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The $P_{33}(1232)$ resonance contribution into the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ from an analysis of the $p(e,e'p)\pi^0$ data at $Q^2 = 2.8$, 3.2, and 4 $(GeV/c)^2$ within dispersion relation approach

I.G.Aznauryan

Yerevan Physics Institute, Alikhanian Brothers St.2, Yerevan, 375036 Armenia
(e-mail addresses: aznaur@jerewan1.yerphi.am, aznaury@cebaf.gov)

S.G.Stepanyan

Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA,
Yerevan Physics Institute, Alikhanian Brothers St.2, Yerevan, 375036 Armenia
(e-mail address: stepanya@cebaf.gov)

Within the fixed-t dispersion relation approach we have analysed the TJNAF and DESY data on the exclusive $p(e,e'p)\pi^0$ reaction in order to find the $P_{33}(1232)$ resonance contribution into the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$. As an input for the resonance and nonresonance contributions into these amplitudes the earlier obtained solutions of the integral equations which follow from dispersion relations are used. The obtained values of the ratio $E2/M1$ for the $\gamma^* N \to P_{33}(1232)$ transition are: 0.039 ± 0.029, 0.121 ± 0.032, 0.04 ± 0.031 for $Q^2 = 2.8$, 3.2, and 4 $(GeV/c)^2$, respectively. The comparison with the data at low Q^2 shows that there is no evidence for the presence of the visible pQCD contribution into the transition $\gamma N \to P_{33}(1232)$ at $Q^2 = 3-4\ GeV^2$. The ratio $S_{1+}^{3/2}/M_{1+}^{3/2}$ for the resonance parts of multipoles is: -0.049 ± 0.029, -0.099 ± 0.041, -0.085 ± 0.021 for $Q^2 = 2.8$, 3.2, and 4 $(GeV/c)^2$, respectively. Our results for the transverse form factor $G_T(Q^2)$ of the $\gamma^* N \to P_{33}(1232)$ transition are lower than the values obtained from the inclusive data. With increasing Q^2, $Q^4 G_T(Q^2)$ decreases, so there is no evidence for the presence of the pQCD contribution here too.

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I. INTRODUCTION

It is known that the information on the Q^2 evolution of the $\gamma^* N \to P_{33}(1232)$ transition form factors may play an important role in the investigation of the energetic scale of the transition to the perturbative region of QCD. Especially important is the information on the ratio $E2/M1$ which being close to 0 at small Q^2 should go to 1 in the pQCD asymptotics. Experimental data on the cross sections of the exclusive reaction $p(e,e'p)\pi^0$ obtained recently at TJNAF at $Q^2 = 2.8$ and 4 $(GeV/c)^2$ [1] and more earlier DESY data at $Q^2 = 3.2$ $(GeV/c)^2$ [2] can be useful for understanding of the place of the region of $Q^2 = 3-4\ GeV^2$ in the transition to the pQCD regime. These data will be analysed in the present paper in order to extract an information on the $\gamma^* N \to P_{33}(1232)$ transition in this region of Q^2.

The investigation of the transition $\gamma^* N \to P_{33}(1232)$, using the experimental data on the pion photo-and electroproduction on the nucleons, is connected with the problem of separation of the resonance and nonresonance contributions in the multipole amplitudes $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$, which carry information on this transition. These amplitudes may contain significant nonresonance contributions, the fact which was clear with obtaining the first accurate data [3,4] on the amplitude $E^{3/2}_{1+}$ at $Q^2 = 0$. The energetic behaviour of this amplitude, in fact, is incompatible with the resonance behaviour. The first investigations of this problem [5-7] showed that it is closely related to the problem of fulfilment of unitarity condition, which for electroproduction amplitudes in the $P_{33}(1232)$ resonance region means the fulfilment of the Watson theorem [8]:

$$M(W,Q^2) = \exp(i\delta^{\frac{3}{2}}_{1+}(W))|M(W,Q^2)|. \tag{1.1}$$

Here $M(W,Q^2)$ denotes any of the multipoles under consideration, and $\delta^{\frac{3}{2}}_{1+}$ is the phase of the corresponding πN scattering amplitude $h^{3/2}_{1+}(W) = \sin(\delta^{3/2}_{1+}(W))\exp(i\delta^{3/2}_{1+}(W))$.

There are different approaches for the extraction of an information on the $\gamma^* N \to P_{33}(1232)$ transition from the pion photo-and electroproduction data with the different forms of the unitarization of the multipole amplitudes. These approaches can be subdivided into the following groups: the phenomenological approaches [5-7,9] including the approaches based on the K-matrix formalism [10,11], the effective Lagrangian approaches [12-16] with different phenomenological form of unitarization of amplitudes, the dynamical approaches [17-23], and the approaches based on the fixed-t dispersion relations [24-26].

In this work our analysis will be based on the solutions for the multipole amplitudes $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$ obtained in Ref. [26] using the fixed-t dispersion relations within the approach of Refs. [27,28]. This approach is very usefull for the extraction of an information on the $\gamma^* N \to P_{33}(1232)$ transition, because it in a natural way reproduces the resonance and noresonance contributions into the multipole amplitudes, and the obtained solutions satisfy unitarity condition (1.1). Let us discuss this in more detail using the simplified version of the dispersion relations for these multipoles with the s-channel cut only, i.e. in the form which is similar to the dispersion relations in the quantum mechanics:

$$M(W,Q^2) = M^B(W,Q^2) + \frac{1}{\pi}\int\limits_{W_{thr}}^{\infty} \frac{Im M(W',Q^2)}{W'-W-i\epsilon}dW' \tag{1.2}$$

Here $M^B(W,Q^2)$ is the contribution of the Born term (i.e. of the nucleon and pion poles) into the multipoles. As it was discussed in more detail in Ref. [26], we can write in the integrand of (1.2) $Im M(W,Q^2) = h^*(W)M(W,Q^2)$ due to the fact that the πN ampltude $h^{3/2}_{1+}(W)$ is elastic up to quite large energies. Thus, the dispersion relation (1.2) transforms into the singular integral equation which has a solution in the following analytical form (see Ref. [27] and the references therein):

$$M(W,Q^2) = M^{part}(W,Q^2) + c_M M^{hom}(W), \tag{1.3}$$

where

$$M^{part}(W,Q^2) = M^B(W,Q^2) + \frac{1}{\pi}\frac{1}{D(W)}\int\limits_{W_{thr}}^{\infty} \frac{D(W')h(W')M^B(W',Q^2)}{W'-W-i\epsilon}dW' \tag{1.4}$$

is the particular solution of the singular equation, generated by the Born term, and

$$M^{hom}(W) = \frac{1}{D(W)} = \exp\left[\frac{W}{\pi}\int\limits_{W_{thr}}^{\infty} \frac{\delta(W')}{W'(W'-W-i\epsilon)}dW'\right] \tag{1.5}$$

is the solution of the homogeneous equation

$$M^{hom}(W) = \frac{1}{\pi}\int\limits_{W_{thr}}^{\infty} \frac{h^*(W')M^{hom}(W')}{W'-W-i\epsilon}dW', \tag{1.6}$$

which enters the solution (1.3) with an arbitrary weight, i.e. multiplied by an arbitrary constant c_M.

The analogy with the quantum mechanics shows that the solution $M^{part}(W,Q^2)$ is the modification of the Born contribution produced by the πN rescattering in the final state (see Ref. [29], Chapter 9). This modification unitarizes the Born contribution which by itself is real:

$$M^{part}(W,Q^2) = \exp[i\delta(W)]\left[M^B(W,Q^2)\cos\delta(W) + e^{a(W)}r(W,Q^2)\right], \tag{1.7}$$

where

$$r(W,Q^2) = \frac{P}{\pi}\int\limits_{W_{thr}}^{\infty} \frac{e^{-a(W')}\sin\delta(W')M^B(W',Q^2)}{W'-W}dW', \tag{1.8}$$

$$a(W) = \frac{P}{\pi}\int\limits_{W_{thr}}^{\infty} \frac{W\delta(W')}{W'(W'-W)}dW'. \tag{1.9}$$

So, $M^{part}(W,Q^2)$ should be considered as nonresonance background to the resonance contribution.

It is natural to identify with the resonance contribution the solution $M^{hom}(W)$, because the dispersion relation (1.2) takes the form (1.6), when only the $P_{33}(1232)$ resonance contribution in the s-channel is taken into account. This solution satisfies the unitarity condition (1.1) too:

$$M^{hom}(W) = \frac{1}{D(W)} = \exp[i\delta(W)]e^{a(W)}. \tag{1.10}$$

From Eq. (1.7) it is seen that $M^{part}(W,Q^2)$ has nontrivial energy dependence. The factor at $\exp[i\delta(W)]$ in $M^{part}(W,Q^2)$ is determined mainly by the first term in the brackets and changes the sign in the vicinity of the resonance. The comparison with the experiment shows that the amplitude $E^{3/2}_{1+}$ at $Q^2 = 0$ is described, in fact, by $M^{part}(W,Q^2 = 0)$ [26]. Hence, this amplitude is mainly of nonresonance nature, and its nontrivial energy dependence is due to the the final state interaction in the Born term.

It is important to note that such type nonresonance contributions exist in all dynamical models [17-23]. They are produced by rescattering effects in the pole terms of these models and have the same type nontrivial energy dependence as (1.7). However, by the magnitudes these contributions are quite different, because their investigations within the models contain many model uncertainties coming from the cutoff procedures, the methods of taking into account off-shell effects and the methods of the treatment of the gauge invariance. These uncertainties are discussed in detail in Refs. [30,31].

It is interesting that in the phenomenological approaches based on the K-matrix formalism [10,11] and in the effective Lagrangian approach of Ref. [15], with the unitarization made by the Noelle method [32] or using the K-matrix ansatz, the nonresonance contributions into the multipoles $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$ have the same kind energy dependence as (1.7). In these cases such energy behaviour of the nonresonance contributions is also connected with the πN interaction in the final state.

In Refs. [24,25] at $Q^2 = 0$ the fixed-t dispersion relations are used in the same way as in Ref. [26]. However, the interpretation of the obtained solutions of the integral equations is different, although the results for the whole amplitudes $M^{3/2}_{1+}, E^{3/2}_{1+}$ are the same as in [26]. In order to extract the $P_{33}(1232)$ resonance contribution in Refs. [24,25] the method of the Speed Plot analysis is used. As a result, ignoring the physical nature of $M^{part}(W)$, the resonance contributions in these parts of the amplitudes are found.

In Sec. II the multipole amplitudes which are included into the fitting procedure in our analysis are listned, and the fitted parameters are specified. In Sec. III the results of our analysis of the TJNAF data at $Q^2 = 2.8$ and 4 $(GeV/c)^2$ [1] and of the DESY data at $Q^2 = 3.2$ $(GeV/c)^2$ [2] are presented. The comparison with theoretical predictions and with the behaviour of the amplitudes, which is characteristic of the pQCD asymptotics, is made.

II. DISPERSION RELATIONS AND PARAMETRIZATION OF MULTIPOLE AMPLITUDES

In our analysis we use the fixed-t dispersion relations for the Ball invariant amplitudes $B_1, B_2, B_3, B_5', B_6, B_8$ [33], which for the reaction $\gamma^* p \to \pi^0 p$ ($B_i^{(\pi^0 p)} = B_i^{(0)} + B_i^{(+)}$) require no subtraction:

$$ReB_i^{(\pi^0 p)}(s,t,Q^2) = R_i^{(p)}\left(\frac{1}{s-m^2} + \frac{\eta_i}{u-m^2}\right) + \frac{P}{\pi}\int_{s_{thr}}^{\infty} ImB_i^{(\pi^0 p)}(s',t,Q^2)\left(\frac{1}{s'-s} + \frac{\eta_i}{s'-u}\right)ds'. \qquad (2.1)$$

Here $s=(k+p_1)^2$, $u=(k-p_2)^2$, $t=(k-q)^2$, $Q^2=-k^2$, k, q, p_1, p_2 are the 4-momenta of virtual photon, pion, initial and final protons, respectively, $\eta_1=\eta_2=\eta_6=1$, $\eta_3=\eta_5'=\eta_8=-1$, $s_{thr}=(m+\mu)^2$, m and μ are masses of the nucleon and the pion, and $R_i^{(p)}$ are the residues in the Born pole terms:

$$\begin{aligned}
R_1^{(\pi^0 p)} &= ge(F_1^{(p)} + 2mF_2^{(p)}),\\
R_2^{(\pi^0 p)} &= -geF_1^{(p)}(Q^2),\\
R_3^{(\pi^0 p)} &= -\frac{ge}{2}F_1^{(p)}(Q^2),\\
R_5'^{(\pi^0 p)} &= \frac{ge}{2}(\mu - Q^2 - t)F_2^{(p)}(Q^2),\\
R_6^{(\pi^0 p)} &= 2geF_2^{(p)}(Q^2),\\
R_8^{(\pi^0 p)} &= geF_2^{(p)}(Q^2),
\end{aligned} \qquad (2.2)$$

where in accordance with the existing experimental data we have:

$$\begin{aligned}
&e^2/4\pi = 1/137, \quad g^2/4\pi = 14.5,\\
&F_1^{(p)}(Q^2) = \left(1 + \frac{g^{(p)}\tau}{1+\tau}\right)G_{dip}(Q^2),\\
&F_2^{(p)}(Q^2) = \frac{g^{(p)}}{2m}\frac{G_{dip}(Q^2)}{1+\tau},\\
&G_{dip}(Q^2) = 1/(1+Q^2/0.71\ (GeV/c)^2),\\
&\tau = Q^2/4m^2, \quad g^{(p)} = 1.79.
\end{aligned} \qquad (2.3)$$

The imaginary parts of the amplitudes $B_i^{(\pi^0 p)}(s,t,Q^2)$ we obtain using their expressions through the intermediate amplitudes f_i (the corresponing formulas are given in our earlier work [26]) which have the following decomposition over multipole amplitudes:

$$\begin{aligned}
f_1 &= \sum\left\{(lM_{l+} + E_{l+})P'_{l+1}(x) + [(l+1)M_{l-} + E_{l-}]P'_{l-1}(x)\right\},\\
f_2 &= \sum[(l+1)M_{l+} + lM_{l-}]P'_l(x),\\
f_3 &= \sum\left[(E_{l+} - M_{l+})P''_{l+1}(x) + (E_{l-} + M_{l-})P''_{l-1}(x)\right],\\
f_4 &= \sum(M_{l+} - E_{l+} - M_{l-} - E_{l-})P''_l(x),\\
f_5 &= \sum\left[(l+1)S_{l+}P'_{l+1}(x) - lS_{l-}P'_{l-1}(x)\right],\\
f_6 &= \sum[lS_{l-} - (l+1)S_{l+}]P'_l(x),
\end{aligned} \qquad (2.4)$$

where $x = \cos\theta$, θ is the polar angle of the pion in the c.m.s. The relations of the amplitudes f_i to the helicity amplitudes and to the cross section are also given in [26].

For the resonance multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ we use as an input the solutions of the integral equations which follow from the dispersion relations for these amplitudes. According to these solutions obtained in Ref. [26] the

resonance multipoles are described as the sums of the particular and homogeneous solutions of the integral equations. The particular solutions which correspond to the nonresonance contributions into the multipoles have the definite magnitudes fixed by the Born terms. The homogeneous solutions corresponding to the resonance contributions have the definite shapes fixed by the homogeneous integral equations which correspond to the dispersion relations for $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ with the zero Born terms. The weights of these solutions are arbitrary and should be found from the requirement of the best description of the experimental data. So, the resonance multipoles bring into our analysis three fitting parameters which are the weights of the resonance contributions in the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$.

In the $P_{33}(1232)$ resonance region a significant contribution into Imf_i for the reaction $\gamma^* p \to \pi^0 p$ can give also the following combinations of the nonresonance multipole amplitudes:

$$\begin{aligned}
&E_{0+}^{(0)} + \frac{1}{3}E_{0+}^{1/2} + \frac{2}{3}E_{0+}^{3/2},\\
&S_{0+}^{(0)} + \frac{1}{3}S_{0+}^{1/2} + \frac{2}{3}S_{0+}^{3/2},\\
&M_{1-}^{3/2}\ and\ S_{1-}^{3/2}.
\end{aligned} \qquad (2.5)$$

This is connected with the fact that the πN phases corresponding to these multipoles are large enough, so, their imaginary parts can be significant. In order to take into account these multipoles in Imf_i, we have calculated their real parts from the Born terms, then the imaginary parts of the multipoles were found using for the corresponding πN phases the following analitical formulas:

$$\begin{aligned}
\delta_{0+}^{1/2} &= \frac{75q}{1+2.5q},\\
\delta_{0+}^{3/2} &= -45q[1 + (2.2q)^2],\\
\delta_{1-}^{3/2} &= -(6.9q)^3,
\end{aligned} \qquad (2.6)$$

where q is the 3-momentum of the pion in the c.m.s. in the GeV units, the phases are in the *degree* units , and all numbers are in the GeV^{-1} units. These formulas describe well experimental data on the phases $\delta_{0+}^{1/2}$, $\delta_{0+}^{3/2}$, $\delta_{1-}^{3/2}$ [34-36] up to $E_L = (W^2 - m^2)/2m = 0.5\ GeV$. At larger energies the smooth cutoff for the contributions of (2.5) was made. We have introduced in our analysis four additional fitting parameters in the form of the coefficients at the combinations (2.5) found in the above described way. These parameters were allowed to vary in the narrow region in the vicinity of 1.

In the description of the data in the $P_{33}(1232)$ resonance region the contributions of the resonances with higher masses, predominantely from the second resonance region, should be taken into account in the dispersion integrals. In the region of $Q^2 = 3-4\ GeV^2$ which we analyse in this work there is no information on the form factors of these resonances, except $S_{11}(1535)$. By this reason we begun our analysis with the DESY data which cover the second resonance region. In this analysis we had additional fitting parameters for the contributions of the amplitudes $M_{1-}^{1/2}, S_{1-}^{1/2}$ for the $P_{11}(1440)$ resonance, of the amplitudes $E_{0+}^{1/2}, S_{0+}^{1/2}$ for the $S_{11}(1535)$ resonance, and of the amplitudes $E_{2-}^{1/2}, M_{2-}^{1/2}, S_{2-}^{1/2}$ for the $D_{13}(1520)$ resonance. The contributions of these amplitudes were described in the Breit-Wigner form according to the parametrization of Ref. [37]. For the multipoles $M_{l+}, M_{l-}, E_{l+}, E_{l-}$ it has the form:

$$M_{B-W}(W,Q^2) = \frac{M\Gamma(W,Q^2)}{M^2 - W^2 - iM\Gamma(W,Q^2)}\left(\frac{q_r}{q}\right)^{l+1}\left(\frac{k}{k_r}\right)^{l'}. \qquad (2.7)$$

For the multipoles S_{l+}, S_{l-} the Breit-Wigner parametrization is:

$$S_{B-W}(W,Q^2) = \frac{M\Gamma(W,Q^2)}{M^2 - W^2 - iM\Gamma(W,Q^2)}\left(\frac{q_r}{q}\right)^{l+1}\left(\frac{k}{k_r}\right)^{l'+1}. \qquad (2.8)$$

Here $l' = l$ for $M_{l+}, M_{l-}, E_{l+}, S_{l+}$, $l' = l-2$ if $l > 1$ for E_{l-}, S_{l-}, and $l' = 1$ for S_{1-}, M and Γ are the masses and the widths of the resonances, k_r, q_r are the photon and pion 3-momenta in the c.m.s. at $W = M$, and

$$\Gamma(W,Q^2)=\Gamma\left(\frac{q}{q_r}\right)^{2l+1}\left(\frac{q_r^2+X^2}{q^2+X^2}\right)^l, \tag{2.9}$$

$X=0.35$. So, in the analysis of the DESY data there are 7 additional fitting parameters which are the coefficients at (2.7,2.8) for the above mentioned multipole amplitudes. These parameters we consider as effective values for the description of the second resonance region, because we did not take into account backgrounds in the multipole amplitudes in this region and did not include into our analysis the resonances from higher resonance regions. Let us note, however, that the value of the amplitude E_{0+} for the resonance $S_{11}(1535)$ obtained in this analysis agrees well with the value known from the analysis of the η electroproduction data.

In the analysis of the TJNAF data, which do not cover the second resonance region, we used the results for the multipoles from this region obtained in the analysis of the DESY data with the Q^2 evolution corresponding to the results of Ref. [38]. Then the small variation of the multipoles was allowed.

III. RESULTS

The data used in our analysis are differential cross sections of π^0 production on protons at $Q^2=2.8$ and 4 $(GeV/c)^2$ [1] and $Q^2=3.2\ (GeV/c)^2$ [2]. A total 751 and 867 points which extend over an invariant mass range $W=1.11-1.39\ GeV/c$ were included in the fit at $Q^2=2.8$ and 4 $(GeV/c)^2$, respectively. At $Q^2=3.2\ (GeV/c)^2$ we have included in the fit 598 points which extend from $W=1.145\ GeV$ to $1.595 GeV$. The reduced χ^2 obtained in the analyses were 1.53, 1.18 and 1.35 at $Q^2=2.8$, 3.2 and 4 $(GeV/c)^2$, respectively. The obtained results for the multipole amplitudes $M_{1+}^{3/2},E_{1+}^{3/2},S_{1+}^{3/2}$ are presented on Fig. 1. On this figure separately the resonance and nonresonance contributions into these amplitudes are also presented. It is seen that the nonresonance contributions play a significant role in the description of the amplitudes, especially for $E_{1+}^{3/2}$ and $S_{1+}^{3/2}$. In the case of $E_{1+}^{3/2}$ the sum of the resonance and nonresonance contributions gives the nontrivial energy dependence of the whole amplitude. At all investigated Q^2, $ImE_{1+}^{3/2}$ changes the sign near the resonance. So, the energy behaviour of this amplitude, similar to the behaviour at $Q^2=0$, has by the form nonresonance character.

In the center of the $P_{33}(1232)$ resonance at $W=m_\Delta$ the resonance contributions into the amplitudes $M_{1+}^{3/2},E_{1+}^{3/2},S_{1+}^{3/2}$ are:

$$\begin{aligned} ImM_{1+}^{3/2}(res)&=0.772\pm0.031,\ 0.523\pm0.021,\ 0.4\pm0.016,\\ ImE_{1+}^{3/2}(res)&=0.03\pm0.022,\ 0.063\pm0.017,\ 0.016\pm0.012,\\ ImS_{1+}^{3/2}(res)&=-0.038\pm0.022,\ -0.052\pm0.022,\ -0.034\pm0.008\end{aligned} \tag{3.1}$$

at $Q^2=2.8$, 3.2 and 4 $(GeV/c)^2$, respectively.

In Fig. 2 our results for the transverse form factor G_T of the $\gamma^*N\to P_{33}(1232)$ transition are presented in comparison with the data obtained from inclusive experiments and partly from exclusive data. These data are taken from Table 5 of Ref. [39] by recalculation for our definition of G_T which is related to the magnetic dipole and electric quadrupole form factors of Ref. [40] by:

$$[G_T(Q^2)]^2=(|G_M^*|^2+3|G_E^*|^2)\left(\frac{m_\Delta+m}{2m}\right)^2. \tag{3.2}$$

At large Q^2 our definition of G_T coincides with the Stoler's definition from Ref. [39] :

$$G_T^2=G_T^2(Stoler)\frac{Q^2}{(m_\Delta-m)^2+Q^2}. \tag{3.3}$$

The form factor G_T defined by Eq. (3.2) is more suitable for the description of low Q^2 data. This form factor is related to the helicity amplitudes of the $\gamma^*N\to P_{33}(1232)$ transition and to the total cross section of the reaction $\gamma^*p\to\pi N$ in the following way:

$$G_T^2=\frac{1}{4\pi\alpha}\left(|A_{1/2}^p|^2+|A_{3/2}^p|^2\right)\frac{2m(m_\Delta^2-m^2)}{(m_\Delta-m)^2+Q^2}, \tag{3.4}$$

$$\sigma(\gamma^*p\to\pi N)=4\pi\alpha G_T^2\frac{(m_\Delta-m)^2+Q^2}{m_\Delta\Gamma(m_\Delta^2-m^2)}. \tag{3.5}$$

It can be expressed through the multipoles $M_{1+}=(2A_{1+}-3B_{1+})/4$ and $E_{1+}=(2A_{1+}+B_{1+})/4$ using Eq. (3.4) and the relations:

$$A_{1+}^{3/2}=-A_{1/2}^p\left(\frac{3km}{8\Gamma\pi qm_\Delta}\right)^{1/2}, \tag{3.6}$$

$$B_{1+}^{3/2}=A_{3/2}^p\left(\frac{km}{2\Gamma\pi qm_\Delta}\right)^{1/2}. \tag{3.7}$$

Our results for G_T in Fig. 2 are lower than other data. This is connected with the fact that they are obtained by taking into account only resonance contributions in the amplitude $M_{1+}^{3/2}$ which gives the main contribution into G_T. Our results confirm the whole tendency of the G_T data to fall more rapidly with increasing Q^2 than $1/Q^4$. Let us remind, that in the pQCD asymptotics G_T behaves as $1/Q^4$ [41–45]. So, there is no evidence for the presence of the pQCD contribution in G_T at $Q^2<4\ GeV^2$.

In Figs. 3,4 our results for the ratios $E_{1+}^{3/2}/M_{1+}^{3/2}$ and $S_{1+}^{3/2}/M_{1+}^{3/2}$ corresponding to the resonance contributions to $M_{1+}^{3/2},E_{1+}^{3/2},S_{1+}^{3/2}$ are presented together with the data at smaller Q^2 [46]. We have presented also the data points at $Q^2=3.2\ GeV^2$ obtained from the DESY data in Ref. [47], assuming that the multipoles $M_{1+}^{3/2},E_{1+}^{3/2},S_{1+}^{3/2}$ are described by the sums of the resonance contributions taken in the Breit-Wigner form and the smooth nonresonance backgrounds.

It is known that the information on the Q^2 evolution of $E_{1+}^{3/2}/M_{1+}^{3/2}$ is important for the investigation of the Q^2 region where the QCD asymptotics begin to work. This is connected with the fact that the transition from the quark model prediction at $Q^2=0$: $E_{1+}^{3/2}/M_{1+}^{3/2}=0$, to the pQCD asymptotics: $E_{1+}^{3/2}/M_{1+}^{3/2}\to1,\ \ Q^2\to\infty$ [41–45], is characterized by a striking change of the behaviour of this ratio. Summarizing our results one can say that the ratio $E_{1+}^{3/2}/M_{1+}^{3/2}$ is positive at $Q^2=2.8-4\ GeV^2$. However, by the magnitude it is small, and the comparison with the data at low Q^2 does not show a visible change in the behaviour of this ratio with increasing Q^2. Therefore, there is no evidence for the presence of the visible pQCD contribution into the transition $\gamma^*N\to P_{33}(1232)$ at $Q^2=2.8-4\ GeV^2$.

In Figs. 3,4 the predictions obtained in the light cone relativistic quark model in Refs. [44,45] and in the relativized versions of the quark model in Refs. [48,49] are presented. It is seen that the predictions of [44,45] are in not bad agreement with the data. We have also presented the predictions from Ref. [50], where an attempt is made to find some approximate formula for the ratio $E_{1+}^{3/2}/M_{1+}^{3/2}$, which connects the quark model prediction at $Q^2=0$ with the pQCD asymptotics. One of the curves, which corresponds to a larger asymptotic value of $A_{1/2}$, describe the data quite well.

Figures 5-10 are presented to show the typical agreement of our results with experimental data.

IV. DISCUSSION

In this work we have analysed the TJNAF [1] and DESY [2] data on the cross sections of the exclusive reaction $p(e,e'p)\pi^0$ at $Q^2=2.8$, 3.2 and 4 $(GeV/c)^2$ and found the $P_{33}(1232)$ resonance contribution into the multipole amplitudes $M_{1+}^{3/2},E_{1+}^{3/2},S_{1+}^{3/2}$. As an input for the resonance and nonresonance contributions into these amplitudes the solutions of the integral equations for the multipoles obtained in Ref. [26] were used. These integral equations follow from the dispersion relations for $M_{1+}^{3/2},E_{1+}^{3/2},S_{1+}^{3/2}$, if we take into account the initarity condition for the multipoles. As it was discussed in the Introduction on the example of the simplified version of the dispersion relations for the multipoles with the s-channel cut only, the solutions of the integral equations for $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ contain two parts which have an interpretation in terms of the resonance and nonresonace contributions into the multipoles. One part is the particular solution of the integral equations generated by the Born term. This part is the modification of the Born contribution, produced by the πN rescattering in the final state; we consider it as the nonresonance background contribution. It has the definite magnitude fixed by the Born term. Other part of the solutions corresponds to the homogeneous parts of the integral equations. We identify it with the resonance contributions. These solutions have the definite shapes fixed by the dispersion relations and arbitrary weights which determine the resonance contributions into $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$. These weights were fitting parameters in our analyses and were found from the experiment.

The dispersion relations for the multipoles $M(W,Q^2) \equiv M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$, which were investigated in Ref. [26], in addition to the integrals over s-channel cut in (1.2) contain also the integrals over u-channel cut. These integrals, in addition to the contribution of $ImM(W',Q^2)$, include contributions of other multipoles. The existing information at $Q^2=0$ allows to estimate these contributions, as well as the high energy contributions into the dispersion integrals. The calculations made in Ref. [26] had shown that at $Q^2=0$ all these contributions can be neglected in comparison with the contribution of the Born term. So, the particular solutions at $Q^2=0$ are determined by the Born term only. The information at $Q^2 \neq 0$ is not enough to estimate the contributions additional to the Born term. The solutions for $M^{part}(W,Q^2)$ at $Q^2 \neq 0$ were obtained in Ref. [26] under assumption that these solutions are also determined by the Born term only. In this work we have used these solutions. In the future, when experimental data in the whole resonance region will be available, the assumption on the dominance of the Born term contributions in the terms, which determine the inhomogeneouty of the integral equations for $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$, will be checked. If it will be found that the additional contributions to the Born term are important, a new analysis in the $P_{33}(1232)$ resonance region, taking into account these additional contributions, will be nessesary.

Let us draw attention to the following point too. The contributions of the diagram, corresponding to the process $\gamma^* N \to P_{33}(1232) \to \pi N$, into the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ we identify with the solutions of the homogeneous parts of the integral equations which follow from the dispersion relations for these amplitudes. The rescattering effects connected with the πN interaction in the final state modify the $\pi N P_{33}(1232)$ vertex in this diagram. A conclusion on the form of this modification can be made using the results of the dynamical model of Ref. [19], if the amplitude $h_{1+}^{3/2}$ of πN scattering is the pure resonance amplitude. According to these results in this case the factor at $1/(W-m_\Delta-i\Gamma/2)$ for $\gamma^* N \to P_{33}(1232) \to \pi N$ is equal to the product of the vertex $\gamma^* N P_{33}(1232)$ and the dressed vertex $\pi N P_{33}(1232)$. The dressed vertex $\pi N P_{33}(1232)$ can be found from experimental data on the width of the $P_{33}(1232) \to \pi N$ decay. This fact was used in the derivation of the relations (3.6),(3.7), which connect the helicity amplitudes $A_{1/2}^p$, $A_{3/2}^p$ and the resonance parts of the amplitudes $A_{1+}^{3/2}$, $B_{1+}^{3/2}$ (i.e. of the amplitudes $M_{1+}^{3/2}$, $E_{1+}^{3/2}$ (3.1)). Our results for the transverse form factor G_T of the $\gamma^* N \to P_{33}(1232)$ transition presented in Fig. 2 are found from Eq. (3.4) using these relations between $A_{1/2}^p$, $A_{3/2}^p$ and $M_{1+}^{3/2}$, $E_{1+}^{3/2}$ (3.1).

The situation is more complicated, if the amplitude $h_{1+}^{3/2}$ contains nonresonance background. In this case it is reasonable to assume, that the ratios of the resonance parts of the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ are equal to the ratios of the verteces $\gamma^* N P_{33}(1232)$ for these amplitudes, i.e. the final state interaction modifies the $P_{33}(1232)$ resonance contributions into $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ in the same way. This assumption is confirmed by the results obtained in Ref. [21] within dynamical model. Therefore, our results for the ratios $E_{1+}^{3/2}/M_{1+}^{3/2}$ and $S_{1+}^{3/2}/M_{1+}^{3/2}$, presented in Figs. 3,4 can be reliably identified with the corresponding ratios for the $\gamma^* N \to P_{33}(1232)$ transition. The same statement is right for the ratios of the multipole amplitudes at different values of Q^2, i.e., for example, for the ratios of G_T at different values of Q^2.

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Figure Captions

Fig. 1 Our results for the imaginary parts of the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$. Dashed curves are the resonance parts of the multipoles corresponding to the $P_{33}(1232)$ resonance contribution; dotted curves are the nonresonance background contributions; full curves are the sums of these contributions; $E_L = (W^2 - m^2)/2m$.

Fig. 2 Experimental data for the transverse form factor of the $\gamma N \to P_{33}(1232)$ transition defined by Eq. (3.2). The data are divided by $3G_{dip}$, where $G_{dip}(Q^2) = 1/(1 + Q^2/0.71\ (GeV/c)^2)$. Data denoted by boxes are taken from Table 5 of Ref. [39] by recalculation for our definition of G_T; data denoted by asterisks are obtained in our analysis.

Fig. 3 Experimental data for the ratio $E_{1+}^{3/2}/M_{1+}^{3/2}$ obtained in our analysis (asterisks) and the data at low Q^2 [46] and at $Q^2 = 3.2 (GeV/c)^2$ from Ref. [47] in comparison with the predictions of Refs. [45] (full line), [48] (dotted line), [49] (dashed line), [50] (dash-dotted lines).

Fig. 4 Experimental data for the ratio $S_{1+}^{3/2}/M_{1+}^{3/2}$ obtained in our analysis (asterisks) and the data at low Q^2 [46] and at $Q^2 = 3.2 (GeV/c)^2$ from Ref. [47] in comparison with the predictions of Refs. [45] (full line), [48] (dotted line), [49] (dashed line).

Fig. 5 Comparison of our results for ϕ distributions with the TJNAF data [1] at $W = 1.235\ GeV$ and $Q^2 = 2.8\ (GeV/c)^2$; $\epsilon = 0.56$.

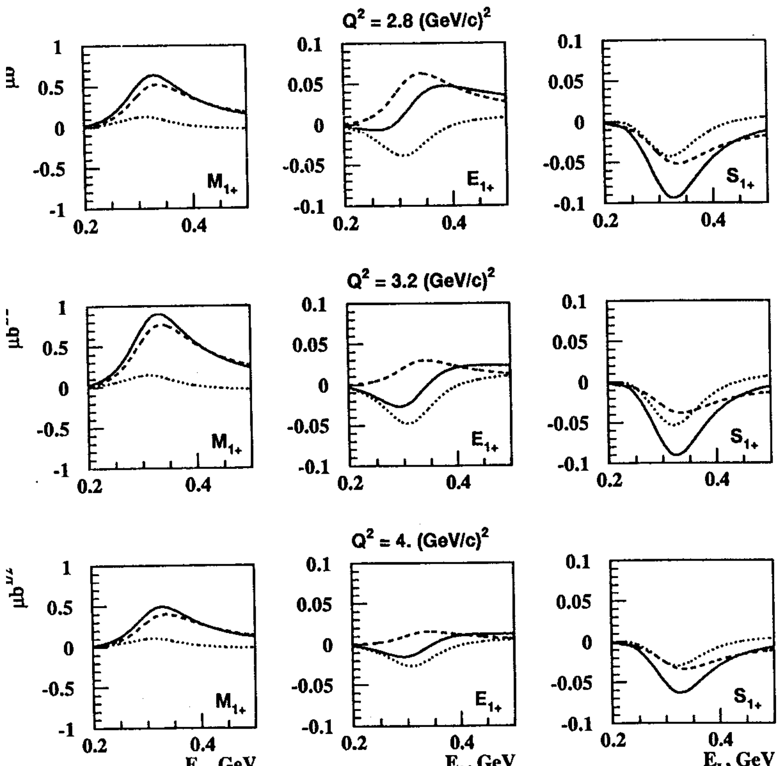
Fig. 6 Comparison of our results for energy dependence of the cross sections with the TJNAF data [1] at $Q^2 = 2.8\ (GeV/c)^2$; $cos\theta = 0.7$, $\epsilon = 0.56$.

Fig. 7 Comparison of our results for angular distributions with the DESY data [2] at $W = 1.235\ GeV$ and $Q^2 = 3.2\ (GeV/c)^2$; $\epsilon = 0.89$.

Fig. 8 Comparison of our results for energy dependence of the cross sections with the DESY data [2] at $Q^2 = 3.2\ (GeV/c)^2$; $\phi = 61.5^\circ$, $\epsilon = 0.89$.

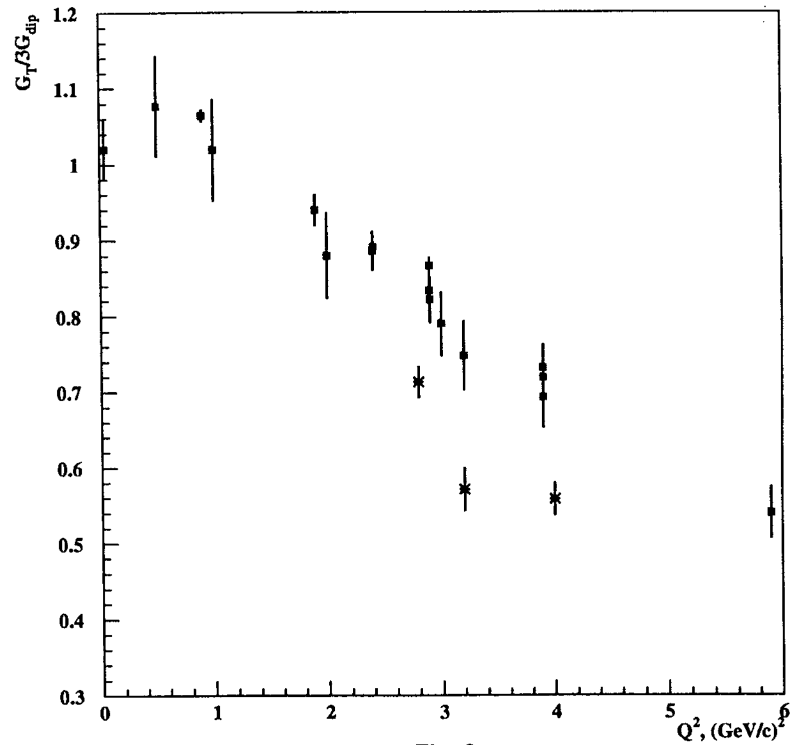
Fig. 9 Comparison of our results for ϕ distributions with the TJNAF data [1] at $W = 1.235\ GeV$ and $Q^2 = 4\ (GeV/c)^2$; $\epsilon = 0.51$.

Fig. 10 Comparison of our results for energy dependence of the cross sections with the TJNAF data [1] at $Q^2 = 4\ (GeV/c)^2$; $cos\theta = 0.7$, $\epsilon = 0.51$.



$Q^2 = 2.8\ (GeV/c)^2$
$Q^2 = 3.2\ (GeV/c)^2$
$Q^2 = 4.\ (GeV/c)^2$
M_{1+}
E_{1+}
S_{1+}
E_L, GeV

Fig.1



$G_T/3G_{dip}$
Q^2, $(GeV/c)^2$

Fig. 2

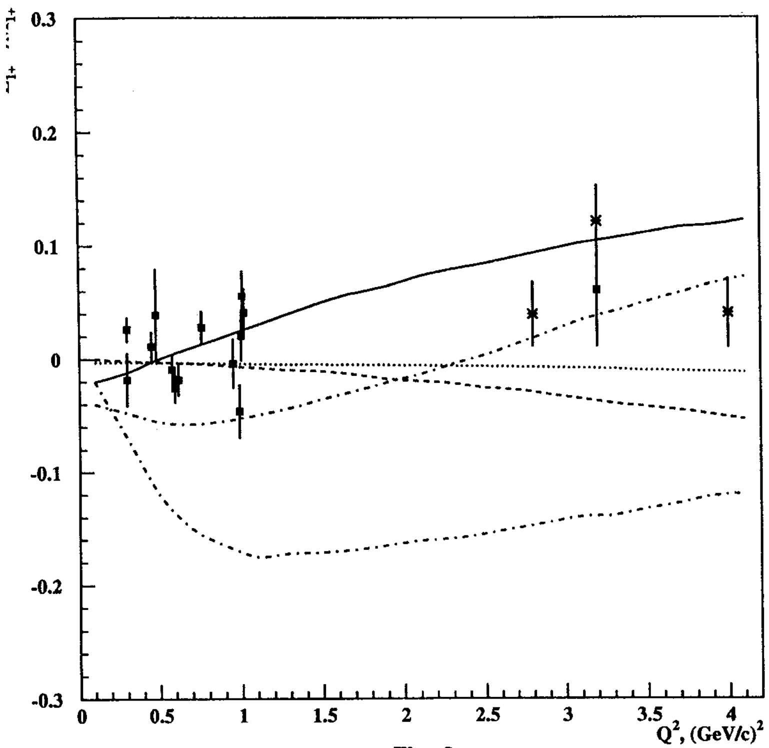

0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
0.5
1
1.5
2
2.5
3
3.5
4
Q^2, $(GeV/c)^2$

Fig. 3

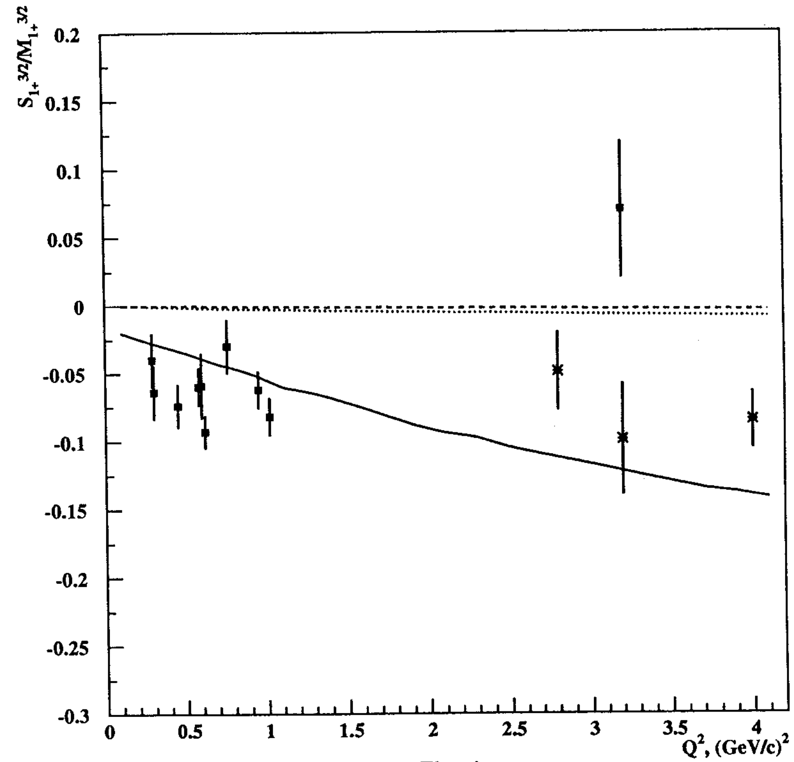

$S_{1+}^{3/2}/M_{1+}^{3/2}$
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
0
0.5
1
1.5
2
2.5
3
3.5
4
Q^2, $(GeV/c)^2$

Fig. 4

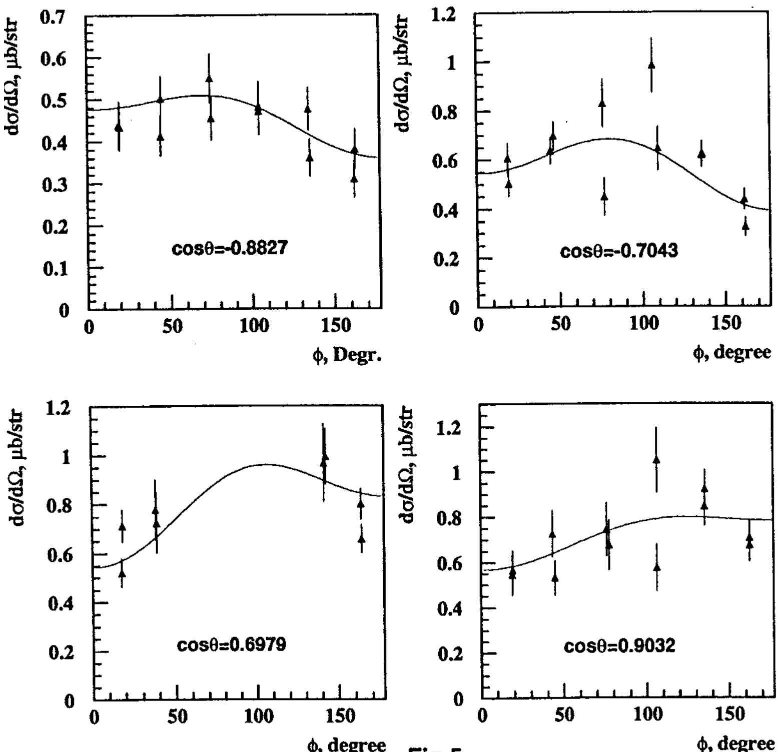

dσ/dΩ, μb/str
φ, Degr.
φ, degree
cosθ=-0.8827
cosθ=-0.7043
cosθ=0.6979
cosθ=0.9032

Fig.5

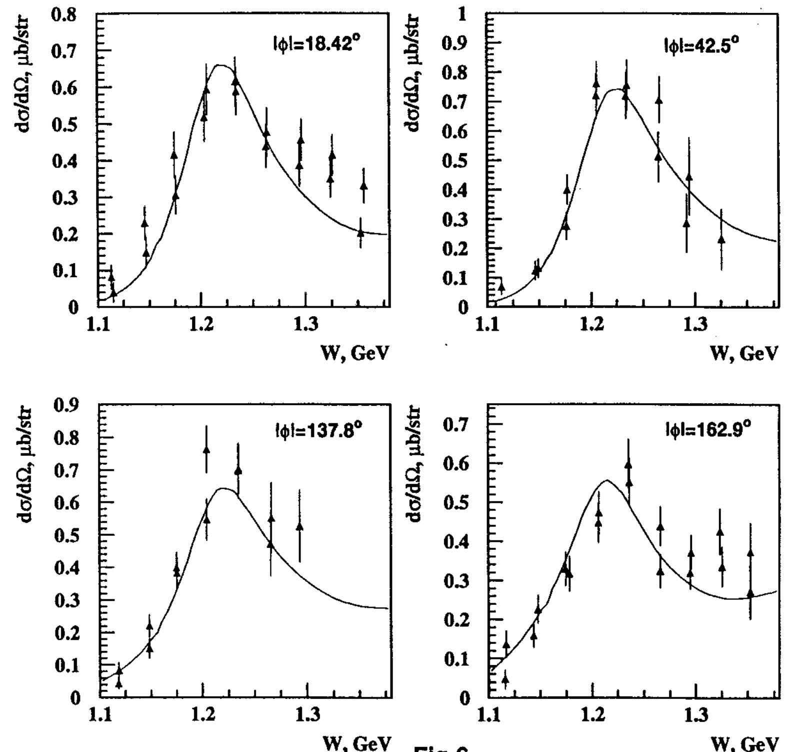
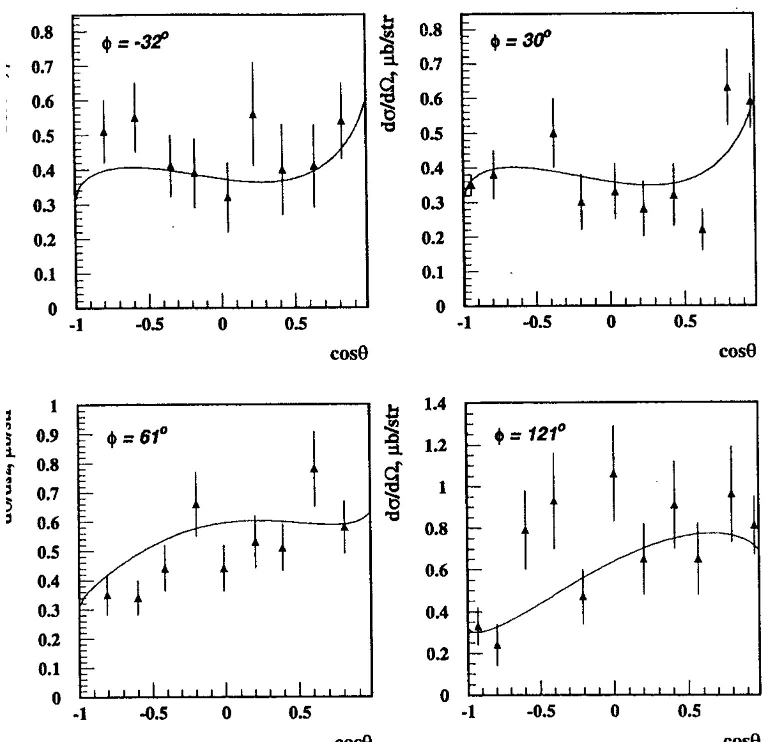

dσ/dΩ, μb/str
W, GeV
|φ|=18.42°
|φ|=42.5°
|φ|=137.8°
|φ|=162.9°

Fig.6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φ = 30°
φ = 61°
φ = 121°
dσ/dΩ, μb/str
cosθ

Fig.7

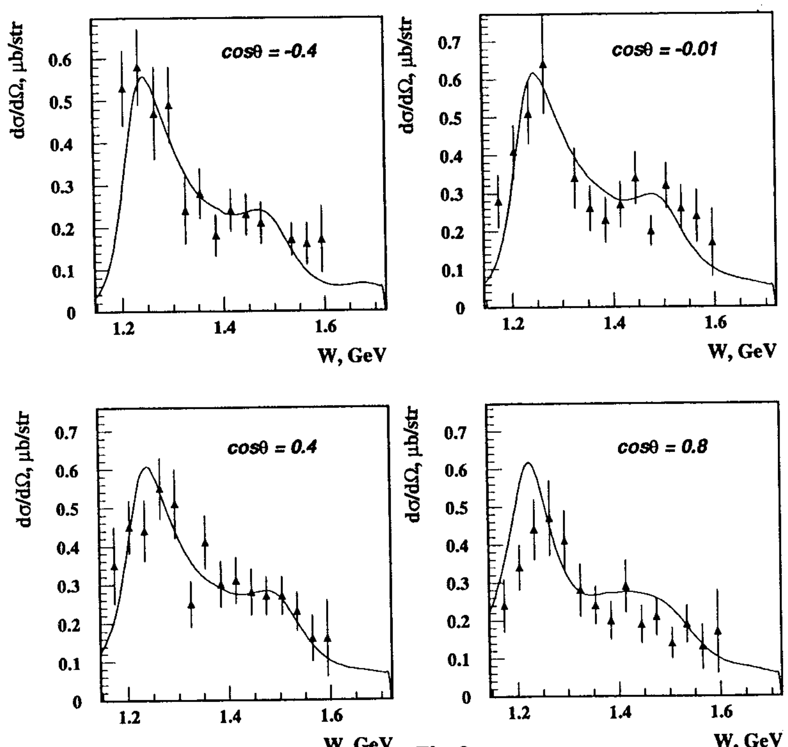
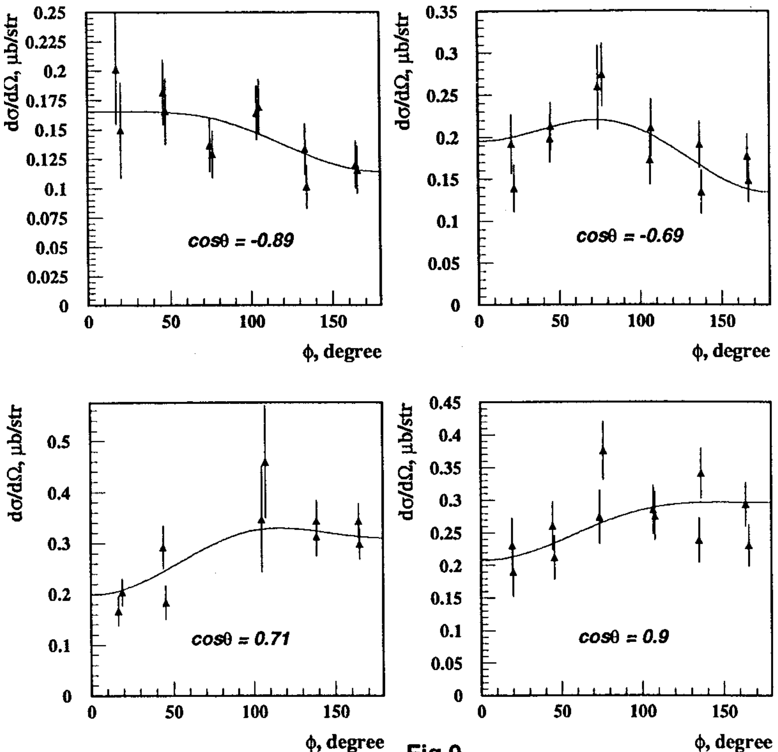
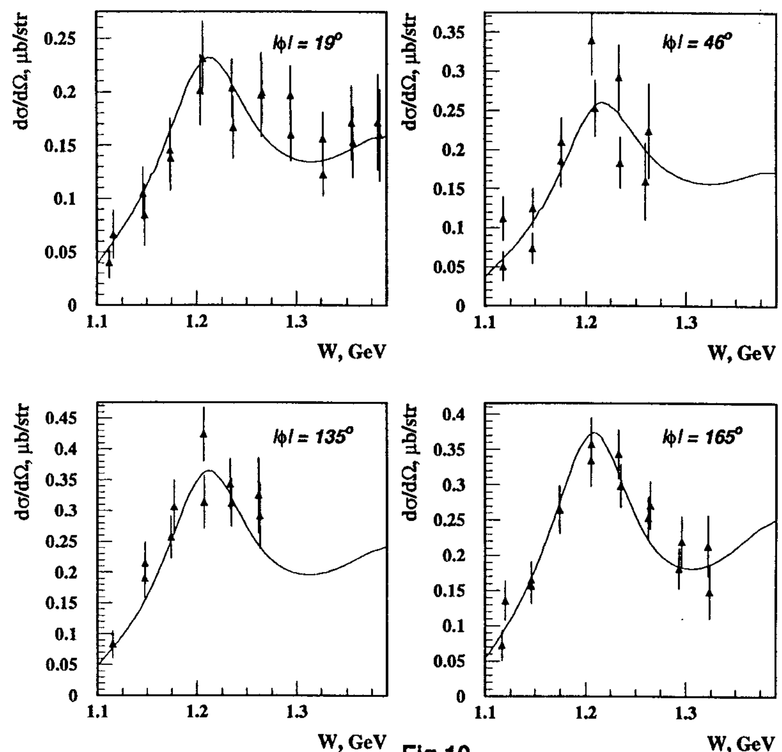

cosθ = -0.4
cosθ = -0.01
cosθ = 0.4
cosθ = 0.8
dσ/dΩ, μb/str
W, GeV

Fig.8



dσ/dΩ, μb/str
cosθ = -0.89
cosθ = -0.69
cosθ = 0.71
cosθ = 0.9
φ, degree

Fig.9



dσ/dΩ, μb/str
|φ| = 19°
|φ| = 46°
|φ| = 135°
|φ| = 165°
W, GeV

Fig.10