The Thomas Jefferson National Accelerator Facility

Theory Group Preprint Series

Additional copies are available from the authors.

The Southeastern Universities Research Association (SURA) operates the Thomas Jefferson National Accelerator Facility for the United States Department of Energy under contract DE-AC05-84ER40150.

JLAB-THY-98-29

Magnetic Radius of the Deuteron

Andrei Afanasev*
North Carolina Central University, Durham, NC 27707, USA
and
Jefferson Lab, Newport News, VA 23606, USA

V.D. Afanasev and S.V. Trubnikov
Kharkov State University, Kharkov 310077, Ukraine

The root-mean squared radius of the deuteron magnetic moment distribution, r_{Md}, is calculated for several realistic models of the NN-interaction. For the Paris potential the result is $r_{Md} = 2.312 \pm 0.010$ fm. The dependence of r_{Md} on the choice of NN model, relativistic effects and meson exchange currents is investigated. The experimental value of r_{Md} is also considered. The necessity of new precise measurements of the deuteron magnetic form factor at low values of Q^2 is stressed.

The root-mean-square radius (RMSR) r_{Md} of the magnetic moment spatial distribution in the deuteron is defined in the usual way as:

$$r_{Md} \equiv < r^2_{Md} >^{1/2} = \left[-\frac{6G'_{Md}(Q^2=0)}{G_{Md}(Q^2=0)} \right]^{1/2} = \left[-\frac{3G'_{Md}(0)}{\mu_d} \right]^{1/2}, \tag{1}$$

where $G_{Md}(Q^2)$ is the deuteron magnetic form factor (DMFF), Q^2 is the modulus of the four-momentum transfer squared, and μ_d is the deuteron magnetic moment. The radius r_{Md} is an independent static property of the deuteron, which is not directly connected to the deuteron charge radius r_{Cd}. Here we see a clear difference between deuteron and nucleon structure. Indeed, if we assume that the scaling law for the nucleon FF's

$$G_{Ep}(Q^2) = \frac{G_{Mp}(Q^2)}{\mu_p} = \frac{G_{Mn}(Q^2)}{\mu_n} \tag{2}$$

is valid for very low values of Q^2 (and we have serious experimental and, especially, theoretical reasons to think so), then the immediate result from eq. (2) for the three nucleonic radii is

$$< r^2_{Ep} > = < r^2_{Mp} > = < r^2_{Mn} >, \tag{3}$$

i.e. the charge and magnetic radii of the proton coincide. In the deuteron case, the theoretical foundations for a scaling law between the charge DFF and magnetic one are absent, so the theoretical values of r_{Cd} and r_{Md} must be considered independent.

Let us first discuss the experimental status of r_{Md}. The results of measurements of $G_{Md}(Q^2)$ for low $Q^2 \lesssim 1$ fm^{-2} are contained in refs. [1] - [4]. These experimental values were approximated by a polynomial of degree n:

$$G_{Md} = 2\mu_d \left[1 - \frac{1}{6} < r^2_{Md} > \cdot Q^2 + \sum_{p=2}^{n} a_p Q^{2p} \right]. \tag{4}$$

The optimal order of the polynomial in eq. (4) appears to be $n = 2$. In this way we obtained an experimental value

$$r_{Md} = 1.90 \pm 0.14 \quad \text{fm}. \tag{5}$$

Note that from experiments on elastic electron–deuteron (ed) scattering, the deuteron charge radius r_{Cd} is determined much better than r_{Md} in eq. (5). The best analysis [5] of experimental data leads to the following result:

$$r_{Cd} = 2.128 \pm 0.011 \text{ fm}. \tag{6}$$

One should also consider the value based on atomic isotope shift measurements of Hänsch et al. [6] which give a value $r_{Cd} = 2.136 \pm 0.005$ fm [7]. Comparing eqs. (5) and (6), we see that the two radii are approximately equal:

---

*On leave from Kharkov Institute of Physics and Technology, Kharkov 310108, Ukraine

$r_{Md} \approx r_{Cd}$, but the precision of the determination of r_{Md} is low as a consequence of the low precision of the available experimental data.

Now let us turn to the theoretical calculation of r_{Md}. The standard expression in the literature [1] for the DMFF in the non-relativistic impulse approximation (NRIA) is

$$G_{Md}(Q^2) = \frac{M_d}{M}\left[2G^S_{MN}(Q^2)\cdot C_S(Q^2) + G^S_{EN}(Q^2)\cdot C_L(Q^2)\right], \qquad (7)$$

where

$$C_S(Q^2) = \int_0^\infty [u^2(r) - \frac{1}{2}w^2(r)]j_0(\frac{1}{2}Qr) + \frac{1}{\sqrt{2}}w(r)[u(r) + \frac{1}{\sqrt{2}}w(r)]j_2(\frac{1}{2}Qr)dr\,,$$
$$C_L(Q^2) = \frac{3}{2}\int_0^\infty w^2[j_0(\frac{1}{2}Qr) + j_2(\frac{1}{2}Qr)]dr\,.$$

In eq. (7), $u(r), w(r)$ are the deuteron radial S–, D– state wave-functions; $j_{0,2}$ are the spherical Bessel functions; $G^S{}_N \equiv \frac{1}{2}(G_{\cdot p} + G_{\cdot n})$ are the isoscalar nucleon FFs; M_d, M are the deuteron and nucleon masses, respectively. From eqs. (1), (7) we have

$$\begin{aligned}< r^2_{Md} > = \frac{1}{\mu_d}\cdot\Big\{&[< r^2_{Mp} > + < r^2_{Mn} >](1 - \frac{3}{2}p_d) + \\ &+(\mu_n + \mu_p)\cdot\int_0^\infty(\frac{1}{4}\cdot u^2 - \frac{1}{10\sqrt{2}}u\cdot w - \frac{7}{40}w^2)r^2\cdot dr + \\ &+\frac{3}{4}p_d\cdot[< r^2_{Ep} > + < r^2_{En} >] + \frac{9}{80}\int_0^\infty w^2\cdot r^2 dr\Big\},\end{aligned} \qquad (8)$$

where, as usual, $p_d = \int_0^\infty w^2(r)dr$.

The results of calculations of r_{Md}, following eq. (8), are listed in the third column of the Table I for several realistic deuteron wave functions (the NN potentials are identified in the first column). Due to lack of experimental information about the magnetic radii of the proton and, especially, the neutron, eq. (2) was used. For charge radii of the proton and neutron the inputs are the well-known experimental values $< r^2_{Ep} >^{1/2} = 0.862 \pm 0.012$ fm [9] and $< r^2_{En} > = -0.1194 \pm 0.0018\ \mathrm{fm}^2$. More recent measurements [8] give a consensus value of $< r^2_{En} > = -0.1140 \pm 0.0026\ \mathrm{fm}^2$.

Before discussing these results, we should estimate the contributions to r_{Md} from relativistic effects (RE) and meson exchange currents (MEC). It's reasonable to think that for $Q^2 \to 0$ the functions which describe the contributions of RE and MEC are small and smooth enough, so that their first derivatives will be negligible.

Indeed, for RE, in the framework of the formalism [10] the formula for the DMFF appears to be [2]

$$G_{Md} = \frac{M_d}{M}\left[\frac{2G^S_{MN}}{\sqrt{1+\tau}}\cdot C_s + (G^S_{EN} + \frac{\tau}{1+\sqrt{1+\tau}}G^S_{MN})\cdot\frac{C}{1+\tau}\right], \qquad (9)$$

where

$$C = C_L + C_1,$$
$$C_1 = \frac{9}{2}\int_0^\infty dr\cdot w^2(r)\cdot\frac{1}{Qr}\int_0^{Qr/2} j_2(\alpha)d\alpha\,,$$
$$\tau = Q^2/4M^2$$

and the other notations were introduced in eq. (7). The RE in eq. (9) are either of Darwin–Foldy (DF) nature (the factor $1/\sqrt{1+\tau}$), or due to nucleon motion (NM) in the deuteron. From eq. (9) we obtain

[1]We have several concerns about the correct formula for $G_{Md}(Q^2)$ in NRIA and plan to discuss it separately. The possible modification of eq. (7) would not influence any calculations in the static limit $Q^2 \to 0$.

[2]As a results of our own calculations, it seems that the relativistic formula for G_{Md} in ref. [10] has several inaccuracies. Here we have corrected it.

$$< r^2_{Md} > = < r^2_{Md} >_{NRIA} + < \Delta r^2_{Md} >_{RE}\,, \qquad (10)$$

$$\begin{aligned}< \Delta r^2_{Md} >_{RE} &= < \Delta r^2_{Md} >_{DF} + < \Delta r^2_{Md} >_{NM} = \\ &= \frac{3}{4M^2} - \frac{9}{16M^2}\cdot p_d(\frac{\mu_n + \mu_p + 2}{\mu_d}),\end{aligned}$$

where the first term in eq. (10) is given by eq. (8). The contribution of RE to the value of r_{Md} is shown in Table I in the fourth column.

As far as the MEC are concerned, it's well-known that the general formalism describing these effects is rather complicated. For an estimate we use the simple parametrization of the MEC contribution to G_{Md} for low Q^2, which was given in [11]:

$$\left(\Delta G_{Md}\right)_{MEC} = \beta_1 e^{-\alpha_1 Q^2}\,, \qquad (11)$$

so the correction to r_{Md} is

$$\left(\Delta r^2_{Md}\right)_{MEC} = 3\frac{\alpha_1\beta_1}{\mu_d}\,.$$

For the Reid soft core potential (RSC), $\beta_1 = 0.0288$ and $\alpha_1 = 0.16\ \mathrm{fm}^2$ and the result is also given in the Table I. No doubt, we could introduce and analyse more refined versions of RE and MEC contributions, but as they are small it may not be necessary.

The main result is evident: the agreement between the calculated theoretical and experimental values of r_{Md} is poor.

So let us discuss in more detail what we have learned. Accepting that the theoretical expressions for r_{Md} are reliable, the main contributions to r_{Md} emerge from the first and second terms in eq. (8). The contributions of the last two terms and the other degrees of freedom (additional to NRIA) are comparable and small. So the theoretical value of r_{Md} depends mainly on the D–state probability in the deuteron, and on the magnetic radii of the neutron and proton. In particular, a small deviation from the scaling law (eqs. (2), (3)) may produce a variation in r_{Md}, which is comparable to that following from a variation of p_d. In this sense it will be very interesting to have results from a direct measurement of the neutron magnetic radius r^2_{Mn} in experiments on neutron–electron scattering (as was done for the determination of $< r^2_{En} >$ in experiments on thermal neutron scattering from atomic electrons). Once the experimental values of r_{Mp} and r_{Mn} are tied down, the theoretical value of r_{Md} will depend mainly on the value of p_d.

Now we make some comments about the determination of experimental value of r_{Md}. The present status of low–Q^2 experiments on elastic ed–scattering at large angles $\Theta_e \sim 180°$ is such that we cannot extract the value of r_{Md} from experimental data with sufficient (at least as compared to eq. (6)) accuracy. Indeed, for low values of Q^2 there are only five experimental points of G_{Md} [1]– [4]. These values were obtained using the old generation of electron accelerators, and the experimental errors in $G_{Md}(Q^2)$ are large. For comparison it may be noted that for determination of the deuteron charge radius many more experimental points were used; moreover for low Q^2 the longitudinal part $A(Q^2)$ of elastic ed–scattering was measured with high accuracy [17]. So for determination of r_{Md} new precise detailed measurements of $G_{Md}(Q^2 \to 0)$ are very desirable.

An improved (as compared to eq. (5)) experimental value of r_{Md} will be useful in the following directions. Firstly, for inclusion of r_{Md} to the standard set of static deuteron properties when investigating a realistic model of the nucleon–nucleon interaction. Secondly, a knowledge of the exact value of r_{Md} is necessary for calculations of the hyperfine splitting of the electronic levels in deuterium (corrections due to the finite size spatial distribution of the deuteron magnetic moment: see, for example, ref. [18]). The hyperfine splitting is the good example of the intersection of high and low energy physics. In principle we may invert the problem and try to extract the value of r_{Md} from experimental data on splitting of the atomic S–levels in deuterium. Lastly, r_{Md} will remain the main piece of information about the magnetic distribution of the neutron, until direct measurements of this quantity in neutron–electron scattering experiments are realized.

ACKNOWLEDGMENTS

We would like to thank Donald Sprung for useful discussions. The work of A.A. was partially supported by the US Department of Energy under contract DE–AC05–84ER40150.

[1] J. Goldemberg, C. Schaerf, Phys. Rev. Lett. **12**, 298 (1964).
[2] B. Grossetete, D. Drickey, P. Lehmann, Phys. Rev. **141**, 1425 (1966).
[3] D. Ganichot, B. Grossetete, D.B. Isabelle, Nucl. Phys. **A178**, 545 (1972).
[4] E.C. Jones,Jr., W.L. Bendel, L.W. Fagg and R.A. Lindgren, Phys. Rev. C **21**, 1162 (1980).
[5] I. Sick and D. Trautmann, Phys. Lett. B **375**, 16 (1996).
[6] K. Pachucki, D. Liebfried, M. Weitz, A. Huber, W. Konig and T.W. Hänsch, J. Phys. B **29**, 177 (1996).
[7] J.L. Friar, J. Martorell and D.W.L. Sprung, Phys. Rev. A **56**, xx (1997).
[8] S. Kopecky, P. Riehs, J.A. Harvey and N.W. Hill, Phys. Rev. Lett. **74**, 2427 (1995).
[9] G. G. Simon, C. Schmidt, F. Borkowski, V. H. Walter, Nucl. Phys. **A333**, 381 (1980). A more recent evaluation of the data can be found in: P. Mergell, Ulf-G. Meißner and D. Drechsel, Nucl. Phys. **A596**, 367 (1996).
[10] M. Gourdin, Nuovo Cim. **28**, 533 (1963); **32**, 493 (1964); **35**, 1105 (1965).
[11] E. Lomon, Ann. Phys. (NY) **125**, 309 (1980).
[12] R. Machleidt, preprint TPI-PP-83-128, Canada, 1983, and private communication.
[13] M. Lacombe, B. Loiseau, J.M. Richard, Phys. Rev. C **21**, 861 (1980).
[14] M.M. Nagels, T.A. Rijken, J.J. de Swart, Phys. Rev. D **12**, 744 (1975).
[15] R.V. Reid, Ann. Phys. (NY) **50**, 411 (1968).
[16] V.I. Kukulin, V.N. Pomerantsev, Progr. Theor. Phys. **88**, 159 (1992).
[17] S. Platchkov, A. Amroun, S. Auffret, J.M. Cavedon, P. Dreux, J. Duclos, B. Frois, D. Goutte, H. Hachemi, J. Martino, X.H. Phan and I. Sick, Nucl. Phys. **A510**, 740 (1990).
[18] S. Azam, Phys. Rev. C **35**, 1132 (1987).

TABLE I. Magnetic radius of the deuteron r_{Md} (in fm)

NN potentials	p_d (%)	nonrelativistic r_{Md} (eq. 8)	with relativistic corrections (eq. 10)
Bonn [12]	4.25	2.346 ± 0.010[a]	2.353 ± 0.010
Paris [13]	5.77	2.312	2.318
Nijmegen [14]	5.92	2.314	2.321
Reid (RSC) [15][b]	6.46	2.295	2.302
Moscow State University [16]	6.74	2.292	2.299
Experimental value is $r_{Md} = 1.90 \pm 0.14$ fm.			

[a]The errors in the other listed values of r_{Md}, due to errors in the nucleon radii, are the same.
[b]For RSC inclusion of the MEC correction leads to the result $r_{Md} = 2.305 \pm 0.010$ fm.