm{N_c}$ is spin-flavor independent up to order $1/\mathrm{N_c}$ corrections and the same for K=0 and K=1 multiplets. This contribution stems from H_T acting on the core, and can be understood by considering an infinitesimal SU(4) transformation on H_T. The infinitesimal transformation of zeroth order generated by $X^{ia} = G^{ia}/\mathrm{N_c}$ produces an energy shift proportional to $[H_T,\ X^{ia}]$, which is of order $1/\mathrm{N_c}$ as a consequence of the commutators $[G^{ia},\ G^{jb}]$ having matrix elements of the order of the spin and isospin of the states. A similar argument implies that the result (15) must respect SU(4) symmetry.

The matrix elements of the rest of the operators are lengthy but straightforward to calculate and read as follows:

$$\langle I'_c;\ I^T\ I^T_3;\ \ell,\ S^{T\prime},\ J^T\ J^T_3 \mid H_{LS} \mid I_c;\ I^T\ I^T_3;\ \ell,\ S^T,\ J^T\ J^T_3 \rangle$$

$$= \Lambda_{LS}\ \delta_{I_c,I'_c}\left[\dim S^T\ \dim S^{T\prime}\right]$$

$$\times \sum_{j=\ell\pm 1/2} \dim j\ \ (j(j+1)-\ell(\ell+1)-3/4)\left\{\begin{array}{ccc} I_c & 1/2 & S^T \\ \ell & J^T & j \end{array}\right\}\left\{\begin{array}{ccc} I^c & 1/2 & S^{T\prime} \\ \ell & J^T & j \end{array}\right\}$$

$$\langle I'_c;\ I^T\ I^T_3;\ \ell,\ S^{T\prime},\ J^T\ J^T_3 \mid H_1 \mid I_c;\ I^T\ I^T_3;\ \ell,\ S^T,\ J^T\ J^T_3\rangle$$

$$= \sqrt{\ell(\ell+1)}\ (-1)^{I^T-J^T+S^T-S^{T\prime}+I_c-I'_c-\ell}\left[\dim S^T\ \dim S^{T\prime}\dim I_c \dim I'_c \dim \ell\right]^{1/2}$$

$$\times \left\{\begin{array}{ccc} I_c & 1 & I'_c \\ 1/2 & I^T & 1/2 \end{array}\right\}\left\{\begin{array}{ccc} S^T & 1 & S^{T\prime} \\ I'_c & 1/2 & I_c \end{array}\right\}\left\{\begin{array}{ccc} \ell & 1 & \ell \\ S^{T\prime} & J^T & S^T \end{array}\right\}$$

$$\langle I'_c;\ I^T\ I^T_3;\ \ell,\ S^{T\prime},\ J^T\ J^T_3 \mid H_2 \mid I_c;\ I^T\ I^T_3;\ \ell,\ S^T,\ J^T\ J^T_3\rangle$$

$$= \frac{3}{2}[\dim S^T\ \dim S^{T\prime}\dim I_c \dim I'_c]^{1/2}\left\{\begin{array}{ccc} I_c & 1 & I'_c \\ 1/2 & I^T & 1/2 \end{array}\right\}\sum_{j=\ell\pm 1/2} \dim j\ j(j+1)$$

$$\times (-1)^{I^T-S^{T\prime}}\left\{\begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & 1 \\ I_c & I'_c & 1 \end{array}\right\}\left\{\begin{array}{ccc} I_c & \frac{1}{2} & S^T \\ \ell & J^T & j \end{array}\right\}\left\{\begin{array}{ccc} S^{T\prime} & \frac{1}{2} & I_c \\ j & J^T & \ell \end{array}\right\} - (I_c \leftrightarrow I'_c, S^T \leftrightarrow S^{T\prime})$$

$$\langle I'_c;\ I^T\ I^T_3;\ \ell,\ S^{T\prime},\ J^T\ J^T_3 \mid H_3 \mid I_c;\ I^T\ I^T_3;\ \ell,\ S^T,\ J^T\ J^T_3\rangle$$

$$= \sqrt{\frac{15}{2}}\ (-1)^{\ell+J^T+I^T-\frac{1}{2}+I'_c+S^{T\prime}+2S^T}$$

$$\times \sqrt{\frac{\ell(2\ell-1)}{(\ell+1)(2\ell+3)}}[\dim \ell \dim S^T\ \dim S^{T\prime}\dim I_c \dim I'_c]^{1/2}$$

$$\times \left\{\begin{array}{ccc} I'_c & 1 & I_c \\ \frac{1}{2} & I^T & \frac{1}{2} \end{array}\right\}\left\{\begin{array}{ccc} 2 & \ell & \ell \\ J^T & S^{T\prime} & S^T \end{array}\right\}\left\{\begin{array}{ccc} S^{T\prime} & S^T & 2 \\ \frac{1}{2} & \frac{1}{2} & 1 \\ I'_c & I_c & 1 \end{array}\right\} \qquad (16)$$

From the above we conclude that spin-flavor symmetry is broken at leading order only in the case of Baryons with non-vanishing orbital angular momentum.

When $\ell > 0$, the multiplicities of states are as follows: in symmetric SU(4) representation, one state for each $\mid \ell - I^T \mid \leq J^T \leq \ell + I^T$; in mixed symmetry representation, one state for $J^T = \ell + I^T + 1$ and for $J^T = min\{\mid \ell - I^T \pm 1 \mid\}$, two for $J^T = \mid \ell - I^T \mid$, and three for the rest.

For illustration, we show the results for a few states in the case of $\ell = 1$ excitations. The states in the totally symmetric SU(4) representation turn out to be affected only by H_3. The (I,J) states (1/2,1/2) and (1/2,3/2) are not shifted, while (3/2,1/2) is shifted by $a_3/23$ and (3/2,3/2) is shifted by $-a_3/40$. Here, a_i parameterizes the energy shift associated with H_i. The mixed symmetry states show a more complicated pattern of shifts. This is due to the fact that the three effective Hamiltonians H_{LS}, H_1 and H_3 do contribute to the shifts, and that the shown (I,J) states have multiplicities 2 or 3. The texture of the mass matrices are the following:

$$
\begin{aligned}
&(1/2,1/2): \\
&\qquad a_{LS}\begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{5}{6} \end{bmatrix} + a_1 \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{5}{6} \end{bmatrix} + a_3 \begin{bmatrix} 0 & -\frac{1}{24\sqrt{2}} \\ -\frac{1}{24\sqrt{2}} & -\frac{1}{12} \end{bmatrix} \\
&(1/2,3/2): \\
&\qquad a_{LS}\begin{bmatrix} \frac{1}{3} & -\frac{\sqrt{5}}{6} \\ -\frac{\sqrt{5}}{6} & -\frac{1}{3} \end{bmatrix} + a_1 \begin{bmatrix} -\frac{1}{3} & -\frac{\sqrt{5}}{6} \\ -\frac{\sqrt{5}}{6} & \frac{1}{3} \end{bmatrix} + a_3 \begin{bmatrix} 0 & \frac{1}{48\sqrt{5}} \\ \frac{1}{48\sqrt{5}} & \frac{1}{15} \end{bmatrix} \\
&(3/2,1/2): \\
&\qquad a_{LS}\begin{bmatrix} -\frac{1}{3} & \frac{\sqrt{5}}{6} \\ \frac{\sqrt{5}}{6} & \frac{1}{3} \end{bmatrix} + a_1 \begin{bmatrix} \frac{1}{3} & \frac{\sqrt{5}}{6} \\ \frac{\sqrt{5}}{6} & -\frac{1}{3} \end{bmatrix} + a_3 \begin{bmatrix} \frac{17}{480} & -\frac{5}{96} \\ -\frac{5}{96} & 0 \end{bmatrix} \\
&(3/2,3/2): \\
&a_{LS}\begin{bmatrix} -\frac{2}{15} & \frac{5}{6\sqrt{2}} & -\frac{3\sqrt{3}}{10\sqrt{2}} \\ \frac{5}{6\sqrt{2}} & -\frac{1}{6} & 0 \\ -\frac{3\sqrt{3}}{10\sqrt{2}} & 0 & -\frac{7}{10} \end{bmatrix} + a_1 \begin{bmatrix} \frac{2}{15} & \frac{5}{6\sqrt{2}} & -\frac{3\sqrt{3}}{10\sqrt{2}} \\ \frac{5}{6\sqrt{2}} & \frac{1}{6} & 0 \\ -\frac{3\sqrt{3}}{10\sqrt{2}} & 0 & \frac{7}{10} \end{bmatrix} + a_3 \begin{bmatrix} -\frac{17}{600} & \frac{1}{96\sqrt{2}} & -\frac{21\sqrt{3}}{800\sqrt{2}} \\ \frac{1}{96\sqrt{2}} & 0 & 0 \\ -\frac{21\sqrt{3}}{800\sqrt{2}} & 0 & -\frac{7}{150} \end{bmatrix}
\end{aligned}
\tag{17}
$$

Quite in general, irrespective of the value of ℓ, the symmetric states are affected only by H_3, while the mixed symmetry states turn out to be insensitive to H_2. Thus, in first order of perturbation theory only three operators are relevant.

So far we have only considered matrix elements between states degenerate in the spin-flavor symmetry limit. Beyond first order of perturbation theory the symmetry breaking produce leading order mixings between states belonging to different spin-flavor multiplets. In this case, other operators besides the ones considered so far become relevant. In particular, we expect that the $\ell = 0$ states will experience spin-flavor breaking through the mixing with states in other towers. The problem of going beyond first order perturbations will be addressed elsewhere.

Finally, concerning the origin of the effective interactions discussed in this section, except for H_{LS}, the other two interactions do not emerge directly from QCD by performing the $1/M_{\rm quark}$ expansion and must, therefore, crucially depend on the structure of the constituent light quark. While the spin-orbit piece is included in analyses of Baryon spectroscopy [19], H_T has been realized by a pion exchange model [20] which gives a-priori big relevance to this interaction, and, the operators H_i, $(i = 1, 2, 3)$ have not been considered as far as we know.

III. CONCLUSIONS

The spin-flavor symmetry breaking level shifts of non-strange excited Baryons have been discussed in the large $\mathrm{N_c}$ limit to leading order in the $1/\mathrm{N_c}$ expansion and first order in perturbation theory. In this approximation, only Baryons with non-vanishing orbital angular momentum do experience leading order spin-flavor breaking in their levels. It was also shown that it is necessary to include only up to two-body operators as higher body operators can be reduced. Similar results will hold for three flavors. Our study is of theoretical interest and we expect that they can be useful in understanding other large $\mathrm{N_c}$ studies of QCD, for instance, through lattice simulations. Finally, it would be interesting to go beyond first order of perturbation theory and also to know how important are the effects of the effective interactions discussed here for the case of real Baryons.

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