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WHERE'S THE GLUE?: A STRONG INTERACTION PUZZLE *

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After recalling why most strong interaction phenomena cannot be studied with quark-gluon perturbation theory, I suggest that a phenomenological form of the old dual string theory may be a good model for the nonperturbative aspects of QCD. I begin by describing some recent progress within this framework in understanding the status of the quark model within QCD. The resulting picture of the ordinary hadrons, hybrids, and glueballs provides a simple framework for studying these elements of hadron spectroscopy.

1 PRELUDE: THE BREAKDOWN OF PERTURBATIVE QCD

We all know that asymptotic freedom guarantees that at sufficiently small distances Quantum Chromodynamics (QCD) becomes a weakly coupled quark-gluon theory which is amenable to a perturbative expansion in the running coupling constant α_s. However, the other side of this coin is that at large distances α_s becomes large so that quark-gluon perturbation theory may break down.

In fact, we now know from numerical studies that QCD predicts confinement: the potential energy between two static quarks grows linearly with their separation r with a constant of proportionality b, called the string tension, that is about 1 GeV/fm. Let me show you that such a result rigorously implies the breakdown of perturbative QCD. Given that confinement is the central feature of strong interaction physics, we are therefore forced to seek new methods for the study of most strong interaction phenomena.

In the pure gluon sector of QCD in which the static potential problem is posed (*i.e.*, QCD with static source and *no* dynamical quarks), the equation for the string tension must take the form

$$b = f_b(g^2)$$

where f_b is some function of the dimensionless coupling constant g^2 since this is the only parameter of pure QCD. This equation is impossible, however, since b has dimensions of $[mass]^2$! The resolution of this paradox lies in the fact that g^2 is not a coupling "constant": according to asymptotic freedom

$$\frac{1}{g^2(Q^2)} = \frac{1}{g_0^2(Q_0^2)} + \frac{11}{16\pi^2}\ln\frac{Q^2}{Q_0^2}$$

where $g(g_0)$ is the effective coupling at momentum transfer $Q^2(Q_0^2)$. Thus QCD is defined by a universal "coupling constant curve" $g^2(Q^2)$ on which g^2 takes all values from zero to infinity, and not a single number. In a given universe with scales external to QCD (like the electroweak electron mass or the masses of the current quarks) this universal curve can be "pegged" to a given normalization at some external scale μ^2, but in pure QCD this is irrelevant: for us the key point is that a particular curve can be defined by choosing a value for $g^2(\mu^2)$ at any normalization point μ^2. This choice then simultaneously gives us a coupling constant $g^2(\mu^2)$ and a scale to give dimension to equation (1):

$$b = \mu^2 f_b(g^2(\mu^2)) \ .$$

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Thus in a pure QCD world, the string tension b and all other dimensionful quantities would have a scale set by the dummy variable μ^2, and all observables would be dimensionless ratios in which this variable cancels out.

Alternatively, we can note that any point μ^2 could have been chosen to define the curve $g^2(Q^2)$ and so

$$\frac{db}{d\mu^2 = 0}$$

or, it i.e.,

$$0 = f_b - \frac{11}{16\pi^2} g^4 \frac{df_b}{dg^2}$$

implying that

$$f_b \propto exp\left[-\frac{16\pi^2}{11g^2}\right] \quad .$$

The essential singularity in g^2 means that the "Feymman diagrammar" is useless for this problem, and that *plane wave quarks and gluons are not a useful starting point for low-energy, confinement-dominated physics.* To make progress in understanding the main phenomena of strong interaction physics, we must therefore either resort to purely numerical methods (*e.g.*, lattice QCD), or we must replace the Feymman diagrammar by new conceptual elements.

2 WHAT IS THE QUARK MODEL?

2.1 ADIABATIC SURFACES

The idea of adiabatic surfaces [1,2] is central to our argument for the quark model approximation to QCD. Consider once again "pure QCD" in the presence of fixed $q_1\bar{q}_2$ or $q_1q_2q_3$ sources. The ground state of QCD with these sources in place will be modified, as will be its excitation spectrum. For excitation energies below those required to produce a glueball, this spectrum will presumably be discrete and continuous as, for example, shown in Figure 1 as a function of the $q_1\bar{q}_2$ spatial separation $\vec{r}$. We call the energy surface traced out by a given level of excitation as the positions of the sources are varied an adiabatic surface.

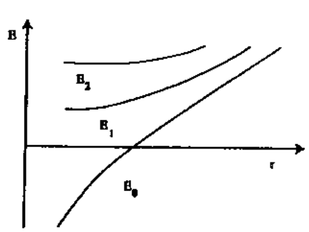

Figure 1: schematic of the low-lying adiabatic surfaces of $q_1\bar{q}_2$ at relative separation r; $E_0(r)$ is the gluonic ground state, $E_1(r)$ the first excited state, *etc.*

Let us now *define* the "quark model limit": the quark model limit obtains when the quark sources move along the lowest adiabatic surface in such a way that they are isolated from the effects of other (excited) surfaces.

Before trying to argue that this definition is relevant, let me first simply note that it has several appealing characteristics:

1) One of the great "mysteries" of the quark model is that it describes the mesons and baryons in terms of a wavefunction which only gives the amplitude for the valence quark variables, even though in QCD the general state vector must also refer to the glue fields. Indeed, in QCD for fixed q_1 and $\bar{q}_2$, for example, there are an infinite number of possible states of the glue so that it is certainly not sufficient to simply specify the state of the quarks. In the "quark model limit", however, although there are an infinite number of possible glue states, for any fixed r there is one lowest-lying one. Moreover, although this lowest-lying state changes as r changes, it is completely determined by the quark coordinates. Thus we see the possibility that the quark model wavefunction had a "secret suppressed subscript" describing the state of the glue: $\psi_0(\vec{r})$. We will argue below that there should be analogous (but as yet undiscovered) worlds $\psi_n(\vec{r})$ for $n > 0$ corresponding to hybrid mesons.

2) The "quark model limit" can easily be seen to be inapplicable to any systems more complicated that $q_1\bar{q}_2$ and $q_1q_2q_3$: such systems will always have adiabatic surfaces which cross so that the condition of isolation cannot be satisfied [1].

3) From deep inelastic scattering we know that only about 50% of the momentum of a high energy proton or pion is carried by its quarks. This seems at odds with the fact that the spectra of mesons and baryons appear to be dominated by the $q\bar{q}$ and qqq degrees of freedom. The adiabatic approximation can resolve this paradox: the virial theorem will store a substantial fraction of the mass of the proton (and therefore of the momentum of a fast-moving proton) in the gluonic fields, but since these fields are determined by the positions of the quarks, they do not enter the low-energy spectroscopy as active degrees of freedom.

With these attractions for motivation, we now proceed with the argument for the relvance of this definition. [2] We first recall a simple molecular physics analogy to this proposed approximation. Diatomic molecular spectra can be obtained in an adiabatic approximation by holding the two relvant atomic nuclei at fixed separation r and then solving the Schrödinger problem for the (mutually interacting) electrons moving in the static electric field of the nuclei. The electrons will, for fixed r, have a ground state and excited states which will eventually become a continuum above energies required to ionize the molecule. The resulting adiabatic surfaces then serve as effective internuclear potentials on which vibration-rotation spectra can be built. Molecular transitions can then take place within states built on a given surface or between surfaces.

In the "quark model limit" the quark sources play the rôle of the nuclei, and the glue plays the rôle of the electrons. From this point of view we can see clearly that conventional meson and baryon spectroscopy has only scratched the surface of even $q_1\bar{q}_2$ and $q_1q_2q_3$ spectroscopy: so far we have only studied the vibration-rotation bands built on the lowest adiabatic surface corresponding to the gluonic ground state. We should expect to be able to build other "hadronic worlds" on the surfaces associated with excited gluonic states [1]: these states correspond to the hybrids first discussed in the bag model (in terms which from this point of view are inappropriate) as $q_1\bar{q}_2g$ and $q_1q_2q_3g$ states [4].

On the basis of this analogy it seems clear that the quark model limit will apply when all of the quarks in a meson or baryon are heavy. This certainly corresponds well with the established success of the "naive" quark model in the $c\bar{c}$ and $b\bar{b}$ sectors. To understand whether such an approximation could be valid even for light quark systems will require going beyond such very general arguments to a detailed dynamical description of the gluonic fields that lie behind these adiabatic surfaces.

2.2 THE FLUX TUBE MODEL

Even after successful numerical calculations within QCD are possible, it will still be useful, and in complex situations essential, to have models which summarise the very complex structure of this theory. The flux tube model [1] is a model for QCD in the non-perturbative regime which emerges from considerations of Hamiltonian lattice QCD.

In the Hamiltonian version of lattice QCD, space (but not time as in most numerical studies) is discretized. In this formulation the lattice spacing "a", without reference to a perturbative expansion, plays the rôle of the regulator mass M. Latticizing the theory also has another advantage: it allows us to set up a strong coupling perturbation expansion in which the expansion parameter for lattice QCD is $1/g$ instead of g. We may expect to be able to learn more about the strongly coupled regime of the theory in terms of such an expansion, and indeed this seems to be the case: for example, confinement is an automatic property of the $g \to \infty$ limit of lattice QCD. Moreover, the natural degrees of freedom of the strong coupling regime are not quarks and gluons, but rather quarks and flux tubes, the latter being more in accord with various qualitative ideas on the nature of confinement in QCD. Of course, space is not coarse-grained (at least not on the scale of 10^{-15} metres), so that to relate lattice QCD to real QCD we must consider the limit $a \to 0$. In this limit $g \to 0$ so that a strong coupling expansion must fail; this is just the other side of the failure of the weak coupling expansion for small Q^2. If, however, it can be shown that the two regimes "match" around g=1, thereby proving that lattice QCD as $a \to 0$ is QCD, then one would nevertheless expect the strong coupling expansion to be useful in many situations where large scales dominate, just as the weak coupling expansion is useful for short distance physics.

The flux tube model thus naturally leads to a simple model for the adiabatic surfaces: the glue between Q and $\bar{Q}$ behaves like a discrete quantum string. The lowest adiabatic surface then corresponds to the ground state of this string, while the low-lying excitations correspond (at least for large r) to the excitation of "phonons" in the string. (At higher mass "topologically excited" strings will create other families of adiabatic surfaces).

This extraordinarily simple picture of the low-lying adiabatic surfaces of QCD has recently received some direct support from Monte Carlo studies of QCD on a space-time lattice. First of all, there is now rather strong evidence for the existence of narrow flux tubes [6]. Measurements on the lattice of the chromoelectric field strengths in the neighbourhood of static $Q\bar{Q}$ colour charges show that this field is strongly aligned and collimated in a way consistent with a thin (discretized) string executing quantum zero-point fluctuations about the $Q\bar{Q}$ axis (see Fig. 2). There is also evidence for a chromomagnetic field (expected by the QCD analogue of Maxwell's law of magnetic field induction) circulating around the $Q\bar{Q}$ axis. A second element of supporting evidence comes from the measurement of the string tension between static sources of differing color charges. The flux tube model predicts that these string tensions should be proportional to the respective quadratic Casimir eigenvalues of $SU(3)$ (*e.g.*, 4/3 for $3\bar{3}$ and 10/3 for $6\bar{6}$), and this behaviour is observed. Perhaps more

important than either of these results are recent *dynamical* results [7] on the excited adiabatic surfaces. These lattice studies find a first excited surface which not only has phonon quantum numbers, but also an energy gap over the ground state surface which is consistent with the $\frac{\pi}{r}$ expected at large r (see Fig. 3).

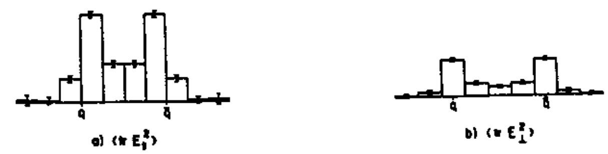

Figure 2: evidence for flux tubes from lattice Monte Carlo studies [5].

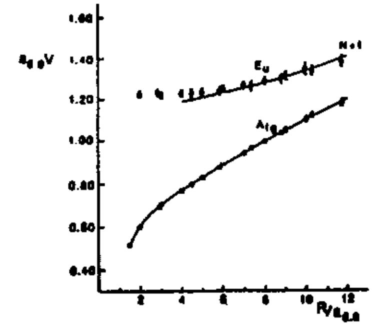

Figure 3: evidence for a phonon-like first excited adiabatic surface from Euclidean lattice Monte Carlo studies [7].

With this evidence in favour of the dynamical picture of the flux tube model for the low-lying adiabatic surfaces, it makes sense to push the model further by asking whether it would really lead to a "quark model limit". Merlin and Paton [8] have thus studied the corrections to the adiabatic limit for quark masses ranging from $m_Q >> b^{\frac{1}{2}}$ to $m_q \sim b^{\frac{1}{2}}$. Such a calculation shows the behaviour one would hope to see: for the low-lying spectrum of the lowest adiabatic surface, non-adiabatic corrections are essentially negligible for $m_Q > 1.5$ GeV, and they remain perturbations even when m_q is as small as 0.3 GeV. (The main correction is actually a "trivial" one: the glue contributes to the moment of inertia of the system. True mixing between adiabatic surfaces is a very small effect.)

Those who have followed or read the history of the string model (either *via* its original hadronic version or through its superstring reincarnation) will realize that while QCD provides a fresh motivation for the flux tube model, the basic picture is very old. It began with the discovery [9] that the Veneziano formula [10], which itself arose from attempts to make Regge theory dual [11], crossong symmetric, and analytic, could be interpreted as containing the dynamics of quarks connected by strings [9]. The string theory of hadrons was eventually abandoned as a *fundamental* theory of hadrons, but I want to remind you that as an *effective* theory of hadrons the string theory had many virtues.

The string theory certainly seems to correspond very well with all of the qualitative features observed in the hadronic world: the confined quark spectrum, Regge phenomenology for cross sections, duality, etc. It was of course designed with these phenomena in mind. What is perhaps more impressive is that it also contains, at least qualitatively, many other features expected in QCD but not yet observed. To me the two most startling such examples are its predictions of hybrid mesons and glueballs. The old string theory had in addition to the known Regge trajectories others called "daughter trajectories". Some of these corresponded to states in which the string degrees of freedom were excited, in remarkably close correspondence to the flux tube model for hybrids to be described below. Glueballs were required in the old string theory for consistency: hadronic reactions were assumed to proceed by an elementary string vertex in which a string breaks, forming a $q\bar{q}$ pair (or the time reversed healing process). When the healing process occurred between the q and $\bar{q}$ of an ordinary meson, a closed loop of string would be formed [12]. Such states can be associated with what we would today call glueballs and indeed correspond to the glueloops of the flux tube model [13]. The exact relationship between QCD and string theory is not yet clear, although there have been several attempts to make a rigorous connection [14].

3 STATUS OF CONVENTIONAL SPECTROSCOPY

Twenty five years after the birth of the quark model, light quark baryon and meson spectroscopy is in a deplorable state. For example, aside from the tensor mesons, of the remaining twelve states of the L=1 meson nonets, only four or five are reasonably well understood: the $J^{PCn} = 0^{++}$ states are in a state of confusion which is probably related to the existence of $qq\bar{q}\bar{q}$ states (see below); the $1^{++} f_1(1420)$ does not seem to fit into its nonet, while the mass of the a_1 (the cornerstone of this nonet) has been called into doubt by recent measurements on $\tau \rightarrow \upsilon_\tau \pi\pi\pi$; and in 1^{+-} the h_1 has only recently been claimed while the $s\bar{s}$ state remains completely undiscovered. Matters are, as one would expect, even worse for the higher mass nonets.

I am not of the opinion that we need to understand everything in low energy spectroscopy perfectly to be satisfied, but given the nearly miraculous dominance of the quark degrees of

freedom in this spectrum, we should at least know enough to verify our understanding of the general picture. Let me therefore provide one example of how we might go about hunting down some of these many missing states: the ω-like $^1D_2$ state with $J^{PC} = 2^{-+}$ (an "η_2"). As with any big game hunting, there are three essentials: 1) a map of the region, 2) a description of the beast, and 3) an appropriate trap. As a good general guide for this type of activity, I recommend [15], which provides spectroscopic predictions for this mass range based on a relativized version of the constituent quark model. In this case such a sophisticated map is not really required, however, since this η_2 is expected to be found within a few tens of MeV (the scale of OZI violation) from its isovector partner, the known π_2 (1680). The main characteristics of the beast which are needed to set a proper trap are, aside from its J^{PC}, its total width and prominent decay modes. The predictions of [16] for these properties are probably not far from the mark; they indicate a total width of about 400 MeV with $a_2\pi$ (70%) and $\rho\pi$ (10%) as the most prominent final states. As for the trap itself: this is not my department. I will nevertheless make one comment. These beasts are generally found hiding in a very dense jungle (i.e., background) populated by many other creatures (i.e., known states). It is very unlikely, therefore, that a simple trap will work. For example, it will almost certainly be necessary to have a J^{PC} filter on the trap (see Figure 4).

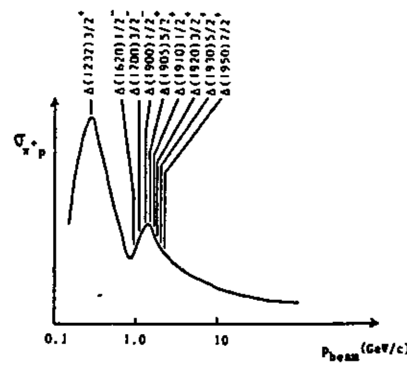

Figure 4: one of the lessons of baryon spectroscopy

3.1 HYBRIDS AND GLUEBALLS

Hybrids, which were originally discussed as $q\bar{q}g$ and $qqqg$ states in the bag model [4], are often still considered in that framework. The flux tube model provides an alternative nonperturbative picture of such states as, for example, $q\bar{q}$ states moving in the adiabatic potential of an excited flux tube.

The lowest order hybrid spectrum predictions of the flux tube model are very simple in character. Since the phonon has ± 1 units of angular momentum about the $q\bar{q}$ (body fixed) axis, good orbital angular momentum states must have total orbital angular momentum ≥ 1 and so have an effective angular momentum barrier in the $q\bar{q}$ relative coordinate. Such states therefore are "doubly excited" relative to the ordinary quarkonia: they have an excited string ($\omega_{gap} \sim 500$ MeV) and they have quarks which are required to have an r=0 node like an ordinary P-wave meson ($\omega_{orbit} \sim 500$ MeV). They are therefore predicted to lie far above the ρ meson at about 1.9 GeV. The ± 1 phonon helicity states combined with the four possible $q\bar{q}$ spin states produce eight nonets of such states with $J^{PC_n} = 2^{+-}_{-+}, 1^{+-}_{-+}, 0^{+-}_{-+}$, and 1^{++}_{--} (C_n is the conjugation of a neutral member of the nonet). Among these are three J^{PC_n} exotic nonets ($2^{+-}, 1^{-+}$, and 0^{+-}) whose existence would be unambiguous evidence for spectroscopy beyond the normal quark model. The crucial missing ingredient required in designing a search for these states is their hadronic couplings. These were supplied in [3] which demonstrated a striking characteristic of these states (common to all the states of all eight nonets): they decouple from ordinary S-wave mesons. There is a simple reason for this [3]: kinematics forbid the phonon helecity from turning into relative angular momentum between the decay products, since the products' relative coordinate vector is parallel to the original $q\bar{q}$ axis. It must therefore be internalized in the angular momentum of one of the decay products.

There are other reasons why even the definitive exotic J^{PC} signals might have escaped detection so far [3]. One is just their rather large masses. Another is that, of the nine candidate states, three are probably too broad to be seen with any clarity. When we turn to the six J^{PC} exotic hybrids which may be narrow enough to stand out as resonances, we encounter in each case simple reasons (usually involving the complexity of their decay modes) why they remain undiscovered.

The flux tube model thus provides in a straightforward way an explanation of why meson hybrids have not yet been discovered and suggests definite ways in which to find them. Baryon hybrids in the model are predicted to be very massive ($m > 2.2 GeV$); since they are undistinguished by any exotic quantum numbers, they will be exceptionally difficult to unravel from the very dense spectrum of ordinary baryons at such masses. Thus, at least with respect to hybrids, the flux tube model explains the hegemony of normal quark model states below 2 GeV.

The flux tube model predictions for the masses of hybrids are, as such things go, relatively straightforward: they are "normalized" by conventional $q\bar{q}$ spectroscopy, and involve no particularly risky approximations. These predictions of the masses of glueloops (also known as glueballs) is trickier: there is no known mass in this sector to set the "zero" and the approximations used to predict [1] the spectrum are shakier. With these caveats (the first leads to an estimated $\pm$ 0.2 GeV uncertainty in the overall mass scale) the model predicts the lowest glueball to be a 0^{++} state at 1.5 GeV with all other states above 2 GeV and the lowest J^{PC} exotic around 2.5 GeV. These predictions used to be in conflict with results from lattice Monte Carlo calculations, but recent results are not so far away from them.

It is clearly too early to be sure, but it now seems possible that — apart from a 0^{++} glueball which will be very hard to disentangle from the very poorly understood ordinary scalar mesons — the "gluonic action" is all in the 2 GeV range. If so, it will be very difficult to see (against what will by that mass be a very dense spectrum of ordinary mesons) unless it has J^{PC} exotic quantum numbers. This suggests concentrating our efforts on finding the hybrids. Such a search will require high statistics to perform the isobar analyses required to prove that any bumps found have exotic J^{PC} (see Figure 4 again).

4 SUMMARY AND CONCLUSIONS

With QCD as the link, all of hadronic spectroscopy is now an indivisable subject. In

this discussion I have tried to present an overview of this subject from the perspective of the flux tube model (which may be viewed as the reincarnation of the old dual string model).

The outstanding issue in the field at the moment is certainly the search for gluonic states, but these searches are hindered by our still sketchy knowledge of the quarkonium spectrum. This plus the growing evidence that the gluonic degrees of freedom are not excited below the 2 GeV mass range suggests looking for J^{PC} exotic hybrids since all such gluonic hadrons will be broad and immersed in a continuum of other broad non-exotic resonances.

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