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STRUCTURE AND DYNAMICS OF LIGHT NUCLEI: A STATUS REPORT

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Structure and Dynamics of Light Nuclei: a Status Report

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The theory of energy spectra, electromagnetic response functions, low-energy electroweak capture reactions, relativistic dynamics, and short-range structure as it relates to light nuclei is reviewed. A selection of results is presented.

I. INTRODUCTION

Nuclear and hadronic systems with only a few active degrees of freedom are important because their properties can be calculated very accurately from the underlying dynamics. Comparison of the predicted properties for these systems with experiment allows us to test our understanding of hadronic structure and dynamics over a wide energy range, from the keV regime of astrophysical relevance, to the MeV regime dominated by the dynamics of interacting nucleons, to the GeV regime where the substructure of the nucleon plays an important role.

The past few years have witnessed dramatic progress in the theory of the energy spectra of nuclei; the response of light nuclei and nuclear matter to external probes, particularly electromagnetic; the electroweak reactions involving few-nucleon systems at very low energy; the relativistic treatment of few-nucleon dynamics; and, finally, the short-range structure of nuclei. This understanding has been largely achieved with the use of either one-boson-exchange (OBE) interactions or interactions based on one-pion-exchange (OPE) supplemented by a phenomenological treatment of the shorter range part. However, apart from OPE, the origin of the effective nuclear force is still uncertain. The size of the heavy mesons, the possible role of crossed-meson-exchange contributions and nucleon resonances, and the partial success of quark-exchange models all make it difficult to understand why the simple OBE model is so successful. The derivation of an effective nuclear force from QCD remains one of the unsolved problems of central importance to this field.

II. ENERGY SPECTRA OF NUCLEI

A. Exact Calculations

Faddeev-Yakubovsky (FY) [1,2], Correlated Hyperspherical Harmonics (CHH) [3–5], and Green's Function Monte Carlo (GFMC) [6] methods have been developed to obtain ground and low-energy scattering states of nuclei. To date, calculations have been carried out for the A=3–4 ground states (FY [2,7], CHH [3,5], and GFMC [6]), several A=6 and 7 bound states (GFMC) [8,9], n+d and p+d scattering states below and above the three-body breakup threshold (FY [1] and CHH [4]), and the P-wave resonances in ^{5}He (GFMC) [8]. Some of these recent calculations use an NN interaction incorporating charge-symmetry- and charge-independence-breaking as well as *pp*, *np* and *nn* Coulomb and magnetic terms [10] to study mass differences between the T=1/2 ^{3}H–^{3}He doublet and the T=1, $J^\pi = 0^+$ ^{6}He–^{6}Li–^{6}Be triplet. Typically the energy eigenvalue can be determined with 1–2% accuracy in these calculations.

B. Accurate Calculations

Variational Monte Carlo (VMC) methods have been used for A=4 low-energy scattering states [11–13]; Variational Cluster Monte Carlo methods, including up to 5–body clusters [14], for ^{15}N [15], ^{16}O [16], and ^{40}Ca [14]. These latter calculations have indicated a rather slow convergence of the cluster expansion in nuclei; for example, the contribution of clusters with $\geq$ 6–nucleons can be of the order of 5% of the total in ^{16}O.

C. Binding Energies

The calculations based on NN interactions have been found to underbind the light nuclei. Inclusion of a three-nucleon interaction (TNI), consisting of a two-pion-exchange part and a phenomenological spin-isospin independent repulsive part, and designed to fit the binding energy of ^{3}H and the empirical equilibrium density of nuclear matter, leads to predictions for the ground-state energies of ^{4}He and ^{6}Li in reasonable agreement with the experimental values (see Tables I and II). However, the low-lying states of nuclei in the p-shell region are underbound by present NN and NNN models. Obviously, this does not imply that the TNI alone is responsible for the binding energy defect in nuclei.

TABLE I. Experimental and quantum Monte Carlo energies of $A = 6, 7$ nuclei in MeV.

$^AZ(J^\pi; T)$	VMC	GFMC	Expt
^{6}He$(0^+; 1)$	-24.38(7)	-27.74(20)	-29.27
^{6}He$(2^+; 1)$	-22.04(8)	-25.45(15)	-27.47
^{6}Li$(1^+; 0)$	-28.04(7)	-31.25(17)	-31.99
^{6}Li$(3^+; 0)$	-24.95(9)	-28.53(32)	-29.80
^{6}Li$(2^+; 0)$	-23.76(9)	-26.82(35)	-27.68
^{7}He$(\frac{3}{2}^-; \frac{3}{2})$	-20.31(11)	-24.22(21)	-28.82
^{7}Li$(\frac{3}{2}^-; \frac{1}{2})$	-32.70(12)	-37.44(28)	-39.24
^{7}Li$(\frac{1}{2}^-; \frac{1}{2})$	-32.39(17)	-36.68(30)	-38.76
^{7}Li$(\frac{7}{2}^-; \frac{1}{2})$	-26.78(17)	-30.7 (5)	-34.61
^{7}Li$(\frac{5}{2}^-; \frac{1}{2})$	-25.89(16)	-27.6 (7)	-32.56

TABLE II. Perturbatively corrected GFMC energy components in MeV and radii in fm for $A = 6, 7$ ground states.

	^{6}He$(0^+; 1)$	^{7}He$(\frac{3}{2}^-; \frac{3}{2})$	^{6}Li$(1^+; 0)$	^{7}Li$(\frac{3}{2}^-; \frac{1}{2})$
$\langle K \rangle$	143.(2)	146.(2)	151.(1)	186.(3)
$\langle v_{18} \rangle$	-169.(2)	-170.(2)	-181.(1)	-223.(3)
$\langle v_{em} \rangle$	0.88(1)	0.86(1)	1.71(1)	1.78(2)
$\langle V_{ijk} \rangle$	-7.3(1)	-7.4(2)	-7.2(1)	-8.9(3)
$\langle r_p^2 \rangle^{1/2}$	1.90(2)	1.91(2)	2.44(1)	2.27(3)
$\langle r_n^2 \rangle^{1/2}$	2.68(2)	3.02(3)	2.44(1)	2.44(3)

The observed splitting between the $p_{1/2}$ and $p_{3/2}$ hole states in ^{15}N is well reproduced [15]; however, that between the $p_{1/2}$ and $p_{3/2}$ resonances in ^{5}He is underestimated in these calculations based on non-relativistic nuclear many-body theory [8]. This splitting is found to be due to the spin-orbit component of the NN interaction, to pion-exchange interactions involving three or more nucleons, and the pion-exchange part of the TNI. It is also believed to be sensitive to relativistic effects.

III. ELECTRON SCATTERING

The ground-state electromagnetic observables of the few-body nuclei have been studied with nuclear charge and current operators including one- and two-body components and the experimental electromagnetic form factors of the nucleon. While the main parts of the two-body current operators are linked to the form of the NN interaction through the continuity equation and can be regarded in this sense "model independent," the most important two-body charge operators are "model dependent" and should be viewed as relativistic corrections. An additional source of uncertainty is that associated with the detailed behavior of the electromagnetic form factors of the nucleon at high momentum transfers. The most common semi-empirical parametrizations of these differ widely, especially in the case of the neutron electric form factor. Nevertheless, the above framework has been shown to provide, at low and moderate values of momentum transfers, a satisfactory description of the deuteron $A(q)$ and $B(q)$ structure functions and threshold electrodisintegration [10,17], the charge [18,20] and magnetic [19,20] form factors of ^{3}H, ^{3}He and ^{4}He (see Figs. 1 and 2), and the two-nucleon distribution functions of the helium isotopes extracted from longitudinal (e,e′) data [21]. The only ground-state observables for which a clear discrepancy exists between theory and experiment are the quadrupole moment and tensor polarization of the deuteron at intermediate values of momentum transfers (q=0.5–1.0 GeV/c), and the ^{3}He magnetic form factor in the first diffraction region. It is worth pointing out that reproducing simultaneously the observed $A(q)$, $B(q)$ and $T_{20}(q)$ structure functions of the deuteron has proven, to date, difficult not only in the non-relativistic appproach outlined above, but also in relativistic approaches based on the Bethe-Salpeter equation [22–24], and on light-front Hamiltonian dynamics [25].

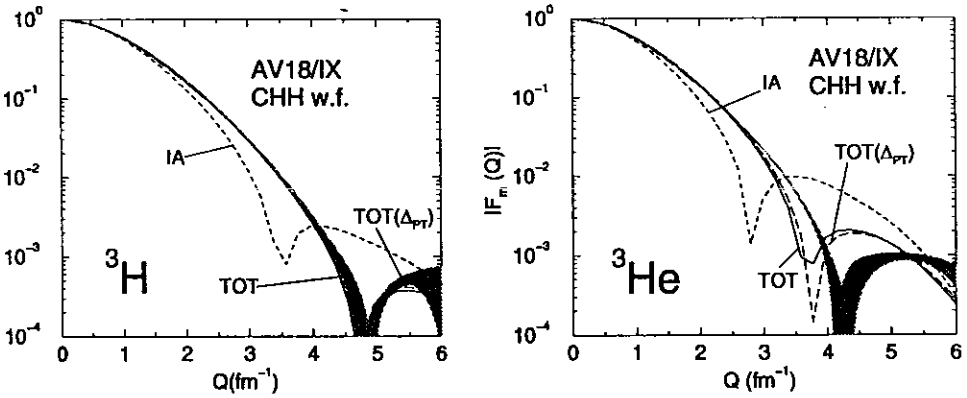

FIG. 1. The ^{3}H and ^{3}He magnetic form factors.

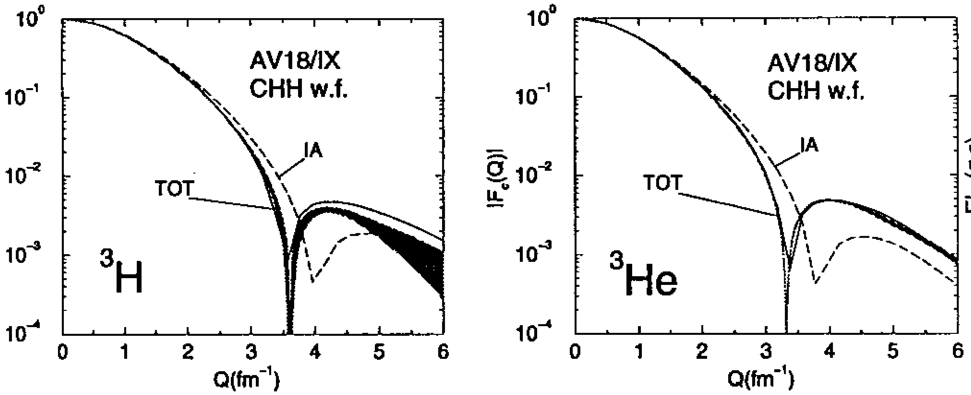

FIG. 2. The ^{3}H and ^{3}He charge form factors.

Experiments with polarized and unpolarized electrons have measured inclusive and exclusive cross sections, longitudinal and transverse response functions, and asymmetry observables at intermediate energy and momentum transfers. The theoretical description of these reactions has also progressed signicantly in the last five years. Essentially exact calculations of the inclusive response functions have now been completed with the Faddeev (A=3) [26,27], GFMC (A=3 and 4) [28,29], and Lorentz-transform [30] methods. Overall good agreement has been found between theory and experiment. The charge-exchange character of the pion-exchange interaction shifts both the longitudinal and transverse strength to higher excitation energies, thus producing a significant quenching of the response in the quasielastic peak. However, in the transverse channel this quenching is more than offset by the two-body pion-exchange current contributions required by gauge invariance and, therefore, the transverse response is enhanced over the entire quasi-elastic spectrum. In particular, the observed ratio of the longitudinal to transverse strength is well predicted [29]. The failure to explain this ratio in calculations based on the naive plane-wave-impulse-approximation (PWIA) had led to speculations of possible in-medium modifications of the nucleon's form factors, the so called "swelling" of the nucleon.

More recently, the longitudinal-transverse and transverse-transverse interference response functions measured in $^3\vec{\rm He}(\vec{e},e')$ experiments have been calculated with the GFMC method. The results of these calculations have indicated the presence in these observables of significant contaminations from final-state-interaction and two-body current contributions [31] (see Fig. 3).

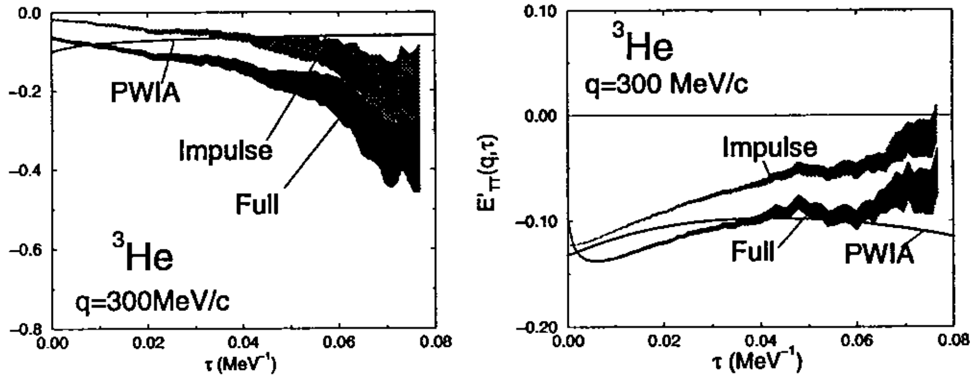

FIG. 3. The longitudinal-transverse and transverse-transverse Euclidean responses of ^{3}He at 300 MeV/c.

IV. LOW-ENERGY ELECTROWEAK REACTIONS ON FEW-NUCLEON SYSTEMS

Some of the low-energy electroweak reactions involving few-nucleon systems, such as ^{1}H(p, $e^+\nu_e$)^{2}H, ^{2}H(p,γ)^{3}He, ^{2}H(d,γ)^{4}He, and ^{3}He(p,$e^+\nu_e$)^{4}He have great astrophysical interest. These same reactions are also very interesting from the standpoint of the many-body theory of strongly interacting systems, since they are sensitive to the ground- and scattering-state wave functions and the electroweak transition operators.

The neutron and proton radiative captures on ^{2}H [32–34], the proton weak captures on ^{1}H and ^{3}He [13,35], and the ^{3}He(n,γ)^{4}He [11,35] and ^{2}H(d,γ)^{4}He [12] reactions have been studied in the past few years with Faddeev, CHH, and VMC ground- and scattering-state wave functions, and with an electroweak transition operator including one- and two-body components. In particular, Δ-isobar degrees of freedom are explicitly included in the nuclear wave functions rather than being eliminated in favor of effective two-body operators acting on the nucleons' coordinates, as is commonly done in perturbative treatments [35]. The Faddeev and CHH calculations of the ^{2}H(p,γ)^{3}He and ^{2}H(n,γ)^{3}H cross sections are in good agreement with the measured values [36]. The four-body capture reactions are particularly sensitive to details of the model for the interactions and currents, as their cross sections vanish in the limit of no tensor force and two-body currents. Discrepancies exist between the variational estimates of the ^{2}H(d,γ)^{4}He, and ^{3}He(n,γ)^{4}He cross sections and the corresponding empirical values [12,35]. It is not clear whether these discrepancies are to be ascribed to deficiencies in the variational wave functions, or to the model for the two-body current operator (or both).

The $^3\mathrm{He}(p,e^+\nu_e)^4\mathrm{He}$ cross section cannot be measured in the energy range relevant for solar fusion. The calculated value is much smaller (by about a factor 6) than that predicted on the basis of shell model descriptions of the initial and final nuclear states, leading to a significantly smaller neutrino flux associated with this reaction [13,35].

V. RELATIVISTIC APPROACHES TO FEW-NUCLEON DYNAMICS

The relativistic dynamics of interacting composite objects, such as nucleons, is non-trivial, and a variety of approaches have been and are presently being developed. These include dynamical schemes based on relativistic Hamiltonians [37], and various quasipotential reductions of the Bethe-Salpeter equation, such as the Blankenbecler-Sugar [38] or Gross [39] equations.

The quasipotential equations rely on one-boson-exchange (OBE) models for the NN interaction, constrained to fit two-body bound and low-energy scattering data [22,40]. Relativistic OBE models can provide a connection between field theory and phenomenological potential models of the nuclear force. Calculations of deuteron elastic and inelastic electromagnetic observables carried out in this framework have been found to be in reasonable agreement with experiment [22–24], although reproducing simultaneously the observed elastic structure functions and tensor polarization has proven to be problematic, as already mentioned. Methods have been found for ensuring that these calculations are gauge invariant, even when phenomenological form factors are used at the OBE vertices [41]. Substantial differences in the form factor predictions remain between the Blankenbecler-Sugar and Gross formulations. Progress is being made in the relativistic calculation of the three-body binding energy from relativistic OBE models [42].

In the relativistic Hamiltonian approach, the nuclear Hamiltonian is written as the sum of relativistic one-body kinetic energies, two- and many-body interactions and their boost corrections [43,44]. The form of the latter are determined by the commutation relations of the Poincaré group, and only the leading terms in an expansion in powers of $(v/c)^2$ are retained. To date, binding energies and ground-state momentum distributions of the A=3 and 4 nuclei have been obtained with the VMC method. Comparison with non-relativistic VMC results calculated with a phase-equivalent two-body interaction suggests that relativistic effects reduce the binding energy of ^{3}H and ^{4}He by a small amount, and do not significantly change the momentum distributions.

VI. SHORT-RANGE STRUCTURE OF NUCLEI

The two outstanding features of the NN interaction are its short-range repulsive and long-range tensor components. These induce spatial-spin-isospin correlations among the nucleons in a nucleus. As a result, the T=0,S=1 two-nucleon densities in nuclei display peculiar toroidal and dumbbell-like shapes depending on whether the pair is in states with S_z=0 and S_z=±1, respectively [45] (see Fig. 4).

The deuteron elastic form factor data do provide experimental evidence for the presence of the dumbbell-like and toroidal structures. The deuteron is a special case, since for it the one- and two-body densities are proportional to each other (this is not so in systems with $A > 2$). Theory predicts that these short-range structures also affect the momentum distributions of $\vec{d}\vec{p}$-, $\vec{d}\vec{d}$-, and $\alpha\vec{d}$-clusters in ^{3}He, ^{4}He and ^{6}Li, as well as produce large asymmetries in the cross sections for the reactions $\vec{d}(e,e'p)n$ and $d(e,e'\vec{p})\vec{n}$ [46]. Experiments aimed at verifying these predictions are currently being planned at TJNAF and elsewhere.

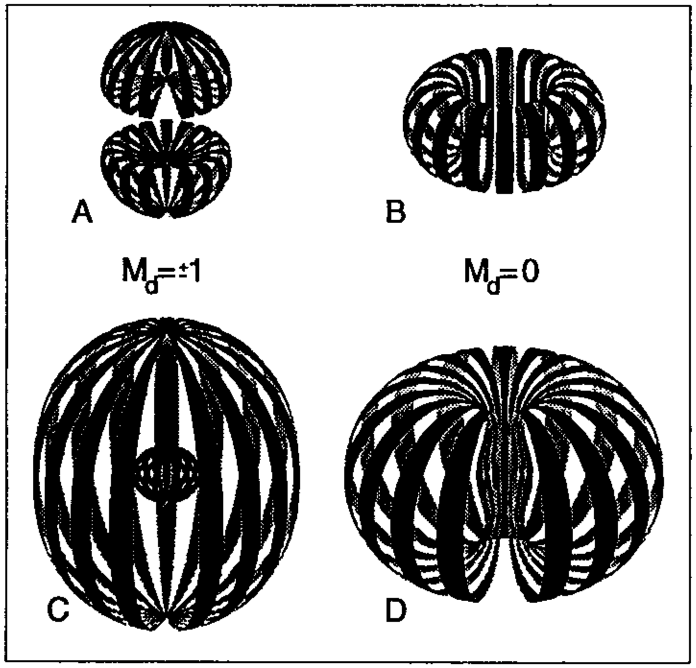

FIG. 4. The one-body density surfaces in deuteron having $\rho_d^{M_d=\pm1}(\mathbf{r}')$=0.24 fm^{-3} (A) and $\rho_d^{M_d=0}(\mathbf{r}')$=0.24 fm^{-3} (B). The surfaces are symmetric about z' axis and have $r' \leq$0.74 fm, i.e., the length of the dumbbell along z' axis as well as the diameter of the outer surface of the torus is 1.48 fm. Sections C and D are for $\rho_d^{M_d=\pm1,0}(\mathbf{r}')$=0.08 fm^{-3}; the maximum value of r' is 1.2 fm.

VII. SUMMARY AND OUTLOOK

The last few years have seen extensive development of relativistic methods for the treatment of few-body systems, and the maturing of our techniques for predicting and calculating the properties of light nuclei using non-relativistic quantum mechanics.

Nuclear many-body theory based on non-relativistic Hamiltonians with two- and three-nucleon interactions and one- and two-body electroweak current operators constructed consistently with these interactions, has been shown, so far, to provide a satisfactory, quantitative description of many nuclear properties that can be reliably calculated. The success achieved within this framework suggests that: i) nucleons are the dominant degrees of freedom in nuclei; ii) meson-, and particularly pion-, degrees of freedom can be eliminated in favor of effective two- and many-body operators involving only nucleonic coordinates; iii) so far, no experimental evidence exists for in-medium modifications of the nucleon's structure, such as its electromagnetic form factors. Clearly, the validity of these conclusions is based on the ability, developed in the past few years, to solve nuclear bound- and scattering-state problems very accurately.

On the relativistic front, covariant two body equations have been solved with realistic OBE interactions, and have been found to give a good description of low energy NN data, and relativistic calculations of deuteron form factors and electromagnetic observables have matured. Quasipotential equations with realistic NN interactions have been solved also for the A=3 system, and it is expected that quantum Monte Carlo methods will allow us to study properties of light nuclei (A$\leq$6) with relativistic Hamiltonians. Comparison between these two different schemes should be helpful in gaining new insights into the relativistic dynamics of interacting nucleons. Electromagnetic properties are of particular interest, since future experiments at TJNAF and other facilities will involve large momentum transfers, and their results may be more sensitive to relativistic effects.

With regard to future prospects, it now appears possible to carry out exact GFMC calculations of the A=7 and 8 ground and low-energy scattering states, and associated electromagnetic observables. Accurate ***ab initio*** microscopic calculations of electroweak reactions, such as ^{3}He(α, γ)^{7}Be, ^{7}Be(p,γ)^{8}B and ^{8}B $\rightarrow \alpha + \alpha + e^+ + \nu_e$, should be feasible. Furthermore, continuing progress in the CHH and FY approaches will allow us to perform exact calculations of electroweak reactions involving four nucleons. This program will lead to the microscopic description of all important reactions occurring in the *pp*-chain for the energy and neutrino production in the Sun. It should also be possible to investigate realistically the problem of electron screening in very low-energy nuclear reactions, and its implication on the extrapolation of the astrophysical factor from the corresponding low-energy data.

At a more fundamental level, the derivation of an effective nuclear force from QCD continues to be an issue of central importance, and a topic for future work. The present failure to correctly reproduce the observed deuteron structure functions and tensor polarization may suggest a breakdown of the"nucleons only" model of few nucleon systems, but more accurate data on T_{20} are needed in order to firmly resolve the issue. These data will be forthcoming in the next few years from experiments currently underway at NIKHEF and in the planning phase at TJNAF. Another relevant discrepancy is that between the spin-longitudinal and spin-transverse response functions measured in quasielastic $(\vec{p}, \vec{n})$ reactions and existing theoretical predictions. Again, forthcoming data on few-body nuclei from IUCF will be very helpful in clarifying the situation.

Although quark models of mesons and baryons (not discussed here) have been developed which give an excellent account of the observed spectrum, their implications have not been fully incorporated into our current models of the nuclear force. In this respect, our ability to calculate six-fermion ground states with relativistic Hamiltonians may allow us to calculate the properties of the deuteron and the NN scattering states directly from the interactions of six constituent quarks. However, any progress in this direction will depend on our ability to define a realistic six-quark Hamiltonian.

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